



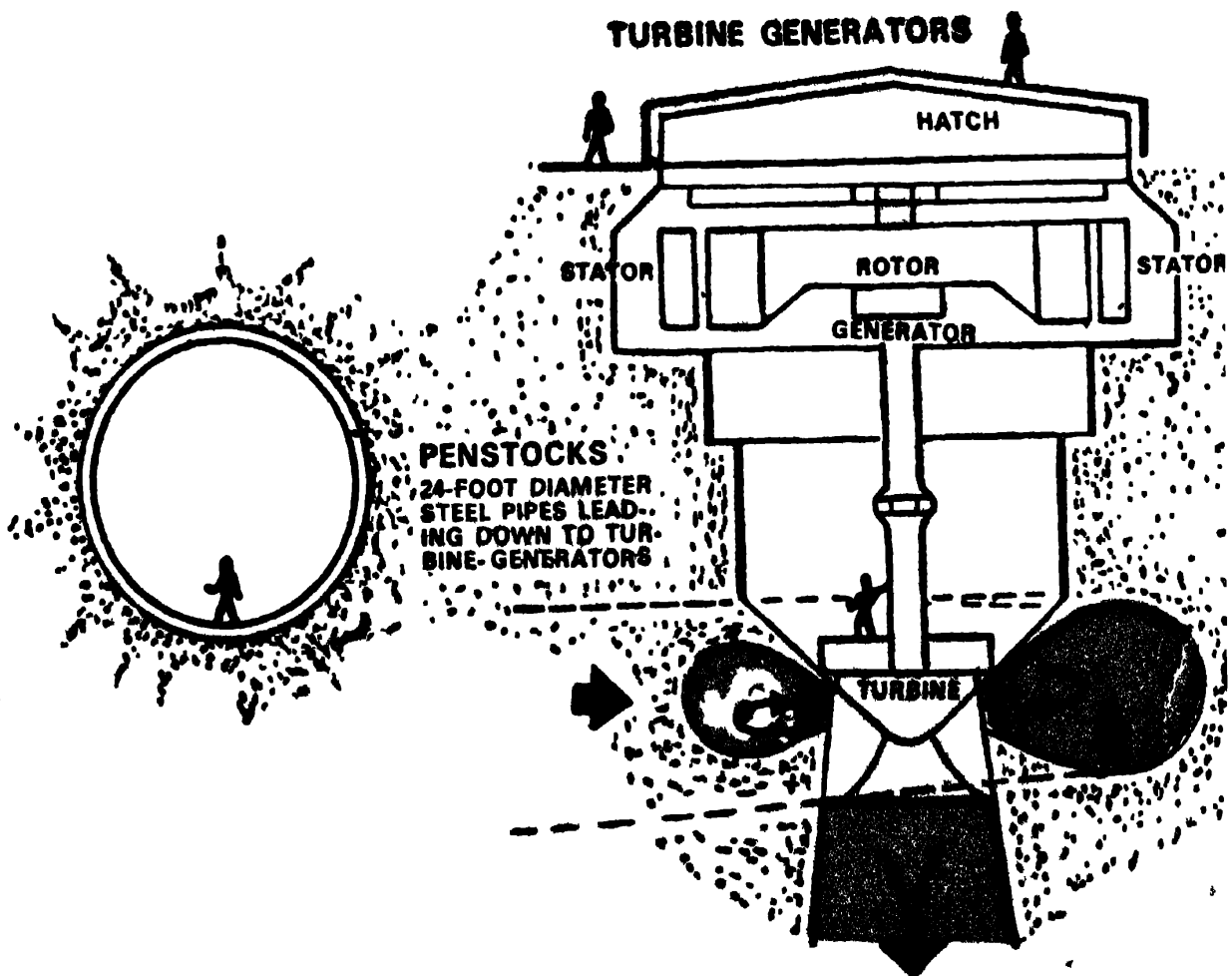






# Hydraulic Machines

Including FLUIDICS





**To**

**My Professors at Home & Abroad**

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# *Foreword*

**to  
First Edition**

Immediately on his return from Zurich, Dr. Jagdish Lal joined the Indian Institute of Technology. He was given the responsibility for organising and equipping the Hydraulic Machines Laboratory. Such a laboratory as distinct from the Hydraulics Laboratory of the Civil Engineering Department, is now considered an essential feature in a modern Technical Institute.

In a rapidly developing country, it is difficult to decide where the responsibility of the hydraulic engineer ends and that of hydraulic machine engineer begins. It is best that each knows as much of the other's job as possible. This book thus covers a field of study which is rapidly gaining in importance.

Dr. Lal has the enthusiasm that we associate with a teacher who remembers the days when he was a brilliant student. This book has grown out of that enthusiasm and has made available to the students in a handy form, knowledge and information which will be of value not only for the completion of their courses but also for practising their profession.

The book emphasises the need of young engineers acquiring great efficiency in using the tool of mathematics. To them the study of the Chapter on Dimensional Analysis is specially recommended.

The book has the merit that theoretical treatment has been well blended with descriptive and practical details, and should add to the reputation of Dr. Lal as a teacher of the subject.

(DR. J.C. GHOSH)

*Late Member, Planning Commission  
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*15th May, 1956*



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# Symbols

|                          |                                                                                                                              |
|--------------------------|------------------------------------------------------------------------------------------------------------------------------|
| <b>A</b>                 | area                                                                                                                         |
| <b>B</b>                 | breadth                                                                                                                      |
| <b>C</b>                 | Venturimeter and nozzle co-efficient                                                                                         |
| <b>C<sub>d</sub></b>     | co-efficient of discharge                                                                                                    |
| <b>D</b>                 | diameter                                                                                                                     |
| <b>D<sub>1</sub></b>     | mean diameter of Pelton runner or mean bucket circle diameter. In general, inlet diameter of turbine runner or pump impeller |
| <b>E</b>                 | energy, voltage and Euler's number                                                                                           |
| <b>F</b>                 | force                                                                                                                        |
| <b>F<sub>x</sub></b>     | dynamic force in horizontal or X-direction                                                                                   |
| <b>F<sub>y</sub></b>     | dynamic force in vertical or Y-direction                                                                                     |
| <b>F<sub>r</sub></b>     | Froude's number                                                                                                              |
| <b>G</b>                 | weight and modulus of rigidity                                                                                               |
| <b>H</b>                 | head                                                                                                                         |
| <b>H<sub>d</sub></b>     | designed head                                                                                                                |
| <b>H<sub>a</sub></b>     | accelerating head                                                                                                            |
| <b>H<sub>mano</sub></b>  | manometric head                                                                                                              |
| <b>H<sub>total</sub></b> | total head                                                                                                                   |
| <b>H<sub>L</sub></b>     | loss of head                                                                                                                 |
| <b>I</b>                 | current                                                                                                                      |
| <b>I<sub>p</sub></b>     | polar moment of inertia                                                                                                      |
| <b>K<sub>s</sub></b>     | non-dimensional factor for specific speed                                                                                    |
| <b>K<sub>v1</sub></b>    | $\frac{v_1}{\sqrt{2gH}}$ = non-dimensional factor for $v_1$ or velocity ( $v_1$ )<br>co-efficient                            |
| <b>K<sub>u1</sub></b>    | $\frac{u_1}{\sqrt{2gH}}$ = speed ratio or non-dimensional factor for $u_1$                                                   |
| <b>L</b>                 | length                                                                                                                       |
| <b>M</b>                 | moment                                                                                                                       |
| <b>M<sub>a</sub></b>     | Mach number                                                                                                                  |
| <b>M<sub>1</sub></b>     | $\frac{M}{H}$ = moment reduced to unit head                                                                                  |
| <b>N</b>                 | speed                                                                                                                        |
| <b>N<sub>1</sub></b>     | $\frac{N}{\sqrt{H}}$ = speed reduced to unit head                                                                            |
| <b>N<sub>s</sub></b>     | specific speed                                                                                                               |
| <b>P</b>                 | power in HP or KW                                                                                                            |

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|            |                                                                                                         |
|------------|---------------------------------------------------------------------------------------------------------|
| $P_a$      | available power or water power                                                                          |
| $P_H$      | power considering the head lost i.e. available power <i>minus</i> power consumed due to head lost       |
| $P_h$      | hydraulic power<br>( $P_a$ <i>minus</i> hydraulic losses)                                               |
| $P_{mech}$ | mechanical losses                                                                                       |
| $P_t$      | brake horsepower or net power developed by a turbine                                                    |
| $P_1$      | $\frac{P}{H^{\frac{5}{4}}} =$ power reduced to unit head                                                |
| $P_{11}$   | $\frac{P}{D^5 \cdot H^{\frac{5}{4}}} =$ power reduced to unit head and unit diameter<br>(runner or jet) |
| $Q$        | quantity of fluid flowing per unit time or discharge                                                    |
| $Q_1$      | $\frac{Q}{\sqrt{H}} = Q$ reduced to unit head                                                           |
| $Q_{11}$   | $\frac{Q}{D^2 \sqrt{H}} = Q$ reduced to unit head and unit diameter<br>(runner or jet)                  |
| $\Delta Q$ | loss of $Q$                                                                                             |
| $R$        | radius                                                                                                  |
| $R_e$      | Reynolds' number                                                                                        |
| $S$        | stroke                                                                                                  |
| $T$        | absolute temperature ; tare                                                                             |
| $V$        | volume                                                                                                  |
| $W$        | work done ; wattage                                                                                     |
| $a$        | area                                                                                                    |
| $b$        | breadth                                                                                                 |
| $c$        | constant                                                                                                |
| $d$        | diameter                                                                                                |
|            | $d_o$ nozzle diameter                                                                                   |
|            | $d_1$ least jet diameter                                                                                |
| $f$        | friction co-efficient                                                                                   |
|            | $f_s$ shear stress                                                                                      |
| $g$        | gravitational acceleration = $9.81 \text{ m/sec}^2$                                                     |
| $h$        | head or height                                                                                          |
| $k$        | constant                                                                                                |
| $l$        | length                                                                                                  |
| $m$        | mass and hydraulic mean depth                                                                           |
| $p$        | intensity of pressure                                                                                   |
| $q$        | partial discharge                                                                                       |
| $r$        | radius                                                                                                  |
| $s$        | distance, specific gravity and stroke                                                                   |
| $t$        | time                                                                                                    |
| $u$        | peripheral or circumferential velocity of water                                                         |

|     |       |                                                   |
|-----|-------|---------------------------------------------------|
| $v$ |       | absolute velocity of water                        |
|     | $v_f$ | velocity of flow                                  |
|     | $v_w$ | velocity of whirl                                 |
| $w$ |       | relative velocity of water                        |
| $x$ |       | horizontal distance                               |
| $y$ |       | vertical distance                                 |
|     |       | centrifugal head developed and height above datum |
|     | $z_1$ | number of vanes of a guide wheel                  |
|     | $z_2$ | number of vanes of a runner                       |

### Greek Letters

|                                          |               |                                                                              |
|------------------------------------------|---------------|------------------------------------------------------------------------------|
| $\alpha$                                 |               | angle between absolute and peripheral velocity ;<br>and angular acceleration |
| $\beta$                                  |               | angle between peripheral ; relative velocity                                 |
| $\gamma$                                 |               | specific weight                                                              |
| $\rho$                                   |               | mass density                                                                 |
| $\eta$                                   |               | efficiency                                                                   |
|                                          | $\eta_h$      | hydraulic efficiency                                                         |
|                                          | $\eta_H$      | head efficiency                                                              |
|                                          | $\eta_{mano}$ | manometric efficiency                                                        |
|                                          | $\eta_{mech}$ | mechanical efficiency                                                        |
|                                          | $\eta_p$      | pump efficiency                                                              |
|                                          | $\eta_Q$      | volumetric efficiency                                                        |
|                                          | $\eta_o$      | overall efficiency                                                           |
| $\sigma$                                 |               | Thoma's cavitation factor                                                    |
| $\lambda$                                |               | fraction                                                                     |
| $\mu$                                    |               | absolute viscosity                                                           |
| $\nu$                                    |               | kinematic viscosity                                                          |
| $\phi$                                   |               | co-efficient                                                                 |
| $\omega$                                 |               | angular velocity                                                             |
| $\alpha, \beta, \theta,$<br>$\phi, \psi$ |               | angles (in general)                                                          |
| suffix 1                                 |               | for inlet                                                                    |
| suffix 2                                 |               | for outlet                                                                   |

# Introduction

**1. Turbo-machines** are devices in which energy is transferred either to, or from, a continuously flowing fluid by the *dynamic action* of moving blades on the runner. The word *turbo* or *turbine* is of latin origin and implies that which spins or whirls around.

The turbomachines may be classified as :

**(a) Open and enclosed machines :**

- (i) Open turbomachines are those which influence an indefinite quantity of fluid *e.g.*, propellers, windmills and unshrouded fans. Such machines come generally under the category of aerodynamics.
- (ii) Enclosed turbomachines in which a *finite* quantity of fluid passes through a casing in unit time.

**(b) Absorption and production of power :**

- (i) Those which *absorb* power to increase the fluid pressure or head *e.g.*, pump, ducted fans and compressors.
- (ii) Those which *produce* power by expanding fluid to a lower pressure or head *e.g.*, hydraulic, steam and gas turbines.

**(c) Type of fluid handled :**

- (i) Those which handle water *e.g.*, pumps and hydraulic turbines.
- (ii) Those which handle steam *e.g.*, steam turbines.
- (iii) Those which handle air or gas *e.g.*, ducted fans, compressors and gas turbines.

This book deals with only the enclosed type hydraulic machines *i.e.*, pumps and hydraulic turbines.

**2. Hydraulic Machines**—Hydraulic machine is a general term used for all devices/machines handling liquids. Hydraulic machines consist of :

(a) **Turbomachines** *e.g.*, pumps and hydraulic turbines generally known as *rotodynamic machines*. The outdated water wheels will also fall under this category.

(b) **Reciprocating machines** *e.g.*, reciprocating pumps. These are known as positive displacement pumps.

(c) **Various water lifting devices** *e.g.*, jet pump, air-lift pump, pulsometer pump and hydraulic ram.

(d) **Pumps transmitting oil** under pressure to operate hydraulic controls and systems e.g., gear pumps, constant delivery and variable delivery pumps. Various appliances and accessories relating to hydraulic systems will also come under this category.

**3. History of the Development of Water Wheel and Water Turbine**—The idea of using water as source of energy existed more than 2,200 years ago. The hydraulic energy was first produced in Asia (China and India) in the form of mechanical energy, by passing water through a water wheel. The old type of water wheel made mainly from wood, still exists in India. Such type of prime movers were taken from the Asian Continent to Egypt and then from there to European countries and America. It is estimated that the water wheel (undershot and overshot) was used in Europe 600 years after its origin in India. The actual design of water wheel was first made by Leonardo da Vinci (1452 to 1519) AD, the great Italian artist, which he did with hand sketches. The theory and the mathematical solutions of such a wheel were drawn by scientists Galileo Galilei and Descartes. The practical experiments for the use of water wheels were carried out by Smearn and Bossut in the year 1759. A French artillery Major Jean Victor Poncelet (1788 to 1867) first designed the water wheel taking in view the theory given by Borda in 1766. This kind of water wheel which the Major designed was mostly manufactured in England in 1828. The first book describing the theory and construction of water wheel came out in 1846. This book was written by Redtenbacher of Karlsruhe (Germany). A Swiss scientist Daniel Bernoulli (1700 to 1782) first wrote a theory for the conversion of water power into other forms of energy in his book "*Hydrodynamica*". Bernoulli's Theorem was given practical application by Segner in Goettingen (Germany) in 1750, who designed a water wheel. Then Leonard Euler (1707 to 1783) from Basle (Switzerland) wrote the theory of hydraulic machines in 1750, which is used even to this day showing the fundamentals of the subject. Bernoulli and Euler discovered most of the mathematical work concerning the problems. In 1824, a French scientist named Burdin designed a radial water wheel with a guide mechanism which could be used in practical field and was the first machine named as water turbine. Burdin could not make much fame of his work, and this turbine was further developed in designs by his student Fourneyron in 1827 which is the first water turbine. The Fourneyron turbines were built in 1843 in the U.S.A. Further development of water turbine is hereunder :

- |                                                                                                    |                   |
|----------------------------------------------------------------------------------------------------|-------------------|
| (a) Henschel—Jonval Turbine                                                                        | 1837              |
| [Axial Flow Turbine]                                                                               | (and 1850 in USA) |
| (b) Girard (French Engineer) Turbine<br>and Howd and Swain (USA) Turbine,<br>[Inward Flow Turbine] | 1850              |
| (c) James Bichens Francis<br>[Inward Flow Turbine]                                                 | 1865              |

The above development has been described for the reaction turbines which were mostly used for driving mills etc. At that time the hydro-power was converted to mechanical power. The problem then arose to produce electrical energy with the help of water turbines by coupling the generator. Since water-power was available in remote places, it became

necessary to carry electricity to far distant towns. This became possible with the high voltage technique. It was required to increase the speed of turbines. This problem involved the factor concerning specific speed (refer Art 4.18) of turbines, which was to be raised. The highest specific speed of Francis Turbine ( $N_s = 520$ ) was obtained by Prof. Dubs of Zurich (Switzerland) in 1914 and the Francis Turbine for specific speed between 280 and 520 was named as Dubs Turbine. The Francis Turbine is designed for a specific speed between 60 to 300. Prof. Kaplan of Bruenn (Germany) worked out a new design in the year 1916 for higher specific speed which

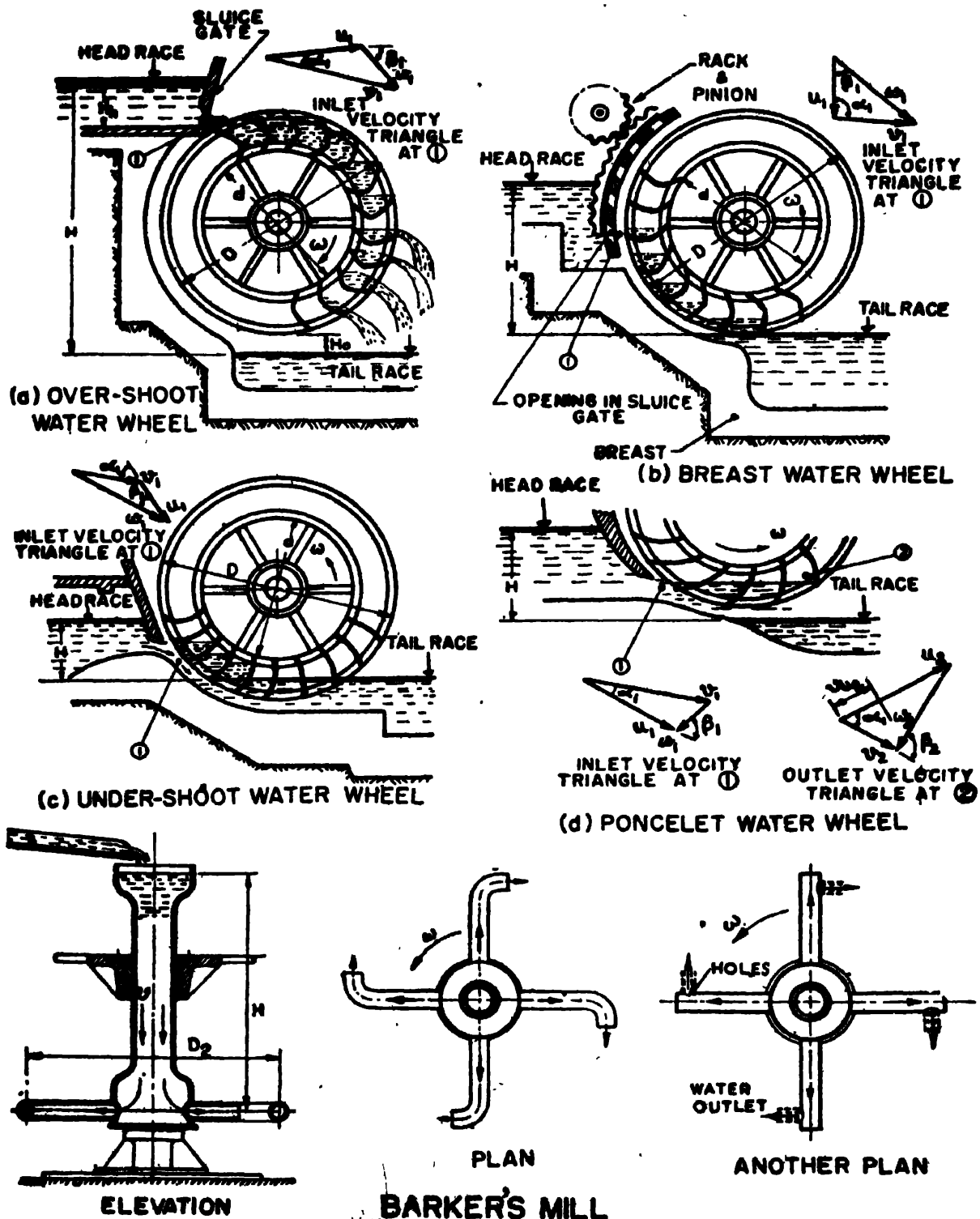


Fig 1. Different Types of Water Wheels (i) Impulse Type (a) to (d)  
(ii) Reaction Type Barker's Mill



## HYDRAULIC MACHINES

is now known as Kaplan Turbine and works for specific speed between 300 to 1000. The above reaction type of turbines are for low and medium heads of water. In a low head Kaplan turbine there are a number of bends at inlet, casing and draft tube, where the water has to turn, leading to head losses. In order to minimise these losses, Tubular Turbine has been developed. The new turbine uses Kaplan runner. It is becoming very popular for low head installations.

Kaplan runner is a propeller provided with rotatable blade mechanism. It is useful for a power station having variable load as with such a mechanism the flow of water is controlled while the turbine is running, giving maximum efficiency at part and over-loads. Since Francis turbine does not have runner with blades rotatable mechanism, it could not give maximum efficiency at part loads. Deriaz runner was involved in 1960 with the concept of that of Kaplan runner to make it useful for part and over-loads.

**Development of Impulse Turbines**—The undershot water wheel worked entirely with impulse of water and hence it was known as impulse wheel. Artillery Major Poncelet improved its design in the middle of 19th century. The later design was given by an American Engineer J. Pelton in 1880 with a tangential flow, which is still used as Pelton Turbine. This type of turbine is also called free jet turbine.

**4. Water Wheels**—A water wheel consists of a central hub and a circular frame with a number of buckets or vanes mounted on the periphery. The water is delivered to the wheel at some point on its circumference striking one or more buckets at a time. Fig 1 shows various types of water wheels having the following details :—

|     | <i>Name of water wheel<br/>(impulse type)</i> | <i>Head<br/>H in m</i> | <i>Diameter<br/>D in m</i> | <i>Speed in<br/>rpm</i> | <i>Overall effi-<br/>ciency %</i> |
|-----|-----------------------------------------------|------------------------|----------------------------|-------------------------|-----------------------------------|
| (a) | Overshoot                                     | 3 to 22                | 3 to 22                    | 4 to 8                  | 65 to 85                          |
| (b) | Breast                                        | 1 to 5                 | 4 to 8                     | 3 to 7                  | 50 to 65                          |
| (c) | Undershoot                                    | less than<br>1.75      | 2 to 4 N                   | 2 to 4                  | 35 to 45                          |
| (d) | Poncelet                                      | 2                      | 2 to 4 H                   | ...                     | 55 to 65                          |

**Jet Reaction water wheel** refer (Fig 1) —Barker's Mill, an adaptation of hydraulic use of Hero's engine, also known as Segner water wheel or Whitelaw's turbine or Scotch turbine, is now used as *lawn sprinkler*. It consists of a vertical pipe mounted in bearings. To the lower end of this pipe are connected two or more radial pipes. The ends of these radial pipes are closed and holes drilled in them. In some cases, the radial pipes

are bent at right angles at the ends. The central pipe is fed with water under pressure and reaction of water escaping from the ports of arms causes them to rotate.

### 5. Obsolete Turbines—

#### (a) Reaction Turbines (refer Fig 2) :

(i) **Fourneyron turbine**—Outward flow type in which the driving force is obtained by the reaction of water flowing radially outward from central pipe and impinging on blades of wheel, thus rotating a shaft passing upward through the elbow. The first set of turbines installed at famous Niagara Falls were of this type.

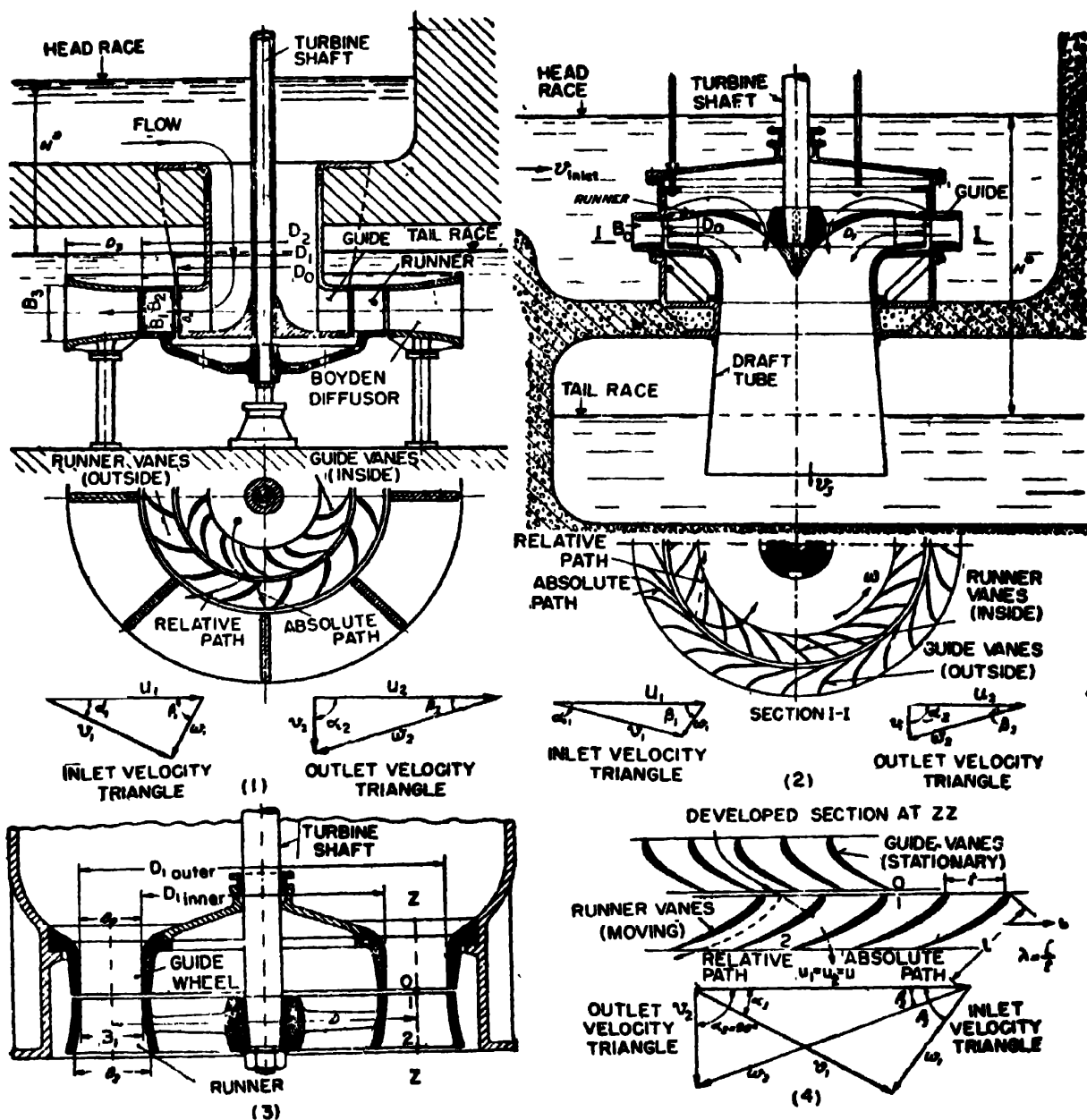


Fig 2. Obsolete Water Turbines

(1) Reaction Type (i) to (ii)

(2) Impulse Type

(ii) **Francis turbine**—Inward flow type in which the inflowing water enters the runner blades from the periphery towards centre. Hewd. was the first to suggest this type of turbine in 1827, but Francis designed and constructed it in 1865.

(iii) **Jonval turbine**—Parallel flow or axial flow in which direction of water flow is parallel to the wheel axis. The runner is horizontal into which the water enters through the guide vanes placed above it.

**(b) Impulse Turbine :**

**Girard turbine**—Axial flow similar to Jonval reaction type (refer iii above) with a difference that guide vanes cover only opposite quadrants instead of whole circumference. The runner will not then run full of water.

# SECTION I

## Principles of Hydraulic Machinery

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### *Dynamic Action of Fluid*

**1.1 Dynamic Force and Power**—A stream of fluid entering in a machine such as a hydraulic or steam turbine, a pump or fan, has more or less a defined direction. A force is always required to act upon the fluid to change its velocity either in direction or in magnitude. Newton's Third Law of Motion states that to every action there is an equal and opposite reaction. According to this law an equal and opposite force is exerted by the fluid upon the body that causes the change. This force exerted by virtue of fluid motion is called a *Dynamic Force* and must be distinguished from hydrostatic pressure. Whereas hydraulic pressure implies no motion, dynamic force always involves a change in velocity and thus a change in momentum.

The major problem in turbomachinery is to find the power developed (or consumed) by (or in) a particular machine. A turbine produces power while a pump, compressor or fan consumes power in order to run. The power is determined from the dynamic force or forces which are being exerted by the flowing fluid on the boundaries of flow passage and which are due to the change of momentum. These are determined by applying "Newton's Second Law of Motion."

Momentum may be linear or angular. In fact angular momentum is moment of linear momentum. Rate of change of linear momentum is equal to the force which is responsible for this change ; while rate of change of angular momentum will be equal to the torque of a fluid mass.

If a fluid particle moves in specified direction (i.e.,  $x$ -direction) and strikes a boundary, change of linear momentum will be involved, giving rise to force. This force will be responsible for the motion of a turbine runner. The force multiplied by the distance moved by the runner per

unit time will give the *power* developed by the machine. This is the case of tangential flow machine which is known as *Pelton turbine*. In case the fluid particles move along a curved path change of angular momentum (i.e. moment of momentum) will be used to determine torque. The torque multiplied by angular velocity of the runner will give the power of the machine. The torque may be positive or negative depending upon whether it is exerted *on the fluid* by the body which is being revolved by some external energy or it is exerted *on the body* by the fluid to revolve it. The positive torque multiplied by angular velocity results in power consumed by a machine such as pump, compressor, blower or fan. The negative torque multiplied by angular velocity will give the power developed by the machine such as turbine, ship and aeroplane propeller including helicopter, windmill and fluid coupling.

**1.2 Newton's Second Law of Motion, Linear Momentum Equation and Impulse Momentum Equation**—The fundamental principle of dynamics is Newton's Second Law of Motion which states that "The rate of change of momentum is proportional to the applied force and takes place in the direction of the force". More precisely this statement may be written as "The resultant external force  $F_x$  acting on the particle of mass  $m$  along any arbitrarily chosen direction  $x$  is equal to the time rate of change of *linear* momentum of the particle in the same direction i.e.,  $x$ -direction."

Momentum of a body is the product of its mass and velocity. Let  $m$  be the mass of fluid moving with velocity  $v$  and let the change of velocity be  $dv$  in time  $dt$ .

$$\therefore \text{change of momentum} = m \cdot dv$$

$$\text{and rate of change of momentum} = m \cdot \frac{dv}{dt}$$

According to the above law,

Dynamic force applied in  $x$ -direction

$$= \text{Rate of change of momentum in } x\text{-direction}$$

$$\text{i.e.,} \quad F_x = m \cdot \frac{dv_x}{dt} \quad \dots(1.1)$$

where the suffix  $x$ , denotes components in  $x$ -direction.

This equation is known as *linear momentum equation* and can also be written as

$$F_x \cdot dt = m \cdot dv_x \quad \dots(1.2)$$

The left hand term of this equation is the product of force and the time increment during which it acts. This is known as *impulse of applied force*. The right hand term is the resulting change in momentum. It should be noted that velocity is a vector quantity, consequently any change in magnitude or in direction or in both will change the velocity, hence momentum.

Equation 1.2 is known as *Impulse-momentum Equation* which states—

"Impulse of dynamic force = resulting change in momentum of body."

**1.3 Limitation of Application of Newton's Second Law of Motion and Concepts of System of Mass and Control Volume, Control Surface**—A system refers to a definite mass of material and all other matter around it is known as its surroundings. The boundaries of the system will form a closed surface and this surface may change with time so that it contains the same mass during which the change takes place. The system may contain an infinitesimal mass or a large finite mass of fluid. For example, fluid contained in a cylinder refers to a system of mass and the end of piston is a system boundary.

Newton's second law of motion (refer Eqn 1.1) is generally applicable to a system. This may be written as

$$\Sigma F_x = m \cdot \frac{dv_x}{dt} \quad \dots(1.1a)$$

where  $m$  is the constant mass of the system.  $\Sigma F_x$  is the algebraic sum (or resultant) of all surface forces as well as body forces such as gravity acting on mass  $m$  in any arbitrary direction  $x$ . These are given in the end of this article.

$v_x$  is the velocity of the centre of mass of the system in  $x$ -direction.

$dv_x$  is the change in  $v_x$  in time  $dt$ .

**Control volume** is a specific region in space, its size and shape entirely arbitrary, however these are made to coincide with solid boundaries.

The boundary of a control volume is its *control surface*. Thus control surface specifies the volume of fluid under consideration. The control will show the sections where the fluid enters and leaves and also where no flow occurs.

For a control volume with fluid entering with uniform velocity  $v_{x1}$  and leaving after time  $t$  with uniform velocity  $v_{x2}$ , Eqn 1.1a gives

$$\Sigma F_x = \frac{m}{t} (v_{x2} - v_{x1}) \quad \dots(1.3)$$

Since the dimensions of  $m/t$  is mass per unit time that is mass flow  $\rho Q$ , the algebraic sum of external forces on the fluid in steady flow is equal to the mass flow multiplied by the velocity change in  $x$ -direction

$$\text{i.e.,} \quad \Sigma F_x = \rho Q (v_{x2} - v_{x1}) \quad \dots(1.4)$$

where  $Q$  is the rate of flow and  $\rho$  the density.

This is *one-dimensional form of steady flow momentum equation*, which states that the algebraic sum of the body forces on the matter within the control volume, plus the forces acting on the control surface, both in the  $x$ -direction is equal to the net out-going flux of momentum in  $x$ -direction. This equation is important in the study of turbomachines as it enables to determine the forces developed by the flow in a fluid machine.

**External forces  $F_x$**  may be of three kinds : pressure forces, inertia forces (body forces) and drag forces.

(a) *Pressure forces* are those acting between the fluid and boundary surfaces or between any two adjacent fluid layers.

(b) *Inertia forces* are those caused by the action of gravity and/or centrifugal effects. These are also known as *body forces*.

(c) *Drag forces* are those existing between boundary surfaces and flow. For example forces acting on model hanging in a wind tunnel with the outside support. These are also known as viscous forces.

**1.4 Applications of Linear-Momentum Equation**—So far there are following two kinds of application of linear-momentum equation :

(a) To determine the forces exerted by the flowing fluid on the boundaries of flow passage due to change of momentum.

(b) To determine the flow characteristics when there is some loss of unknown quantity of energy in the flow system such as sudden enlargement of a pipe cross-section and hydraulic jump in an open channel flow.

In this book we are concerned with the applications under (a) above which are given as under :—

(i) *Forces caused by a fluid jet striking a surface*—The surface may be of plate, vane or blade of turbomachines. Such vanes may be flat or curved shape, single or series of them in number and held normal or inclined to the axis of jet. The linear momentum equation is applied to find the power developed by a tangential flow runner of a Pelton turbine which has a series of curved blades (refer Plate 1) on which the water jet impinges.

(ii) *Jet Propulsion and Rocket Mechanics*—Propulsion through a fluid, air or water, is developed by reaction of the jet issuing from a body. The reaction is determined by applying Newton's Law of Action and Reaction (Third Law of Motion), which acts in the opposite direction to that of velocity of jet. The propulsion of boat in sea is an example of jet propulsion which is obsolete now. However, *rocket mechanics* is the modern application of jet propulsion.

Turbojet, turboprop and ram jet also work on the jet propulsion principle. These are however not included in this book.

(iii) *Propeller of Marine and Airships including Helicopter*—The propeller is revolved by a primemover (steam or gas turbine, or diesel engine) in a fluid (air or water). The action of propeller is to change the momentum of fluid in which it is submerged, developing a thrust for the propulsion of ship. The naval ship, air ship and helicopter work on this principle.

*Difference between the propeller and the jet* is that the former exerts a thrust by imparting a small increase in velocity to a relatively large mass of fluid, whereas a jet generally discharge a much smaller mass of fluid at a much higher speed.

(iv) *Force Caused by Flow Round a Pipe-Bend*—Pipe bend or a reducer will produce a change in velocity  $v_x$  of fluid flowing in  $x$ -direction in a pipe line. The change may be in magnitude or direction or both causing forces exerting on the pipe. This necessitates the bend or elbow to be anchored. The design of *anchor blocks* will require the knowledge of force analysis at the pipe bend.

### 1.5 Dynamic Force Exerted by Fluid Jet on Stationary Flat Plate :

(a) **Plate Normal to Jet** (refer Fig 1.1)—A fluid jet issues from a nozzle and strikes a flat plate with a velocity  $v$ . The plate is held stationary and perpendicular to the centre line of the jet.

Let  $Q$  = quantity of fluid falling on the plate, in  $\text{m}^3/\text{sec}$   
 $= \frac{\text{volume}}{\text{time}}$

then  $\gamma \cdot Q$  = weight of fluid in  $\text{kg}/\text{sec}$  ;

$\gamma$  = specific weight of fluid in  $\text{kg}/\text{m}^3$  ;

and  $\frac{\gamma \cdot Q}{g} = \rho Q$  = mass of fluid per sec.

Velocity of fluid in  $x$ -direction before striking the plate =  $v$  m/sec

Final velocity of fluid in  $x$ -direction after striking the plate =  $0$  m/sec

Dynamic force on the fluid by the plate = Change of momentum/sec  
 $= \text{mass striking the plate/sec} \times \text{change of velocity normal to the plate}$

Applying Eqn 1.4

$$\Sigma F_x = \rho Q (v_{x_2} - v_{x_1})$$

The change of velocity is the difference between its final and initial values. Then

$$-F_x = \rho Q (0 - v) = -\rho Q \cdot v$$

The minus sign on right hand side of the equation indicates that the velocity is decreasing, while this sign used with  $F_x$  indicates that the force is acting in the negative direction of  $x$ -axis. The force exerted on the fluid by the plate is thus

$$-F_x = -\rho Q \cdot v \quad (1.5)$$

Here the plate is responsible for changing the velocity of jet, therefore the force is exerted by the plate on the fluid. Since the final velocity is less than the initial velocity, the force exerted on the fluid by the plate is a retarding force, thus it acts in the opposite direction to that of flow.

Now the force exerted by the fluid on the plate is given by 'Newton's Law of Action and Reaction' which will be equal and opposite, namely :

$$F_x = \rho Q \cdot v \quad \dots(1.5a)$$

Half of the quantity of water  $Q$  falling on the stationary vertical plate will move upward and remaining half downward, along the surface of the plate.

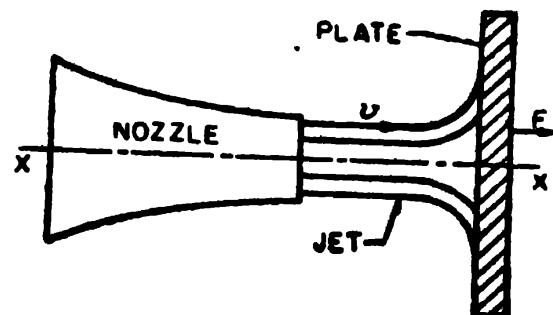


Fig 1.1 Fluid Jet on Stationary Vertical Plate



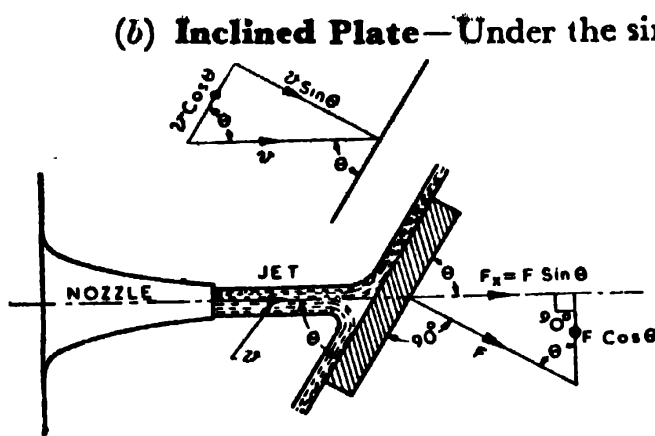


Fig 1.2 Fluid Jet on Stationary Inclined Plate

$F = \text{mass striking the plate per sec} \times \text{change of velocity normal to the plate.}$

Initial velocity of fluid normal to the plate  $= v \sin \theta$ . This is the normal component of velocity  $v$  as shown in Fig 1.2. The final value of this velocity is zero. Then, applying Eqn 1.5.

$$F = \rho Q \cdot v \sin \theta \quad \dots(1.6)$$

Component of this force  $F$  in the direction of jet (refer Fig 1.2) is

$$F_x = F \cdot \sin \theta = \rho Q \cdot v \cdot \sin^2 \theta \quad \dots(1.6a)$$

No work is done in the above cases.

**Determination of division of flow along the two directions of inclined plate** (refer Fig 1.3)—Let  $F_x$  be the force along the inclined surface of plate, and  $Q_1$  and  $Q_2$  the quantities of flow along the surface as shown. As there is no change in elevation of pressure before and after the impact, the magnitude of velocity leaving the plate will remain the same.

Applying linear momentum equation, neglecting losses due to impact, no force is exerted on the fluid by the plate in  $s$ -direction,

$$\begin{aligned} \therefore F_s &= 0 = \rho \cdot Q_1 v \cos \theta \\ &= \rho Q_1 v - \rho Q_2 v \end{aligned}$$

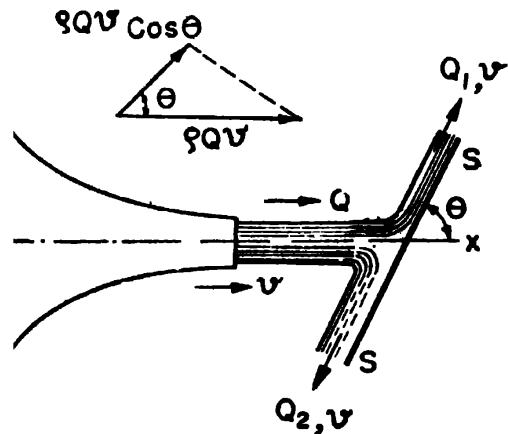


Fig 1.3 Division of Flow Along Surface of Inclined Fixed Plate

Where  $Q_1$  and  $Q_2$  are the rates of flow moving upward and downward along the plate respectively.

$$\text{or} \quad Q \cos \theta = Q_1 - Q_2 \quad \dots(1)$$

$$\text{From Equation of Continuity } Q = Q_1 + Q_2 \quad \dots(2)$$

Solving for  $Q_1$  and  $Q_2$  from (1) and (2),

$$Q_1 = \frac{1}{2} Q (1 + \cos \theta) \quad \text{and} \quad Q_2 = \frac{1}{2} Q (1 - \cos \theta) \quad \dots(1.7)$$

**Problem 1.1** Find the force exerted by a 5 cm diameter water jet directed against a flat plate held normal to the axis of stream. The velocity of the jet is 35 m/sec.

**Solution**

$$d = 5 \text{ cm} \quad v = 35 \text{ m/sec} \quad \gamma = 1,000 \text{ kg/m}^3 \quad g = 9.81 \text{ m/sec}^2$$

$$\text{Density of fluid } \rho = \frac{\gamma}{g} = \frac{1,000}{9.81} \text{ kg sec}^2 \text{m}^{-4}$$

Rate of flow  $Q = a \cdot v$

$$= \frac{\pi}{4} \times \left(\frac{5}{100}\right)^2 \times 35 \text{ m}^3/\text{sec}$$

Applying Eqn 1.5,

Dynamic force exerted by the fluid on the plate  $F_s = \rho Q \cdot v$

or

$$F_s = \frac{1,000}{9.81} \times \left\{ \frac{\pi}{4} \times \left(\frac{5}{100}\right)^2 \times 35 \right\} \times 35 \text{ kg}$$

$$= \frac{1,000 \times \pi \times 0.05 \times 0.05 \times 35 \times 35}{4 \times 9.81} \text{ kg}$$

or Force = 245 kg      Answer

**Problem 1.2** A 2.5 cm diameter water jet exerts a force of 90 kg in the direction of flow on a flat plate which is held inclined at an angle of  $30^\circ$  with the axis of stream. Find the rate of flow.

**Solution**

$$d = 2.5 \text{ cm} \quad F_s = 90 \text{ kg} \quad \theta = 30^\circ \quad \sin 30^\circ = 0.5$$

Applying Eqn 1.6a, Dynamic force exerted by the fluid on the plate in the direction of jet,

$$F_s = F \cdot \sin \theta = \rho Q v \cdot \sin^2 \theta$$

$$= \rho \cdot Q \cdot \frac{Q}{a} \cdot \sin^2 \theta$$

or

$$90 \times 1,000 = \frac{1}{981} \times \frac{Q^2}{\frac{\pi}{4} (2.5)^2} \times 0.5^2$$

$$Q = \sqrt{\frac{90 \times 1,000 \times 981 \times \pi \times 2.5^2}{4 \times 0.5^2}}$$

$$= 41,640 \text{ cm}^3/\text{sec}$$

or  $Q = 41.64 \text{ litres/sec}$       Answer

If the plate were held normal to the axis of jet instead of being inclined at  $30^\circ$ , the rate of flow  $Q$  to exert the same force would obviously be 20.82 litres/sec, as  $\sin \theta$  will then be unity and  $Q$  varies inversely with  $\sin \theta$ .

**Problem 1.3** A square plate weighing 14 kJ and of uniform thickness and 30 cm edge is hung so that it can swing freely about the upper horizontal edge. A horizontal jet 2 cm diameter and having a velocity of 15 m/sec impinges on the plate. The centre line of the jet is 15 cm below the upper edge of the plate, and when the plate is vertical the jet strikes the plate normally and at its centre. (a) Find what force must be applied at the lower edge of the plate in order to keep the plate vertical.

(b) If the plate is allowed to swing freely, find the inclination to the vertical which the plate will assume under the action of the jet.

(London University)

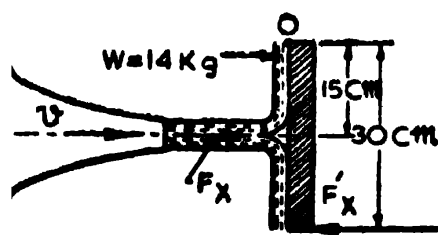
**Solution**

Fig 1.4

$$(a) W = 14 \text{ kg}$$

Square plate 30 cm × 30 cm

$$d = 2 \text{ cm} ; v = 15 \text{ m/sec}$$

Dynamic force exerted by the fluid on the plate at its centre in X-direction,

$$F_s = \rho Q \cdot v \text{ (refer Eqn 1.5)}$$

or

$$F_s = \frac{1,000}{9.81} \left\{ \frac{\pi}{4} \times \left( \frac{2}{100} \right)^2 \times 15 \right\} \times 15$$

$$= 7.2 \text{ kg}$$

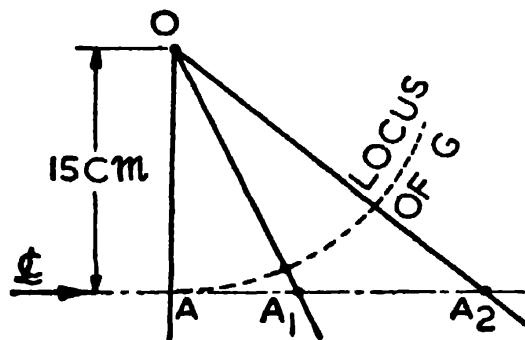


Fig 1.5 (a)

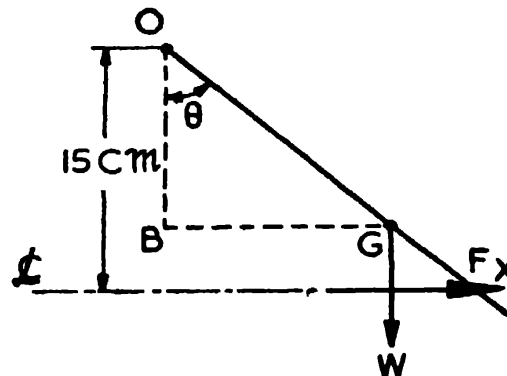


Fig 1.5 (b)

Let  $F'_s$  be the force applied horizontally at the bottom edge of the plate (refer Fig 1.4) to keep the plate in vertical position. Taking moments about O, the upper edge of plate,

$$F'_s \times 30 = 7.2 \times 15$$

or

$$F'_s = \frac{7.2 \times 15}{30} = 3.6 \text{ kg} \quad \text{Answer}$$

(b) If the plate is allowed to swing freely, the vertical distance from the pivot O to the centre line of the jet remains fixed at 15 cm (refer Fig 1.5 a), however the distance from O to the point where it strikes the plate, varies with the angle of inclination  $\theta$ . Thus with angle  $\theta_1$ , the distance is  $OA_1$  and with  $\theta_2$  it changes to  $OA_2$ .

Let the actual inclination be  $\theta$ ,

$$\text{then distance } BG = OG \sin \theta = 15 \sin \theta \quad (\text{refer Fig 1.5b})$$

Now there are two forces acting on the plate—

Force  $F_s$  acting at 15 cm from O.

and force  $W$  acting at a distance  $15 \sin \theta$  from O.

If the plate is to be in equilibrium, the moment of these forces about O must balance each other.

$$\therefore F_s \times 15 = W \times 15 \sin \theta$$

$$\text{or} \quad \sin \theta = \frac{F_s}{W} = \frac{7.2}{14} = 0.515$$

$$\text{or} \quad \theta = \sin^{-1} 0.515 = 31^\circ \quad \text{Answer}$$

**1.6 Force on Moving Flat Plate**—Let the plate in Fig 1.1 move with a velocity  $u$  in the same direction as the jet after the jet with velocity  $v$  has struck the plate, then the change in velocity is  $(u-v)$ . Also the quantity of water striking the plate per second is given by cross-sectional area multiplied by the velocity of jet *relative* to plate which is  $w$ .

i.e.,  $Q = a \cdot w = a \cdot (v-u)$

where  $w$  = velocity of jet relative to the motion of plate ;  
 $v$  = absolute velocity of jet.

Again, Force exerted on *the fluid* by the vane

= mass striking/sec  $\times$  change of velocity

$$F_u = \rho Q (u-v)$$

$\therefore$  Force exerted by the fluid *on the vane*

$$F_u = \rho Q (v-u) = \rho a (v-u)^2 \quad \dots(1.8)$$

Here the distance between plate and nozzle is constantly increasing by  $u$  m/sec. A single moving plate is, therefore, not a practical case. If, however, a *series of plates* (refer Fig 1.6) were so arranged that each plate appeared successively before the jet in the same position and always moving with a velocity  $u$  in the direction of jet, then whole flow from the nozzle is utilised by the plates. Thus weight of water striking the plate would be  $\rho a v$

and  $F = \rho a \cdot v (v-u) \quad \dots(1.9)$

Work done on the plates =  $F \cdot u$   
 $= \rho Q (v-u)u \quad \dots(1.10)$

Kinetic energy of jet =  $\frac{1}{2} m v^2 = \frac{1}{2} \rho Q \cdot v^2$

where  $m$  is the mass of fluid.

$\therefore$  Efficiency of system,  $\eta = \frac{\text{Work obtained}}{\text{Energy input}}$   
 $= \frac{\rho Q (v-u)u}{\frac{1}{2} \rho Q v^2} = \frac{2(v-u)u}{v^2} \quad \dots(1.11)$

For  $\eta_{max}$ ,  $\frac{d\eta}{du} = 0$

i.e.,  $\frac{d}{du}(vu - u^2) = 0$  or  $v - 2u = 0$

$\therefore u = \frac{v}{2}$

Substituting the value of  $u$  in Eqn 1.11

$$\eta_{max} = \frac{2 \left( v - \frac{v}{2} \right) \cdot \frac{v}{2}}{v^2} = \frac{1}{2} \text{ or } 0.5 \text{ or } 50\% \therefore \dots(1.12)$$

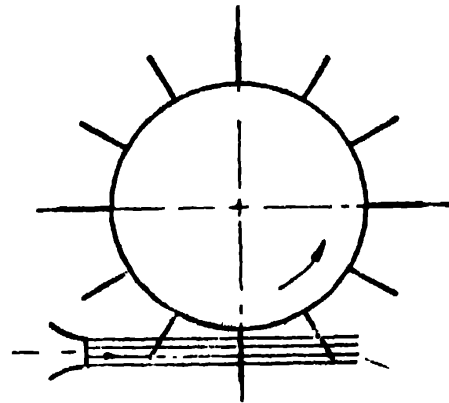


Fig 1.6 Fluid Jet on a Series of Moving Plates

**Problem 1.4** A jet of water 8 cm in diameter moving with a velocity of 12 m per sec strikes a flat vane which is normal to the axis of the stream. (a) Find the force exerted by the jet if the vane moves with a velocity of 5 m per sec. (b) Determine the force exerted by the jet if instead of one flat vane, there is a series of vanes so arranged that each vane appears successively before the jet in the same position and always moving with a velocity of 5 m per second.

**Solution**

$$d = 8 \text{ cm} \quad v = 12 \text{ m/sec} \quad u = 5 \text{ m/sec}$$

$$\therefore \text{Cross-sectional area of jet, } a = \frac{\pi}{4} \times (0.08)^2 \text{ m}^2$$

(a) Force exerted by jet on a single vane

$$F_s = \rho a (v-u)^2 \quad (\text{refer Eqn 1.8})$$

$$= \frac{1,000}{9.81} \times \frac{\pi}{4} \times (0.08)^2 \times (12-5)^2$$

$$= 25.1 \text{ kg} \quad \text{Answer}$$

(b) Force exerted by jet on a series of vanes

$$F = \rho a \cdot v (v-u) \quad (\text{refer Eqn 1.9})$$

$$= \frac{1,000}{9.81} \times \frac{\pi}{4} \times (0.08)^2 \times 12(12-5)$$

$$= 43 \text{ kg} \quad \text{Answer}$$

**1.7 Fluid Jet on Curved Plate—**

(a) **Stationary Plate**—The jet impinges on a curved plate Fig 1.7 at an angle  $\alpha_1$  and is deviated to an angle  $\alpha_2$  both angles being measured

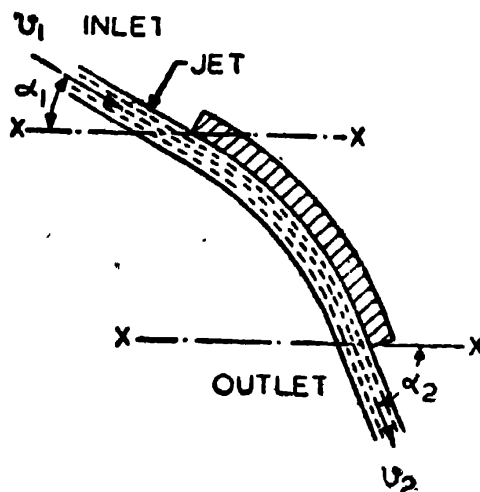


Fig 1.7 Jet Falling on Stationary Curved Plate with Acute Discharge Angle

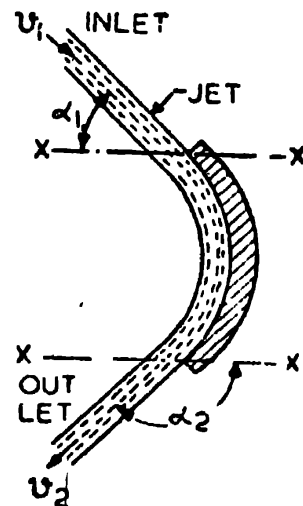


Fig 1.8 Jet Falling on Stationary Curved Plate with Obtuse Discharge Angle

with respect to X-direction. Let  $v_1$  and  $v_2$  be the velocities of jet at inlet and outlet respectively. The velocity of jet at inlet  $v_1$  and the velocity of jet at outlet  $v_2$  will be same as long as there is no friction on the plate.

Velocity of jet at inlet in  $X$ -direction  $= v_1 \cos \alpha_1$

Velocity of jet at outlet in  $X$ -direction  $= v_2 \cos \alpha_2$

$\therefore$  Force exerted *on the jet* by the plate in  $X$ -direction can be determined by applying Linear Momentum Equation—

$$F_x = \frac{m}{t} \times \text{change of velocity in } X\text{-direction (refer Eqn 1.3)}$$

The change of velocity is the difference between its final and initial values.

$$\therefore F_x = \rho Q (v_2 \cos \alpha_2 - v_1 \cos \alpha_1) \quad \dots(1.13)$$

and force exerted *on the plate* by the jet in  $X$ -direction

$$F_x = \rho Q (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \quad \dots(1.14)$$

where  $Q = av_1 =$  quantity of water per second falling on the plate.

$$\therefore F_x = \rho \cdot a \cdot v_1 (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \quad \dots(1.15)$$

If the curvature of the plate at outlet is such that outlet angle  $\alpha_2$  is more than  $90^\circ$  (refer Fig 1.8), then the second term in the bracket of Eqn 1.15 (i.e.,  $v_2 \cos \alpha_2$ ) will be negative.

Hence in order to get more force, the curvature of the plate should be such that the outlet angle  $\alpha_2$  is obtuse.

**(b) Single Moving Plate**—The jet having an inlet velocity  $v_1$  impinges on a moving curved plate at an angle  $\alpha_1$  (refer Fig 1.9) with respect to  $X$ -direction. The curvature of the plate at the point where the jet strikes may or may not make the same angle  $\alpha_1$  with  $X$ -direction. Let the angle of curvature of the plate at inlet with the reversed direction of motion of plate i.e.,  $-u_1$  be  $\beta_1$  (refer Fig 1.9). The plate is moving with a velocity  $u$  in  $X$ -direction. As soon as the jet falls over the plate,

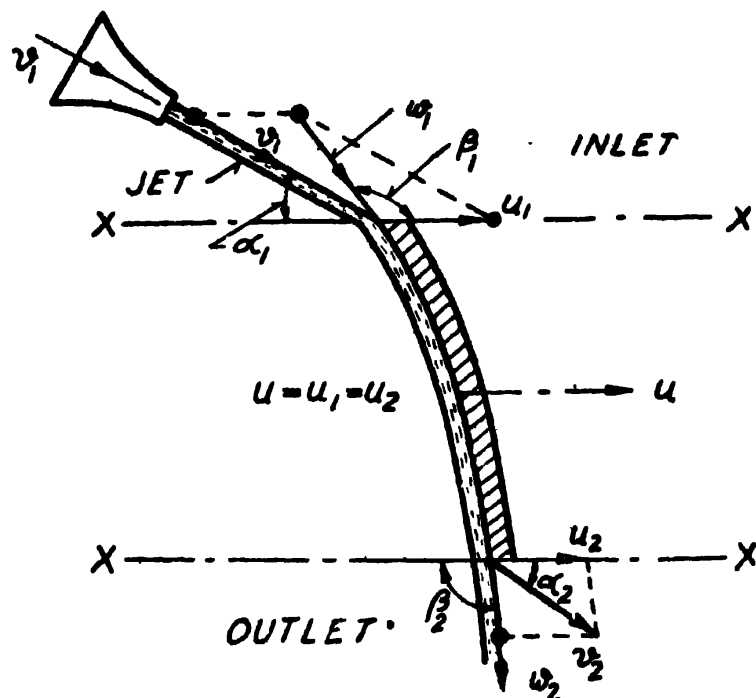


Fig 1.9 Jet Falling on Single Moving Curved Plate with Acute Discharge Angle  $\alpha_2$

the velocity of jet remains no more equal to  $v_1$ , but it becomes the velocity of jet relative to the motion of the plate. This velocity is denoted by  $w_1$ . Its direction will be tangential to the point of inlet. Its magnitude is determined by subtracting  $u$  from  $v_1$  vectorially by Law of Parallelogram of Velocities. The jet glides over the curved surface of the plate with a velocity  $w_1$ . Thus when the jet leaves the plate, its relative velocity will remain

equal to  $w_1$  provided there is no decrease in velocity due to friction on the surface of flow. Let the velocity of jet relative to the plate motion at outlet be denoted by  $w_2$ , then  $w_1 = w_2$ . Let the plate at outlet be inclined at an angle  $\beta_2$ . This angle  $\beta_2$  is measured from the reversed direction of motion of plate at outlet i.e.,  $-u_2$ . Now the absolute velocity of water at outlet  $v_2$  will be the vector sum of the following two velocities—

- (i) Velocity of water with which the water leaves the plate i.e.,  $w_2$ ,
- (ii) Velocity with which the plate moves in X-direction i.e.,  $u$ .

$$\therefore \vec{v}_2 = \vec{w}_2 + \vec{u} \quad \dots(1.16)$$

The magnitude and direction of absolute velocity  $v_2$  is determined by applying Law of Parallelogram of Forces (refer Fig 1.9). The angle which the absolute velocity of water  $v_2$  makes with X-direction or with the direction of motion, is denoted by  $\alpha_2$ .

Force exerted by the jet on the plate in X-direction or in the direction of motion, is determined by applying Linear-Momentum Equation—

$$F_x = \frac{m}{t} \times \text{change of velocity in X-direction} \dots(\text{refer Eqn 1.3 a})$$

$$\text{or } F_x = \rho Q (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \quad \dots(1.17)$$

$$\text{where } Q = a (v_1 - u) \quad \dots(1.18)$$

The water issues from the jet at the rate of  $av_1$ . However, the plate moves away from the jet with velocity  $u$ , therefore, the velocity of the jet falling upon the plate is reduced by  $u$ , or the velocity with which the jet falls upon the plate  $= v_1 - u$ .

This is the velocity of water relative to the motion of the plate i.e.,  $w_1$

$$F_x = \rho \cdot a(v_1 - u) (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \quad \dots(1.19)$$

$$\text{For } \alpha_2 > \frac{\pi}{2}, \cos \alpha_2 < 0 \quad (\text{refer Fig 1.10})$$

Then the second term in the bracket of Eqn 1.19 (i.e.,  $v_2 \cos \alpha_2$ ) will be negative. Hence in order to get more force, the curvature of plate should be such that  $\alpha_2$  is obtuse.

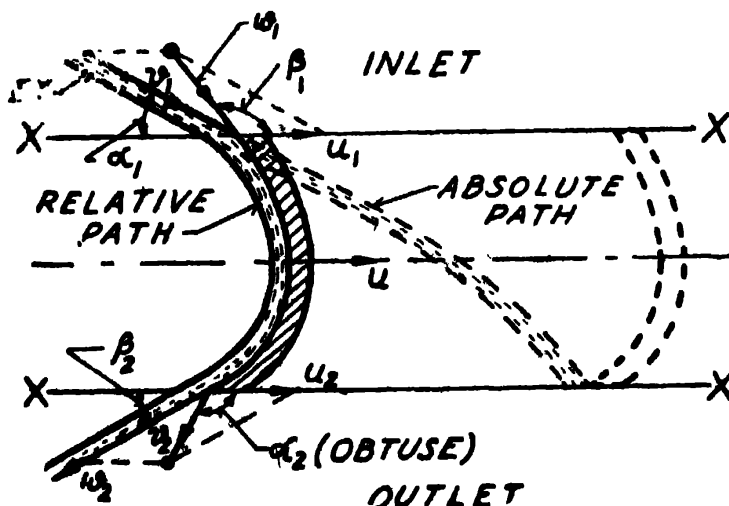


Fig 1.10 Jet Falling on Moving Curved Plate with Obtuse Discharge Angle  $\alpha_2$

**1.8 Absolute Path of Water**—When the jet strikes the moving plate, its position is given by full lines (refer Fig 1.10). As the plate moves with velocity  $u$ , it reaches the position shown by dotted lines when the jet leaves it. Now there are two paths traced by water jet, one over the plate surface which is relative to the motion of plate and therefore appears to be moving with the

plate; and the other is known as *absolute path* which appears to be stationary with respect to earth.

To determine the absolute path of water particle (refer Fig 1.11), take any six points (0 to 5) from the inlet to the outlet of the plate.

Take the distances  $Sw_{0-1}$ ,  $Sw_{0-2}$ ,  $Sw_{0-3}$  etc., along the curved path of the plate from the point of entrance 0 to points, 1, 2, 3 etc. These are the distances traversed by the water particle with  $w$ , the velocity of water relative to the motion of the plate in times  $t_1$ ,  $t_2$ ,  $t_3$ , etc., respectively. Now take the distances  $Su_{0-1}$ ,  $Su_{0-2}$ ,  $Su_{0-3}$  etc., in the horizontal direction from points 1, 2, 3 etc., respectively. These are the distances travelled by the plate moving with  $u$ , its peripheral velocity, in time  $t_1$ ,  $t_2$ ,  $t_3$  etc., respectively. Join the points  $Su_{0-1}$ ,  $Su_{0-2}$ ,  $Su_{0-3}$  etc., taken in horizontal direction, with a curve which indicates the absolute path of water particles.

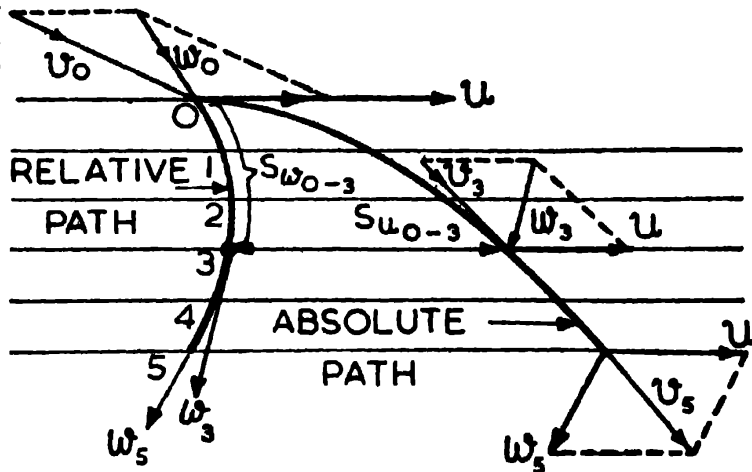


Fig 1.11 Determination of Absolute Path of Water Particle from its Relative Path

The direction of absolute velocity of water at any point will be tangential to the absolute path of water. Similarly the direction of relative velocity of water at any point will be tangential to the relative path of water. The direction of the peripheral velocity of plate is always horizontal. The direction of all the three velocities  $u$ ,  $v$  and  $w$  being known, the velocity triangle can be drawn at any point of the path. The velocity triangles have been shown at points 0, 3 and 5 in Fig 1.11.

**Problem 1.5** A circular water jet having a cross-sectional area of  $25 \text{ cm}^2$  moves with a velocity of  $35 \text{ m/sec}$  and strikes tangentially a curved plate. The angle of curvature of plate at outlet is  $120^\circ$  with the  $X$ -direction. Assuming the plate to be frictionless, find the force exerted by the jet on the plate in  $X$ -direction :

- when the plate is stationary,
- when the plate is moving in the direction of jet with the velocity of  $15 \text{ m/sec}$ .

### Solution

Cross-sectional area of jet  $a = 25 \text{ cm}^2$

Velocity of jet at inlet  $v_1 = 35 \text{ m/sec}$

Angles  $\alpha_1 = 0^\circ$ ,  $\alpha_2 = 120^\circ$

Quantity of water falling on the plate per sec

$$Q = av_1 = \frac{25}{100 \times 100} \times 35 = 0.0875 \text{ m}^3/\text{sec}$$



(a) JET FALLING UPON STATIONARY PLATE—Velocity of jet at inlet in  $X$ -direction  $= v_1 \cos \alpha_1 = v_1 \cos 0^\circ = v_1 = 35 \text{ m/sec}$ .

As the plate is stationary, the velocity of water at outlet will be the same as that at the inlet i.e.,  $v_2 = v_1$ , because there is no friction on the plate (refer Fig 1.8). The velocity  $v_2$  at the outlet makes an angle  $\alpha_2$  with  $X$ -direction.

$\therefore$  Velocity of jet at outlet in  $X$ -direction

$$= v_2 \cos \alpha_2 = 35 \cos 120^\circ = -35 \cos 60^\circ = -17.5 \text{ m/sec}$$

Force exerted by the jet on the plate in  $X$ -direction  $=$  mass of water/sec  $\times$  negative change of velocity of water in  $X$ -direction

or  $F_x = \rho (a v_1) (v_1 \cos \alpha_1 - v_2 \cos \alpha_2)$  —(refer Eqn 1.15)

$$= \frac{1,000}{9.81} \times \left( \frac{25}{100^2} \times 35 \right) \{ 35 - (-17.5) \}$$

$$= \frac{1,000}{9.81} \times \frac{875}{10,000} \times 52.5 = 469 \text{ kg} \quad \text{Answer}$$

(b) JET FALLING UPON MOVING SINGLE PLATE—Peripheral velocity of plate  $u = 15 \text{ m/sec}$ .

Velocity of water relative to the motion of plate at inlet  $w_1 = v_1 - u$  (Vector subtraction of  $v_1$  and  $u$  as shown in Fig 1.12 or Fig 1.15a)

$$= 35 - 15 = 20 \text{ m/sec to the right}$$

Quantity of water falling on the plate/sec,

$$Q = a (v_1 - u) = \frac{25}{100 \times 100} \times 20 = 0.05 \text{ m}^3/\text{sec}$$

Velocity of jet at inlet in  $X$ -direction

$$= v_1 \cos \alpha_1 = v_1 = 35 \text{ m/sec}$$

As the plate is moving with a velocity  $u$ , the water will be discharg-

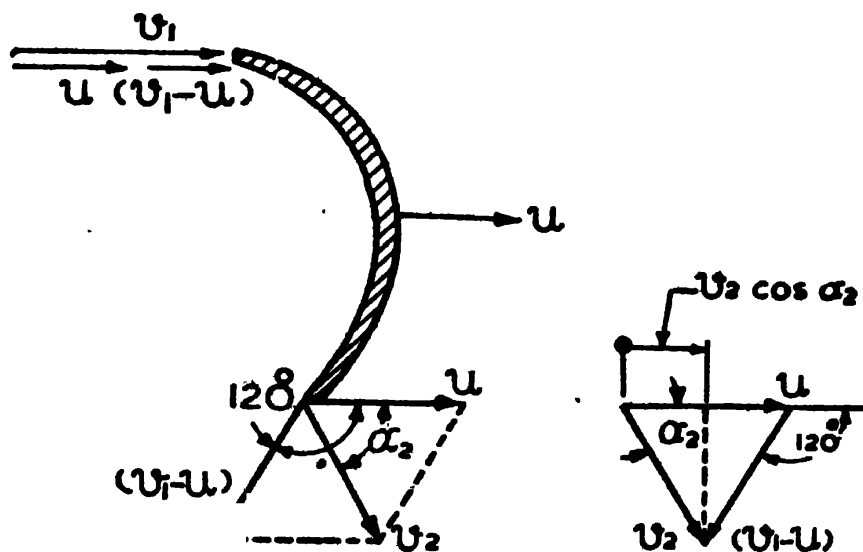


Fig 1.12

ing out with a velocity relative to the motion of the plate i.e.,  $v_1 - u = w_1$ . Since there is no friction on the plate, this velocity  $v_1 - u = w_1 = w_2$ .

It makes an angle  $120^\circ$  with  $X$ -direction or with the direction of the motion of plate. Hence velocity of water at outlet  $v_2$  will be vector sum of the following two velocities—

(i) Velocity of water with which the water is discharging out  
 $w_1 = v_1 - u$ ,

(ii) Velocity of the plate  $= u$

$\rightarrow \quad \longrightarrow \quad \rightarrow$

$$\therefore v_2 = (v_1 - u) + u \quad \text{—(refer Eqn 1.16)}$$

This can be determined by Law of Parallelogram of Forces as shown in Fig 1.12. The angle which the velocity of water  $v_2$ , or the resultant makes with  $X$ -direction is  $\alpha_2$ .

Velocity of jet at outlet in  $X$ -direction

$$= v_2 \cos \alpha_2 = u - (v_1 - u) \cos (180^\circ - 120^\circ) = 15 - 20 \times 0.5 \\ = 5 \text{ m/sec}$$

Force exerted by the jet on the plate in  $X$ -direction = mass of water per sec  $\times$  negative change of velocity of water in  $X$ -direction

$$\therefore F_x = \rho a (v_1 - u)(v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \quad \text{(refer Eqn 1.19)}$$

$$= \frac{1,000}{9.81} \times \frac{25}{100^2} \times 20 \times (35 - 5)$$

$$= 153 \text{ kg} \quad \text{Answer}$$

### 1.9 Fluid Jet on Moving Curved Surface of a Turbine Blade—

Fig 1.13 shows the sectional plan of a double hemispherical blade or bucket as it is sometimes called. Such buckets form the runner of water turbine of Pelton type, shown in Plate 1. Each bucket consists of two hemispherical cups separated by a sharp edge at the centre. The water jet

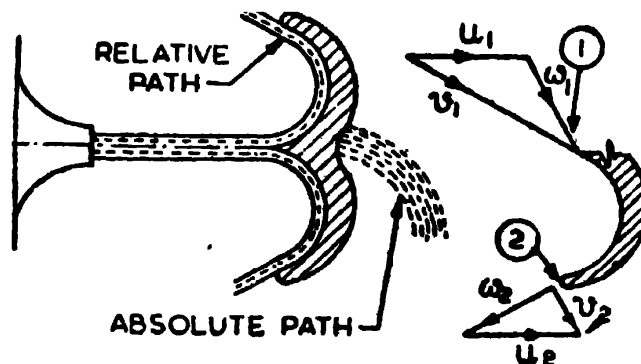


Fig 1.13 Plan of a Double Hemispherical Turbine Bucket  
 (See Plate 1 for Pelton Runner)

impinges at the centre of bucket and is divided by the sharp edge, without shock, into two parts moving sideways in opposite directions. The jet is thus deflected backward when leaving the bucket (refer Fig 1.13). The theoretical angle through which the jet deflects is  $180^\circ$ , but due to some practical difficulties, the angle of deflection is made equal to about  $160^\circ$  (refer Fig 1.13).

The moving curved surface of turbine blade is similar to the moving curved plate described already under Art 1.7b. The flow over the curved surface has been shown in Fig 1.10 as the outlet curvature of the plate  $\beta_2$  is obtuse in case of turbine blade.

The blade moves with a peripheral or circumferential velocity of  $u = \omega \cdot r$ , where  $\omega$  is the angular velocity of the wheel to which the blade is fixed and  $r$  is the radius of an arc of a circle drawn from the centre of the wheel to the point where the jet impings. Now the radius  $r$  is same at the point of inlet and at the point where the jet discharges because both the points of inlet and outlet are in the *same* horizontal plane. Hence circumferential velocity of the blade at inlet is same

$$\text{i.e.,} \quad u = u_1 = u_2 \quad \dots(1.20)$$

**1.10 Velocity Diagrams for Turbine Blades**—Fig 1.14 shows the typical velocity triangles or diagrams of turbine blades. They have been taken from Fig 1.10 and are drawn.—

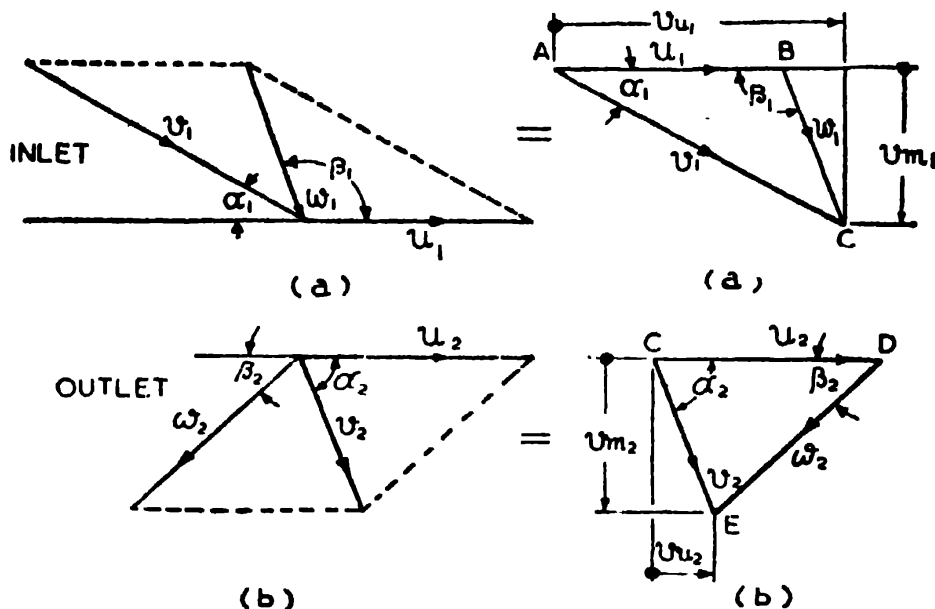


Fig 1.14 Typical Velocity Triangles for the Flow over Turbine Blade  
(a) Inlet (b) Outlet

#### List of Symbols Used in Velocity Diagrams or Triangles

- $u_1$  and  $u_2$  = Circumferential or peripheral velocities of vanes at inlet and outlet respectively.
- $w_1$  and  $w_2$  = Velocities of jet relative to motion of vane at inlet and outlet respectively.
- $v_1$  and  $v_2$  = Absolute velocities (i.e., relative to earth) of jet at inlet and outlet respectively.
- $\alpha_1$  and  $\alpha_2$  = Angles between  $v_1$  and  $u_1$  and  $v_2$  and  $u_2$  respectively i.e., angles of jet with the direction of motion of vane.
- $\beta_1$  and  $\beta_2$  = Angles between  $w_1$  and  $-u_1$  and  $w_2$  and  $-u_2$  respectively i.e., angle of vane tips. These angles are measured between  $w$  and  $u$  reversed.

The absolute velocities  $v_1$  and  $v_2$  can be resolved in two components :

(a) Tangential components  $v_1 \cos \alpha_1$  and  $v_2 \cos \alpha_2$  are represented by symbols  $v_{u_1}$  and  $v_{u_2}$  respectively. These components are parallel to the direction of motion of vane i.e.,  $u$  and are therefore responsible for doing the work. Therefore they are termed as *Velocities of Whirl*.

(b) Radial or axial components  $v_1 \sin \alpha_1$  and  $v_2 \sin \alpha_2$  are represented by symbols  $v_{m_1}$  and  $v_{m_2}$ . These components are perpendicular to the direction of motion of vane and hence they do not do any work on the blades. These components cause the water to flow through turbine blade and are therefore, called the *Velocities of Flow*.

**Drawing of Velocity Triangles**—The velocity is a vector quantity, therefore the velocity triangle is a vector diagram.

**Inlet** (refer Fig 1.14 a)—Draw  $AC = v_1$ , the absolute velocity of water at inlet at an angle of  $\alpha_1$  to the wheel tangent. Draw  $\overline{AB} = u_1$ , the peripheral velocity of wheel in the horizontal direction. Join  $\overline{BC}$  which gives  $w_1$ , the velocity of water relative to wheel motion at inlet, making angle  $\beta_1$  with wheel tangent.

$$\begin{array}{ccc} \rightarrow & \rightarrow & \rightarrow \\ w_1 & = & v_1 + u_1 \end{array}$$

Resolve the absolute velocity of water at inlet into two components  $v_{u_1}$ , the velocity of whirl at inlet which is the tangential component, and  $v_{m_1}$ , the velocity of flow which is the normal and radial component.

Mark the directions of the velocities with arrows as shown in Fig 1.14a.

**Outlet** (refer Fig 1.14b)—Draw  $\overline{CD} = u_2$ , the peripheral velocity of wheel at outlet in the horizontal direction. Draw  $\overline{DE} = w_2$ , the relative velocity of water at outlet, at an angle  $\beta_2$  to  $u_2$ . Join  $\overline{CE}$  which gives  $v_2$  the absolute velocity of water at outlet making an angle  $\alpha_2$  to the wheel motion.

$$\begin{array}{ccc} \rightarrow & \rightarrow & \rightarrow \\ v_2 & = & w_2 + u_2 \end{array}$$

Resolve the absolute velocity of water at outlet into two components  $v_{u_2}$ , the velocity of whirl, the tangential component, and  $v_{m_2}$ , the velocity of flow which is the normal or radial component.

Mark the direction of velocities with arrow as shown in Fig 1.14b. The velocity of whirl at outlet  $v_{u_2}$  may be positive or negative, depending upon the angle  $\alpha_2$  being acute or obtuse respectively.

**1.11 Work Done on Tangential Flow Turbine Runner**—The tangential flow turbine runner (Pelton wheel) is shown in Plate 1. It consists of a number of double hemispherical type blades shown in Fig 1.13. The jet of water issues from a nozzle and impinges on a few blades of runner at a time with an absolute velocity of water  $v_1$ . Thus with this arrangement, each blade, of the wheel appears successively before the jet in the same position. The wheel revolves with an angular velocity  $\omega$ . If  $r$  is the

radius of this point on the blade where the jet impinges, the peripheral velocity of this point is  $u = \omega \cdot r$ . The velocity  $u$  is tangent to the wheel motion and direction of jet. The velocity of water which is responsible for doing the work on the blades, is the velocity of whirl. This is the component of the absolute velocity of water in the direction of motion.

Force exerted by jet on the vanes in the direction of motion = mass of water per sec  $\times$  negative change of velocity in the direction of motion = mass of water per sec  $\times$  negative change of velocity of whirl

$$\text{or } F_u = \rho Q (v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \quad \text{---(1.21)}$$

This equation is the same as that already derived under Eqn 1.14 as well as under Eqn 1.17. However because of series of blades,

in this case  $Q = a \cdot v_1$

$$\therefore F_u = \rho \cdot (a \cdot v_1) (v_{u_1} - v_{u_2}) \quad \text{---(1.22)}$$

The factor  $(v_{u_1} - v_{u_2})$  is the algebraic difference of the velocities of whirl at inlet and outlet and can be obtained from velocity triangles (refer Fig 1.14). If the water is discharged in the direction of the motion of blade,  $v_{u_2}$  will be positive (refer Fig 1.9), which means that the change of velocity of whirl will be equal to  $(v_{u_1} - v_{u_2})$ . However, in order to get more force, the discharge angle  $\alpha_2$  should be obtuse which makes the direction of discharging water opposite to the blade motion. This makes  $v_{u_2}$  negative. Then the change of velocity of whirl will be equal to the arithmetic sum of  $v_{u_1}$  and  $v_{u_2}$ .

Work done on blade per sec = Force exerted by the jet on the blades in the direction of motion  $\times$  distance moved by the blades per sec.

$$\text{W.D} = \rho Q (v_{u_1} - v_{u_2}) u \quad \text{...(1.23)}$$

$$\text{Horse power developed by the wheel} = \rho Q (v_{u_1} - v_{u_2}) \frac{u}{75} \quad \text{...(1.24)}$$

$$\text{Work done on the blades per kg of water} = \frac{(v_{u_1} - v_{u_2}) \cdot u}{g} \quad \text{...(1.25)}$$

$$\text{Work done on the blades per unit mass of water} = (v_{u_1} - v_{u_2}) u \quad \text{...(1.26)}$$

Energy supplied to the blades per kg of water

= Kinetic energy of jet at entrance per kg of water

$$= \frac{v_1^2}{2g}$$

$$\therefore \text{Efficiency of turbine runner } \eta = \frac{\text{Output}}{\text{Input}}$$

$$\text{or } \eta = \frac{\text{Work done on blades per kg of water}}{\text{Energy supplied to the blades per kg of water}}$$

$$= \frac{(v_{u1} - v_{u2}) \cdot u}{\frac{g}{v_1^2} \cdot \frac{1}{2g}} = \frac{2u(v_{u1} - v_{u2})}{v_1^2} \quad \dots(1.27)$$

**Problem 1.6** A jet of water 5 cm in diameter impinges on a curved vane and is deflected through an angle of  $175^\circ$ . The vane moves in the same direction as that of the jet with a velocity of 35 m per sec. The rate of flow is 170 lit/sec. Determine the component of force on the vane in the direction of motion. How much would be the Horse Power developed by the vane and what would be the water efficiency? Neglect friction.

Solve the problem if, instead of one vane, there is a series of vanes fixed to a wheel.

**Solution**

$$d = 5 \text{ cm} \quad u = 35 \text{ m/sec} \quad Q = 170 \text{ lit/sec} = 0.17 \text{ m}^3/\text{sec}$$

$$v_1 = \frac{Q}{a} = \frac{0.17}{\frac{\pi}{4} \times \left(\frac{5}{100}\right)^2} = 86.8 \text{ m/sec}$$

Assume  $\alpha_1 = 0$  and  $\beta_1 = 0$ ;  $\beta_2 = 180^\circ - 175^\circ$  (refer Fig 1.15b)

Velocity of jet in direction of motion at inlet,  $v_1 \cos \alpha_1 = v_{u1} = v_1$  and velocity of jet in direction of motion at outlet,  $v_2 \cos \alpha_2 = v_{u2}$

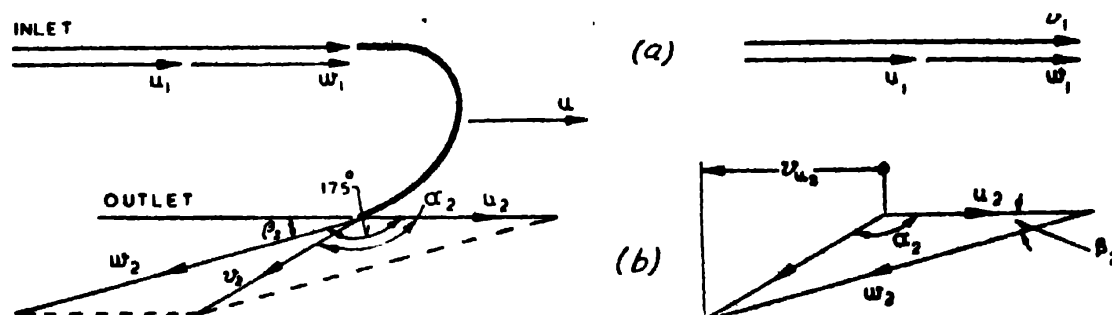


Fig 1.15 Velocity Triangles (a) Inlet, (b) Outlet

Draw velocity triangles at inlet and outlet of vane (refer Fig 1.15)

Considering the inlet velocity triangle first—

$$v_1 = u_1 + w_1$$

$$\therefore w_1 = v_1 - u_1 = 86.8 - 35 = 51.8 \text{ m/sec.}$$

Now  $w_1 = w_2$ , assuming that no loss occurs along the vane from inlet to outlet. Further the vane is moving with a velocity of 35 m/sec. The radius of arc of a circle drawn from the centre of the wheel to the point where the jet impinges and to the point where the jet leaves the vane, is the same, therefore  $u = \omega r = u_1 = u_2$ .

From outlet velocity triangle (refer Fig 1.15b)

$$\begin{aligned} v_2 \cos \alpha_2 &= v_{u2} = u_2 - w_2 \cos \beta_2 \\ &= 35 - 51.8 \cos 5^\circ \\ &= 35 - 51.8 \times 0.9962 = 35 - 51.6 \\ &= -16.6 \text{ m/sec} \end{aligned}$$

Force exerted by the jet on the vane in the direction of motion, (refer Eqn 1.19)

$$\begin{aligned} F_u &= \rho \cdot a \cdot (v_1 - u_1)(v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \\ &= \frac{1,000}{9.81} \times \frac{\pi}{4} \times \left( \frac{5}{100} \right)^2 \times 51.8 \times (86.8 + 16.6) \\ &= 1,075 \text{ kg Answer} \end{aligned}$$

$$\text{HP developed by jet} = \frac{F_u \cdot u}{75} = \frac{1,075 \times 35}{75} = 502 \text{ HP Answer}$$

$$\begin{aligned} \text{Water efficiency} &= \frac{\text{HP developed} \times 75}{\text{K.E. of jet } \left( \frac{1}{2} m \cdot v_1^2 \right)} \\ &= \frac{502 \times 75}{\frac{1}{2} \times \frac{1,000 \times 0.17}{9.81} \times 86.8^2} \\ &= 0.575 \text{ or } 57.5\% \quad \text{Answer} \end{aligned}$$

(b) A series of vanes fixed to a wheel, instead of one vane only—

$$\begin{aligned} F_u &= \rho (a \cdot v_1)(v_1 \cos \alpha_1 - v_2 \cos \alpha_2) \\ &= \frac{1,000}{9.81} \times 0.17 \times (86.8 + 16.6) \\ &= 1,790 \text{ kg Answer} \end{aligned}$$

$$\text{HP developed by jet} = \frac{F_u \cdot u}{75} = \frac{1,790 \times 35}{75} = 835 \text{ HP Answer}$$

$$\begin{aligned} \text{Water efficiency} &= \frac{\text{HP developed}}{\text{K.E. of jet}} \times 75 = \frac{835 \times 75}{\frac{1}{2} \times \frac{1,000 \times 0.17}{9.81} \times 86.8^2} \\ &= 0.959 \text{ or } 95.9\% \quad \text{Answer} \end{aligned}$$

**Problem 1.7** A 7.5 cm diameter water jet having a velocity of 15 m per sec impinges on the bucket of a wheel. The axis of the jet coincides with the axis of the bucket. The bucket is a part of sphere and has a radius of 20 cm, the depth being 10 cm. Determine the force exerted by the jet on bucket when :

(a) the bucket is fixed,

(b) the bucket is moving in the same direction as the jet with a velocity such that the work done per second by the jet on bucket would be maximum,

(c) there is a series of buckets in place of only one, and moving with a velocity such that the efficiency is maximum.

Find the horse power in each case and also give the values of the maximum efficiency.

**Solution**

$$\begin{aligned} d &= 7.5 \text{ cm} & v_1 &= 15 \text{ m/sec} \\ \text{radius of bucket} &= 20 \text{ cm} & \text{depth} &= 10 \text{ cm} \\ \text{area of jet} & a = \frac{\pi}{4} \times \left( \frac{7.5}{100} \right)^2 = 0.00442 \text{ m}^2 \\ Q &= a \cdot v_1 = 0.00442 \times 15 = 0.0663 \text{ m}^3/\text{sec} \end{aligned}$$

From Fig 1.16

$$OC = OD = \text{radius of bucket} \\ = 20 \text{ cm}$$

$$BE = 10 \text{ cm}$$

$$OB = 10 \text{ cm}$$

$$\text{and } BD = \sqrt{20^2 - 10^2} \\ = \sqrt{300} \\ = 17.32 \text{ cm}$$

angle of jet with the axis at outlet—

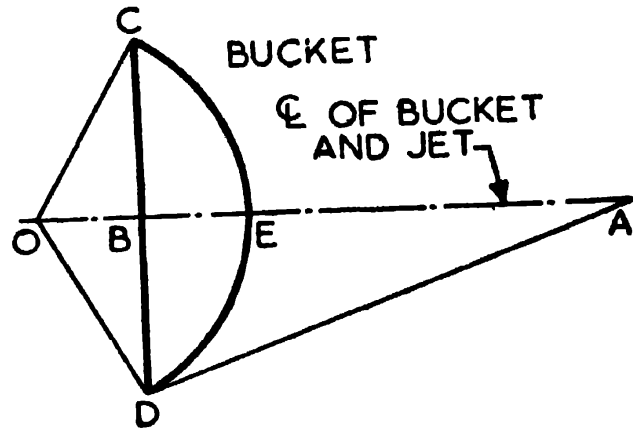


Fig 1.16 Bucket made from a Sphere

$$\beta_2 = \angle BAD = \angle ODB$$

$$= \sin^{-1} \frac{OB}{OD} = \sin^{-1} \frac{10}{20} = \sin^{-1} 0.5$$

$$\therefore \beta_2 = 30^\circ$$

(a) Change of velocity in direction of jet

$$= v_1 - (-v_1 \cos 30^\circ) = v_1 + v_1 \cos 30^\circ$$

Mass of water moving per sec =  $\rho Q$

$$\therefore \text{Force on fixed bucket} = \rho Q (v_1 + v_1 \cos 30^\circ)$$

$$= \frac{1,000 \times 0.0663}{9.81} \times 15 (1 + 0.866) = 189 \text{ kg}$$

Horse Power is **zero**, because the bucket is fixed and no work is done. Hence efficiency is also **zero**.

(b) The bucket is moving with the velocity  $u$ , in the same direction as that of jet. Drawing the velocity diagrams (Fig 1.15).

$$v_{u1} = v_1 = 15 \text{ m/sec}$$

and  $w_1 = w_1 \dots \dots$  (assuming no loss on the bucket)

$$\therefore w_2 = w_1 = v_1 - u$$

$$= 15 - u$$

From outlet velocity diagram (refer Fig 1.15)

$$v_{u2} = u - w_2 \cos 30^\circ \\ = u - (15 - u) \cos 30^\circ \\ = u - (15 - u) \times 0.866 \\ = 1.866 u - 13$$

Force exerted on the bucket,

$$F_x = \rho a (v_1 - u) (v_{u1} - v_{u2})$$

$$\therefore \text{Work done per sec, } W = \rho a (v_1 - u) (v_{u1} - v_{u2}) \cdot u$$

$$\text{or } W = \rho a (15 - u) (15 + 13 - 1.866 u) u \\ = \rho a [15 \times 28 - \{(15 \times 1.866) + 28\}u + 1.866 u^2] u$$

$$\text{For maximum efficiency, } \frac{dW}{du} = 0$$



$$\therefore \rho a \cdot \frac{d}{du} (15 \times 28 u - 56 u^2 + 1.866 u^3) = 0$$

$$\text{or } 15 \times 28 - 2 \times 56 u + 3 \times 1.866 u^2 = 0$$

$$\text{or } u^2 - \frac{2 \times 56}{3 \times 1.866} u + \frac{15 \times 28}{3 \times 1.866} = 0$$

$$\text{or } u^2 - 20u + 75 = 0$$

$$\therefore u = \frac{20 \pm \sqrt{20^2 - 4 \times 75}}{2}$$

$$= \frac{20 \pm \sqrt{100}}{2}$$

$$= \frac{20 \pm 10}{2} = 15 \text{ or } 5 \text{ m/sec}$$

If  $u = 15$  m/sec, work done = 0, i.e., minimum, and for maximum work done,  $u = 5$  m/sec.

$$\begin{aligned} \text{Now } F_s &= \rho a (v_1 - u) (v_{u_1} - v_{u_2}) \\ &= \frac{1,000}{9.81} \times 0.00442 \times (15 - 5)(28 - 1.866 \times 5) \\ &= \frac{1,000}{9.81} \times 0.00442 \times 10 \times 18.67 \\ &= 83.5 \text{ kg} \end{aligned}$$

$$\begin{aligned} \text{Horse power} &= \frac{F_s \cdot u}{75} = \frac{83.5 \times 5}{75} \\ &= 5.57 \text{ HP} \end{aligned}$$

$$\begin{aligned} \text{Efficiency} &= \frac{\text{Work done/sec}}{\text{Energy supplied}} = \frac{83.5 \times 5}{\frac{1}{2} \times \frac{1,000 \times 0.0663}{9.81} \times 15^3} \\ &= \frac{417.5}{759} \\ &= 0.55 \text{ or } 55\% \end{aligned}$$

(c) If there is a series of buckets,

$$\text{Force exerted on the buckets, } F_s = \rho \cdot a \cdot v_1 \cdot (v_{u_1} - v_{u_2})$$

$$\text{and work done per sec} = \rho \cdot a \cdot v_1 \cdot (v_{u_1} - v_{u_2}) \cdot u$$

as all the water impinges with a velocity of  $v_1$ .

$$\therefore W = \frac{1,000 \times 0.0663}{9.81} (15 + 13 - 1.866 u)u$$

$$\text{For maximum efficiency, } \frac{dW}{du} = 0$$

$$\text{i.e., } 28 - 2 \times 1.866 u = 0$$

$$\text{or } u = \frac{28}{2 \times 1.866} = 7.5 \text{ m/sec}$$

(i.e., for maximum efficiency,  $u_1 = \frac{1}{2} v_1$ )

$$\therefore \text{Force exerted, } F_s = \frac{1,000 \times 0.0663}{9.81} \times (28 - 1.866 \times 7.5)$$

$$= \frac{1,000 \times 0.0663 \times 14}{9.81} = 94.5 \text{ kg}$$

$$\therefore \text{Horse power} = \frac{F_s \cdot u}{75} = \frac{94.5 \times 7.5}{75} = 9.45 \text{ HP}$$

$$\text{Efficiency} = \frac{\text{Work done/sec}}{\text{Energy supplied}} = \frac{94.5 \times 7.5}{\frac{1}{2} \times \frac{1,000 \times 0.0663}{9.81} \times 15^2}$$

$$= \frac{708}{759}$$

$$= 0.935 \quad \text{or} \quad 93.5\%$$

Answers—

| Force       | Horse Power | Efficiency |
|-------------|-------------|------------|
| (a) 189 kg  | 0 HP        | 0 %        |
| (b) 83.5 kg | 5.57 HP     | 55 %       |
| (c) 94.5 kg | 9.45 HP     | 93.5 %     |

### 1.12 Jet Propulsion—

**Principle**—An orifice of cross-sectional area  $a$  is fixed to a tank full of water (refer Fig 1.17). The jet issues out from the orifice with a large velocity  $v$ . Let the quantity of water coming out of orifice per sec be  $Q$ . Applying linear momentum equation (refer Eqn 1.1), the force exerted upon the fluid to change its velocity from 0 to  $v$  is

$$F = \rho Q (v - 0), = \rho Qv \quad \text{--- (1.28)}$$

$$\text{where } Q = av$$

$$v = K_v \cdot \sqrt{2gh}$$

$K_v$  = the co-efficient of velocity ;

$h$  = the head over the centre line of orifice.

According to Newton's Law of Action and Reaction, an equal and opposite force ( $-F$ ) is exerted upon the tank. If the tank is provided with wheels, it will start moving towards the left with a velocity  $u$  (say). Thus  $F$  is the *propulsive* force and it may be used to propel the crafts such as air craft, rocket ship or submarine to which the orifice or nozzle is fitted.

**Propulsion of Ships**—The principle explained above may be applied to drive the ship through the water. The ship carries pumps which take water from the surroundings of the vessel and discharge by forcing through the orifice at the back of the ship.

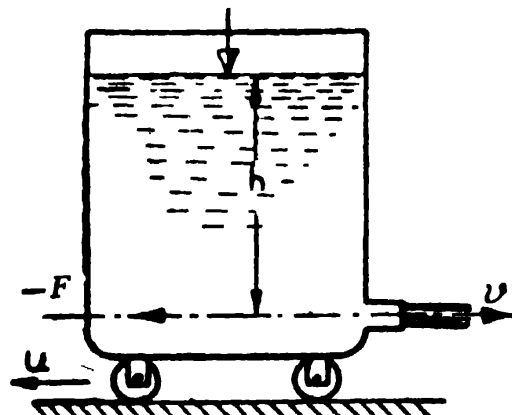


Fig 1.17 Principle of Jet Propulsion

Let  $u$  = the velocity of ship ;  
 $v$  = the velocity of issuing jet ;  
 $w_1$  = the velocity of water issuing from the jet relative to the motion of ship.

Then  $w_1 = u + v$

( $\because u$  and  $v$  act in opposite directions refer Fig 1.18)

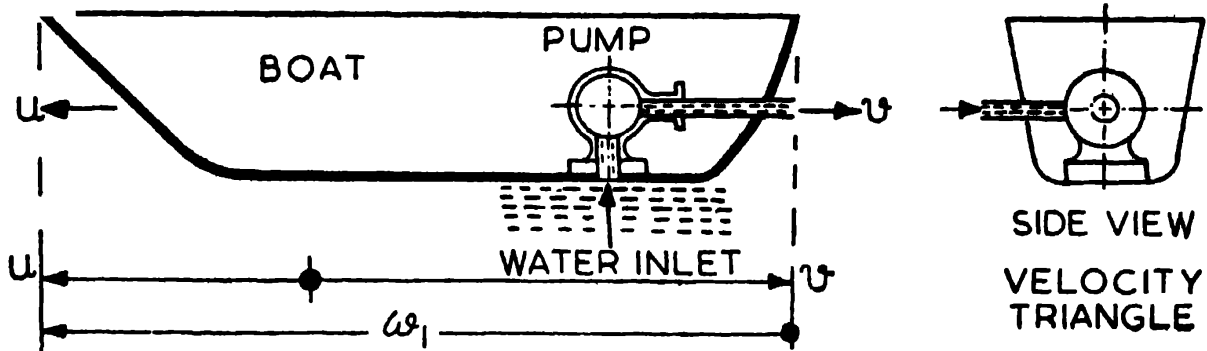


Fig 1.18 Jet Propulsion of Boat

Head of water supplied by the pumps or energy supplied per kg of water  $= \frac{w_1^2}{2g}$ .

Momentum of jet per sec  $= \rho \cdot Q \cdot v$

Momentum of water entering the ship per sec  $= 0$ .

( $\because$  It is at rest).

$\therefore$  Rate of change of momentum  $= \rho \cdot Q \cdot v$

This is equal to the *Dynamic Force* exerted on the fluid. An equal in magnitude and opposite in direction will be the force exerted on the ship by the fluid. This will be  $-\rho Q v$  and responsible for propelling the ship.

Work done on the ship by the jet/sec  $= \rho \cdot Q \cdot v \cdot u$  ... (1.29)

but  $w_1 = u + v$  or  $v = w_1 - u$

$\therefore$  Work done on the ship by the jet per sec  $= \rho Q (w_1 - u) \cdot u$  ... (1.29a)

Energy supplied per sec  $= \text{mass/sec} \times (\text{energy supplied per kg of water})$

$$= \gamma \cdot Q \cdot \frac{w_1^2}{2g} = \frac{1}{2} \rho Q w_1^2$$

where  $\gamma = \rho \cdot g$

$\therefore$  Efficiency of the system  $\eta$

$$= \frac{\text{Work done on the ship by the jet per sec}}{\text{Energy supplied per sec}} \\ = \frac{\rho Q (w_1 - u) \cdot u}{\frac{1}{2} \rho \cdot Q \cdot w_1^2} = \frac{2(w_1 - u) \cdot u}{w_1^2} \quad \dots (1.30)$$

For maximum efficiency  $\frac{d\eta}{du} = 0$

$$\therefore \frac{d}{du} (w_1 \cdot u - u^2) = 0$$

or  $w_1 - 2u = 0$

or  $u = \frac{w_1}{2} \quad \dots(1.31)$

Substituting the value of  $u$  in Eqn 1.30

$$\eta_{\text{max}} = \frac{2 \left( w_1 - \frac{w_1}{2} \right) \cdot \frac{w_1}{2}}{w_1^2} = \frac{1}{2} \text{ or } 50\% \dots(1.32)$$

In some cases the water enters the ship through a pipe having an open end at the front of ship. As the water is at rest and the ship moves with a velocity  $u$ , the water will enter the pipe with velocity  $u$  relative to the motion of the ship (refer Fig 1.19).

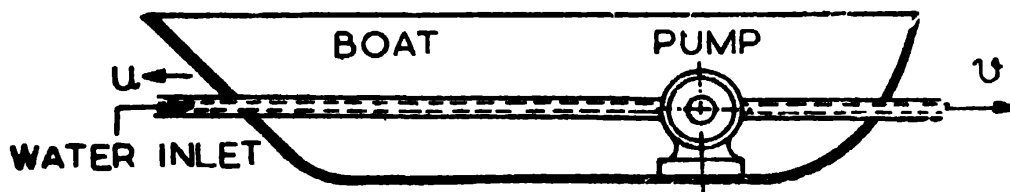


Fig 1.19 Jet Propulsion of Boat

$$\begin{aligned} \therefore \text{Energy supplied by the water per sec} &= \gamma \cdot Q \left( \frac{w_1^2}{2g} - \frac{u^2}{2g} \right) \\ &= \frac{1}{2} \rho Q (w_1^2 - u^2) \end{aligned}$$

instead of  $\gamma \cdot Q \cdot \frac{w_1^2}{2g}$  as in the previous case. Thus now the energy supplied is less than the previous case.

Now the work done on the ship by the jet per sec

$$= \rho Q (w_1 - u) u \quad \dots(\text{refer Eqn 1.29a})$$

i.e., same as in previous case.

$$\begin{aligned} \therefore \text{Efficiency } \eta &= \frac{\rho \cdot Q (w_1 - u) u}{\frac{1}{2} \rho Q (w_1^2 - u^2)} = \frac{2(w_1 - u) \cdot u}{(w_1 + u)(w_1 - u)} \\ &= \frac{2u}{(w_1 + u)} \quad \dots(1.33) \end{aligned}$$

The principle of jet propulsion is similar to that of rocket, therefore, the jet propulsion of ships can be known as water rocket. In practice screw propulsion is employed to move the ships and jet propulsion is no longer used, except in the case of life boats to a limited extent only.

**Problem 1.8** A jet propelled boat, moves at 25 km/hour. The jet having a cross-sectional area 225 sq cm, has a relative velocity of 10 m/sec. Find the BHP required to work the pumps, assuming the orifices face the direction of motion of boat. (Panjab University)

Find also the efficiency of the jet propulsion.

**Solution**

$$\begin{aligned}\text{Velocity of boat } u &= 25 \text{ km/hour} \\ &= \frac{25 \times 1,000}{3,600} = 6.95 \text{ m/sec}\end{aligned}$$

$$\begin{aligned}\text{Velocity of water issuing from the jet relative to the motion of boat,} \\ w_1 &= 10 \text{ m/sec}\end{aligned}$$

$$\begin{aligned}\therefore \text{ Discharge of water by the jet } Q \\ &= a w_1 \\ &= \frac{225}{100 \times 100} \times 10 = 0.225 \text{ m}^3/\text{sec}\end{aligned}$$

$$\begin{aligned}\text{BHP of the pump} &= \text{energy supplied by the water/sec} \\ &= \frac{\gamma Q}{75} \left( \frac{w_1^2}{2g} - \frac{u^2}{2g} \right) \\ &= \frac{1,000 \times 0.225}{75 \times 19.62} (10^2 - 6.95^2) \\ &= \frac{1,000 \times 0.225 \times 51.5}{75 \times 19.62} = 7.88 \text{ HP} \quad \text{Answer}\end{aligned}$$

Efficiency of jet propulsion,  $\eta$

$$\begin{aligned}&= \frac{2u}{w_1 + u} = \frac{2 \times 6.95}{10 + 6.95} = \frac{13.90}{16.95} \\ &= 0.822 \quad \text{or} \quad 82.2\% \quad \text{Answer.}\end{aligned}$$

**1.13 Rocket Mechanics**—A rocket is driven forward by the principle of jet propulsion. The jet consisting of burnt gases, comes out of the nozzle fitted in the bottom of the rocket (refer Fig 1.20). The gases are produced inside the body of rocket by burning the fuel with the oxidizing agent (or oxidant) which is *also* carried by the rocket. Thus no further air is required and the rocket can move in any kind of space—atmosphere or vacuum. As the fuel and oxidizing agent are burnt and consumed, the mass of the rocket does not remain constant. Thus the simple equation of Linear Momentum will not be applied to find the propulsive force. Moreover the rocket is accelerating and its directions are also changed as it moves further.

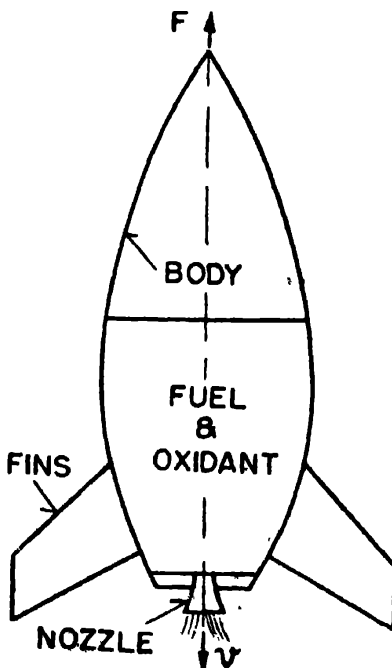


Fig 1.20 Propulsive Force of a Rocket

**Jet Engine** is a device which carries the fuel but not the oxidant. The fuel is burnt by the air taken from the atmosphere. Jet engine may be of different kinds ; turbojet, turbo-prop and ramjet.

**Turbojet** is a jet engine consisting of a compressor, a combustion chamber and a jet pipe (refer Fig 1.21). The compressor takes air from the front and compresses it and moves it further to the combustion chamber where the fuel is burnt producing combustion gases. These gases pass through the gas turbine to run it for driving the compressor. Thus the gas turbine takes only a portion of energy of hot gases and the remaining gases exhaust through the jet pipe producing the propulsive force. A turbojet can take off from the ground.

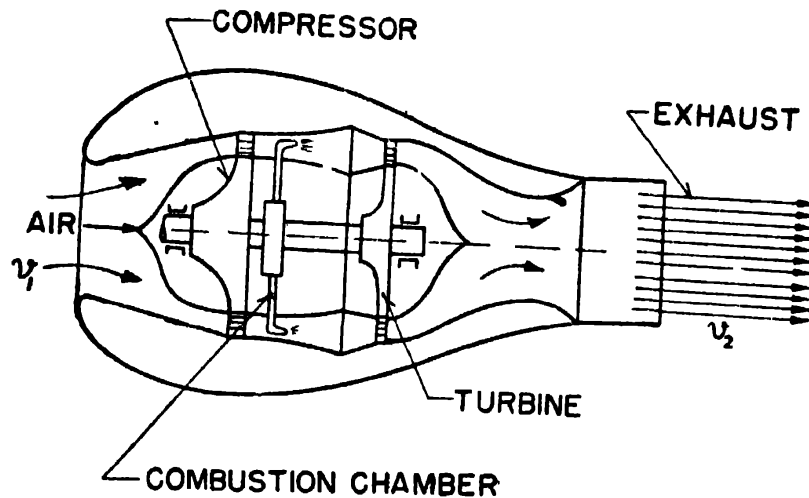


Fig 1.21 Working Principle of Turbojet

**Turbo-prop**—It is a turbojet provided with a propeller. The gas turbine drives both the compressor and the propeller. The propulsive force is given by the ejection of the gases as well as the propeller. The theory of development of propulsive force by the propeller is explained in the next article (refer Art 1.14).

**Ramjet** (refer Fig 1.22) is a high speed jet engine without any compressor and turbine. It must be brought up to a high speed by rocket or some other means. The air is then scooped from the front, converting its

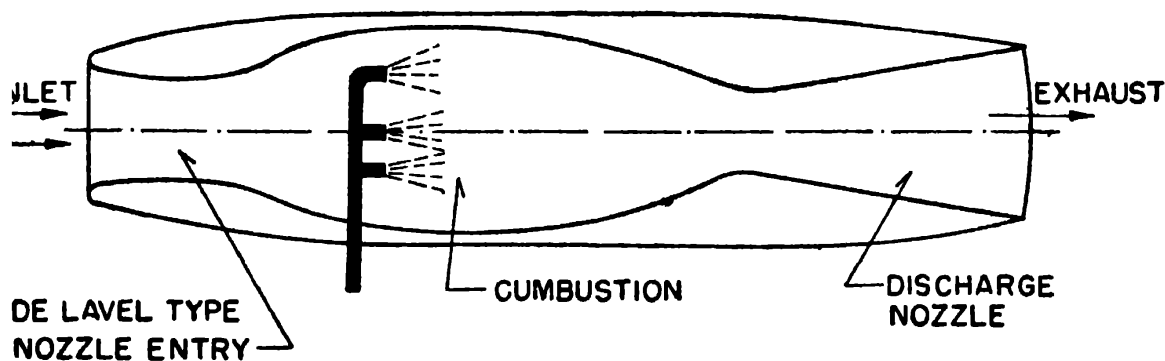


Fig 1.22 Ramjet

kinetic energy into pressure energy by enlarging flow cross-section. The pressurised air then goes to the combustion chamber where combustion takes place with the fuel supplied. The burnt gases come out of the jet pipe developing the propulsive force.

**1.14 Propulsion of Marine and Air Ships Including Helicopter**—*Momentum theory* of a propeller is applied to find the propulsive force. The propeller uses the torque of the driving shaft in order to develop axial thrust. This is possible by increasing the momentum of fluid (air or water) in which it is submerged. The body of fluid affected by the propeller is called *slip stream* (refer Fig 1.23).

At section 1, the flow is taken as stationary and is accelerated towards the propeller because the pressure reduces on the upstream side. When

the fluid passes through the propeller, its pressure is greatly increased. This accelerates the flow reducing cross-section at section 4.

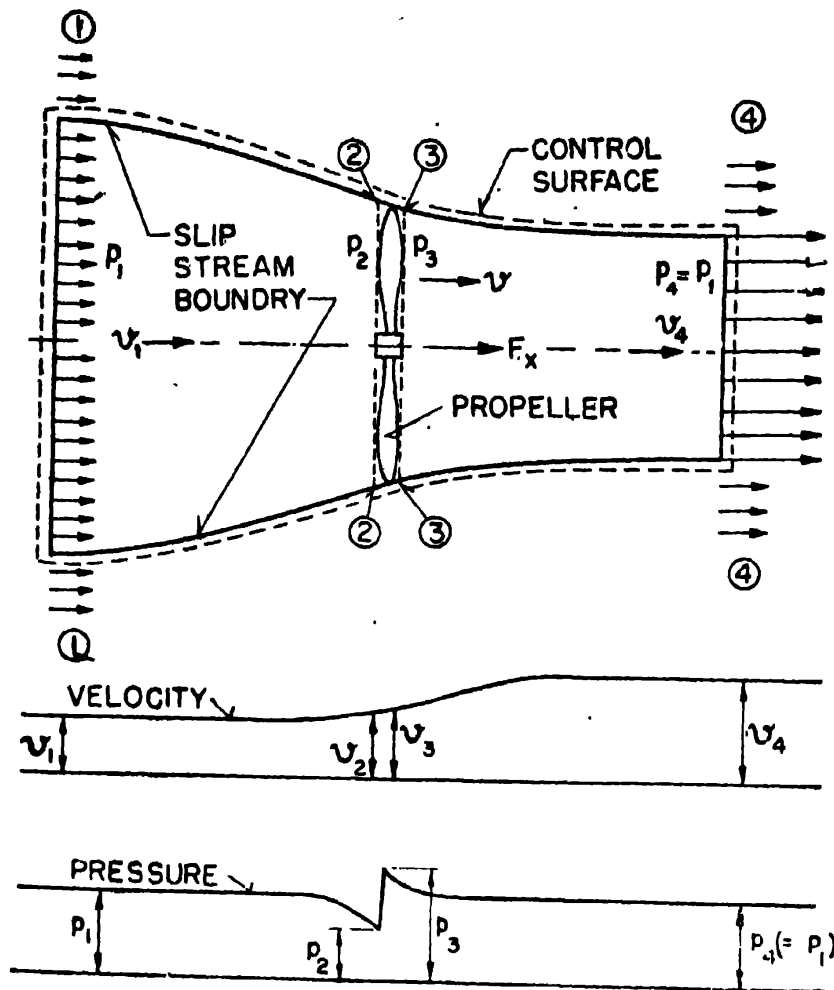


Fig 1.23 Momentum Theory for the Propulsion of Marine and Airships including Helicopter

Consider the control surface along the slip stream boundary and the two sections 1 and 4, the pressure all round this volume is same for the ideal propeller (i.e., a disc). In actual practice there is decrease of pressure at the impeller inlet and a sudden increase at its outlet, but at section 4, the pressure again comes down to that of pressure at section 1 as shown in Fig 1.23. For ideal fluid there is no shear force. As the fluid approaches the propeller, its velocity is increased to  $v_2$  and then to  $v_3$  and  $v_4$  after it passes through the propeller as shown in Fig 1.23. Due to change of velocity there is a change of momentum producing force.

Thus force (or thrust) exerted on the fluid by the propeller in  $x$ -direction is given by the application of momentum equation as

$$F_s = \rho Q(v_4 - v_1) = (p_3 - p_2) A$$

where

$Q = A \cdot v$  = rate of flow through the slipstream ;

$A$  = cross sectional area swept by propeller ;

and

$$v = \frac{1}{2}(v_1 + v_4)$$

The useful work done by the propeller moving through still fluid

$$W = F_s \cdot v_1 = \rho Q$$

The propeller advances through the stationary (assumed) flow with velocity  $v_1$ .

This is the power output of the propeller, taken from external source and imparted by the fluid. Thus this is the power consumed.

In order to find the efficiency of propeller, the total work done by the propeller is determined which is equal to the useful work (refer Eqn 1.35) plus the kinetic energy given to the slipstream which is wasted.

Total work done by the propeller or total power

$$= \rho Q (v_4 - v_1)v_1 + \frac{1}{2} \rho Q (v_4 - v_1)^2$$

$$\text{and efficiency } \eta = \frac{\text{Power output}}{\text{Power input}} = \frac{v_1}{v_1 + \frac{1}{2} (v_4 - v_1)} \quad (1.36)$$

*Note*—All the above equations (refer Eqn 1.34 to 1.36) are derived by considering the propeller to be an ideal one which may be called an *actuator disc*. In other words the propeller has infinite number of blades. The rotary motion of the propeller does not play any role and represents waste of energy.

**Wind mill** is a prime mover. Its principle of working is similar to a propeller, but it takes away energy *from the fluid* instead of giving energy to it. The flow pattern of windmill is reverse to that of the propeller, widening the slipstream as the fluid passes the disc.

**1.15 Forces due to Deviated Flow**—The jet impinging on vanes already dealt with in Art 1.5 is a case of deviated flow. In this article the forces caused by flow when it deviates in the pipe-line bend or reducer will be considered.

**(a) Forces caused by Flow Round a Pipe-Bend and Reducer**—In a pipe line flow the static pressure may vary from one section to another due to which some forces are called into play. For such a flow in addition to static pressure, the weight of pipe and fluid is another force of gravity, acting vertically downward. Whenever the flow takes place through a pipe bend its direction changes. In case the bend is acting as reducer, the cross-sectional area of flow also varies. These result in introduction of forces which are required to be determined. Derivation of such forces are given in Author's book on Fluid Mechanics.

The dynamic force component in positive  $x$ -direction exerted on flowing fluid by the bend, is given by

$$F_x = \rho Q (v_2 \cos \theta_2 - v_1 \cos \theta_1) \quad \dots(1.37)$$

where  $Q$  = volumetric and rate of flow in conduit i.e., discharge ;

$\rho Q$  = mass of fluid flowing per second.

Suffices 1 and 2 refer to inlet and outlet of bend.

In a similar manner the dynamic force in positive  $y$ -direction exerted on flowing fluid by the bend, is given by

$$F_y = \rho Q (v_2 \sin \theta_2 - v_1 \sin \theta_1) \quad \dots(1.38)$$

The term  $\rho Q v$  is sometimes known as *momentum flux*.

Resultant force acting on the flowing fluid,

$$F = \sqrt{F_x^2 + F_y^2} \quad \dots(1.39)$$



Direction of resultant force with the horizontal i.e., angle  $\theta_r$  is found from  $\tan \theta_r = \frac{F_y}{F_x}$  ... (1.40)

The forces  $F_x$  and  $F_y$  calculated above are components of dynamic force exerted by the bend i.e., the boundary of the conduit, on the free body of flowing fluid due to rate of change of momentum. From Newton's third law of motion—"Action and reaction are equal and opposite", the flowing fluid will also exert a force equal in magnitude but opposite in direction on the bend i.e., on the boundary of conduit. The later force is generally required to be determined for the design of pipe bends.

In the derivation of Eqns 1.37 and 1.38 the only external force considered was the dynamic force exerted by the boundary of conduit on the free body of flowing fluid between sections 1-1 and 2-2. In addition to this force, there might be many other external forces e.g., (a) static pressure forces at both ends of bend ; (b) the weight of fluid enclosed in the bend. All such external forces will have to be considered while calculating the total resultant force and its direction acting on the boundary of pipe bend or conduit.

Considering all the above forces, the component of total force +ve in  $x$ -direction acting on the fluid in the control volume, is given by

$$p_1 \cdot A_1 \cdot \cos \theta_1 - p_2 \cdot A_2 \cdot \cos \theta_2 + F_x \quad \dots (1.41)$$

Similarly the component of total force +ve in  $y$ -direction acting on the fluid in the control volume—

$$p_1 \cdot A_1 \cdot \sin \theta_1 - p_2 \cdot A_2 \cdot \sin \theta_2 + F_y \quad \dots (1.42)$$

From the above  $x$ - and  $y$ -components the magnitude of total force acting on the fluid can be calculated. The total force exerted by the fluid on the bend is equal and opposite in direction to the above.

**Practical Uses—**(1) A pipe line of large diameter, carrying water for Town Water Supply or Water Power Plants is required to be bent in order to lay it on the ground of different elevations. At the pipe bend large forces are brought into play in different directions, for which *Anchor Blocks* of reinforced concrete are built round the bend to withstand these by using Eqns 1.41 and 1.42. Such a force will be maximum when the pipe has a  $90^\circ$ -bend.

The strength of anchor depends upon the method of joining the bend to the pipeline. If the bend is rigidly secured for example bolted, a part of the load is transferred to the pipeline. If the bend is connected to the pipeline by expansion joint, then anchor will have to take up the total load.

It may be pointed out that the static pressure is always required to be determined to get the total pressure acting on the anchorage of the pipe bend.

$$\begin{aligned} \text{If} \quad \alpha &= 180^\circ, \quad \text{than} \quad F_x = \rho Q v (1+1) \\ F_x &\text{ is maximum and equal to } 2 \rho Q \cdot v = 2 \rho a v^2 \quad \dots (1.43) \\ \text{and} \quad F_y &= \text{zero} \quad (\because \sin 180^\circ = 0) \end{aligned}$$



**PLATE 1 (a) Tangential Flow Runner of a Pelton Turbine of Cypress Plant. This runner weighs 7 tonnes. The Runner bowl and back are being grounded and polished Manufactured by J. M. Voith, Heidenheim (West Germany)**



**PLATE 1 (b) Water Jet Impinging on Pelton Buckets (Escher Wyss)**

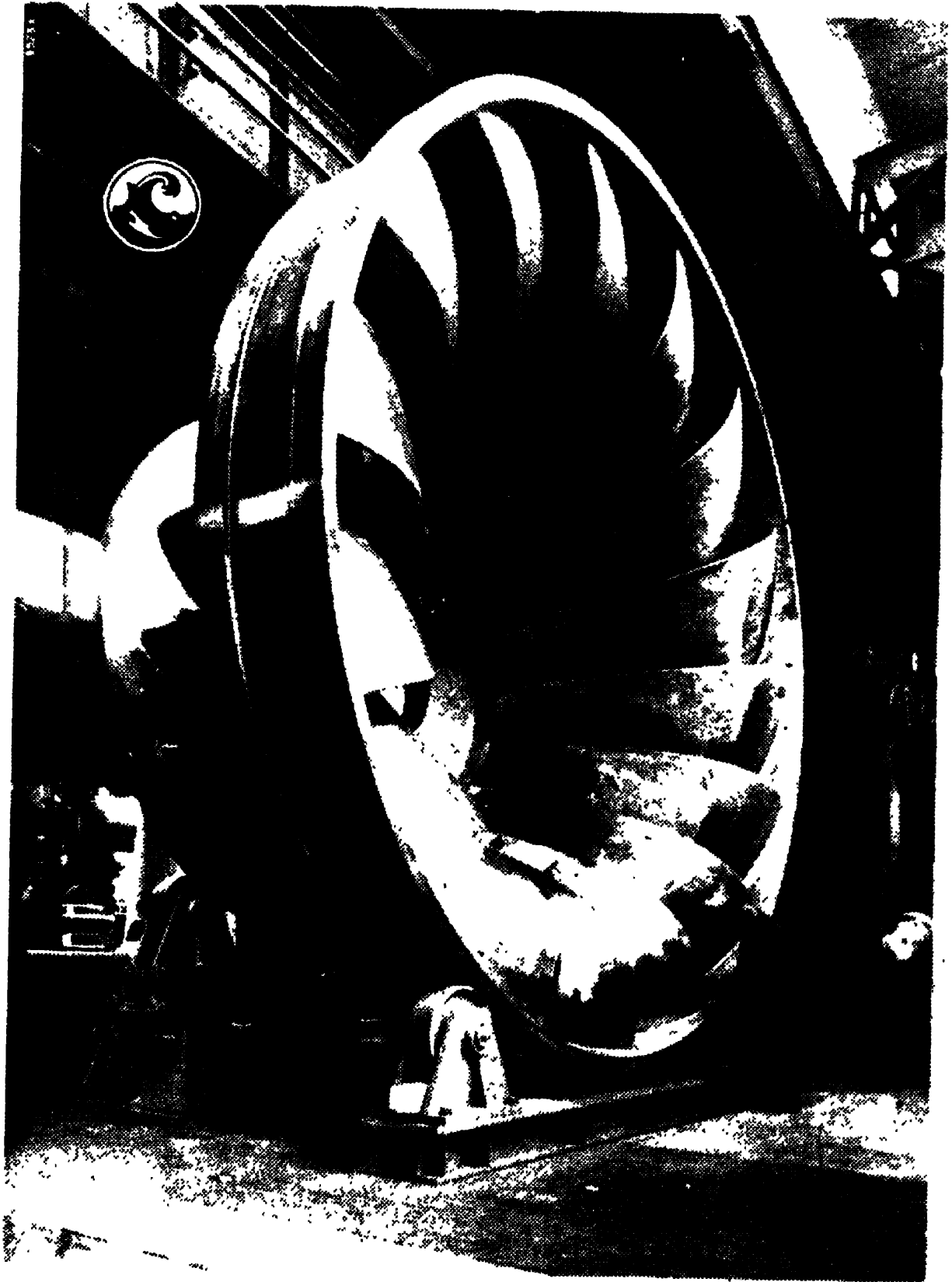


**PLATE 2. Radial Flow Runner of a Francis Turbine. This is the largest alloy steel runner of the world. Max runner dia 5.38 m. Height 2.6 m. made out of 13% chrome cast steel grounded and precision polished, manufactured by G. M. Voith, Heidenheim, West Germany.**

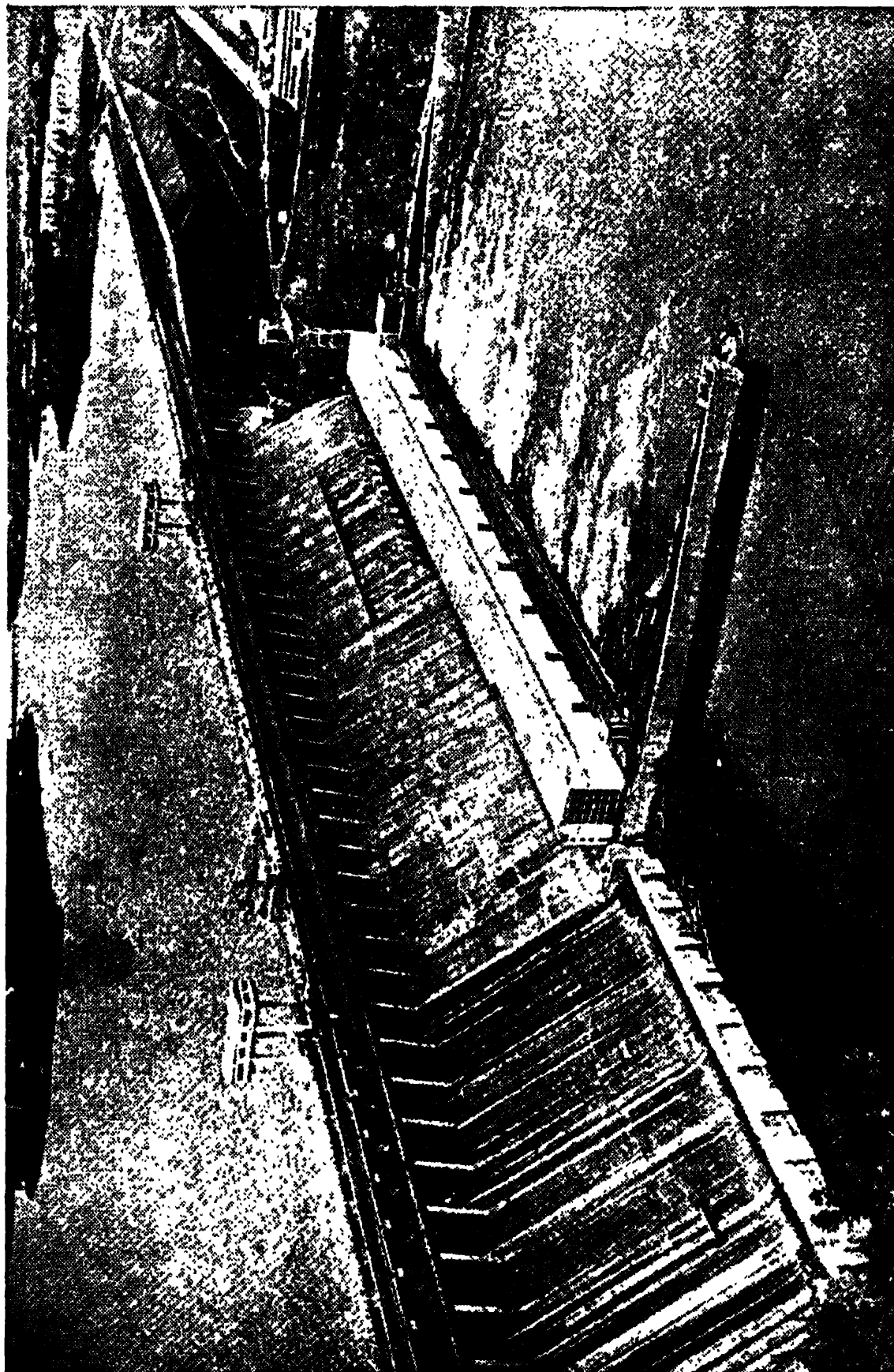


PLATE 3. Dam for Bratsk Hydro-electric Project, East Siberia USSR-Largest in the world

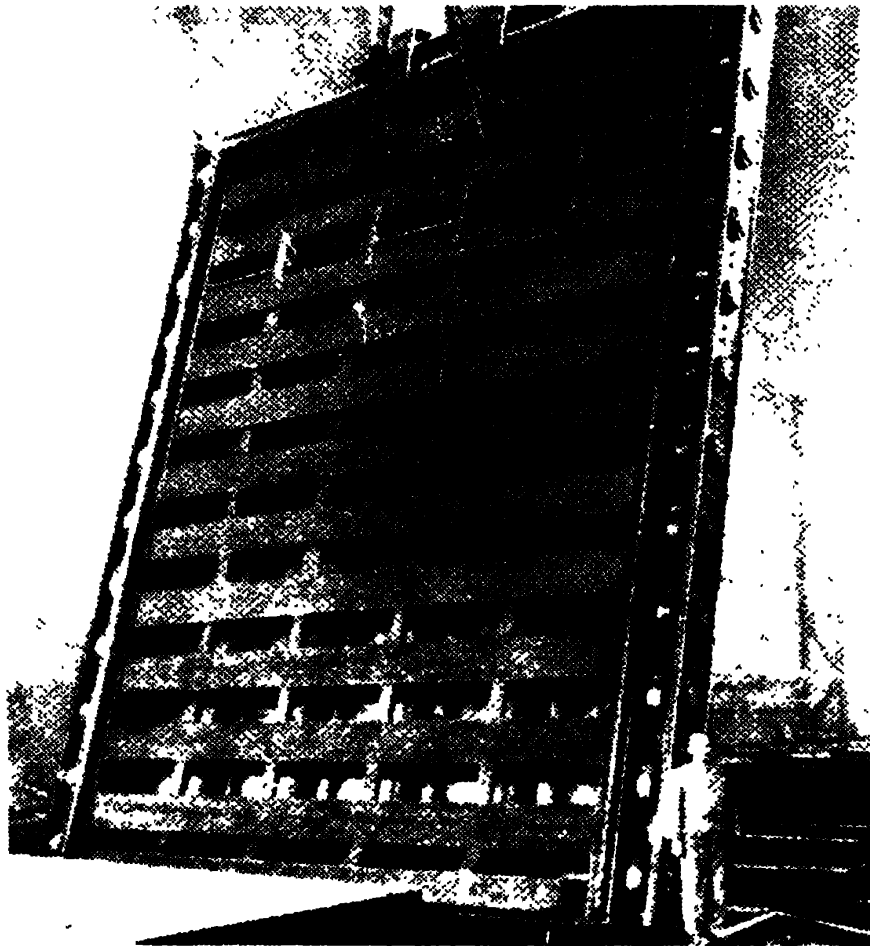


**PLATE 4.** Roller Gate for 25m Water depth in Power Station Pedreia. Brazil Manufacturers : Voith Heidenheim. West Germany Clear opening 4 m wide x 6.5 high. The gate leaf supplied in three sections and then rivetted on site.

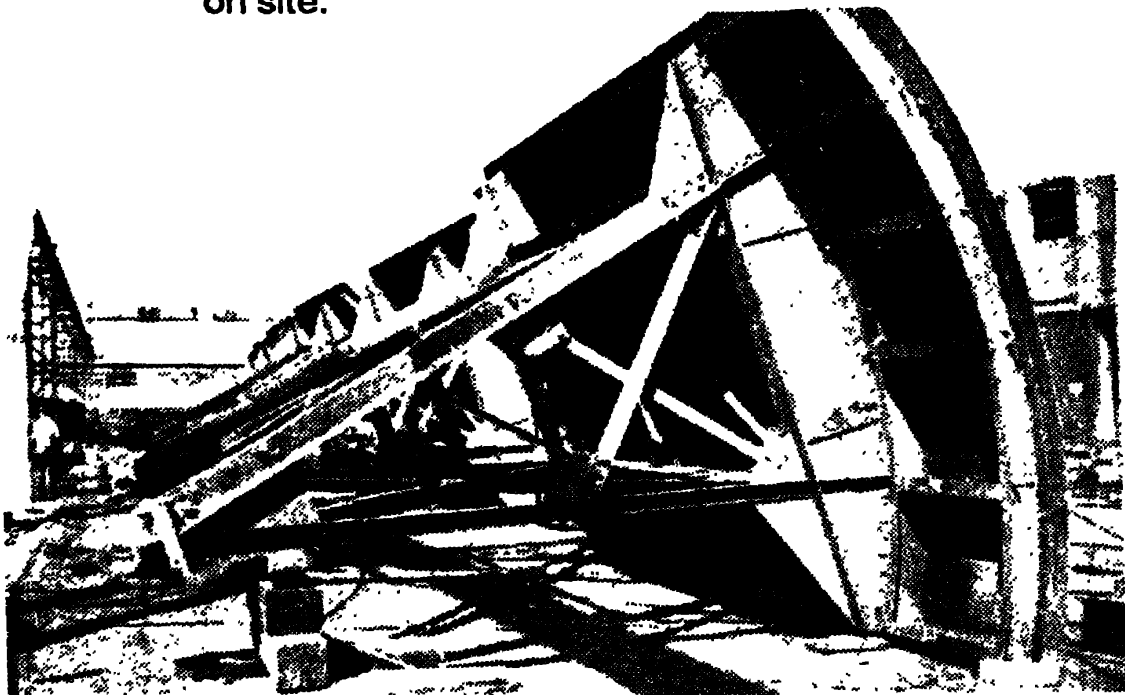
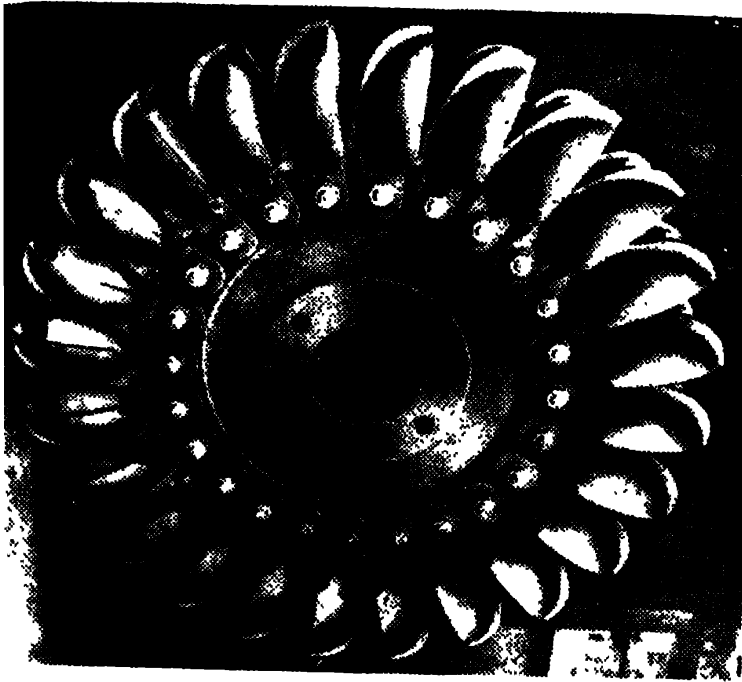


**PLATE 5** Tainter Gate of Staudamm Dam, Glen Canyon USA, 16 m High Width 12.2



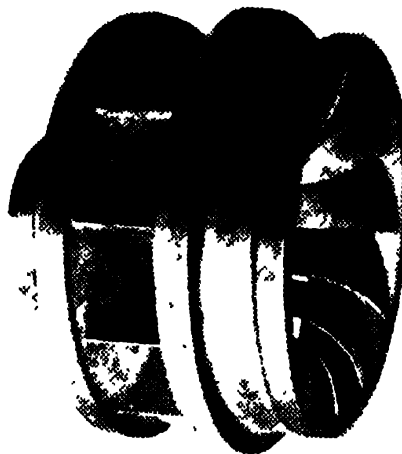
(a) Pelton Runner,  $N_s = 15$



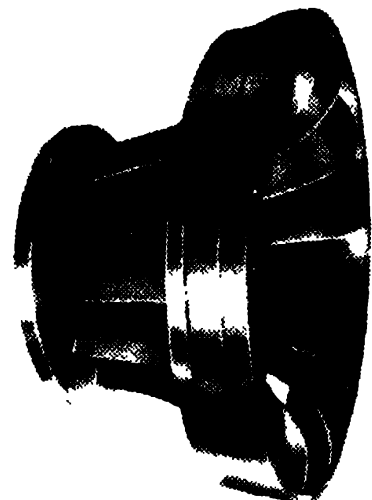
(b) Pelton Runner,  $N_s = 25$



(c) Francis Runner  $N_s = 75$



(d) Francis Runner  $N_s = 150$



(e) Francis Runner  $N_s = 300$



(f) Dubs Runner,  $N_s = 400$    5-Blades   5-Blades   3-Blades  
 $N_s = 450$    600   1,000

(G) KAPLAN RUNNERS

PLATE 6 Different Types of Water Turbine Runners

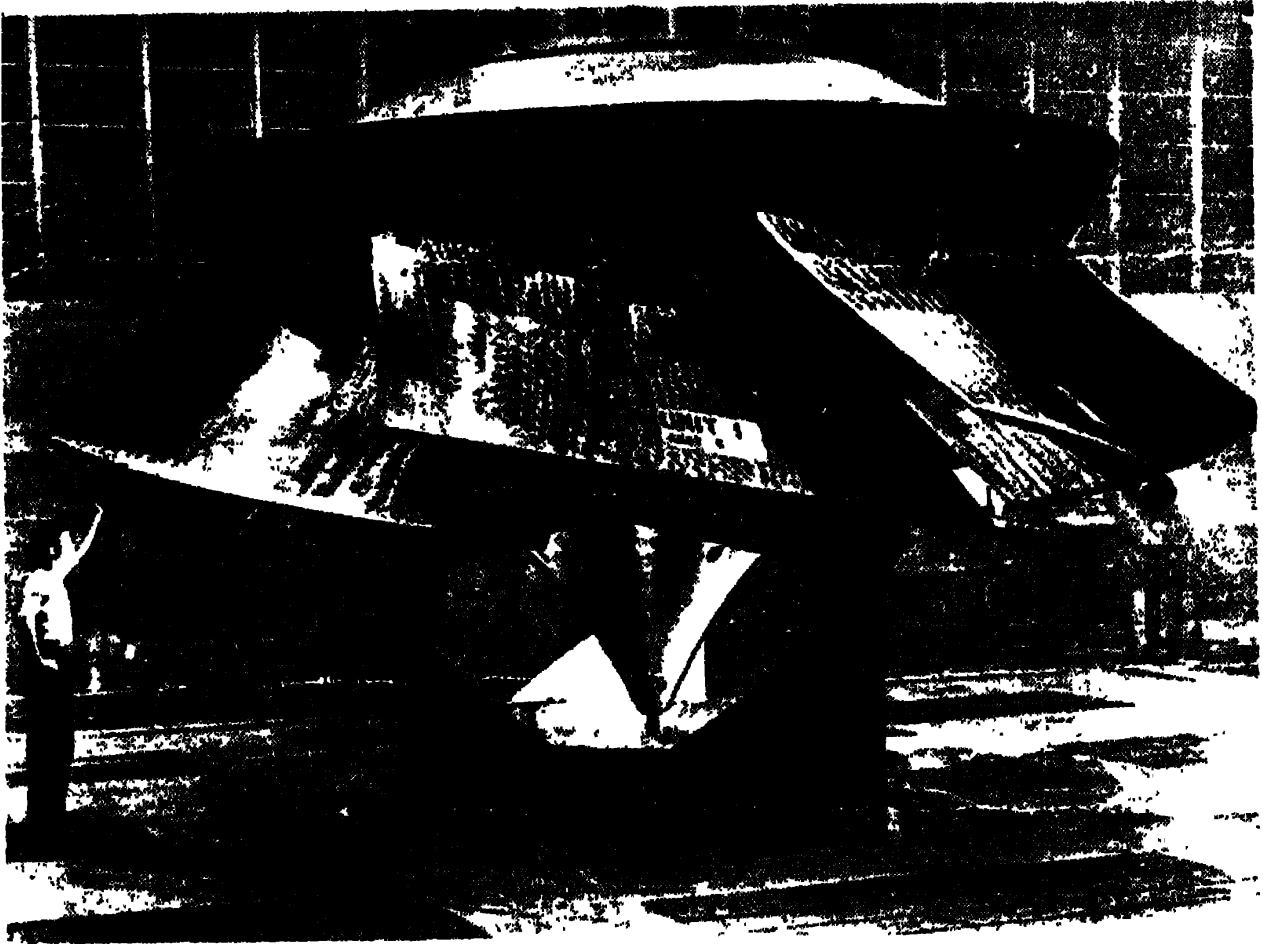


PLATE 7. Runner of the Deriaz Pump turbine with Blades Ney open

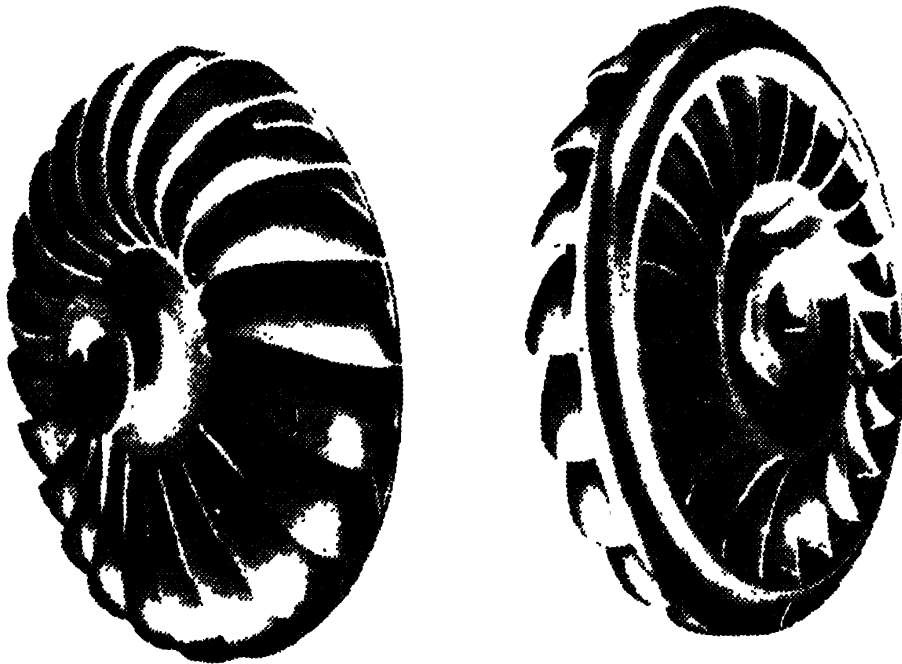


PLATE 8. Turgo Impulse water Turbine Runner (Manufactured by Gilbert Gilkes & Gordon Ltd. Kendal, England)

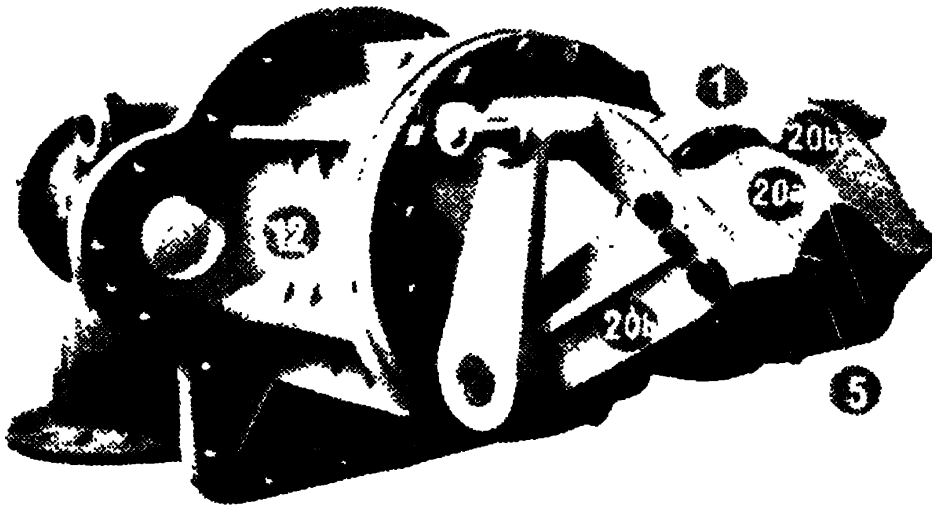


**PLATE 9. Deflector Pelton Turbine (Manufactured by Escher Wyss)**



**PLATE 10. Seewer Jet Diffuser of Pelton Turbine (Manufactured by English Electric Co.)**



**PLATE 11. Buckets of Runner of Pelton Turbine (Manufactured by Escher Wyss)**





**PLATE 12. Jet Impinging on Pelton Bucket (Escher Wyss)**



**PLATE 13. Demction of Pelton Tubire Bucket due to Sand Erosion & Corrosion (Escher Wyss)**

(2) Fig 1.24 shows a moving vane of double hemispherical type used for Pelton turbines (refer Chapter 5). The water stream impinges at the centre of the vane and deflects through an angle of  $180^\circ$ . At the same time the water stream is symmetrically divided at the centre of vane, the  $Y$ -components of dynamic forces ( $F_y$ ), one of them acting towards the

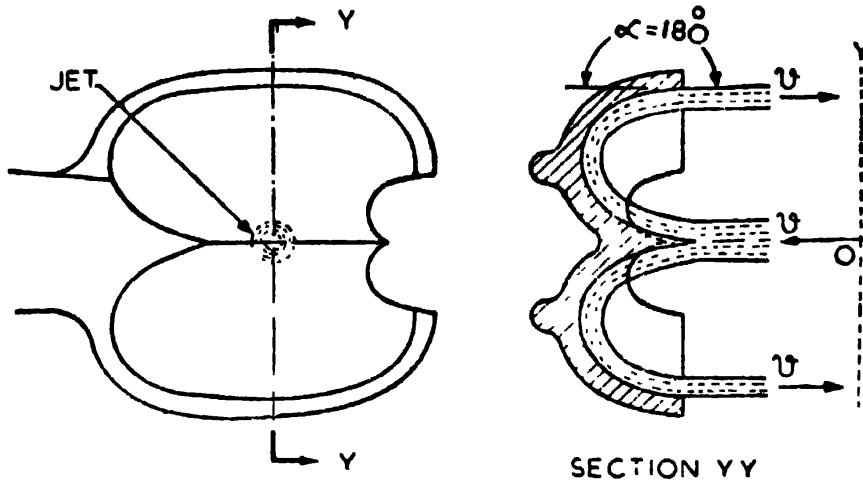


Fig 1.24 Jet on a Moving Vane of Double Hemispherical Type

bottom and the other acting towards the top, neutralise each other. Thus the only force acting on the vane, is  $F_x$ .

**1.16 Angular Momentum (or Moment of Momentum) Equation**—If a fluid particle of mass  $m$  is moving along a curved path (refer Fig 1.25) such that its distance from the axis of rotation (i.e., fixed centre) changes with time, then radial distance will be different at different positions of the particle. In Fig 1.25  $r_1$ ,  $r_2$  and  $r_3$  are the distances at positions 1, 2 and 3 respectively. If at any instant, the velocity of particle is  $v$  and the radial distance from the axis of rotation is  $r$ , then momentum of particle in a direction normal to radius  $r$ ,  $= m \cdot v_n$

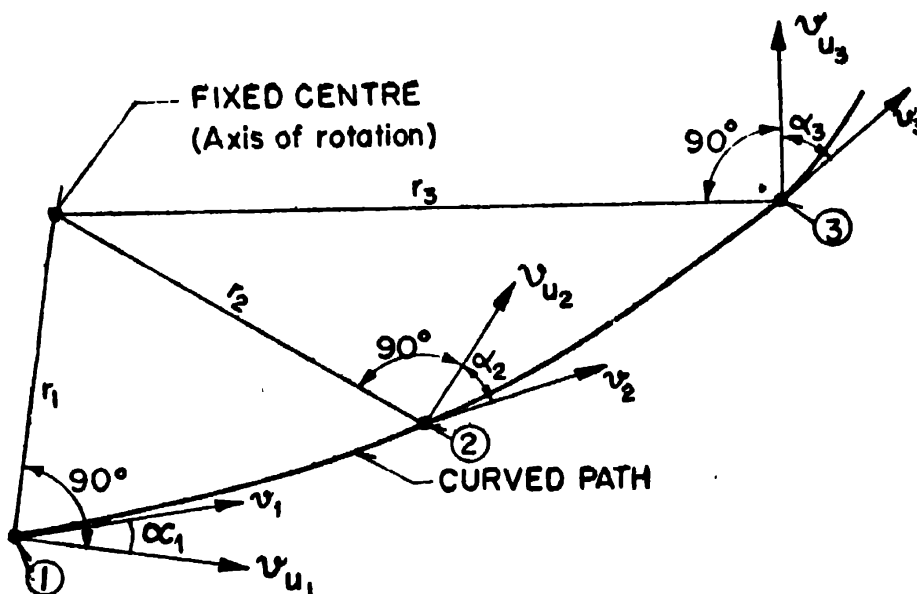


Fig 1.25 Moment of Momentum

and moment of momentum of particle about the axis of rotation

$$= m \cdot v_u \cdot r \quad \dots(1.44)$$

where  $v_u$  = component of velocity in a direction normal to radius  $r$ .

If  $\alpha$  is the angle made by velocity component  $v_u$  with the tangent drawn, the component of velocity normal to radius of the curved path is given by

$$v_u = v \cos \alpha \quad \dots(\text{refer Fig 1.25})$$

$$\therefore \text{Moment of momentum of particle} = m \cdot v \cos \alpha \cdot r \quad \dots(1.44 a)$$

This moment of momentum is also known as *angular momentum* to distinguish it from linear momentum.

$$\text{Angular momentum of particle at point 1} = m \cdot v_1 \cos \alpha_1 \cdot r_1$$

$$\text{Angular momentum of particle at point 2} = m \cdot v_2 \cos \alpha_2 \cdot r_2$$

$\therefore$  Change of angular momentum of particle

$$= m (v_2 \cos \alpha_2 \cdot r_2 - v_1 \cos \alpha_1 \cdot r_1)$$

Let this change take place in time  $t$ , then rate of change of angular momentum of particle

$$= \frac{m}{t} (v_2 \cos \alpha_2 \cdot r_2 - v_1 \cos \alpha_1 \cdot r_1)$$

Now rate of change of angular momentum is equal to the torque—

$$\therefore T_p = \frac{m}{t} (v_2 \cos \alpha_2 \cdot r_2 - v_1 \cos \alpha_1 \cdot r_1)$$

Where  $\frac{m}{t} = \rho Q$  = mass of fluid flowing per unit time.

$$\therefore T_p = \rho Q (r_2 v_2 \cos \alpha_2 - r_1 v_1 \cos \alpha_1) \quad \dots(1.45)$$

In case of circular path  $r_1 = r_2 = r$  i.e., same for all positions and  $\alpha_1$  and  $\alpha_2 = 0$ , then

$$T_p = \rho Q r (v_2 - v_1) \quad \dots(1.46)$$

The torque shown by Eqn 1.45 is exerted *on the fluid* by the body which is being revolved by some external agency. This is the case of a *centrifugal pump runner*. Therefore suffix  $p$  is used to indicate torque of pump runner. The negative value of this torque will be exerted by the fluid *on the body* in order to revolve it, which happens in case of a turbine. This can be written as  $T_t$  and

$$\begin{aligned} T_t &= -\rho Q (r_2 v_2 \cos \alpha_2 - r_1 v_1 \cos \alpha_1) \\ &= \rho Q (r_1 v_1 \cos \alpha_1 - r_2 v_2 \cos \alpha_2) \quad \dots(1.47) \end{aligned}$$

The above is the Equation of moment of momentum or angular momentum. This can be obtained for any control volume. Fig 1.26 shows the control volume enclosing the rotor of a generalised turbomachine.

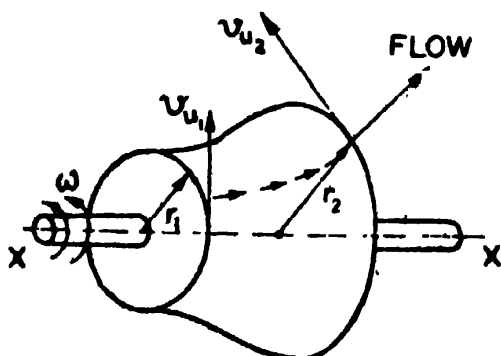


Fig 1.26 Control Volume for a generalised Turbomachine

Fig 1.26 shows the control volume enclosing the rotor of a generalised turbomachine. Swirling fluid enter the control volume at radius  $r_1$  with velocity  $v_{u1}$  ( $= v_1 \cos \alpha_1$ )—the peripheral component of velocity and acting normal to the radius  $r_1$  of curved path. The fluid leaves the vane at radius  $r_2$  with velocity  $v_{u2}$  ( $= v_2 \cos \alpha_2$ )—the peripheral component of velocity and acting normal to the

## DYNAMIC ACTION OF FLUID

radius  $r_2$  of the curved vane. This type of turbomachine is a centrifugal pump or a compressor. In case the fluid enters at point 2 and leaves at point 1, then the turbomachine will become a Francis turbine.

**1.17 Radial Flow Over Turbine Blade**—Radial flow is one in which a particle of water during its flow through the vanes of rotating runner, remains in the plane normal to the axis of rotation, in such a way that its position changes only with respect to its distance from the axis of rotation. In tangential flow (Art 1.9 to 1.11), the distance of water particle from the axis of rotation remains same i.e.,  $r = \text{constant}$ , therefore, the action of stream on the vane is determined by computing the force exerted by the moving fluid by applying Linear Momentum Equation (refer Eqn 1.1). However, in case of radial flow the distance  $r$  of the water particle from the axis of rotation varies along its path of flow, which has to be taken into account. Hence, instead of finding the force by Linear Momentum Equation, the action of stream on the vanes for radial flow is determined by the evaluation of turning moment or torque produced on the vanes. The torque is equal to the Rate of Change of Angular Momentum.

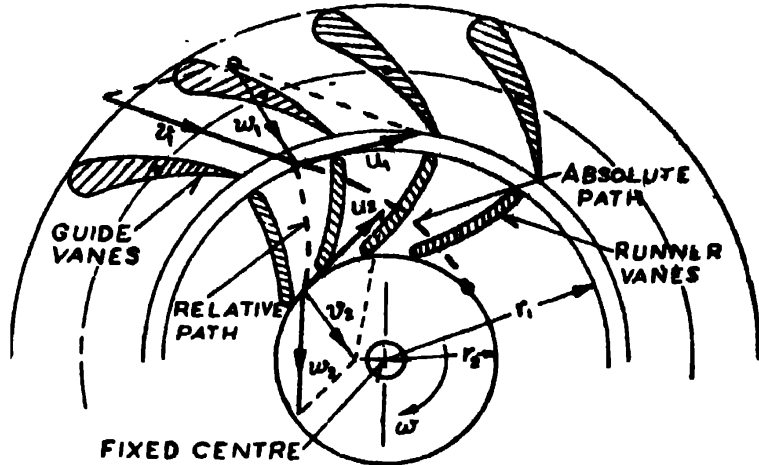


Fig 1.27 Section Through a Radial Flow Turbine

### 1.18 Work Done/sec or Power Produced by Radial Runner—

Fig 1.27 shows the section through the radial flow turbine. The water from the pipes first reaches the guide vanes which guide the water to the runner vanes of turbine. The guide vanes are stationary and the runner revolves about a fixed centre with angular velocity  $\omega$ . The flow over the runner vanes is radial.

Let an elementary mass of water  $m$  enters the rotating vane of the runner with velocity  $v_1 \cos \alpha_1$  at radius  $r_1$  and leaves with velocity  $v_2 \cos \alpha_2$  at radius  $r_2$  in time  $t$ . Draw the velocity triangles at inlet and outlet of turbine vane as shown in Fig 1.28.

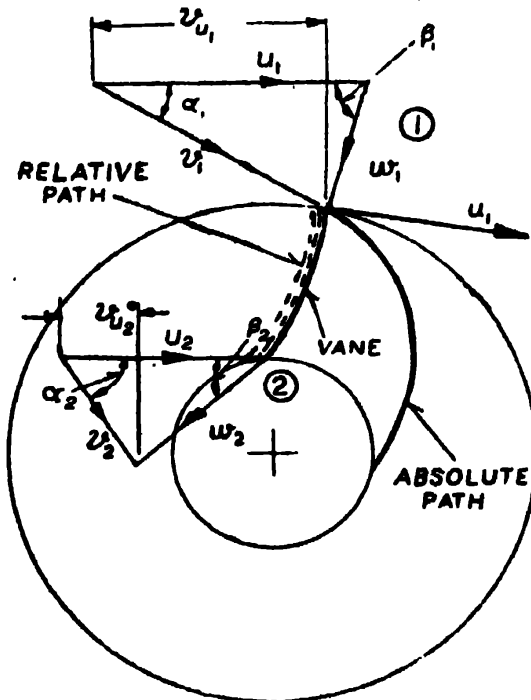


Fig 1.28 Velocity Triangles at Inlet (Pressure Side) and Outlet (Suction Side) of Vane

Angular momentum at inlet

$$= m (v_1 \cos \alpha_1) r_1$$

Angular momentum at outlet

$$= m (v_2 \cos \alpha_2) r_2$$

$\therefore$  Change of angular momentum

$$\text{in time } t = m (r_2 v_2 \cos \alpha_2 - r_1 v_1 \cos \alpha_1)$$

$\therefore$  Rate of change of angular momentum

$$\frac{m}{t} (r_2.v_2 \cos \alpha_2 - r_1.v_1 \cos \alpha_1)$$

but  $\frac{m}{t} = \rho Q = \text{mass of flowing fluid per unit time.}$

$\therefore$  Rate of change of angular momentum

$$= \rho Q (r_2.v_2 \cos \alpha_2 - r_1.v_1 \cos \alpha_1)$$

which is equal to the torque  $T_p$ ,

$$\therefore T_p = \rho Q (r_2.v_2 \cos \alpha_2 - r_1.v_1 \cos \alpha_1) \quad \dots(\text{refer Eqn 1.45})$$

This is the torque acting on the fluid by the vanes. The torque acting on the vanes by the fluid will therefore be

$$T_s = \rho Q r_1 (v_1 \cos \alpha_1 - r_2.v_2 \cos \alpha_2) \quad \dots(\text{refer Eqn 1.47})$$

Work done/sec by the runner or Power produced

$$= \text{Torque on the runner} \times \text{angular velocity of runner}$$

$$\text{or } P = T_s \cdot \omega$$

$$= \rho Q (r_1.v_1 \cos \alpha_1 - r_2.v_2 \cos \alpha_2) \cdot \omega \quad \dots(1.48)$$

$$\text{but } u_1 = \omega r_1 \text{ and } u_2 = \omega r_2 \quad \dots(1.49)$$

where  $u_1$  and  $u_2$  are the peripheral velocities of vanes at inlet and outlet.

Work done/sec or Power produced by runner

$$P = \rho Q (u_1.v_1 \cos \alpha_1 - u_2.v_2 \cos \alpha_2) \quad \dots(1.50)$$

$$\text{or } P = \rho Q (u_1 \cdot v_{u_1} - u_2 \cdot v_{u_2}) \quad \dots(1.50 a)$$

The units of  $P$  are m-kg/sec.

This is the *Fundamental Equation of Fluid Machines*. As mentioned above, this is applicable to the turbine as well as to the pump runner. In such a case point 1 will denote the *pressure side* and point 2 the *suction side* of the turbine or pump (refer Fig 1.28). Thus the value of power  $P$  will be positive in case of turbine as it is delivering power and this value is negative in case of pump which is consuming power.

The fluid may leave the vanes of *turbine runner* with an absolute velocity in a direction

- (a) against the motion of wheel,
- (b) same as motion of wheel,
- or (c) radially.

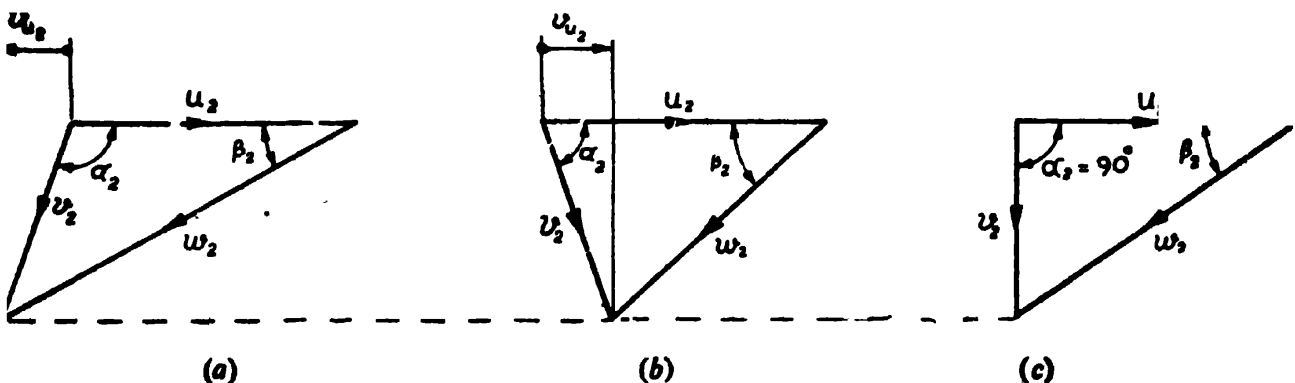


Fig 1.29 Outlet Velocity Triangles

In case (a),  $\alpha_2 > 90^\circ$ ,  $v_{u_2}$  is negative and

substituting this value in Eqn 1.50 a

$$\text{Work done/sec or Power} = \rho Q (v_{u_1} \cdot u_1 + v_{u_2} \cdot u_2) \text{ m}\cdot\text{kg/sec} \dots (1.51)$$

In case (b),  $\alpha_2 < 90^\circ$ ,  $v_{u_2}$  is positive,

$$\therefore \text{Work done/sec or Power} = \rho Q (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2) \text{ m}\cdot\text{kg/sec} \dots (1.52)$$

In case (c),  $\alpha_2 = 90^\circ$ ,  $v_{u_2} = 0$

$$\therefore \text{Work done/sec or Power} = \rho Q (v_{u_1} \cdot u_1) \text{ m}\cdot\text{kg/sec} \dots (1.52)$$

**Problem 1.9** Water enters an inward flow turbine at an angle of  $22^\circ$  to the tangent to the outer rim and leaves the turbine radially. The speed of the wheel is 300 rpm and the velocity of flow is constant at 3 m/sec, find the necessary angle of the blades when the inner and outer diameters of the turbine are 30 cm and 60 cm respectively. If width of wheel at inlet is 15 cm, calculate HP developed. Thickness of blade may be neglected. (AMIE)

**Solution**

$$\alpha_1 = 22^\circ$$

$$\alpha_2 = 90^\circ \text{ (radial discharge).}$$

$$N = 300 \text{ rpm}$$

$$v_{m_1} = 3 \text{ m/sec} = v_{m_2}$$

( $\because$  Velocity of flow remains constant)

$$D_1 = 60 \text{ cm} = 0.6 \text{ m (inward flow turbine)}$$

$$D_2 = 30 \text{ cm} = 0.3 \text{ m}$$

$$B_1 = 15 \text{ cm} = 0.15 \text{ m}$$

$$\text{Peripheral velocity of wheel at inlet, } u_1 = \frac{\pi D_1 N}{60}$$

$$\text{or } u_1 = \frac{\pi \times 0.6 \times 300}{60} = 9.42 \text{ m/sec}$$

$$\text{Peripheral velocity of wheel at outlet, } u_2 = \frac{\pi D_2 N}{60}$$

$$u_2 = \frac{\pi \times 0.3 \times 300}{60} = 4.71 \text{ m/sec}$$

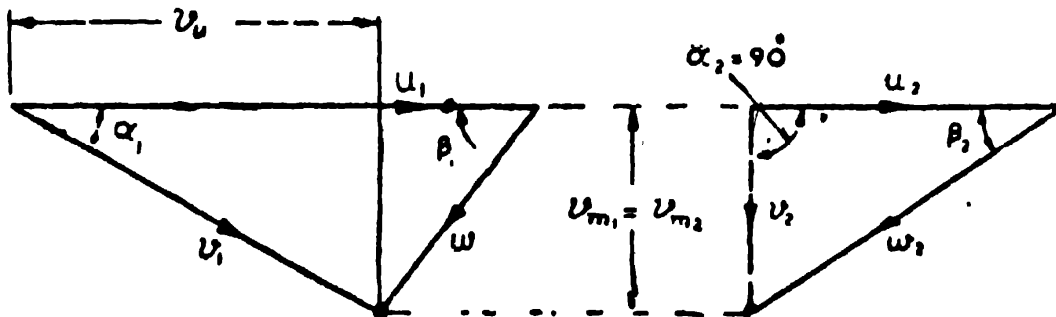


Fig 1.30 Inlet and Outlet Velocity Triangles

Draw the velocity triangle at inlet with  $u_1$ ,  $\alpha_1$  and  $v_{m_1}$ . It is required to find out the blade angle  $\beta_1$  from the triangle. ... (refer Fig 1.30)

$$\tan \beta_1 = \frac{v_{m_1}}{u_1 - v_{u_1}} = \frac{v_{m_1}}{u_1 - v_1 \cos \alpha_1} \dots (1)$$

But  $v_{m1} = v_1 \sin \alpha_1$

$$\therefore v_1 = \frac{v_{m1}}{\sin \alpha_1} = \frac{3}{\sin 22^\circ} = \frac{3}{0.3746} = 8.01 \text{ m/sec}$$

Substituting  $v_1$ ,  $v_{m1}$ ,  $\alpha_1$  and  $u_1$  in Eqn (1)

$$\tan \beta_1 = \frac{3}{9.42 - 8.01 \times \cos 22^\circ} = \frac{3}{9.42 - 7.42} = 1.5$$

$$\therefore \beta_1 = 56^\circ - 23' \quad \text{Answer}$$

Similarly to find the blade angle at outlet  $\beta_2$ , draw the velocity triangle at outlet with  $u_2$ ,  $w_2$  and  $v_{m2}$  (refer Fig 1.30)

$$\tan \beta_2 = \frac{v_{m2}}{u_2} = \frac{3}{4.71} = 0.636$$

$$\therefore \beta_2 = \tan^{-1} 0.636 = 32^\circ - 30' \quad \text{Answer}$$

Work done/sec by the turbine runner

$$= \rho Q (v_{u1} \cdot u_1 - v_{u2} \cdot u_2) \quad \dots (\text{refer Eqn 1.50 a})$$

but  $\alpha_2 = 90^\circ, \therefore v_{u2} = 0$

$$\therefore \text{Work done/sec} = \rho Q (v_{u1} \cdot u_1) \quad \dots (\text{refer Eqn 1.53})$$

$$\text{HP of turbine} = \frac{1}{75} \cdot \rho Q (v_{u1} \cdot u_1)$$

But  $Q = a_1 v_{m1}$

and  $a_1$ , the area of flow  $= \pi D_1 \cdot B_1$

$B_1$ , the width of wheel at inlet  $= 0.15 \text{ m}$

$$\therefore a_1 = \pi \times 0.6 \times 0.15 = 0.283 \text{ m}^2$$

$$\therefore Q = 0.283 \times 3 = 0.849 \text{ m}^3/\text{sec}$$

$$\begin{aligned} v_{u1} &= v_1 \cos \alpha_1 \\ &= 8.01 \times \cos 22^\circ = 8.01 \times 0.9272 \\ &= 7.42 \text{ m/sec} \end{aligned}$$

$$\begin{aligned} \text{Hence HP of turbine} &= \frac{1,000 \times 0.849}{9.81 \times 75} \times 7.42 \times 9.42 \\ &= 80.9 \text{ HP} \quad \text{Answer} \end{aligned}$$

**Problem 1.10** In an inward flow turbine the water comes out of guide vanes and falls with a velocity of 35 m/sec on the runner consisting of a series of curved blades. The speed of the runner is 300 rpm. The vanes have inlet and outlet diameters of 1.8 m and 0.9 m respectively. The angle which the guide vanes make with the periphery of the wheel is  $30^\circ$ .

The water after doing work on the runner discharges with an absolute velocity of 3 m/sec at an angle of  $120^\circ$  to the wheel tangent. Find the water horse power if the rate of flow is 300 lit/sec. Determine the best angle of the blades.

**Solution**

$$v_1 = 35 \text{ m/sec} \quad N = 300 \text{ rpm} \quad D_1 = 1.8 \text{ m} \quad D_2 = 0.9 \text{ m}$$

$$\alpha_1 = 30^\circ \quad \alpha_2 = 120^\circ \quad v_2 = 3 \text{ m/sec} \quad Q = 300 \text{ lit/sec}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 1.8 \times 300}{60} = 28.3 \text{ m}$$

$$v_{u_1} = v_1 \cos \alpha_1 = 35 \times \cos 30^\circ = 35 \times 0.866 = 30.4 \text{ m/sec}$$

$$v_{m_1} = v_1 \sin \alpha_1 = 35 \times \sin 30^\circ = 35 \times 0.5 = 17.5 \text{ m/sec}$$

Draw inlet velocity triangle (refer Fig 1.31)

$$\tan (180^\circ - \beta_1) = \frac{v_{m_1}}{v_{u_1} - u_1} = \frac{17.5}{30.4 - 28.3} = \frac{17.5}{2.1} = 8.33$$

$$\therefore (180^\circ - \beta_1) = 83.3^\circ \quad \text{or } \beta_1 = 96.7^\circ \quad \text{Answer}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.9 \times 300}{60} = 14.1 \text{ m/sec}$$

$$v_{u_2} = v_2 \sin \alpha_2 = 3 \times \cos 120^\circ = -1.5 \text{ m/sec}$$

$$v_{m_2} = v_2 \sin \alpha_2 = 3 \times 0.866 = 2.598 \text{ m/sec}$$

Draw outlet velocity triangle (refer Fig 1.31)

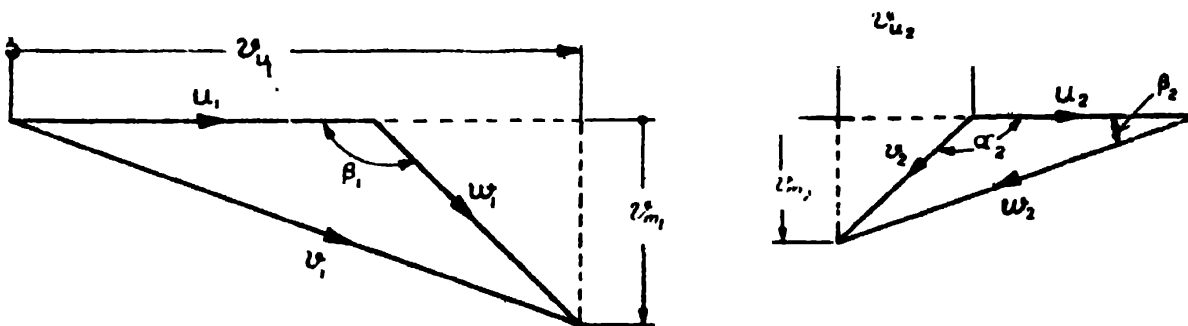


Fig 1.31 Inlet and Outlet Velocity Triangle

$$\tan \beta_2 = \frac{v_{m_2}}{u_2 - v_{u_2}} = \frac{2.598}{14.1 + 1.5} = \frac{2.598}{15.6} = 0.167$$

$$\therefore \beta_2 = 9^\circ - 28' \quad \text{Answer}$$

$$\text{Work done by water on the turbine} = \rho Q (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2) \quad \dots(\text{refer Eqn 1.50a})$$

$$\therefore \text{Hydraulic or Water Horse power} = \frac{\rho Q}{75} (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2)$$

$$= \frac{1,000}{9.81} \times \frac{0.3}{75} \{ 30.4 \times 28.3 - (-1.5) \times 14.1 \}$$

$$= \frac{300}{9.81 \times 75} \times (861 + 21.18) = 360 \text{ HP} \quad \text{Answer}$$



**UNSOLVED PROBLEMS**

- 1.1 Define Dynamic Force. How is it distinguished from hydrostatic pressure ?
- 1.2 Derive Linear-Momentum and Impulse-Momentum Equations.
- 1.3 Define system of mass, control volume and control surface.
- 1.4 What are the external forces acting on a control surface ?
- 1.5 State the applications of linear-momentum equation.
- 1.6 Differentiate between tangential and radial flows.
- 1.7 What is the difference between the force of jet when it impinges on a single moving flat plate and the force of jet when it strikes on a series of moving plates ?
- 1.8 Explain the flow over the double hemispherical vane of Pelton turbine.
- 1.9 How is the absolute path of jet falling on a moving vane determined ?
- 1.10 How are the velocity diagrams for the flow over the moving vanes drawn ?
- 1.11 What are 'Velocity of Whirl' and 'Velocity of Flow' and why are they so named ?
- 1.12 Show that when a jet of water impinges on a series of curved vanes, maximum efficiency is obtained when the vane is semi-circular in section and the velocity of the vane is half that of the jet.  
(Mysore University)
- 1.13 What is jet propulsion ? How do the ships move with this principle ?
- 1.14 Explain the working of the followings with the help of a sketch: rocket, jet engine, turbojet, turbo prop and ramjet.
- 1.15 Explain momentum theory for the propulsion of marine and air ships as well as helicopter.
- 1.16 Explain how the windmill works.
- 1.17 How is the force on the bend of a pipe-line running full of water determined ? What precautions are taken if this force is very large ?
- 1.18 Define angular momentum and explain how it is used to determine the torque and work done by a vane in case of radial flow runner.
- 1.19 What is Fundamental Equation of Fluid Machines ? How is it applied to both turbines and pumps ?
- 1.20 How is the Fundamental Equation of Fluid Machines affected if angle  $\alpha_2$  is acute, obtuse or complimentary ?

**Numericals**

- 1.21 A jet of water 5 cm in diameter exerts a force of 300 kg on a flat plate held normal to the jet's path ; what is the rate of discharge ?  
(76 lit/sec)
- 1.22 A jet of water 5 cm in diameter is moving at 15 m per second. It strikes a flat plate which is inclined at  $30^\circ$  to the jet. Find the force on the plate in the direction of jet, when

(a) the plate is stationary,

(b) the plate is moving at 3 m per second in the direction of jet.

(11.25 kg ; 7.2 kg) (Roorkee University)

1.23 A jet of water 5 cm in diameter strikes a curved vane at rest with a velocity of 3 cm/sec and is deflected through  $35^\circ$  from its original direction. Neglecting friction, compute the resultant force on the blade in magnitude and direction. (332.8 kg ;  $22.5^\circ$ ) (AICTE)

1.24 A free jet whose sectional area is 20 sq cm and whose velocity is 25 metres/sec impinges tangentially on a smooth vane which diverts its direction through  $120^\circ$ . What is the magnitude and direction of the resultant force on the vane ? (220 kg ;  $29^\circ - 55'$ )

1.25 A rectangular plate weighing 5 kg is suspended vertically by a hinge on the top horizontal edge. The centre of gravity of the plate is 10 cm from the hinge. A horizontal jet of water of 2.5 cm diameter whose axis is 15 cm below the hinge impinges normally on the plate with a velocity of 5 m per sec. Find the horizontal force applied at the centre of gravity to maintain the plate in its vertical position.

Find the change of velocity of the jet if the plate is deflected through an angle of  $30^\circ$  and the same horizontal force continues to act at the centre of gravity of the plate.

(1.875 kg ; 2.95 m/sec) (Annamalai University)

1.26 A square plate of uniform thickness is hinged about its upper edge, which is horizontal. The length of each side is 0.4 m and the plate weighs 12.5 kg. A horizontal jet of water 0.02 m in diameter impinges on the plate at a point 0.2 m below the upper edge so that when the plate is vertical the jet strikes it normally at its centre point. The jet has a velocity of 15 m/sec. Determine the force which must be applied at the lower edge in order to keep the plate vertical.

If the plate is allowed to swing freely find the inclination to the vertical which the plate assumes under the action of the jet.

In both cases assume that after impact the jet flows over the surface of the plate. (3.6 kg ;  $35^\circ$ ) (AMI Mech E-Lond)

1.27 A water jet issues from a nozzle 5 cm diameter, strikes a plate with a velocity of 30 m/sec. The plate makes an angle of  $60^\circ$  to the horizontal. If frictional loss is such as to reduce the velocity of the stream leaving the body to 25 m/sec, find :

(a) the component of force in the direction of jet,

(b) the component of force normal to jet,

(c) the magnitude and direction of resultant of force exerted by water.

[(a) 105 kg ; (b) 130 kg ; (c) 167 kg at  $51^\circ - 12'$  with direction of jet.]

1.28 A jet of water issues from a nozzle 0.5 cm diameter and impinges axially on an approximately hemispherical cup which is observed to turn it through a total angle of  $160^\circ$ . Quantity of water coming out of nozzle is 10 kg in 40 seconds and the force exerted on the vane is 0.5 kg. Determine the ratio of the velocity of water as it leaves the vane to the velocity of approach. (0.576)

- 1.29 A jet of water 5 cm diameter having a velocity of 20 m/sec, glides without shock into a series of smooth curved vanes moving in a direction parallel to the jet and away from it with a velocity of 8 m/sec. The water is turned round through an angle of  $15^\circ$  as it passes over the vane. Find the pressure exerted on the vanes in the direction of rotation, and HP developed.

If for any reason the blade is held stationary, what will be the pressure on it ? (89 kg ; 9.5 HP ; 149 kg) (*Bombay University*)

- 1.30 A series of curved vanes (entrance angle  $30^\circ$  and exit angle  $15^\circ$ ) deflect a jet of water 10 sq cm in area, moving at 50 m/sec and inclined at  $15^\circ$  to the line of motion of the vane, Find :

- (a) the velocity of the vane to avoid shock at entry,
- (b) the magnitude and direction of the resultant force on the vane and the force in the direction of motion,
- (c) the magnitude and direction of velocity at exit.

[(a) 25.8 m/sec ; (b) 243 kg at  $7.5^\circ$  and 241 kg in the direction of motion ; (c) 6.7 m/sec at  $82.5^\circ$ ]

(*Bombay University*)

- 1.31 A jet of water having a velocity of 35 m/sec impinges on a series of vanes moving with a velocity of 20 m/sec. The jet makes an angle of  $30^\circ$  to the direction of motion of vanes when entering and leaves at an angle of  $120^\circ$ . Draw the triangle of velocities at inlet and outlet and find :

- (a) the angles of vane tips so that water enters and leaves without shock,
- (b) the work done per kg of water entering the turbine,
- (c) efficiency of the turbine. ( $120.1^\circ$  ;  $1.6^\circ$  ; 61.1 m·kg/sec ; 98.6%)

(*AMIE and Madras University*)

- 1.32 A motor-boat with jet propulsion draws 300 lit/sec through orifices amid ships and discharges it astern through orifices having an effective area of  $0.05 \text{ m}^2$ . If the boat travels at 4.5 m/sec, find the propelling force. (45.8 kg) (*AMI Mech E*)

- 1.33 A locomotive going at 65 kmph scoops up water from a trough. The tank is 2.5 m above the mouth of the scoop, and the delivery pipe has an area of 325 sq cm. If half the available head is wasted at entrance, find the velocity at which the water is delivered into the tank and the number of tons lifted from a trench 500 m long. What, under these conditions, is the increased resistance ; and what is the minimum speed of train at which the tank can be filled ?

(10.68 m/sec ; 9.61 tonne, 35.6 km/hr) (*London University*)

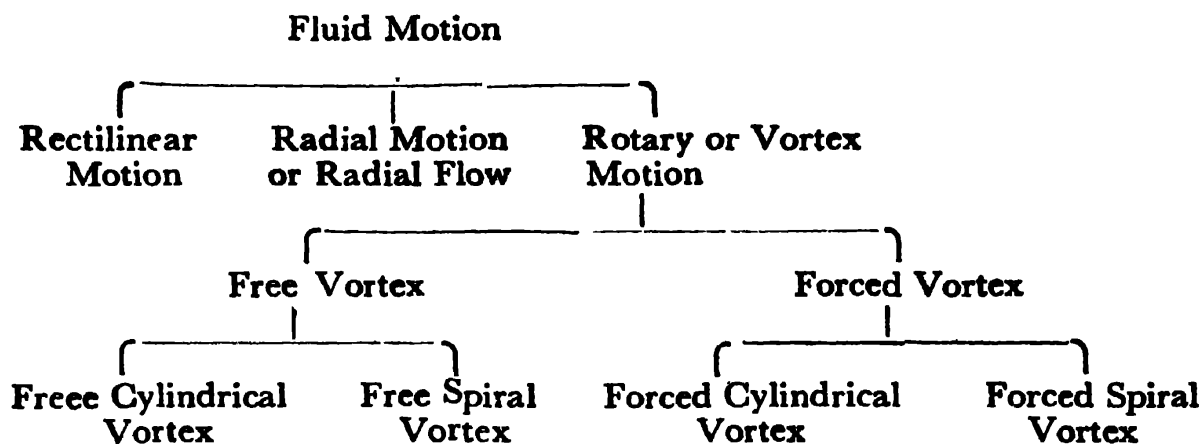
- 1.34 In a jet propelled-boat, the resistance to the motion is 400 kg. The boat is provided with two jets each having an area 0.02 sq metres. The water is drawn through orifices amid ship. If the boat travels at a speed of 6 metres/sec, determine the quantity of water to be pumped per second and the efficiency of jet propulsion in this case.

(0.534 cu metres, 49.5%)

- 1.35 A jet aircraft flying at a speed of 800 km/hour takes in air at its front and the exhaust gases leave the nozzle at the rear. The air used by the jet is at the rate of 20 kg/sec and the fuel injected in the combustion chamber is  $\frac{1}{40}$ th of the air. The mixture leaves the aircraft with a velocity of 400 m/sec. Find the thrust on the aircraft due to change of momentum.  
(Hint :  $W_1 = 20$  kg/sec and  $W_2 = 20.5$  kg/sec) (377 kg)
- 1.36 In an inward flow turbine, the peripheral velocity of the wheel is 20 m/sec. The velocity of the whirl at inlet is 15 m/sec, and radial velocity of flow is 2 m/sec. If the flow is  $0.68 \text{ m}^3/\text{sec}$  and hydraulic efficiency 80%, find the head available and the horse-power of the turbine. Assume radial discharge.  
(38.2 m ; 275 HP) (Roorkee University)
- 1.37 An inward flow reaction turbine develops 260 HP at an overall efficiency of 78% under a head of 70 m. The peripheral speed of vanes at inlet is 35 m/sec. Width of wheel at inlet is  $\frac{1}{6}$ th the corresponding diameter. Velocity of flow remains constant at 5 m/sec. Outlet diameter of vanes is  $\frac{3}{4}$  inlet diameter. If inlet angle of runner vane is  $90^\circ$  to the tangent, determine the guide blade discharge angle and runner vane blade outlet angle. Velocity of whirl at exit is zero.  
( $11^\circ-20'$  ;  $14^\circ-57'$ ) (Delhi University)
- 1.38 Guide vanes of a Francis turbine make an angle of  $15^\circ$  with tangent to the wheel, the peripheral speed of which is 12 m/sec. The absolute velocity of water at inlet is 13.5 m/sec. Determine the proper values of vane angles at inlet and outlet, the inner dia of the wheel being half the outer. The water leaves the wheel in outer direction and velocity of flow is constant throughout. ( $73.3^\circ$  ;  $30^\circ-6'$ ) (AMIE)
- 1.39 An inward flow turbine works under a total head of 36 m. The velocity of the wheel periphery at inlet is 18 m/sec. The outlet pipe of the turbine is 30 cm in dia, and the turbine is supplied with 250 lit/sec. The radial velocity of flow through the wheel is the same as velocity in the outlet pipe. Neglecting friction, determine :  
(a) the vane angle at inlet,  
(b) the guide blade angle and  
(c) the HP of the turbine. ( $69^\circ-27'$  ;  $10^\circ-23'$  ; 118 HP) (AMIE)

## Whirling Fluid

**2.1 Types of Fluid Motion**—A fluid may possess at any instant any one or more types of motion given below—



(1) **Rectilinear Motion** of fluid is one-dimensional in which stream lines are parallel.

(2) **Radial Flow** is defined as one in which the fluid flows in radial direction in such a way that pressure and velocity at any point change with respect to distance of that point from the central axis only. In this case the velocity of flow is in radial direction and the flow may take place radially inward to or outward from the centre. Flow is two dimensional and streamlines are not parallel.

(3) **Rotary or Vortex Motion**—A mass of fluid in rotation about a fixed axis is called *vortex*.

Rotary motion of fluid is also called *vortex motion*. In this case the rotating fluid particles have velocity in tangential direction. Thus *vortex motion* is defined as motion in which the whole fluid mass rotates about an axis. The vortex motion is of two types : free vortex and forced vortex.

(a) **Free Vortex** is that in which the fluid mass rotates without any external impressed contact force. The whole fluid mass rotates either due to fluid pressure itself or the gravity or due to rotation previously imparted. Energy is not expended by any outside source.

The free vortex motion is also called *potential vortex* or *irrotational vortex*.

**Examples :** Common examples of free vortex are :

(1) Flow of liquid in a centrifugal pump casing after it has left the impeller.

(2) Flow of water in a turbine casing before it enters the guide vanes.

(3) Flow around a circular bend.

(4) Rotary flow observed in shallow vessel, such as wash basin or bath tub while draining liquid through the outlet at the bottom.

(5) Whirlpool in a river.

(b) **Forced Vortex** is one in which the fluid mass is made to rotate by means of some external agency. The external agency is generally the mechanical power which imparts a constant torque on the fluid mass. The torque rotates the fluid mass either by stirring the liquid contained in the vessel or by spinning the vessel containing the liquid, about a vertical axis ; consequently the whole fluid mass revolves with constant angular velocity  $\omega$ . Thus in forced vortex, there is always expenditure of energy.

The forced vortex motion is also called *flywheel* vortex or *rotational* vortex.

**Examples :** Rotation of liquid inside the impeller of a centrifugal pump and inside the runner of a hydraulic turbine.

Free or forced vortex is further characterised as cylindrical free or forced vortex ; or spiral free or forced vortex.

(i) **Cylindrical Vortex**—In cylindrical free or forced vortex, the fluid mass rotates in concentric circles. Hence it is also known as circulatory flow.

(ii) **Spiral Vortex**—When cylindrical vortex is superimposed over the radial flow described above, the resulting vortex is known as spiral vortex.

**Example :** Motion of liquid in spiral casing (*i.e.*, *volute*) of a centrifugal pump.

**2.2 Radial Motion or Radial Flow**—Radial flow can be produced by constraining a stream of fluid to flow between two parallel circular plates, the openings being at the centre and the periphery. Fig 2.1 shows two parallel circular plates. The lower plate has an opening at the centre of radius  $r_1$ . Suppose a pipe of same radius  $r_1$  is connected to the opening. If liquid is inserted to the pipe it will flow radially outward between the two parallel plates. If the liquid is pushed in at the periphery between the two plates, it will flow radially inward to the centre of the pipe.

If  $b$  is the distance between two parallel circular plates (refer Fig 2.1) and  $v_m$  the radial velocity at a point distance  $r$  from the centre, then

$$\text{area across flow} = 2\pi r b$$

If the quantity of fluid flowing per second is  $Q$ , then by applying 'Equation of Continuity'.

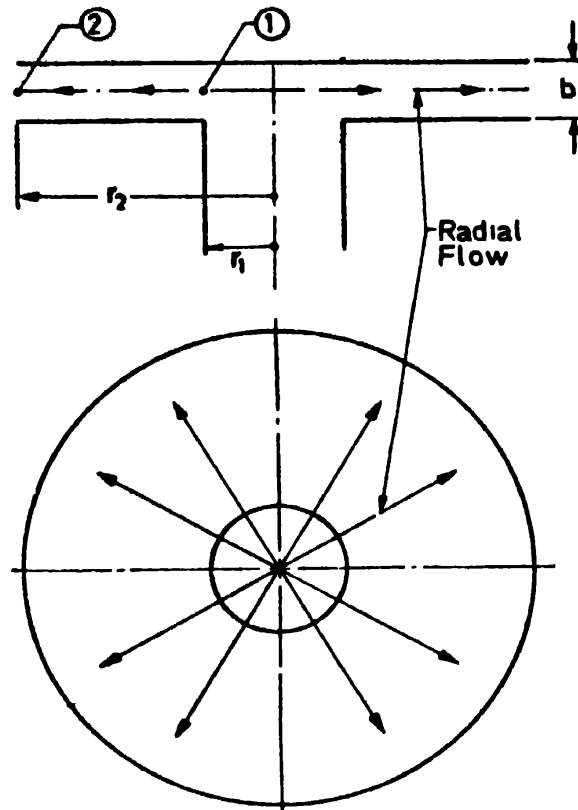


Fig 2.1 Radial Flow

$$Q = (2\pi r b) \cdot v_m \quad \dots(2.1)$$

where

$$Q = \text{constant}$$

A.

$$r b v_m = \text{constant}$$

Since  $b$  remains constant in case of radial flow,

$$r v_m = \text{constant} \quad \dots(2.2)$$

or

$$v_m \propto \frac{1}{r} \quad \dots(2.2a)$$

i.e., Radial velocity of flow is inversely proportional to the radius of circular parallel plates through which the radial flow takes place.

**Determination of Pressure Difference**—Since there is no addition or subtraction of energy, and if losses are neglected, the Bernoulli's equation can be applied between any two points. Let points 1 and 2 denote the entrance to and the exit from the plate. Let  $p_1$  and  $p_2$  be the pressures and  $v_{m1}$  and  $v_{m2}$  be the radial velocities at points 1 and 2 respectively.

Then from Bernoulli's theorem—

$$\frac{p_1}{\gamma} + z_1 + \frac{v_{m1}^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_{m2}^2}{2g}$$

If plates are horizontal, then  $z_1 = z_2$ , the above equation reduces to

$$\frac{p_1}{\gamma} + \frac{v_{m1}^2}{2g} = \frac{p_2}{\gamma} + \frac{v_{m2}^2}{2g}$$

or

$$\frac{p_1 - p_2}{\gamma} = \frac{v_{m2}^2 - v_{m1}^2}{2g} \quad \dots(2.3)$$

$$= \frac{v_{m2}^2}{2g} \left( 1 - \frac{r_1^2}{r_2^2} \right) \quad \dots(2.3a)$$

Thus the pressure variation caused due to change of radial velocity is proportional to radius  $r$ .

**Flow Along Curved Path**—Consider a small fluid element moving in a horizontal plane along a curved path of radius  $r$ , with a constant angular velocity  $\omega$ . Let the length of the element in the plane of paper be  $dr$  and its breadth be  $b$  in the plane normal to the paper (refer Fig 2.2). The element subtends an angle  $d\theta$  at the centre of revolution.

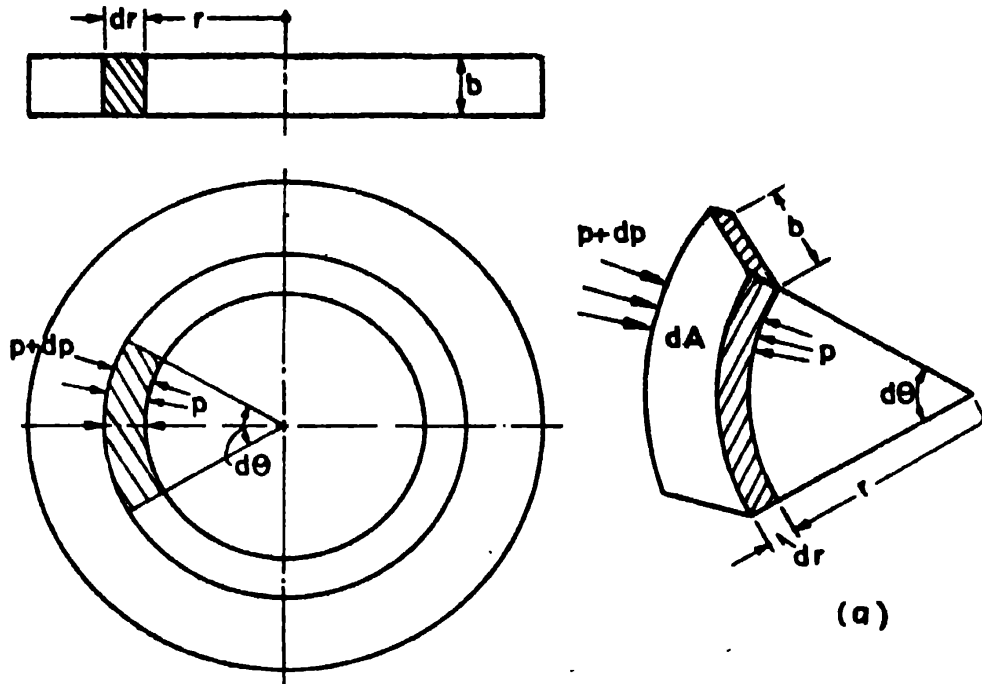


Fig 2.2 The Flow Along a Curved Path

Since the fluid element is moving with constant angular velocity  $\omega$ , the tangential velocity  $u = \omega r$  will also be constant and consequently tangential acceleration will be equal to zero. Hence there will not be any pressure difference in tangential direction.

As far as the direction of tangential velocity is concerned, it is changing continuously due to the motion in a curved path, with the result a normal or radial acceleration  $\omega^2 r$  acts towards the centre. It follows that a pressure force must exist along radial direction in any horizontal plane to cause this acceleration.

$$\text{Area of fluid element} \quad dA = rd\theta \cdot b$$

$$\text{Mass of fluid element} \quad = \rho \cdot dA \cdot dr$$

The fluid element is subjected to

$$(i) \text{ pressure force on inward face} = p \cdot dA$$

$$(ii) \text{ pressure force on outward face} = (p+dp)dA$$

$$\text{and } (iii) \text{ radial acceleration force (or centripetal force acting towards centre)} = \rho \cdot dA \cdot dr (\omega^2 r)$$

Applying Newton's second law of motion—

$\Sigma F$  in any direction = mass  $\times$  acceleration in the direction as that of force

$$p \cdot dA - (p+dp)dA = -\rho (dA \cdot dr)(\omega^2 r)$$



$$\text{or} \quad -dp \, dA = -\rho dA \cdot dr \cdot \omega^2 r$$

$$\text{or} \quad dp = \rho \omega^2 r \, dr \quad \dots(2.4)$$

Writing this equation in terms of tangential velocity

$$u = \omega r$$

$$dp = \rho \frac{u^2}{r} \cdot dr \quad \dots(2.5)$$

This is the *Equation of Flow in a curved path*. If the flow is in a straight line then  $r = \infty$ , then  $dp$  in Eqn 2.5 will be equal to zero. This means that for rectilinear flow the pressure remains constant in directions transverse to the direction of motion.

**2.4 Forced Vortex**—When a fluid is forced to rotate as a solid body i.e., by rotating the containing vessel by some external agency or by stirring the liquid, it will get a motion known as forced vortex. Thus forced vortex occurs when constant torque is maintained on the fluid. The two types—cylindrical and spiral forced vortices will now be discussed.

#### Cylindrical Forced Vortex—Mathematical Analysis

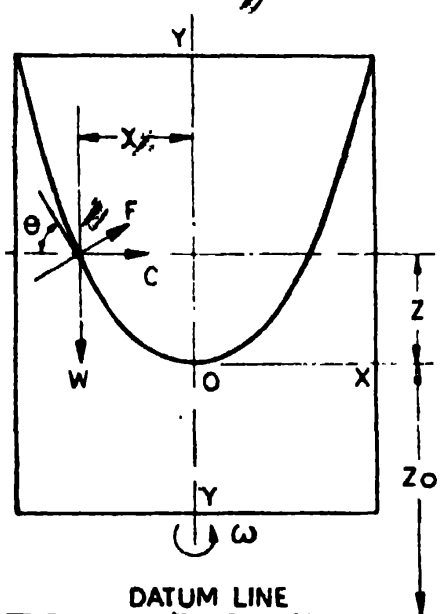


Fig 2.3 Mathematical Analysis of Forced Vortex

Liquid is rotated in a cylindrical container by supplying energy from an external source. Let there be a vertical section through the axis of such a vessel (refer Fig 2.3).

Let the section of liquid surface be any curve  $y = f(x)$  and let  $A$  be any point on it given by  $(x, y)$  measured from an origin at the lowest point on the curve. It follows from symmetry that the latter point lies on the axis.

Consider the equilibrium of a small mass  $m$  of liquid at a point  $A$  on free surface. It is subjected to the following forces :

(i) Weight of liquid mass acting vertically downward,  $W = mg$

(ii) Centripetal force acting horizontally,  $C = m \omega^2 \cdot x$

and (iii) Fluid pressure force acting always normal to the surface or tangent drawn.

Let the tangent at  $A$  makes an  $\angle \theta$  with  $x$ -axis.

For equilibrium, equating components of forces, in the direction of tangent at  $A$

$$m \cdot \omega^2 \cdot x \cos \theta = m \cdot g \cdot \sin \theta$$

$$\text{or} \quad \tan \theta = \frac{\omega^2 \cdot x}{g}$$

From elementary knowledge of calculus,  $\tan \theta$ , the slope of the tangent to curve  $y = f(x)$  at point  $(x, y)$  is  $\frac{dy}{dx}$  :

$$\therefore \frac{dy}{dx} = \frac{\omega^2 \cdot x}{g}$$

$$\text{Integrating, } y = \frac{\omega^2 \cdot x^2}{2g}$$

$$\text{or } x^2 = \frac{2g}{\omega^2} \cdot y \quad \dots(2.6)$$

This evidently is the equation of a parabola of the second order touching the origin and symmetrical about  $x$ -axis.

The surface of liquid in a forced vortex is, therefore, the surface generated by the revolution of a parabola i.e., the *Paraboloid of Revolution*.

In engineering practice the mathematical symbols  $x$  and  $y$  are often replaced by  $r$  and  $z$  respectively, then

$$r^2 = \frac{2g}{\omega^2} \cdot z \quad (\text{where } z = \text{head of particle})$$

$$\text{or } z = \frac{\omega^2 \cdot r^2}{2g} = \frac{u^2}{2g} \quad \dots(2.7)$$

(where  $u = \omega \cdot r$  circumferential or tangential velocity of particle)

If  $z_0$  is the height of zero point measured from some datum line, it will become a more general relation,

$$z = \frac{\omega^2 \cdot r^2}{2g} + z_0 = \frac{u^2}{2g} + z_0$$

Difference of head between two points 1 and 2,

$$z_2 - z_1 = \frac{u_2^2 - u_1^2}{2g} \quad \dots(2.8)$$

This is the head developed by the centrifugal action and is known as the *Centrifugal Head*.

A more general relation is

$$\left( \frac{p_2}{w} - \frac{p_1}{w} \right) + (z_2 - z_1) = \frac{\omega}{2g} (r_2^2 - r_1^2) = \frac{u_2^2 - u_1^2}{2g} \quad \dots(2.9)$$

**2.5 Forced Vortex in Open Vessel**—Liquid is free to form a paraboloid. Let  $z_1$  be the depression of level at centre due to formation of forced vortex (refer Fig 2.4)

Since the total volume remains the same,

volume of liquid in vessel before rotation = volume of liquid in vessel after rotation.

$$V_0 + \pi r^2 \cdot z_1 = V_0 + \frac{1}{2} \pi r^2 \cdot z$$

(where  $V_0$  = initial volume up to centre of depression)

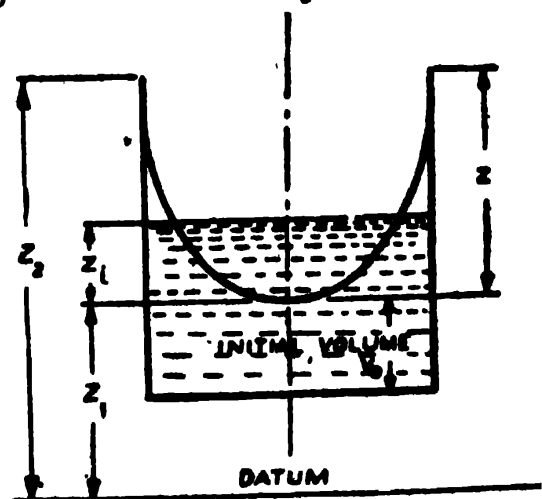


Fig 2.4 Forced Vortex in Open Vessel

$$\therefore z_t = \frac{z}{2} = \frac{z_2 - z_1}{2}$$

$$\text{Since } r^2 = \frac{2g}{\omega^2} \cdot z \quad r^2 = \frac{2g}{\omega^2} \cdot 2 \cdot z_t$$

$$\therefore z_t = \frac{r^2 \cdot \omega^2}{4g} = k \cdot \omega^2 \quad \dots(2.10)$$

(where  $k$  is a constant for the vessel)

Thus the depression of vortex below the original level of still water is proportional to square of angular velocity of rotation,  $z_t \propto \omega^2$ .

**Problem 2.1** An open cylindrical vessel 30 cm in internal diameter and 1 m long stands vertically. It is filled with water upto a height of  $\frac{3}{4}$  m from the bottom. Find the speed of rotation about its vertical axis, when the water is just going to spill out of vessel.

**Solution**

$$d = 30 \text{ cm}; \quad l = 1 \text{ m}$$

$$z_{\text{empty}} = 1 - \frac{3}{4} = \frac{1}{4} \text{ m}$$

$$\text{Centrifugal head, } z = \frac{u^2}{2g} = \frac{\omega^2 \cdot r^2}{2g} = \frac{\left(\frac{2\pi N}{60}\right)^2 \cdot r^2}{2g} \quad \dots(\text{refer Eqn 2.7})$$

At the point of spilling, the volume of empty space in the vessel before rotation = volume of the paraboloid.

$$\text{i.e., } \pi r^2 z_{\text{empty}} = \frac{1}{2} \pi r^2 \cdot z$$

$$\therefore z_{\text{empty}} = \frac{1}{2} z$$

$$\text{or } z = 2 z_{\text{empty}} = 2 \times \frac{1}{4} = \frac{1}{2} \text{ m}$$

$$\text{But } z = \frac{\left(\frac{2\pi N}{60}\right)^2 \cdot r^2}{2g} = \frac{\left(\frac{2\pi \times N}{60}\right)^2 \times \left(\frac{0.3}{2}\right)^2}{2 \times 9.81}$$

$\therefore$  Substituting for  $z$ ,

$$\frac{2}{3} = \frac{\left(\frac{2\pi \times N}{60}\right)^2 \times (0.15)^2}{19.62}$$

$$\text{or } N = \sqrt{\frac{2 \times 3,600 \times 19.62}{3 \times 2 \times 2 \times \pi \times \pi \times 0.15 \times 0.1}}$$

$$= \sqrt{53,200}$$

$$= 230 \text{ rpm} \quad \text{Answer}$$

**Problem 2.2** A cylinder 15 cm in dia and 37.5 cm high containing water is rotated about its vertical axis at a speed of 320 rpm, so that a portion of water spills out :

- If the cylinder is now brought to rest, what would be the depth of water in it? (AMIE)
- If now the speed is increased to 600 rpm, how much water will be left in the vessel?

**Solution**

$$d = 15 \text{ cm} = 0.15 \text{ m}, \quad r = \frac{0.15}{2} \text{ m} = 0.075 \text{ m}$$

$$N = 320 \text{ rpm}, \quad h = 37.5 \text{ cm} = 0.375 \text{ m}$$

$$\text{Angular velocity of vessel, } \omega = \frac{2\pi N}{60} = \frac{2\pi \times 320}{60} \\ = 33.5 \text{ radians/sec}$$

$$\text{Peripheral velocity of vessel, } u = \omega r = 33.5 \times 0.075 = 2.513 \text{ m/sec}$$

$$\therefore \text{Centrifugal head generated by rotation, } z = \frac{u^2}{2g} = \frac{(2.513)^2}{19.62} \\ = 0.322 \text{ m}$$

Volume of water which spills out, will be the volume of the paraboloid of revolution.

$$\text{Volume of paraboloid of revolution} = \frac{1}{2} \pi r^2 \cdot z \\ = \frac{1}{2} \times \pi \times (0.075)^2 \times 0.322 = 0.002845 \text{ m}^3$$

Volume of vessel when full with water

$$= \frac{\pi}{4} d^2 \cdot h = \frac{\pi}{4} \times (0.15)^2 \times 0.375 = 0.00665 \text{ m}^3$$

$$\therefore \text{Remaining water} = 0.006650 - 0.002845 \\ = 0.003805 \text{ m}^3$$

$\therefore$  (a) Depth of water after the vessel has come to rest

$$= \frac{\text{Volume}}{\frac{\pi}{4} d^2} = \frac{0.003805}{\frac{\pi}{4} \times (0.15)^2} = 0.2145 \text{ m} \\ = 21.45 \text{ cm} \quad \text{Answer}$$

(b) Centrifugal head generated when the speed is 600 rpm,

$$= \frac{\left( \frac{2\pi r N}{60} \right)^2}{2g} = \frac{(2\pi \times 0.075 \times 600)^2}{3,600 \times 19.62} = 1.128 \text{ m}$$

The shape of the free surface will be as shown in Fig 2.7.

The volume of water spilling out will now be equal to the hatched volume in Fig 2.5. This can be determined as follows:

Let  $x$  be the radius of parabola at the bottom of vessel, then

Applying Eqn 2.6 and replacing  $y$  by  $h$ , where  $h = 112.8 - 37.5 = 75.3 \text{ cm}$

$$= \frac{\sqrt{2gh}}{\omega} = \frac{\sqrt{19.62 \times 0.753}}{2\pi \times 600 / 60} = 0.0613 \text{ m}$$

$\therefore$  Volume of water spilled  
(Total volume of paraboloid) — (volume of paraboloid below the bottom of vessel)

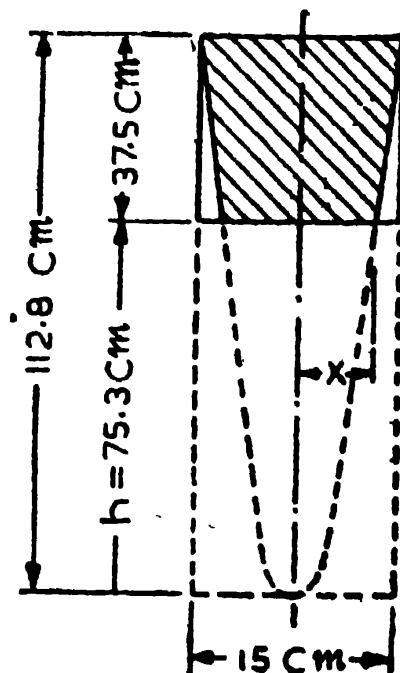


Fig 2.5 Paraboloid of Revolution with a Speed of 600 rpm

$$\begin{aligned}
 &= \left\{ \left( \frac{1}{2} \times \pi \times (0.075)^2 \times 1.128 \right) - \left( \frac{1}{2} \times \pi \times (0.0613)^2 \times 0.753 \right) \right\} \\
 &= \frac{\pi}{2} (0.00634 - 0.00283) \\
 &= \frac{\pi}{2} \times 0.00351 = 0.0055 \text{ m}^3
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Remaining water} &= \left\{ \frac{\pi}{4} \times (0.15)^2 \times 0.375 - 0.0055 \right\} \\
 &= 0.00665 - 0.0055 = 0.00115 \text{ m}^3 \quad \text{Answer}
 \end{aligned}$$

**2.6 Forced Vortex in Closed Vessel**—Let the fluid touch the lid when the angular velocity is  $\omega$ , (refer Fig 2.6). If the speed is increased even further say to  $\omega'$ ,

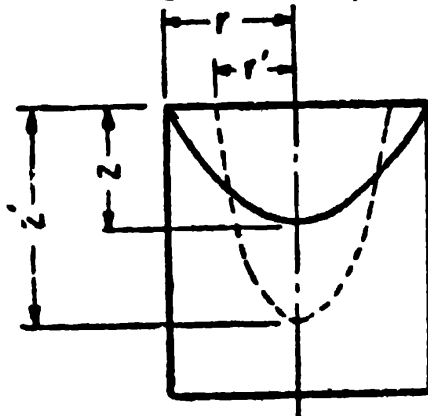


Fig 2.6 Forced Vortex in Closed Vessel

$$(r')^2 = \frac{2g}{(\omega')^2} \cdot z'$$

$$\text{and } r^2 = \frac{2g}{\omega^2} \cdot z$$

Since the volume of liquid remains the same, the volume of air enclosed also remains the same.

$$\therefore \frac{1}{2} \cdot \pi \cdot r^2 \cdot z = \frac{1}{2} \cdot \pi \cdot (r')^2 \cdot (z')$$

$$\therefore r^2 \cdot z = r'^2 \cdot z' = \frac{2g}{(\omega')^2} (z') \cdot (z')$$

$$\text{Hence, } (z')^2 = \frac{r^2 z}{2g} \cdot (\omega')^2 \quad \dots(2.11)$$

$$= m^2 \cdot (\omega')^2$$

$$\text{or } z' = m \cdot \omega' \quad (\text{where } m \text{ is a constant}) \quad \dots(2.11a)$$

Pressure on the lid is equal to the head to which liquid would have risen in the absence of the lid.

**Problem 2.3** A closed cylindrical vessel 25 cm in internal diameter is completely filled with water and is rotated about its vertical axis at 1,450 rpm. Determine the difference of pressure between its circumference and the centre in metres of water having the specific weight of 1,000 kg/m<sup>3</sup>. If the vessel now contains air of specific weight 1.2 kg/m<sup>3</sup>, what would be the pressure difference in metres of fluid? Give value in kg/m<sup>3</sup> in both the cases.

**Solution**

$$d = 25 \text{ cm} \quad \therefore r = 12.5 \text{ cm} = 0.125 \text{ m}$$

$$N = 1,450 \text{ rpm} \quad \therefore \omega = \frac{2\pi N}{60} = \frac{2\pi \times 1,450}{60} = 152 \text{ radians/sec}$$

Difference of pressure and centrifugal head

$$\frac{p}{\gamma_{\text{water}}} = \frac{u^2}{2g} = \frac{\omega^2 \cdot r^2}{2g} = \frac{(152 \times 0.125)^2}{19.62} \text{ m}$$

$$= 18.4 \text{ m of water} \quad \text{Answer}$$

Difference of pressure if the fluid is air

$$= \frac{p}{\gamma_{\text{air}}} = \frac{u^2}{2g} = \frac{\omega^2 r^2}{2g} = \frac{(152 \times 0.125)^2}{19.62} \text{ m}$$

$$= 18.4 \text{ m of air} \quad \text{Answer}$$

Hence the difference of pressure is *same* if it is expressed in metres of fluid handled.

Difference of pressure in  $\text{kg/cm}^2$ , if the vessel contains water

$$= \left( \frac{p}{\gamma_{\text{water}}} \right) \cdot \gamma_{\text{water}} = \frac{18.4 \times 10^3}{100^3} \text{ kg/cm}^2$$

$$= 1.84 \text{ kg/cm}^2 \quad \text{Answer}$$

Difference of pressure in  $\text{kg/cm}^2$ , if the vessel contains air

$$= \left( \frac{p}{\gamma_{\text{air}}} \right) \cdot \gamma_{\text{air}} = \frac{18.4 \times 1.2}{100^3} \text{ kg/cm}^2$$

$$= 0.00221 \text{ kg/cm}^2 \quad \text{Answer}$$

**Problem 2.4** A closed cylindrical vessel 15 cm in internal diameter and 1 m long, stands vertically. It is filled with water upto a height of  $\frac{2}{3}$  m from the bottom, and is rotated about its vertical axis. Find the revolving speed of the vessel when :—

(i) the top of the cup formed by water just touches the top lid of the vessel, and (ii) the bottom of the cup formed by water just touches the bottom lid of the vessel.

**Solution**

$$d = 15 \text{ cm} \quad l = 1 \text{ m}$$

$$z_1 = 1 - \frac{2}{3} = \frac{1}{3} \text{ m}$$

(i) When top of cup just touches the lid (refer Fig 2.7) :

Volume of hollow cup = Volume of empty space in the cylinder before rotation.

$$\text{i.e., } \frac{1}{2} \cdot \pi \cdot r^2 \cdot z = \pi \cdot r^2 \cdot z_1$$

$$\therefore z = 2z_1 = 2 \times \frac{1}{3} = \frac{2}{3} \text{ m}$$

Centrifugal head developed by rotation,

$$z = \frac{\omega^2 r^2}{2g} = \left( \frac{2\pi N}{60} \right)^2 \times r^2$$

$$\text{or } \frac{2}{3} = \left( \frac{2\pi N}{60} \right)^2 \times \frac{(0.075)^2}{19.62}$$

$$\therefore N = \sqrt{\frac{2 \times 3,600 \times 19.62}{3 \times 2 \times 2 \times \pi \times \pi (0.075)^2}}$$

$$= \sqrt{228,600}$$

$$= 460 \text{ rpm} \quad \text{Answer}$$

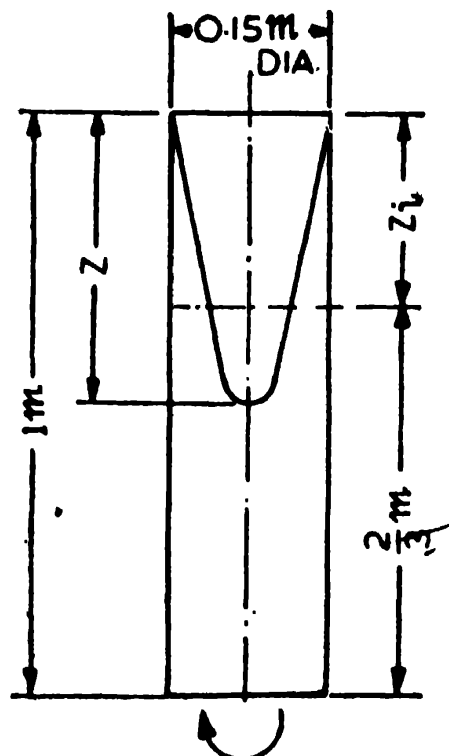


Fig 2.7 Top of Cup Touching The Lid of Closed Vessel

(ii) When the bottom of cup touches the bottom of vessel (refer Fig. 2.8) — Volume of the paraboloid = volume of the empty space before rotation.

$$\text{i.e., } \frac{1}{2} \cdot \pi \cdot (r')^2 \cdot (z') = \pi \cdot r^2 \cdot z_1$$

In this case the speed of the vessel is increased to  $\omega'$

$$\therefore (r')^2 = \frac{2g}{(\omega')^2} \cdot (z')$$

Substituting the value of  $r'$  in the above equation

$$\frac{1}{2} \cdot \pi \left\{ \frac{2g}{(\omega')^2} \cdot z' \right\} \cdot z' = \pi \cdot r^2 \cdot z_1$$

$$\therefore (\omega')^2 = \frac{g \cdot (z')^2}{r^2 \cdot z_1}$$

$$\text{or } \left( \frac{2\pi N'}{60} \right)^2 = \frac{g \cdot (z')^2}{r^2 \cdot z_1}$$

$$\begin{aligned} \therefore N' &= \sqrt{\frac{g \cdot (z')^2}{r^2 \cdot z_1} \cdot \left( \frac{60}{2\pi} \right)^2} \\ &= \sqrt{\frac{9.81 \times 1^2}{(0.075)^2 \times \frac{1}{2}} \times \frac{3,600}{4\pi^2}} \\ &= \sqrt{486,000} \end{aligned}$$

$$= 696 \text{ rpm} \quad \text{Answer}$$

The results obtained from the above numerical problem are very interesting. The speed in the first case is two-third the speed in the second case when the vessel is originally two-third full with water. Similarly, it can be shown that if the vessel is originally three-fourth full, the speed in the first case would be half the speed in the second case. Also if the vessel is half full, the speed in both cases would be the same.

Forming an equation :

$$\text{Since } \frac{N'}{N} = \frac{\frac{60}{2\pi r} \cdot z' \cdot \sqrt{\frac{g}{z_1}}}{\frac{60}{2\pi r} \times \sqrt{2z_1 \cdot g}} = \frac{z'}{\sqrt{2z_1 \cdot z_1}} = \frac{z'}{\sqrt{4z_1^2}} = \frac{z'}{2z_1}$$

$$\left( \text{provided } \frac{z_1}{z} \leq \frac{1}{2} \right)$$

$$\text{or } \frac{z_1}{z} = \frac{N}{2N'}$$

$$\text{or } 1 - \frac{z' - z_1}{z'} = \frac{1}{2} \left( \frac{N}{N'} \right)$$

$$\text{Putting } \frac{z' - z_1}{z'} = \frac{x}{100}$$

which is percent full of water

and  $\frac{N}{N'} = y$ , the ratio of speeds

$$\text{then } \frac{1}{100} \left( 1 - \frac{1}{2y} \right)$$

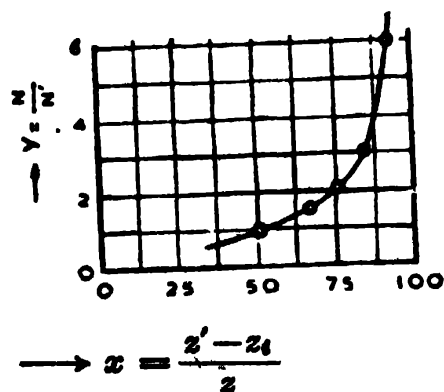


Fig 2.9 Variation of Centrifugal Head with Change of Container Speed

Tabulating and drawing in Fig 2.19

|     |    |      |    |      |      |          |
|-----|----|------|----|------|------|----------|
| $x$ | 50 | 66.7 | 75 | 83.3 | 21.7 | 100      |
| $y$ | 1  | 1.5  | 2  | 3    | 6    | $\infty$ |

**2.7 Principle of a Centrifugal Pump**—If the liquid is rotated with a sufficiently high velocity so as to enable it to rise beyond the walls of the container and if more liquid is constantly supplied at the centre by some suitable means, the tendency of the liquid would be to flow out as illustrated in Fig 2.10. Such a system in principle is a *Centrifugal Pump*.

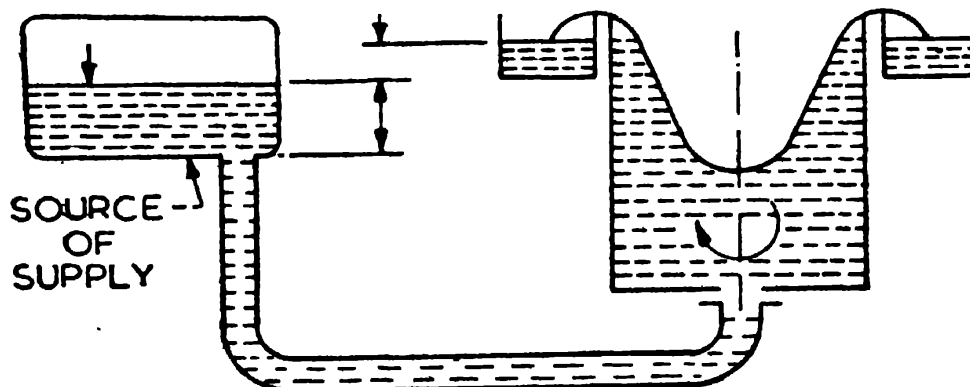


Fig 2.10 Principle of Centrifugal Pump

If on the other hand, liquid flowed into the tank over the rim from an outside source at a higher elevation and were drawn out at the centre, the flow being inward, the system will constitute a *Francis Water Turbine* in principle.

**2.8 Spiral Vortex**—In runners of turbines and pumps the flow of fluid is usually a combination of

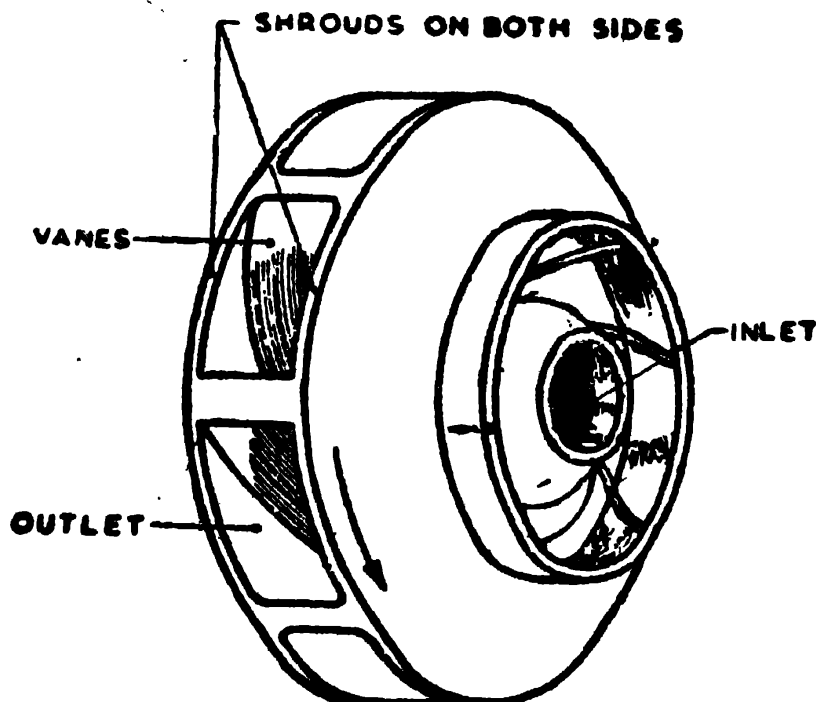


Fig 2.11 Impeller of a Centrifugal Pump



(i) Circulatory or cylindrical forced flow i.e., flow in concentric circles and (ii) Radial flow i.e., flow involving a change of distance from the axis of rotation (refer Art 2.2).

The path resulting from the superimposition of these two motions is of the form of spiral.

The impeller of a centrifugal pump consists of two plates called shrouds between which are fixed a number of vanes (refer Fig 2.11) — Water enters through an opening provided at the centre and leaves at the periphery. Vanes are made of spiral shape to enable water to have both circulatory and radial flows inside the impeller.

The pressure difference between any two points in spiral forced vortex taken on the same horizontal plane, is found by adding the pressure difference due to radial flow and that due to cylindrical forced vortex both considered separately i.e.,

$$\frac{p_2}{\gamma} - \frac{p_1}{\gamma} = \frac{1}{2g} (v_{m_1}^2 - v_{m_2}^2) \quad \text{for radial flow (refer Eqn 2.3)}$$

$$\text{and} \quad \frac{p_2}{\gamma} - \frac{p_1}{\gamma} = \frac{\omega^2}{2g} (r_2^2 - r_1^2) \\ = \frac{u_2^2 - u_1^2}{2g}$$

for cylindrical forced vortex whose points 1 and 2 are in the same horizontal plane.

∴ Pressure difference due to spiral forced vortex,

$$\frac{p_1 - p_2}{\gamma} = \frac{v_{m_1}^2 - v_{m_2}^2}{2g} + \frac{u_2^2 - u_1^2}{2g} \quad \dots(2.12)$$

**Problem 2.5** A centrifugal pump impeller has an inner diameter of 50 cm. Its outer diameter is twice the inner diameter. Find the speed of the impeller in rpm at which the lifting will commence against a head of 15 metres.

**Solution**

$R_1$  the inner radius of impeller = 0.5 m

$R_2$  the outer radius of impeller = 1.0 m

Centrifugal head,  $z_1 = \frac{\omega^2}{2g} (R_2^2 - R_1^2)$

$$\text{or} \quad 15 = \frac{\left(\frac{2\pi N}{60}\right)^2}{2 \times 9.81} \times (1^2 - 0.5^2)$$

$$\text{or} \quad N = \sqrt{\frac{15 \times 19.62 \times 3,600}{4\pi^2 \times 0.75}} \\ = 190 \text{ rpm} \quad \text{Answer}$$

**Problem 2.6** Find the head developed by the impeller in Problem 2.5 if the radial velocities of water at inlet and outlet are 7.5 m/sec and 5.4 m/sec respectively.

**Solution**  $R_1 = 0.5 \text{ m}$   $R_2 = 1.0 \text{ m}$

Head developed by centrifugal action = 15 m

$$v_{m_1} = 7.5 \text{ m/sec} \quad v_{m_2} = 5.4 \text{ m/sec}$$

Head developed by impeller

$$\begin{aligned} &= \frac{v_{m_1}^2 - v_{m_2}^2}{2g} + \frac{u_2^2 - u_1^2}{2g} \dots (\text{refer Eqn 2.12}) \\ &= \frac{(7.5)^2 - (5.4)^2}{2 \times 9.81} + 15 = 1.38 + 15 \\ &= 16.38 \text{ m} \quad \text{Answer} \end{aligned}$$

**2.9 Free Cylindrical Vortex**—Consider a mass ‘ $m$ ’ of fluid moving in a circular path with a tangential velocity component  $v_u$  at a distance  $r$  from the centre. As no external torque is applied in free vortex, corresponding time rate of change of angular momentum is zero, i.e.,

$$\text{Torque} \quad T = \frac{d(m v_u r)}{dt} = 0$$

$$\text{Since mass ‘} m \text{’ is constant, } \frac{m \cdot d(v_u r)}{dt} = 0$$

On integration, this gives

$$v_u \cdot r = \text{constant} = C \quad \dots (2.13)$$

where  $C$  is a constant and is known as *strength of vortex*

$$\therefore v_u = \frac{C}{r} \quad \dots (2.14)$$

$$\text{or} \quad v_u \propto \frac{1}{r} \quad \dots (2.14a)$$

i.e., tangential velocity component  $v_u$  is inversely proportional to distance  $r$ .

In case of forced vortex,

$$\begin{aligned} \text{tangential velocity, } u &= \omega r = (\text{constant}). \quad r = C_1 \text{ (say).} \\ &= C_1 \cdot r \end{aligned}$$

and in radial flow (refer Eqn 2.2),  $v_m r = \text{constant } C_2 \text{ (say)}$

$$\therefore v_m = \frac{C_2}{r}$$

The only difference between free vortex motion and radial motion is that in case of free vortex, the velocity is in tangential direction i.e., velocity of whirl, while in radial motion the velocity is in radial direction.

Since in free vortex no energy is imparted to or extracted from the fluid, the total head at any point is constant throughout. This means that Bernoulli’s theorem holds good in this case, if there are no losses.

$$\text{Hence} \quad \frac{p}{\gamma} + z + \frac{v_u^2}{2g} = \text{constant}$$

or 
$$\frac{p_1}{\gamma} + z_1 + \frac{v_{u1}^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_{u2}^2}{2g}$$

Pressure difference between any two points, thus, can be determined by applying Bernoulli's equation.

Discharge is obtained by integration as follows :

$$Q = \int v_u dA = \int \frac{C}{r} dA$$

where  $dA$  is expressed in terms of  $r$ .

**Difference of pressure between any two points in free vortex** can also be determined by applying Eqn 2.5 and integrating it after substituting the value of radial acceleration  $= \frac{v_u^2}{r}$

$$dp = \rho \frac{v_u^2}{r} \cdot dr = \frac{\gamma}{g} \cdot \frac{v_u^2}{r} \cdot dr$$

This gives the variation of pressure in the radial direction for the fluid mass rotating in a horizontal plane.

Substituting  $v_u = \frac{C}{r}$  for free vortex and integrating the above equation between two points on the same horizontal plane (i.e.,  $z_1 = z_2$ ) but at radii  $r_1$  and  $r_2$ .

$$\begin{aligned} \int_{p_1}^{p_2} dp &= \frac{\gamma}{g} \left( \frac{C}{r} \right)^2 \frac{1}{r} \cdot dr = \frac{\gamma \cdot C^2}{g} \int_{r_1}^{r_2} r^{-3} \cdot dr \\ \therefore p_2 - p_1 &= \frac{\gamma}{g} \cdot C^2 \left( \frac{r_2^{-2}}{-2} - \frac{r_1^{-2}}{-2} \right) = \frac{\gamma \cdot C^2}{2g} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) \\ \text{or} \quad -\frac{p_2}{\gamma} - \frac{p_1}{\gamma} &= \frac{C^2}{2g} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) \quad \dots(2.15) \\ &= \frac{C^2}{2g} \cdot \frac{1}{r_1^2} \left\{ 1 - \left( \frac{r_1}{r_2} \right)^2 \right\} \\ &= \frac{v_{u1}^2}{2g} \left\{ 1 - \left( \frac{r_1}{r_2} \right)^2 \right\} \quad \dots(2.15a) \end{aligned}$$

Since there is no vertical acceleration, the pressure varies hydrostatically in vertical direction for points 1 and 2 which are not in the same horizontal plane, but are at the same radial distance,

$$\frac{p_2}{\gamma} - \frac{p_1}{\gamma} = z_1 - z_2$$

Hence the pressure difference between any two points 1 and 2 which are at radii  $r_1$  and  $r_2$  and elevations  $z_1$  and  $z_2$  respectively, will be given by—

$$\frac{p_2}{\gamma} - \frac{p_1}{\gamma} = \frac{C^2}{2g} \left( \frac{1}{r_1^2} - \frac{1}{r_2^2} \right) + (z_1 - z_2) \quad \dots(2.16)$$

This is a *general equation for free cylindrical vortex*

This Eqn 2.16 can also be written in the form of

$$\frac{p_2 - p_1}{\gamma} = -\frac{1}{2g} (v_{u1}^2 - v_{u2}^2) + (z_1 - z_2) \quad \dots(2.17)$$

or 
$$\frac{p_1}{\gamma} + z_1 + \frac{v_{u1}^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_{u2}^2}{2g} = \text{constant}$$

On replacing  $v_u$  by  $v$

$$\frac{p_1}{\gamma} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 + \frac{v_2^2}{2g} = \text{constant}$$

This is nothing but the Bernoulli's equation.

It is to be noted that Eqn 2.8 for forced cylindrical vortex while arranging in the pattern of Bernoulli's equation, yields to

$$\frac{p_1}{\gamma} + z_1 - \frac{\omega^2 r_1^2}{2g} = \frac{p_2}{\gamma} + z_2 - \frac{\omega^2 r_2^2}{2g}$$

On writing  $\omega r_1 = v_1$  and  $\omega r_2 = v_2$ , this gives

$$\frac{p_1}{\gamma} + z_1 - \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + z_2 - \frac{v_2^2}{2g} \quad \dots(2.18)$$

This shows that Bernoulli's equation is not applicable to forced vortex because in this case some external energy is imparted to the fluid mass.

It is important to note that Eqn 2.17 for free cylindrical vortex and Eqn 2.18 for forced cylindrical vortex differ only in velocity-head terms.

**2.10 Spiral Free Flow**—Similar to that of forced vortex, the spiral flow in the case of free vortex will be the super-imposition of radial flow over circulatory flow. For such a flow the distance between two plates  $b$  (refer Fig 2.13) will vary, thus

$$R \cdot b \cdot v = \text{constant}$$

or 
$$v = \frac{\text{constant}}{R \cdot b} \quad \dots(2.19)$$

This shows that the pressure variation caused due to the change in velocity will not be the same as in the previous case.

One way to find out the actual pressure difference is to calculate separately for each kind of flow by applying Eqn 2.2 a and 2.14 a. This is employed when the factors influencing the two flows are different.

Direct computation of pressure difference from the change in absolute velocity  $v$  is also possible, when the total head for either type of flow is the same.

Since both  $v_u$  and  $v_m$  have been shown to vary inversely as  $R$ , other factors remaining the same angle  $\alpha$  must remain constant (refer Fig 2.12). Then the resultant path is an equiangular or *logarithmic spiral*.

Angle  $\alpha$  is determined as

$$\tan \alpha = \frac{v_m}{v_u} \quad \dots(2.20)$$

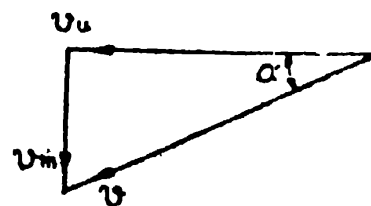


Fig 2.12 Velocity Triangle for Spiral Flow

If a structurally essential obstacle to the flow of a stream of fluid is so designed that the natural path of a fluid is a spiral, resistance to its motion would be least. A practical example of such a motion is in the volute of a centrifugal pump.

**Problem 2.7** The impeller of a centrifugal pump is surrounded by a casing which is made up of two parts—vortex chamber and volute. The outer diameter of the impeller is 50 cm and its width at outlet is 5 cm. The outer diameter of the vortex chamber is 70 cm. The impeller discharges water into the vortex chamber with a velocity of 15 m/sec at an angle of  $20^\circ$  with the tangent of rotation. The width of volute is 10 cm when the diameter is 85 cm. Determine :

- the magnitude and the direction of velocity of water when it leaves the vortex chamber and enters the volute ;
- the rise in pressure in the vortex chamber ;
- the magnitude and direction of the velocity of water at a point 42.5 cm from the centre ; and
- the rise in pressure from the outlet of the vortex chamber to a point in the volute 42.5 cm from the centre.

#### Solution

Let point 2 denote the position of impeller outlet as well as the vortex chamber inlet, point 3 denote the position of vortex chamber outlet as well as the volute inlet, and point 4 denote the position of volute outlet, (refer Fig 2.13).

$$\begin{array}{lll} D_2 = 50 \text{ cm} & D_3 = 70 \text{ cm} & D_4 = 85 \text{ cm} \\ b_2 = 5 \text{ cm} & b_3 = 5 \text{ cm} & b_4 = 10 \text{ cm} \\ v_2 = 15 \text{ m/sec} & & \end{array}$$

(a) The water is constrained to flow between two parallel plates in vortex chamber between points 2 and 3. It is a case of free vortex. Applying Eqn 2.14a for circulatory flow.

$$v_u \propto \frac{1}{r}$$

and applying Eqn 2.2a for radial flow.

$$v_m \propto \frac{1}{r}$$

$\therefore \alpha = \text{constant}$  (refer Fig 2.12),

$$\text{thus } v \propto \frac{1}{r}$$

$$\therefore v_2 \cdot R_2 = v_3 \cdot R_3$$

$$\text{or } v_3 = \frac{v_2 \cdot R_2}{R_3}$$

$$= \frac{15 \times 25}{35} = 10.7 \text{ m/sec}$$

$$\text{and } \alpha_3 = \alpha_2 = 20^\circ$$

(b) Rise in pressure in the vortex chamber between points 2 and 3.

$$\begin{aligned} &= \frac{p_2 - p_3}{\gamma} = \frac{v_2^2 - v_3^2}{2g} \\ &= \frac{(15)^2 - (10.7)^2}{2 \times 9.81} = 5.62 \text{ m} \end{aligned}$$

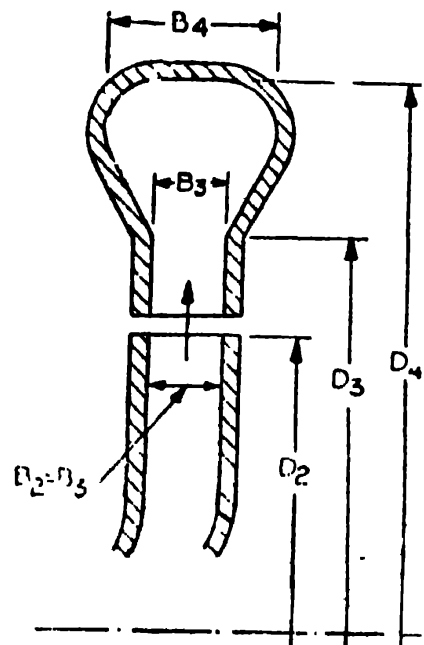


Fig 2.13 Section through Centrifugal Pump Impeller with Vortex Chamber and Volute Casing

(c) Direction of motion changes as the water flows through the volute between points 3 and 4, thus angle  $\alpha$  does not remain constant.

Because it is free vortex,  $v_u \propto \frac{1}{r}$  ... (refer Eqn 2.14a)

$$\begin{aligned}\therefore v_{u_4} &= \frac{v_{u_3} \cdot R_3}{R_4} = \frac{v_3 \cdot \cos \alpha_3 \cdot R_3}{R_4} \\ &= \frac{10.7 \cos 20^\circ \times 35}{42.5} = 8.3 \text{ m/sec}\end{aligned}$$

Since  $Q$  remains constant,

$$2\pi \cdot R_3 \cdot b_3 \cdot v_{m_3} = 2\pi \cdot R_4 \cdot b_4 \cdot v_{m_4} \quad \text{(refer Eqn 2.1)}$$

$$\begin{aligned}\therefore v_{m_4} &= \frac{R_3 \cdot b_3 \cdot v_{m_3}}{R_4 \cdot b_4} = \frac{R_3 \cdot b_3 \cdot v_3 \cdot \sin \alpha_3}{R_4 \cdot b_4} \\ &= \frac{35 \times 5 \times 10.7 \times 0.3420}{42.5 \times 10} = 1.5 \text{ m/sec}\end{aligned}$$

$$\begin{aligned}\therefore v_4 &= \sqrt{v_{u_4}^2 + v_{m_4}^2} \quad \text{... (refer Fig 2.12)} \\ &= \sqrt{(8.3)^2 + (1.5)^2} = 8.45 \text{ m/sec}\end{aligned}$$

$$\tan \alpha_4 = \frac{v_{m_4}}{v_{u_4}} = \frac{1.5}{8.3} = 0.182$$

$$\therefore \alpha_4 = \tan^{-1} (0.182) = 10^\circ - 19'$$

(d) Rise in pressure from point 3 to point 4

$$\frac{p_4 - p_3}{\gamma} = \frac{v_3^2 - v_4^2}{2g} = \frac{(10.7)^2 - (8.45)^2}{2 \times 9.81} = 2.2 \text{ m}$$

#### Answers

(a) Magnitude of velocity of water when it leaves the vortex chamber and enters the volute, i.e., at point 3

$$= 10.7 \text{ m/sec}$$

Direction of velocity of water at point 3

$$= 20^\circ \text{ with the tangent of rotation}$$

(b) Rise in pressure in vortex chamber between points 2 and 3

$$= 5.62 \text{ m}$$

(c) Magnitude of velocity of water at point 4

$$= 8.3 \text{ m/sec}$$

Direction of velocity of water at point 4

$$= 10^\circ - 19' \text{ with the tangent of rotation}$$

(d) Rise in pressure between points 3 and 4

$$= 2.2 \text{ m}$$

#### UNSOLVED PROBLEMS

2.1 Distinguish between rectilinear, radial and rotary (vortex) motions. Differentiate between Free Vortex and Forced Vortex. Cite examples.

- 2.2 Drive 'Equation of Flow' in a curved path, and prove that for rectilinear flow the pressure remains constant transverse to the direction of motion.
- 2.3 Liquid in a deep vessel is stirred. Prove that it forms two types of curves, the forced vortex in the centre and the free vortex on the outside.
- 2.4 What is centrifugal head ? Derive an equation for the same.
- 2.5 Prove that in an open cylinder the depression of vortex below the original level of still water is proportional to the square of angular velocity of rotation.
- 2.6 Explain with the help of a sketch the working principle of a centrifugal pump.
- 2.7 What is spiral vortex ? Give an example to illustrate such a flow.
- 2.8 Prove that in free vortex, the tangential velocity as well as the radial velocity is inversely proportional to  $r$ , the distance from the centre.
- 2.9 Explain the nature of flow in a vortex chamber and volute of a centrifugal pump.

### Numericals

- 2.10 A glass tube 5 cm in diameter, open at the top, containing a liquid, rotates about its axis which is vertical, at 700 rpm. What is the depression of the lowest point of the surface below the surface of liquid when at rest ?

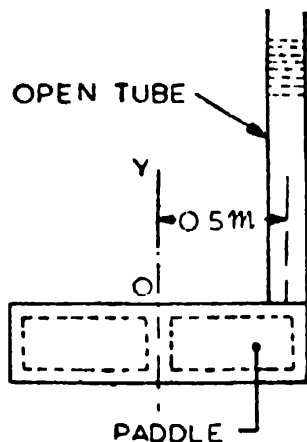


Fig 2.14

- (8.55 cm) (*London University*)
- 2.11 The closed cylindrical container shown in Fig 2.14 is filled with oil having a specific gravity of 0.85. By means of paddles, the oil is rotated so that each particle has an angular velocity of 100 rpm about  $O$ . To what height above  $O$  will the oil rise in the open tube ?
- (1.397 m)
- 2.12 An open cylindrical tank 1 m in diameter and 1.5 m deep is filled with water and rotated about its axis at 60 rpm. How much liquid is spilled and how deep is the water at the axis ?

At what speed should the tank be rotated in order that the centre of the bottom of the tank has zero depth of water ?

(0.2 m<sup>3</sup> ; 1.997 m ; 103.5 rpm)

- 2.13 A closed cylinder, 1 metre in diameter and 2 m high is mounted on a vertical rotating shaft. The cylinder is filled with water upto three-fourth of its height. Show that the ratio of speed to  $y$ , where  $y$  is the distance from the lowest point of the paraboloid to the top of the cylinder, is constant between certain limits of speed. Find out these limiting speeds. (84.5 rpm ; 169 rpm). (*Gujarat University*)
- 2.14 A closed cylindrical vessel with 0.4 metre diameter, is rotated about its axis which is vertical. The vessel is 2 m long and is filled, with

liquid upto a height of 1.5 m. Find the maximum speed in rpm at which the vessel can be rotated so that the bottom of the vessel is wholly covered with the thin layer of the liquid. (422.8 rpm)

- 2.15 A closed cylinder 30 cm diameter and 0.25 cm deep is completely filled with water. It is rotated about its axis which is vertical, at 240 revolutions per minute. Calculate the total pressure of water on each end. (25.6 kg ; 25.78 kg) (*Roorkee University*)
- 2.16 A centrifugal pump has an inner radius of 25 cm and outer radius of 50 cm. It is not fitted with vortex chamber or guide vanes. Determine the speed in rpm at which lifting will commence against a head of 7.5 m. (268 rpm) (*AMIE*)
- 2.17 Calculate the least inner and outer diameters of the impeller of a centrifugal pump to just start delivering water to a height of 30 cm. The inside diameter of impeller is half that of the outside diameter and a manometric efficiency is 0.8. Pump runs at 1,000 rpm. (30 cm ; 60 cm)
- 2.18 Water enters a centrifugal pump impeller with the resultant or absolute velocity in an axial direction. The impeller rotates at 1,500 rpm and has an outer diameter of 45 cm. The relative velocity at exit is radial. The pump efficiency is 85% and the capacity is 85 lit/sec. Neglect losses in the pump shaft bearings. What is the pump head and the useful power input to the fluid ? (64 m ; 85.3 HP)
- 2.19 The external and internal diameters of a centrifugal pump impeller are 30 cm and 16 cm respectively. It discharges 56 lit/sec of water when running at 1,200 rpm. The net areas at inlet and at outlet are  $0.035 \text{ m}^2$  and  $0.0225 \text{ m}^2$  respectively. The angle of discharge  $\beta_1 = 18^\circ$  and angle of entrance  $\alpha_1 = 90^\circ$ . Determine the head generated in the water passing through the impeller.
- If the discharge is reduced to 42 lit/sec and speed is maintained, determine the head created by the impeller. (21.6 m ; 25.6 m)
- 2.20 In a free cylindrical vortex of water, the tangential velocity at a radius of 10 cm from the axis of rotation is found to be 7 m/sec and the intensity of pressure is  $3 \text{ kg/cm}^2$ . Find the intensity of pressure at a radius of 20 cm from the axis. ( $3.187 \text{ kg/cm}^2$ ) (*AMIE*)
- 2.21 A pump impeller is 37.5 cm diameter and it discharges water with component velocities of 2 and 12 m/sec in the radial and tangential directions respectively. The impeller is surrounded by a concentric cylindrical chamber with parallel sides, the outer diameter being 45 cm. If the flow in this chamber is a free spiral vortex, find the component velocities of the water on leaving and increase in pressure if there is no loss. (1.67 m/sec ; 10 m/sec ; 2.29 m of water)
- 2.22 A pump impeller discharges water at a speed of 25 m/sec, the angle of discharge being  $10^\circ$  with the tangent to the periphery. The diameter of the impeller is 0.6 m and width at periphery 10 cm. The impeller is surrounded by a chamber with outer dia of 0.75 m and width at outer side equal to 12 cm. If the flow in the chamber is free spiral, find the radial and tangential velocities of water when it leaves the chamber, and also find pressure increase due to flow in the chamber. (8.9 m/sec, 16.4 m/sec., 17.7 m)



## *Unit and Specific Quantities, Specific Speed*

**3.1 Introduction.** Two similar hydraulic machines designed for different specifications are required to be compared.

It is possible that a hydraulic machine may not give the desired results for which it has been designed. Such a machine is costly to manufacture and once it is made, it is difficult to change its components. Therefore it is required to predict the performance of a prototype hydraulic machine before it is manufactured. This is done by making its model. Experiments are first performed on models and from their results the performance of the prototype machine is predicted. Chapter on "Dimensional and Model Analysis" given in Author's Fluid Mechanics will be useful for the purpose. Here prototype and its model are two similar machines having different specifications which are to be compared. Concept of 'Unit and Specific Quantities' is a prerequisite for the comparison of hydraulic machines.

**3.2 Unit and Specific Quantities**—The rate of flow, speed, power etc., of hydraulic machines are all functions of the working head which is one of the most fundamental of all quantities that go to determine the flow phenomena associated with machines such as turbines and pumps. To facilitate correlation, comparison and use of experimental data, these quantities are usually reduced to unit heads. Each is expressed as a function of head and its value corresponding to a unit value of head is determined. These reduced quantities are known as unit quantities\* *e.g.*, unit flow, unit speed, unit force, unit power and unit torque etc. Thus two similar turbines having different data can be compared by reducing the data of both turbines under unit head.

For similar reasons it is also convenient to use some specific quantities. A specific quantity is obtained by reducing any quantity to a value corresponding to unit head and some unit size. The later dimension is the inlet diameter of runner in case of reaction turbines and least jet diameter in Pelton turbines. When two different turbines are to be compared, it can be done by reducing their data to specific quantities.

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\*This should not be confused with numerically unit quantities. Thus one  $\text{m}^3/\text{sec}$  is not unit flow in this sense of the term nor is one rpm a unit speed. Following pages will make it clear.

Thus specific flow is the rate of flow corresponding to unit head and unit diameter. Similarly specific power is the power corresponding to a unit head and unit diameter.

The term '*specific*' is, however, used in a slightly different sense in connection with speed. The specific speed of a turbine is defined as the speed of a geometrically similar turbine working under a unit head and developing unit power. The specific speed of a pump is the speed of a geometrically similar pump working against a unit head and raising unit quantity of water.

The concept of specific speed is very important in the study of turbines and pumps. It is the modern basis of scientific classification of turbines and pumps. (refer Chapters 4 and 12 on Hydro Electric Plants and Centrifugal Pumps). It does not follow that a given machine can actually operate with unit and specific quantities.

For instance it would be more precise to say that unit flow is the flow in a geometrically similar machine working under unit head, and the same applies to other unit and specific quantities.

The condition of similarity is essential to justify the assumption that the following co-efficients  $k_1, k_2, k_3, \dots$  etc., are all constants.

Expressions† will now be derived for unit and specific quantities.

### 3.3 Unit Quantities—

#### (a) Unit Rate of Flow—

Rate of flow = cross-sectional area  $\times$  velocity of flow

Symbolically,  $Q \propto v_{m_0}$

$$\text{But } v_{m_0} = K_{v_{m_0}} \cdot \sqrt{2g \cdot H}$$

Where  $H$  is the head and  $K_{v_{m_0}}$  some velocity co-efficient.

$$\therefore Q \propto \sqrt{H}$$

$$\text{or } Q = k_1 \cdot \sqrt{H}$$

Now by definition unit rate of flow  $Q_1$  is the value of  $Q$  when  $H = 1$

$$\therefore Q_1 = k_1 \cdot \sqrt{1} = k_1$$

i.e.,  $Q_1$  is numerically equal to  $k_1$

$\therefore$  Numerically, unit rate of flow

$$Q_1 = \frac{Q}{\sqrt{H}} \quad \dots(3.1)$$

#### (b) Unit Speed—

Let  $N$  rpm be the speed of the turbine then linear or peripheral velocity of runner at inlet,

$$u_1 = \frac{\pi \cdot D_1 \cdot N}{60}$$

†The following expression have been derived for turbines but those for flow, speed and power hold good for pumps also.

Also  $u_1 = K_{u_1} \sqrt{2gH}$

$\therefore N \propto u_1 \propto \sqrt{H}$

or  $N = k_2 \cdot \sqrt{H}$ ,

where  $k_2$  is some co-efficient.

Now, by definition, unit speed

$$N_1 = k_2 \sqrt{1} = k_2$$

$\therefore$  Unit speed is numerically equal to  $k_2$

or numerically,

$$N_1 = k_2 = \frac{N}{\sqrt{H}} \quad \dots(3.2)$$

(c) **Unit Power—**

The available horse-power of a turbine

$$P_a = \frac{\gamma \cdot Q \cdot H}{75} \text{ HP}$$

and brake horse-power

$$P_b = \eta_t \frac{\gamma \cdot Q \cdot H}{75}$$

...(where  $\eta_t$  = turbine overall efficiency)

or in general, horse-power

$$P \propto Q \cdot H$$

But  $Q \propto \sqrt{H}$

$\therefore P \propto H \cdot \sqrt{H}$

or  $P = k_3 \cdot H^{\frac{3}{2}}$ ,

where  $k_3$  is some co-efficient.

Now, by definition, unit power

$$P_1 = k_3 \cdot (1)^{\frac{3}{2}} = k_3$$

or numerically,

$$P_1 = k_3 = \frac{P}{H^{\frac{3}{2}}} \quad \dots(3.3)$$

(d) **Unit Force—**

The force exerted by jet on Pelton runner at its periphery is given by

$$F = \frac{\gamma \cdot Q}{g} (v_{u_1} - v_{u_2})$$

$$\text{i.e., } F \propto Q \cdot v_u$$

$$\text{But } Q \propto \sqrt{H}$$

$$\text{and } v_u \propto \sqrt{H}$$

$$\therefore F \propto H$$

$$\text{or } F = k_4 \cdot H$$

Now, by definition, unit force on periphery of runner,

$$F_1 = k_4 \cdot 1 = k_4$$

$\therefore$  Numerically,

$$F_1 = k_4 = \frac{F}{H} \quad \dots(3.4)$$

(e) **Unit Torque—**

Torque or turning moment on runner = force at periphery  $\times$  radius.

Symbolically,

$$T = F \cdot R \quad \text{or} \quad T \propto F$$

$$\text{But } F \propto H$$

$$\therefore T \propto H$$

$$\text{or } T = k_5 \cdot H$$

Now by definition, unit torque

$$T_1 = k_5 \cdot 1 = k_5$$

$\therefore$  Numerically,

$$T_1 = k_5 = \frac{T}{H} \quad \dots(3.5)$$

### 3.4 Specific Quantities—

#### (a) **Specific Rate of Flow or Specific Flow—**

For a reaction turbine,

$$Q = (\pi \cdot D_o \cdot B_o) \cdot v_{m_o}$$

The dimensions  $B_o$  and  $D_o$  generally have linear relations with  $D_1$  the runner diameter at inlet, and, therefore, since

$$v_{m_o} \propto \sqrt{H}$$

$$Q \propto D_1^3 \cdot \sqrt{H}$$

$$\text{or } Q = k_6 \cdot D_1^3 \cdot \sqrt{H}$$

Now, by definition, specific rate of flow

$$Q_{11} = k_6 \cdot 1^3 \times \sqrt{1}$$

Numerically,

$$Q_{11} = k_6 = \frac{Q}{D_1^3 \cdot \sqrt{H}} \quad \dots(3.6)$$

For a Pelton turbine,

$$Q = \frac{\pi}{4} \cdot d_1^3 \cdot v_1 \quad \text{i.e.,} \quad Q \propto d_1^3 \cdot \sqrt{H}$$

where  $d_1$  = the least diameter of water jet falling on turbine runner

$$\therefore Q_{11} = \frac{Q}{d_1^3 \cdot \sqrt{H}} \quad \dots(3.6a)$$

(b) **Specific Power—**

Power,  $P \propto Q \cdot H$

Since  $Q \propto D_1^3 \cdot \sqrt{H}$  for a reaction turbine

$$\therefore P \propto D_1^3 \cdot H^{\frac{3}{2}}$$

$$\text{or } P = k_7 \cdot D_1^3 \cdot H^{\frac{3}{2}}$$

Now, by definition, the specific power

$$P_{11} = k_7 \cdot 1^3 \times 1^{\frac{3}{2}}$$

Numerically,

$$P_{11} = k_7 = \frac{P}{D_1^3 \cdot H^{\frac{3}{2}}} \quad \dots(3.7)$$

Similarly for a Pelton Turbine,

$$P_{11} = \frac{P}{d_1^3 \cdot H^{\frac{3}{2}}} \quad \dots(3.7a)$$

(c) **Specific Force of Jet on Periphery of Runner—**

$$F = \rho \cdot Q (v_{u1} - v_{u2})$$

$$\text{or } F \propto Q \cdot v_u$$

$$\text{But } Q \propto d_1^3 \cdot \sqrt{H} \text{ and } v_u \propto \sqrt{H}$$

$$\therefore F \propto d_1^3 \cdot H \quad \text{or} \quad F = k_8 \cdot d_1^3 \cdot H$$

$\therefore$  By definition, the specific force

$$F_{11} = k_8 \cdot 1^3 \times 1$$

or numerically,

$$F_{11} = k_8 = \frac{F}{d_1^3 \cdot H} \quad \dots(3.8)$$

(d) **Specific Torque—**

Torque = Peripheral Force  $\times$  Radius of Runner.

$$\text{Symbolically, } T \propto F$$

$$\text{or } T \propto d_1^3 \cdot H$$

$$\text{or } T = k_9 \cdot d_1^3 \cdot H$$

By definition, the specific torque,

$$T_{11} = k_9 \cdot 1^3 \times 1$$

$$\text{Numerically, } T_{11} = k_9 = \frac{T}{d_1^3 \cdot H} \quad \dots(3.9)$$

Alternatively,  $T = \frac{P}{\omega}$

and since  $\omega$  is the angular velocity and is proportional to  $\sqrt{H}$

and  $P \propto D_1^2 \cdot H^{\frac{3}{2}}$

$$\therefore T \propto \frac{D_1^2 \cdot H^{\frac{3}{2}}}{H^{\frac{1}{2}}} \quad \text{or} \quad T \propto D_1^2 \cdot H$$

$$\therefore \text{Specific torque, } T_{11} = \frac{T}{D_1^2 \cdot H}$$

**3.5 Specific Speed of a Turbine**—Unlike other specific quantities, specific speed is the speed of a geometrically similar turbine working under unit head and delivering unit brake horse-power.

It has been shown previously that

$$u_1 = \pi \cdot D_1 \cdot N$$

and  $u_1 \propto \sqrt{H}$

$$\therefore D_1 \propto \frac{\sqrt{H}}{N}$$

Also it has been shown earlier that brake horse-power

$$P_t \propto Q \cdot H,$$

where  $Q \propto D_1^2 \cdot \sqrt{H}$

$$\therefore P_t \propto D_1^2 \cdot H^{\frac{3}{2}}$$

Substituting for  $D_1$ ,

$$P_t \propto \frac{H}{N^2} \cdot H^{\frac{3}{2}} \quad \text{i.e., } P_t \propto \frac{H^{\frac{5}{2}}}{N^2}$$

$$\text{or } N \propto \sqrt{\frac{H^{\frac{5}{2}}}{P_t}} \quad \text{or } N = N_s \cdot \frac{H^{\frac{5}{4}}}{\sqrt{P_t}}$$

$$\text{where } N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} \quad \dots(3.10)$$

If  $P_t = 1$  and  $H = 1$ , then numerically  $N_s = N$

$N_s$  is, therefore, by definition, the specific speed of turbine.

**Problem 3.1** Each of the two Francis turbines installed at Talaiya Power House (D. V. C.) works under a maximum head of 23.4 m producing 2,800 HP at 250 rpm. Determine :

- the HP and rpm of this turbine under a head of one metre,
  - the rpm of this turbine under one metre head and producing one HP,
- and (c) the HP of this turbine working under one metre head and running at 1 rpm.

**Solution**

$$H = 23.4 \text{ m}; \quad P_t = 2,800 \text{ HP}; \quad N = 250 \text{ rpm}$$

$$(a) (i) \frac{P_t}{P_t'} = \left( \frac{H}{H'} \right)^{\frac{3}{2}} \quad \dots (H' = 1 \text{ m})$$

$$\therefore P_t' = P_t \cdot \left( \frac{H'}{H} \right)^{\frac{3}{2}} = 2,800 \times \left( \frac{1}{23.4} \right)^{\frac{3}{2}} = 25.2 \text{ HP Answer}$$

$$(ii) \frac{N}{N'} = \left( \frac{H}{H'} \right)^{\frac{1}{2}} \quad \dots (H' = 1 \text{ m})$$

$$\text{or } N' = N \cdot \left( \frac{H'}{H} \right)^{\frac{1}{2}} = 250 \times \left( \frac{1}{23.4} \right)^{\frac{1}{2}} = 51.6 \text{ rpm Answer}$$

$$(b) N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{250 \times \sqrt{2,800}}{(23.4)^{\frac{5}{4}}} = 256 \text{ rpm Answer}$$

(c)  $N_s$  must be same—

$$P_t = \left( N_s \cdot \frac{H^{\frac{5}{4}}}{N} \right)^2 = \left( \frac{256 \times 1^{\frac{5}{4}}}{1} \right)^2 = 65,500 \text{ HP Answer}$$

**3.6 Specific Speed of a Pump**—Specific speed of a pump is the speed of a geometrically similar pump delivering a unit quantity of water against a unit head.

For pumps also  $u_1 = \pi D_1 N$  and  $u_1 \propto \sqrt{H}$

$$\therefore D_1 \propto \frac{\sqrt{H}}{N}$$

Further  $Q \propto D_1^2 \cdot H^{\frac{1}{2}}$

Substituting for  $D_1$ ,

$$Q \propto \frac{H}{N^2} \cdot H^{\frac{1}{2}}$$

$$\text{or } N \propto \sqrt{\frac{H^{\frac{3}{2}}}{Q}} \quad \text{or } N = N_s \cdot \frac{H^{\frac{3}{4}}}{\sqrt{Q}}$$

$$\therefore N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}} \quad \dots (3.11)$$

When  $Q = 1$ , and  $H = 1$ , then numerically,  $N_s = N$

$\therefore N_s$  is, by definition, the specific speed of a pump.

**UNSOLVED PROBLEMS**

3.1 What are “Unit Quantity” and “Specific Quantity” ?

3.2 Explain (a) Unit speed, (b) Unit discharge and (c) Unit power of a hydraulic turbine. Derive expressions for each of them.

(Jadavpur University)

3.3 Explain (a) Specific flow, (b) Specific power and (c) Specific torque of a hydraulic turbine. Derive expressions for each of them.

- 3.4 What is specific speed and how does it differ in definition from the other specific quantities ?
- 3.5 Deduce an expression for the specific speed of a hydraulic turbine and explain how it is useful in practice.
- 3.6 Deduce an expression for the specific speed of a pump. How does it differ from that of a turbine ?

### Numericals

- 3.7 A 1 : 15 scale model of a propeller turbine 5 m diameter is found to develop 3.25 HP with a flow of  $0.3 \text{ m}^3/\text{sec}$  under a head of 1.15 m while rotating at 500 rpm. Calculate the corresponding HP, flow, head and speed of prototype, neglecting frictional effect.  
(17.25 m ; 129 rpm ;  $261 \text{ m}^3/\text{sec}$  ; 42,200 HP)
- 3.8 Derive an expression for the specific speed of a hydraulic turbine.

A turbine develops 5,000 horse-power under a head of 110 m at 100 rpm. What is its specific speed ? What would be its normal speed and output under a head of 81 m ?

(19.85 ; 68.2 rpm ; 2,330 HP) (*Gujarat University*)



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## *Hydro Electric Plants*

**4.1 Introduction**—The purpose of a Hydro-Electric Plant is to harness power from water flowing under pressure. As such it incorporates a number of water driven prime-movers known as Water Turbines.

Water flowing under pressure has two forms of energy—kinetic and potential. The kinetic energy depends on the mass of water flowing and its velocity while the potential energy exists as a result of the difference in water level between two points which is known as “head”. The water or hydraulic turbine, as it is sometimes named, converts the kinetic and potential energies possessed by water into mechanical power. The hydraulic turbine is, thus, a prime mover which when coupled to a generator produces electric power. The projects designed to produce electric power from water are known as “Hydro-Electric Projects”. Hydro-Electric Projects may not be used exclusively for power generation. Sometimes, they are the off-shoot of flood control and irrigation projects in which case they are known as “Multipurpose Projects”, for example, Bhakra Nangal and Damodar Valley Corporation (D.V.C.) projects. The former is useful mainly for power and irrigation whereas the latter is primarily concerned with flood control.

Hydro-electric power can be developed wherever, water continuously flowing under pressure is available. Dams constructed across flowing rivers divert the riverine bounty through the turbines giving rise to such useful power. But this is not all. Water that collects in natural or artificial lakes in the high and huge mountains, due to the melting of snow and due to heavy monsoon rain, can be led down to the turbines through large pipes known as *penstocks*.

The table 4.1 gives the total energy consumption and the resources of hydro electric power of some of the important countries of the world.

It shows that India has developed only 14.4% of its water power potential upto March 1969 as per information supplied by Central Water and Power Commission, Government of India.

**4.2 Heads and Rate of Flow or Discharge**—Head is the difference in elevation between two levels of water. The head of a hydro-electric plant is entirely dependent on the topographical conditions. Head can be characterised as :

- (a) gross head, and
- (b) net or effective head.

TABLE 4.1

**Total Energy Consumption & Hydro-Electric Resources**

| Country  | Energy Consumption        |                                                     |                                      |                             | Hydro-Electric Resources    |                             |
|----------|---------------------------|-----------------------------------------------------|--------------------------------------|-----------------------------|-----------------------------|-----------------------------|
|          | Population<br>$\div 10^6$ | Total<br>Consumption<br>$\div 10^6$ kwh<br>per year | Electric Consumption                 |                             | Potential<br>$\div 10^6$ KW | Developed<br>$\div 10^6$ KW |
|          |                           |                                                     | Total<br>$\div 10^6$ kwh<br>per year | kwh<br>per head<br>per year |                             |                             |
| U.S.A.   | 178                       | 8,800,000                                           | 544,000                              | 3,060                       | 100                         | 18                          |
| U.S.S.R. | 220                       | 4,400,000                                           | 166,000                              | 755                         | 60                          | 1.7                         |
| U.K.     | 52.7                      | 1,750,000                                           | 64,000                               | 1,215                       | 0.5                         | 0.27                        |
| France   | 45                        | 550,000                                             | 41,000                               | 912                         | 4.5                         | 4.5                         |
| Norway   | 3.6                       | 100,000                                             | 22,000                               | 6,125                       | 7.5                         | 2.9                         |
| India    | 537                       | 1,200,000                                           | 9,100                                | 17                          | 41                          | 5.91                        |

(a) **Gross Head** is defined as the difference in elevation between the head race level at the intake and the tail race level at the discharge side, naturally, both the elevations have to be measured simultaneously. The gross head may vary as both the elevations of water do not remain the same at all times. It is essential to know the maximum and minimum as well as the normal values of the gross head. The normal value would be that for which the plant works most of the time. In rainy season the flood may raise the elevation of tail race, thus, reducing the gross head. On the other hand at the time of draught the same may be increased.

(b) **Net or Effective Head** is the head obtained by subtracting from gross head all the losses in carrying water from the head race to the entrance of the turbine. The losses are due to friction occurring in tunnels, canals and penstocks which lead the water into the turbine. Net or effective head is, therefore, the true pressure difference between the entrance to the turbine casing and the tail race water elevation. The measurement of the pressure head at the entrance to the turbine casing must be corrected for the velocity head at that point, and the tail-water elevation corrected for residual velocity head.

**Rate of Flow or Discharge** of water is the quantity of water used by the water turbine in unit time and is, generally, measured in  $\text{m}^3/\text{sec}$  (i.e. cumec) or lit/sec.

The maximum, minimum and the normal rates of flow and flow distribution throughout the various seasons ought to be known.

#### 4.3 Essential Components of Hydro-Electric Power Plant—

- Storage reservoir,
- Dam with its control works,
- Waterways with their control works,
- Power house with turbo and other machinery,
- Tail race, and
- Generation and transmission of electric power.

(a) **Storage Reservoir**—The water available from a catchment area is stored in a reservoir, so that it can be utilised to run the turbines for

producing electric power according to the requirements throughout the year. The storage reservoir may be natural or artificial.

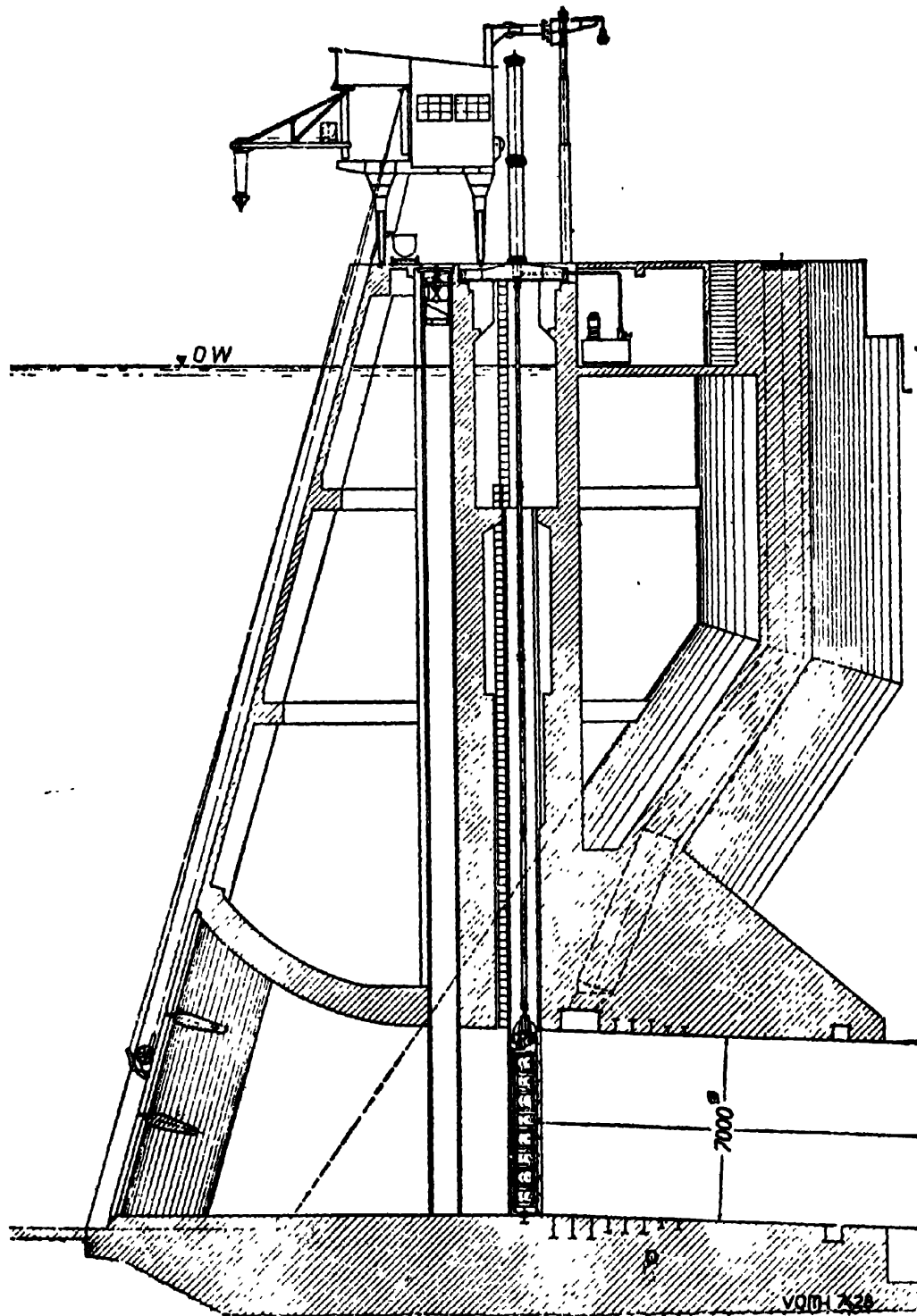


Fig 4.1 Section of Dam of Rio Negro Power Station Uruguay, equipped with 4 roller gates each  $7 \times 7$  m clear opening, water depth 35 m, closing time 30 sec, manufactured by Voith, W. Germany. When head race water level is adequately lowered the gate leaf can be shifted to unloading chamber below servomotor by means of hydraulic drive.

Trash Rack Cleaning Machine working is also shown. The cleaner moves on the rack bars while the machine carrying the cleaner moves on rails fitted on the top of the dam. This machine is also fitted with stop log (which obstruct the water passage) lifting arrangement (i.e., crane arrangement).

*Natural reservoir* or lake may be found in high and huge mountains. Water is taken off from one end of the reservoir through a tunnel built by cutting the mountain.

*Artificial reservoir* is made by constructing dam on one or two openings of a valley in order that considerable amount of water is stored.

(b) **Dam with its Control Works**—Dam is a structure erected on suitable site to provide for the storage of water and to create head. Dam may be built to make an artificial reservoir from a valley or it may be erected in a river to control the flowing water. Dam may be of different types such as earth dam, masonry dam, wooden dam or steel dam. Masonry dams are very common. Plate 3 shows the dam for Bratsk Hydro electric Project in Eastern Siberia USSR claimed to be the largest in the world.

Structures and appliances to control the supply of water from the storage reservoir through the dam, are known as *Control Works or Head Works*. The principal elements of control works are :

- (i) *gates and valves* of various types and their operating mechanisms ;
- (ii) structures necessary for their operation ;
- and (iii) devices for the protection of gates and hydraulic machines.

The different types of gates are—

Plain sliding gates, wheeled gates, (refer Fig 4.1 and Plate 4), sluice gates, crest gates (i.e., flush boards or shutters), tainter gates, (refer Plate 5), rolling or drum gates, radial gates etc.

The different types of valves are butterfly and needle valves.

The devices for the protection of gates and hydraulic machines consist of :

- (i) *Trashracks* (refer Fig 4.1 to 4.7)—They are made up of a row of rectangular cross sectional structural steel bars placed across the entire intake opening in an inclined position. They are used to obstruct debris from going into the intake.
- (ii) *Debris cleaning device* fitted on the trashrack (refer Fig 4.1).
- (iii) *Heating element* against ice troubles.

(c) **Waterways with their Control Works**—Waterway is a passage through which the water is carried from the storage reservoir to the power house. It may consist of tunnels, canals, flumes, forebays, pipes (i.e., penstocks). *Tunnel* (refer Fig 4.2) is a water passage made by cutting the mountain to save distance. *Forebay* (refer Fig 4.5) is an enlarged section of a canal spread out to accommodate the required width of intake.

Its function is to store temporarily, the water rejected by the plant when its load is reduced and to meet the instantaneous increased demand of water due to sudden increase in load.

*Penstocks* are pipes of large diameter carrying water under pressure from the storage reservoir to the turbine. Penstocks are subjected to water hammer pressure due to fluctuation of turbine load. For long penstocks the water hammer effect is reduced by providing surge tank.

The control works for the tunnels, canals, flumes, forebays and pipes, may be different types of gates and valves described above. In addition to these, *surge tank* is used where water is carried through penstocks. A surge tank (refer Fig 4.2) is a reservoir fitted at some opening made on a

long pipe line to receive the rejected flow when the pipeline is suddenly closed by a valve at its steep end. The surge tank, therefore, controls the pressure variations resulting from the rapid changes in pipe line flow thus eliminating water hammer effects. For further details, refer Chapter 8.

(d) **Power House** is a building to house the turbines, generators and other accessories for operating the machines.

(e) **Tail Race** is a waterway to conduct the water discharged from the turbines to a suitable point where it can be safely disposed of or stored to be pumped back into the original reservoir.

(f) **Generation and Transmission of Electric Power**—It consists of electrical generating machines, transformers, switching equipments and transmission lines.

**4.4 Classification of Water Power Plants**—Water power development schemes or *hydro-electric plants* or *hydel plants* may be classified according to the following three basis—

(a) The *head* under which they work—High head, low head and medium head plants.

(b) Their *functions*—Run off river plants, storage plants and pumped storage plants.

(c) The *load capacity*—Base load plants and peak load plants.

(d) The *source of energy*—River (or reservoir) power plants and tidal power plants.

All the above plants under *a*, *b* and *c* are river (or reservoir) power plants. Though tidal power plant is a water power plant, but it exploits energy of earth's rotation.

**4.5 High Head Water Plants**—Such plants work under heads ranging from 25 to 2,000 metres. Water is usually stored up in lakes on high mountains during the rainy season or during the season when the

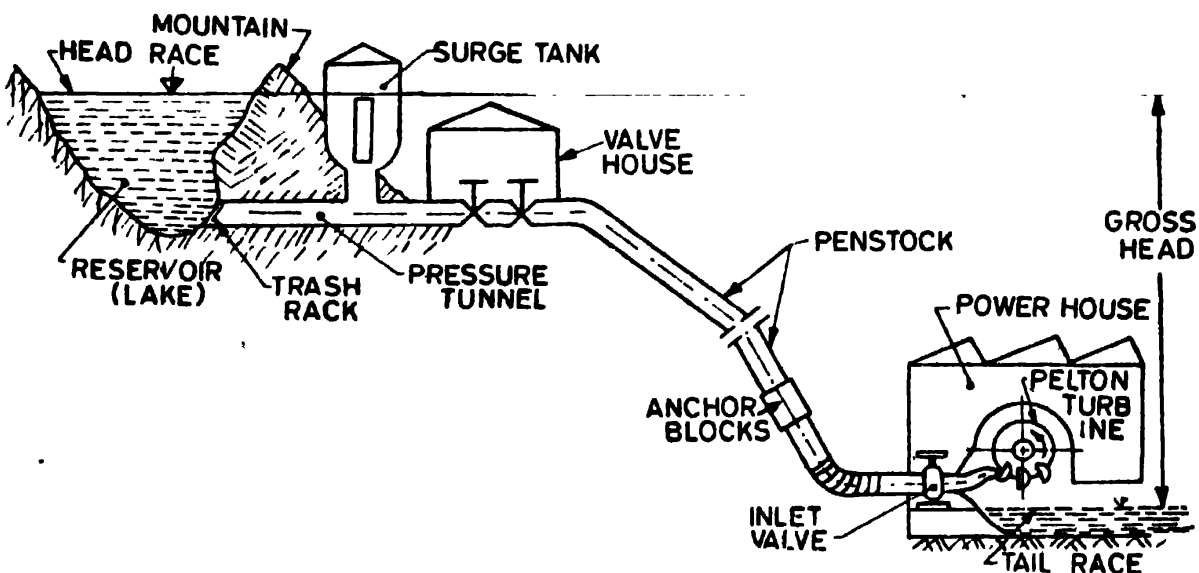


Fig 4.2 High Head Water Power Plant Layout

snow melts. The rate of flow should be such that water can last throughout the year. From one end of the lake, tunnels are constructed which lead the water into smaller reservoirs known as *forebays*. The forebays distribute the water to penstocks through which it is lead to the turbines. These forebays help to regulate the demand of water according to the load.

on the turbines. In cases where it is not possible to construct forebays, *surge tanks* are built. When the load on the turbines decreases, the excess quantity of water surges into the surge tank preventing a sudden pressure rise. Surge tank occupies much less space than a forebay. Fig 4.2 illustrates water flowing through the tunnel to the surge tank. Situated near the surge tank is a *Valve House* housing electrically driven sluice type

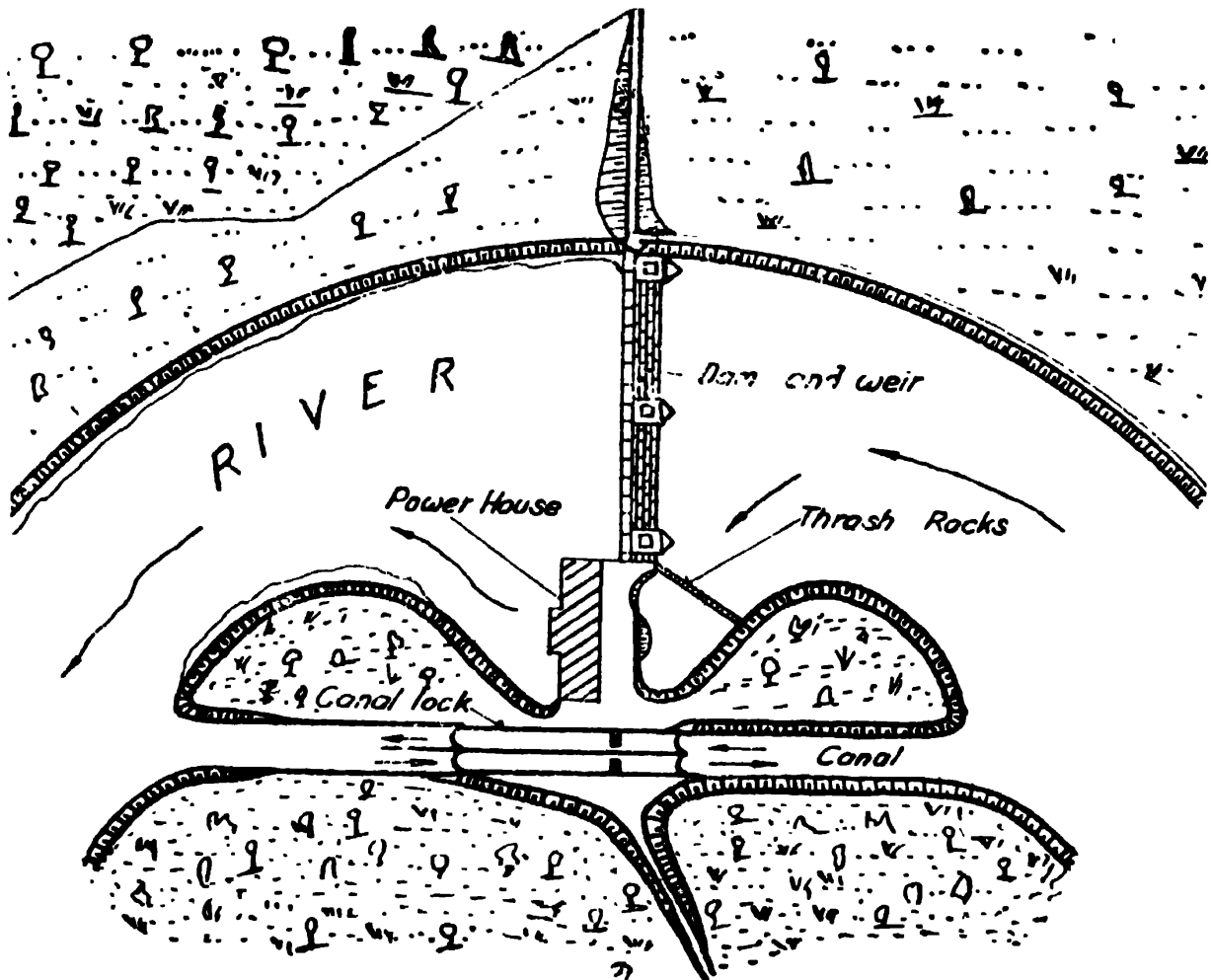


Fig 4.3 Low Head Water Power plant Layout

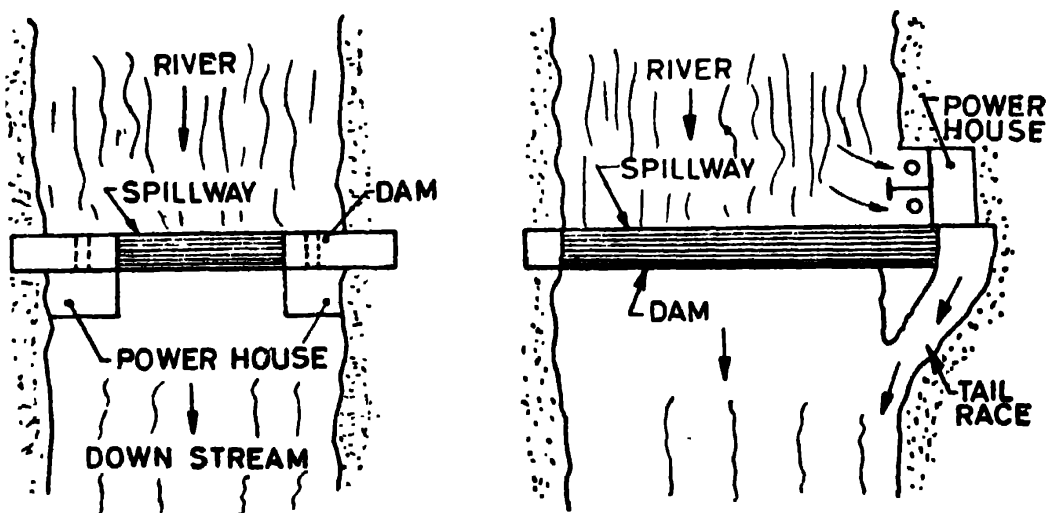


Fig 4.4 a & b Low Head Water Power Plant Layouts

valves meant for the control of flow in the penstocks. The valves also help to close the penstock and, thus, facilitate cleaning and repairing.

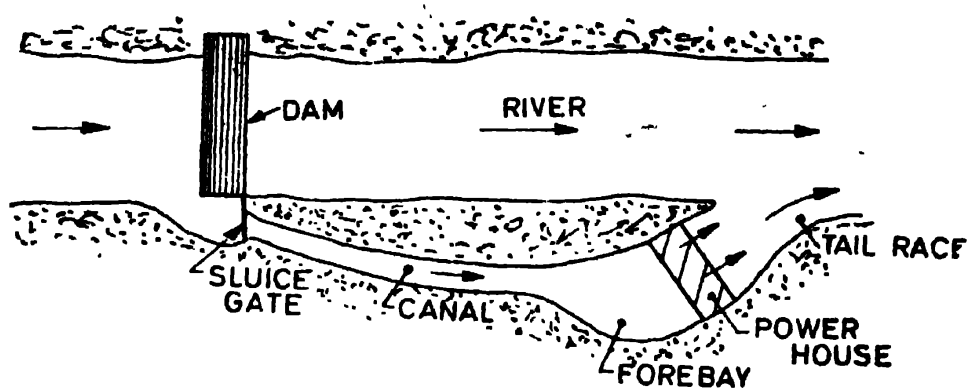


Fig 4.4 c Low Head Water Power Plant Layout

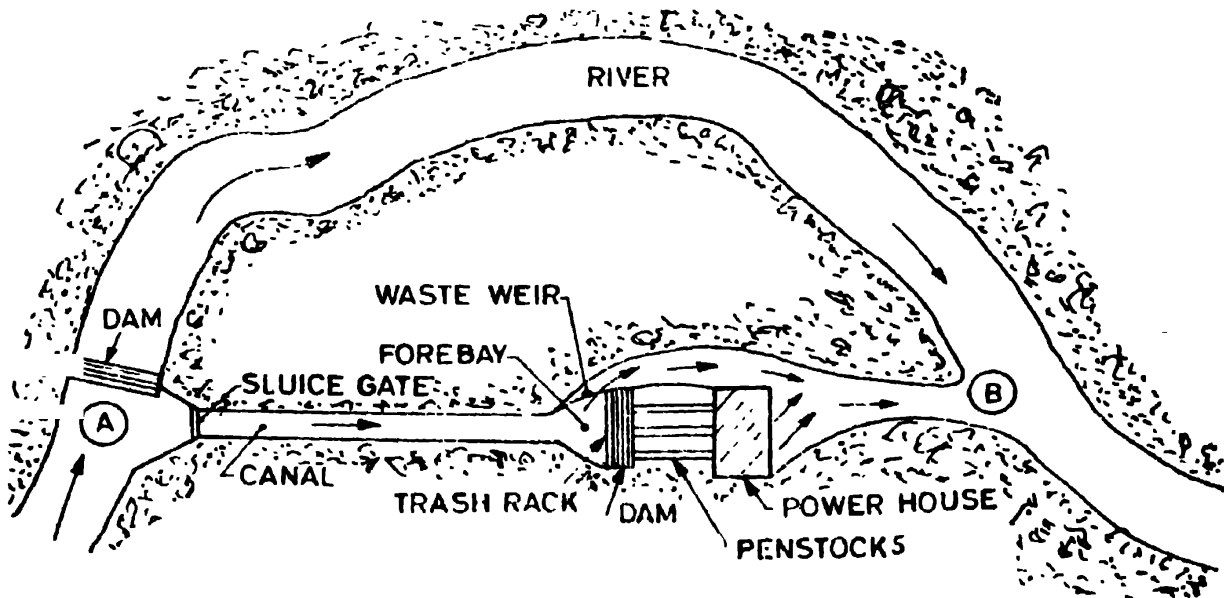


Fig 4.5 Canal Water Power Plant Layout

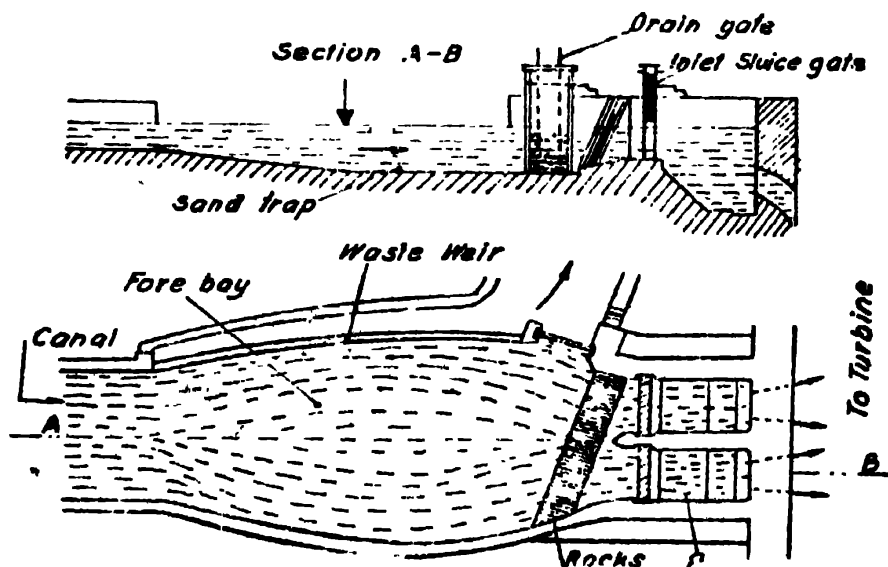


Fig 4.6 Details of Low Head Water Plant, Canal Type

The total distance from the water source to the turbine inlet may, sometimes, be even 15 to 20 km. The water is discharged after passing through the turbines into the tail race from where it can be taken for irrigation. It is sometimes fed to fish ponds.

**4.6 Low Head Water Power Plants** work upto 80 metres of head. These plants usually consist of a dam across a river. A sideways stream diverges from the river at the dam. Over this stream the power house is constructed. Later this channel joins the river further downstream. Fig 4.3 illustrates a typical low head station together with a canal-lock for easy navigation of ships from one side of the dam to the other. Fig 4.4 shows three different layouts for such a type of plant.

**4.7 Canal Water Power Plant**—These also fall under low head plants. If there is a river having a bend (refer Fig 4.5) whose level *A* is higher than at *B*, a straight canal joining *A* and *B* is constructed. A dam near *A* diverts the water into the channel. The main sluice gates are erected near *A* to control the rate of flow of water into the canal as well as the forebay. The power house is shown on the right hand side of the forebay. On leaving the turbines, installed in the power house, the water is lead back into the river by the canal at *B*. The river water generally contains floating or suspended matter like grass, leaves, or small twigs which may bring the turbine to a standstill if permitted to pass through. To separate such foreign bodies in the water, it is passed through the trashracks. Fig 4.6 shows the details of the plant of Fig 4.5.

When the turbine load decreases and there is less demand for water by the turbines, the excess flow instead of swelling the canal is diverted over the *waste weir* into the river, thus by-passing the turbines. A drain gate on the side of waste weir facilitates cleaning and repairing of canal.

**4.8 Medium Head Water Power Plant**—If the head available in low head plant is above 25 metres a penstock joining the forebay to the turbine inlet is built as shown in Fig 4.7. This plant would now be known as a medium head plant, which may also include a high head plant working under heads less than 300 metres.

The following figures give a rough idea of the heads under which the various types of plants work :—

- (a) High head power plants—100 m and above,
- (b) Medium head power plants—30 to 500 m,
- (c) Low head power plants—25 to 80 m.

It is to be noted that the figures given above overlap each other. Therefore, it is difficult to classify the plants directly on the basis of head alone. The basis, therefore, technically adopted is the *specific speed* of the turbine used for a particular plant, as explained later.

#### **4.9 Run-off-River, Storage and Pumped Storage Plants—**

(a) **Run-off-river plants**—Essentially run-off-river plant is one which does not store any water and uses the water as it flows. Thus water is not stored during flood periods and also when the turbines use less water due to low loads with the result the water is wasted. The utility of such a plant is limited.



Run-off-river plants may be without water pondage or with water pondage. The plants with pondage are provided with a weir or barrage to accommodate sufficient storage to take care of load fluctuations.

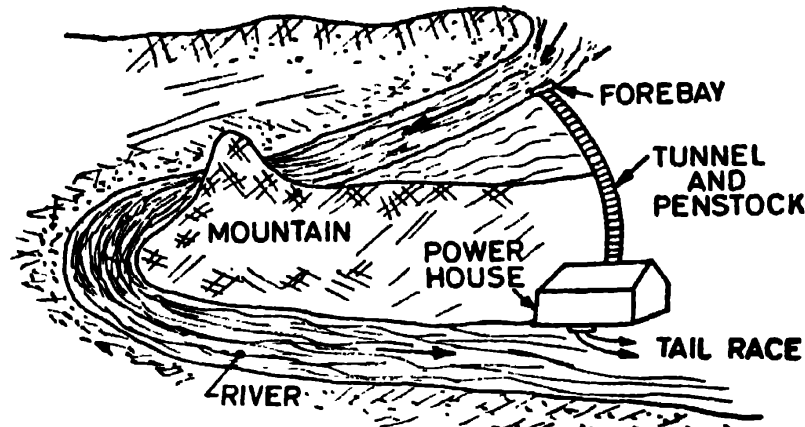


Fig 4.7 (a) Medium Head Water Plant Layout

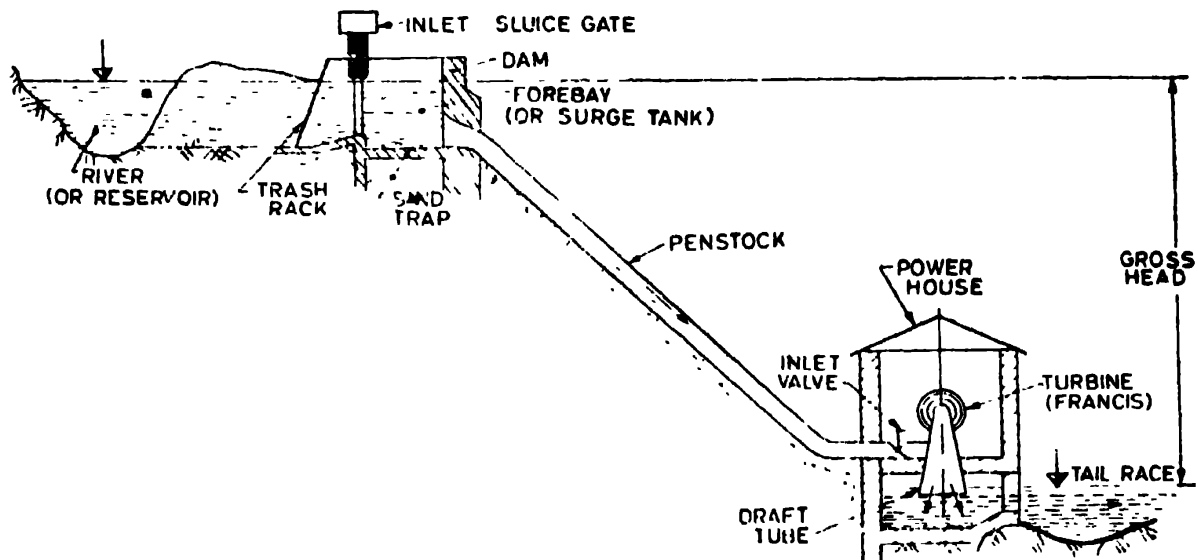


Fig 4.7 (b) Medium Head Water Plant Layout

(b) **Storage Plants** have storage reservoir provided by constructing a dam across the river if the plant is low head or by closing the opening of a valley by a dam in case of a high head plant. The storage is collection of water which takes care of fluctuations of water supply during flood and draught periods, as well as load fluctuations, thus increasing the utility of plant. Such a plant is also useful as peak load plant.

(c) **Pumped Storage Plant** caters for the peak load of the power station. It stores power by pumping back a portion of water from the tail race back to the reservoir during off-peak hours, when excess power can be available for this purpose. The pumping is done by the same turbine-generator (producing power) which now acts as a pump-motor set. In certain cases where it is not possible to use turbine as pump, a separate pump is coupled to the same shaft. The stored water is utilised during peak load periods on demand. Such plants are built where there is a continuous shortage of water. The details of such plants are given in chapter 10.

#### 4.10 Base-load and Peak-load Plants

(a) **Base load Plants** are those which cater for the base load of the system. Such plants are required to supply a constant power when connect-

ed to the grid. Thus they run without stop and are often remote-controlled with which least staff is required for such plants. Run-off-river plants without pondage may sometimes work as base load plants, but the firm capacity in such cases will be very much less.

(b) **Peak-load Plants** are those which can supply the power during peak loads. Some of such plants supply the power during average load but also supply peak load as and when it is there; whereas other peak load plants are required to work during peak load hours only. The run-off-river plants may be made for the peak load by providing pondage.

**4.11 Tidal Power Plant**—It taps a new source of energy viz, energy of the earth's rotation as distinct from other energy sources i.e., river, thermal, wind, solar and atomic.

Tides flood vast coastal areas and raise the sea level along the sea shores by amounts varying from a few centimetres to several metres. The highest tide anywhere on earth has been 19.6 m in the Bay of Fundy at Port Moncton (New Brunswick, Canada). In Bay of Bengal the height of tide has been upto 9.3 m and in Bombay 9 m. The range of tide depends on the position of the point on the globe, shape of shore line and depth of ocean. The tidal current penetrates very far up in some rivers, maximum so far has been 1,400 km up the Amazon River.

Though we live to-day in atomic age, still engineers are thinking again to harness tidal power, generally known as *blue coal* inspite of various difficulties. It had been proved most uneconomical to get power out of tidal energy and all the projects undertaken before World War II were abandoned. With the evolution of pump-turbine units as well as tubular and bulb turbines (refer Chapter 7), it has been seen recently that it is worth-exploiting tidal energy also. In France 240,000 kw Rance Tidal Power Project has been completed and started. In other countries like USSR, USA and UK, the construction work is in progress. One billion kw of tidal power is dissipated by friction and eddies alone which is slightly less than the power potentials of all the rivers of the world.

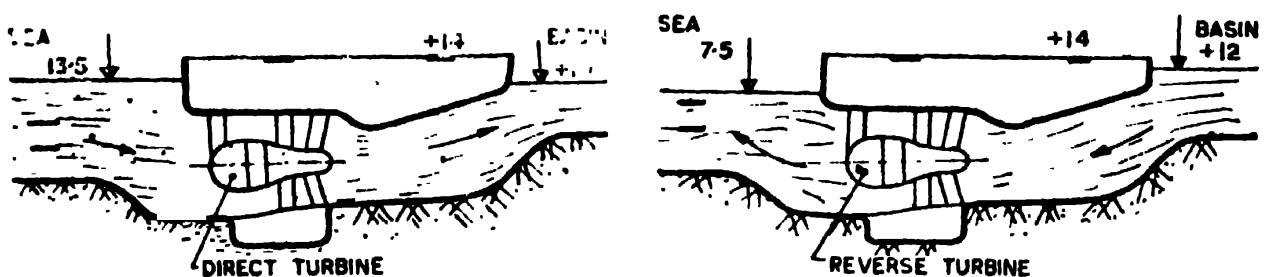


Fig 4.8 (a) and (b) Tidal Power Plant Layouts

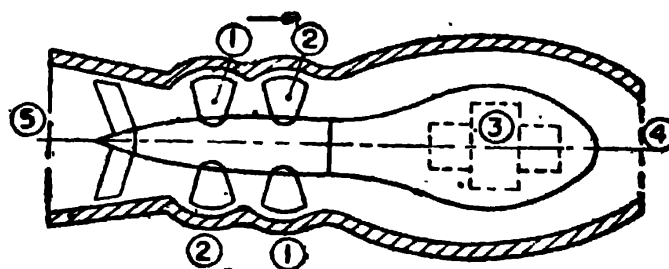


Fig 4.8 (c) Section Through Bulb Turbine Used For Tidal Power Plant

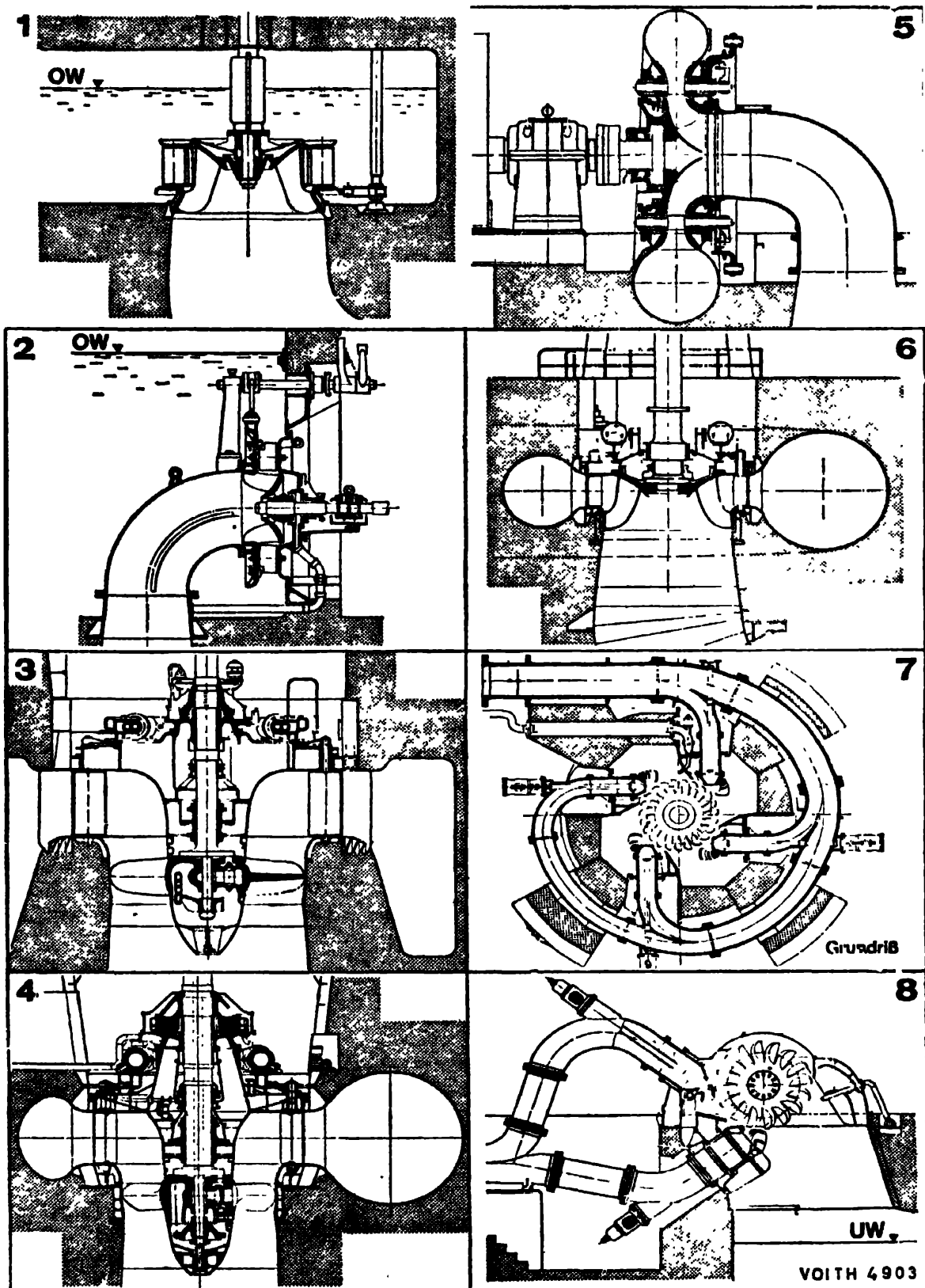


**Fig 4.9 Type of Construction of Water Turbines**

**Manufactures : J.M. Voith, GMBH, West Germany**

- 1 & 2** Open Flume Francis Turbines, Vertical and Horizontal types respectively ;  
**5 & 6** Closed (Spiral casing). Francis Turbines, Horizontal and Vertical types respectively ;  
**3 & 4** Kaplan Turbines with Concrete Built-in Casing and Steel Spiral Casing respectively ;  
**7 & 8** Pelton Turbines,  
**Plan of Four Jet / Vertical Type, and Elevation of two Jet Horizontal Type Turbines respectively**  
**OW = Head Race ; UW = Tail Race**

The tide advances for 6 hours and 12 minutes onto the land and then retreats during the next 6 hr 12 min. The diurnal variation of tide has a period of 24 hours 50 minutes, the lunar day being 50 minutes longer than the solar day. A turbine operating under the head formed by the flood and ebb will therefore have to change periodically its direction of rotation. Fig 4.8 shows working of a tidal power plant.

**4.12 Classification of Modern Water Turbines** can be made according to :

- (i) the head and the quantity of water available,
- (ii) the name of the originator,
- (iii) the action of water on the moving blades,
- (iv) the direction of flow of water in the runner,
- (v) the disposition of turbine shaft,
- and (vi) the specific speed  $N_s$  or non-dimensional factor  $K_s$ .

The last one is the only scientific classification.

**4.13 Classification of Turbines Depending Upon the Head and the Quantity of Water Available :**

- (a) **Impulse turbine** requires high head and small quantity of flow.
- (b) **Reaction turbine** requires low head and high rate of flow.

Actually there are two types of reaction turbines, one for medium head and medium flow and the other for low head large flow.

Similar to the classification of Water Power Plants (refer Art 4.4) it is quite difficult to classify the turbines according to head and discharge, as for the same head or discharge all types of turbines can be employed.

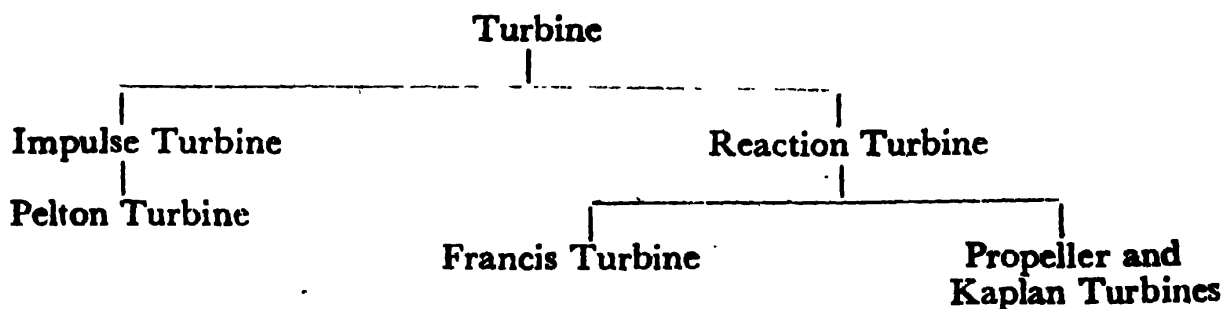
**4.14 Classification After the Names of Originators** (refer Fig 4.9)

(a) **Pelton Turbine**—named in honour of Lester Allen Pelton (1829-1908) of California (USA) is an impulse type of turbine; used for high head and low discharge. He patented it in 1889.

(b) **Francis Turbine**—named after James Bichens Francis (1815-92) who was born in England and later went to USA, is a reaction type of turbine for medium high to medium low heads and medium small to medium large quantities of water.

(c) **Kaplan Turbine**—named in honour of Dr Victor Kaplan (1876-1934) of Bruenn (Germany) is a reaction type of turbine for low heads and large quantities of flow.

**4.15 Classification According to Action of Water or Moving Blades :**



(a) **Impulse Turbine**—The water is brought in through the penstock ending in a single nozzle. The whole pressure energy of water is transformed into kinetic energy. The water coming out of the nozzle in the form of a free jet is made to strike on a series of buckets mounted on the periphery of a wheel. The water is delivered to the wheel on a part of its circumference filling or striking only a few of the buckets at a time. The wheel revolves in open air i.e., there is no difference of pressure in the water at the inlet to the runner and at the discharge. Therefore the casing of an impulse turbine has no hydraulic function to perform. It is necessary only to prevent splashing and to lead the water to the tail race, and also act as a safeguard against accidents.

This turbine is also known as a free jet turbine.

In such type of turbine, it can be written that

$$p_1 = p_2 \quad ; \quad v_1 \gg v_2$$

and  $w_1 \approx w_2$  (neglecting losses in buckets).

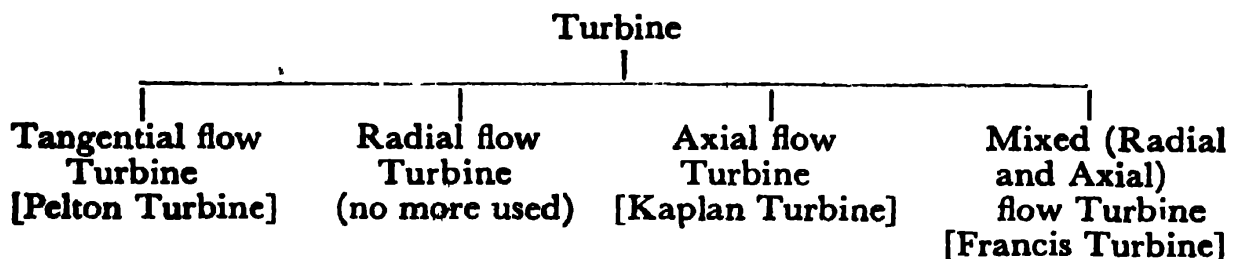
(b) **Reaction Turbine**—The reaction turbine operates with its wheel submerged in water. The water before entering the turbine has pressure as well as kinetic energy. All pressure energy is not transformed into kinetic energy as in case of impulse turbine. The moment on the wheel is produced by both kinetic and pressure energies. The water leaving the turbine has still some of the pressure as well as the kinetic energy. The pressure at the inlet to the turbine is much higher than the pressure at the outlet. Thus, there is a possibility of water flowing through some passage other than the runner and escape without doing any work. Hence a casing is absolutely essential due to the difference of pressure in reaction turbine.

For this type of turbine, it can be mentioned that

$$p_1 \gg p_2 \quad ; \quad v_1 > v_2 \quad \text{and} \quad w_1 < w_2.$$

Propeller turbines are mainly Kaplan Turbines but Moody, Nagler, and Bell turbines may be found in the market. The main difference between Kaplan turbine and the other type of propeller turbines is that the former has adjustable runner blades.

#### 4.16 Classification According to Direction of Flow of Water in the Runner :



In tangential flow turbine of Pelton type the water strikes the runner tangential to the path of rotation. This path is the centre line of buckets which is, sometimes, known as pitch circle diameter or mean diameter of wheel.

Radial flow turbine has already been explained in Introduction. In axial flow turbine water flows parallel to the axis of the turbine shaft. In inward flow turbine, the water flows from the periphery towards the centre of the wheel.

In mixed flow turbine, water enters radially and emerges out axially, so that the discharge is parallel to the axis of the shaft. Modern Francis turbine have mixed flow runners.

**4.17 Classification According to Disposition of Turbine Shaft**—Turbine shaft may be either vertical or horizontal. In modern turbine practice, Pelton turbines usually have horizontal shafts whereas the rest, especially the large units, have vertical shafts.

**4.18 Classification According to Specific Speed**—Turbines fall under a natural order when classified according to their specific speeds (refer Fig 4.10). The specific speed of a turbine is the speed of a geometrically similar turbine that would develop one Brake Horse Power (BHP) under a head of one metre.

$$\text{Specific speed } N_s = \frac{N \cdot \sqrt{P_t}}{H \cdot \sqrt[5]{H}} = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} \quad (\text{Refer Eqn 3.10})$$

where  $N$  = the normal working speed of turbine in RPM  
 $P_t$  = the turbine output in BHP ;  
 $H$  = the net or effective head in metres.

TABLE 4.2

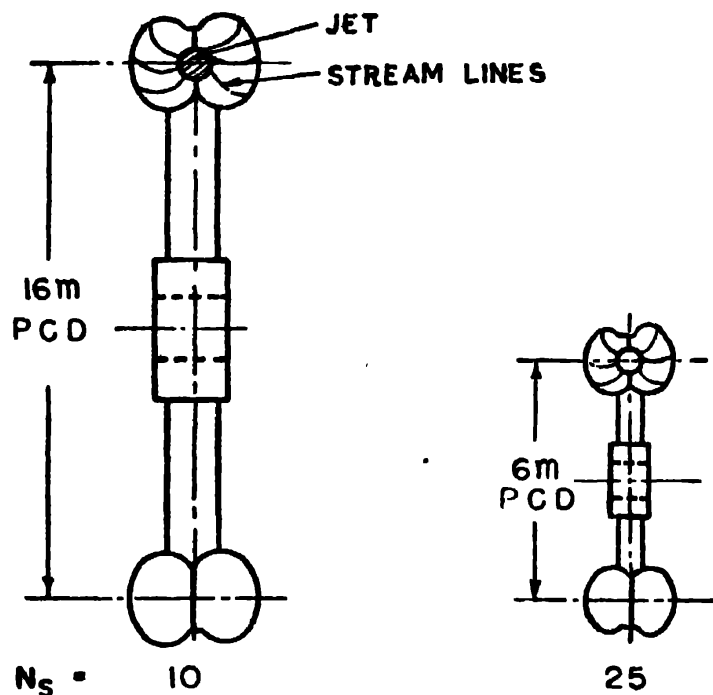
**Values of Specific Speed  $N_s$  and Non-dimensional Factor  $K_s$  of Turbines**

| Type of Turbine | Type of Runner | Specific Speed $N_s$ | $K_s$         |
|-----------------|----------------|----------------------|---------------|
| Pelton          | Slow           | 10 to 20             | 0.098 to 0.39 |
|                 | Normal         | 20 to 28             | 0.39 to 0.76  |
|                 | Fast           | 28 to 35             | 0.76 to 1.2   |
| Francis         | Slow           | 60 to 120            | 3.5 to 14     |
|                 | Normal         | 120 to 180           | 14 to 31.5    |
|                 | Fast           | 180 to 300           | 31.5 to 88    |
| Kaplan          | —              | 300 to 1,000         | 88 to 980     |

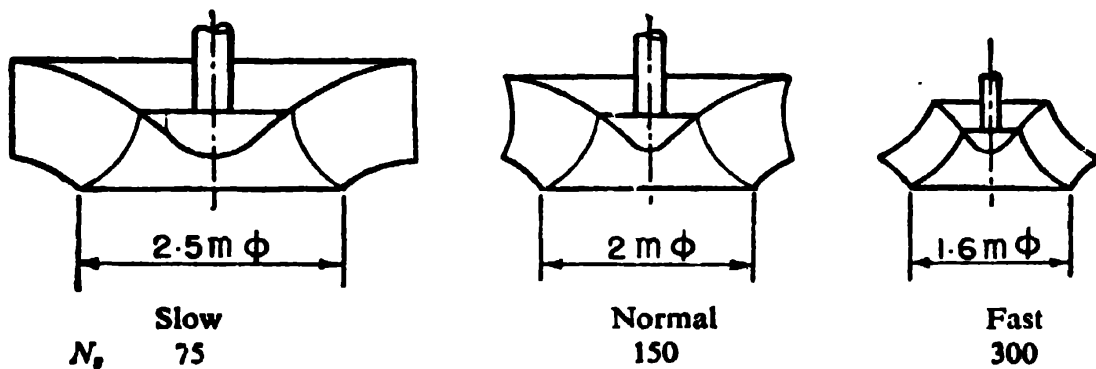
Pelton wheels having a specific speed of 10 to 35 are those designed for one nozzle. The gap between the specific speed relating to Pelton and Francis turbine i.e. 35 to 60 is bridged by equipping Pelton wheel with more than one nozzles which, of course is the general practice. Messrs Gilbert Gilkes and Gordon Ltd. Kendal, (England) have patented another turbine known as *Turgo-Impulse Wheel* (refer Plate 8) which covers the required range of specific speed.

M/s Komplex, Budapest, Hungary manufacture **Banki** turbines (refer Art 5.18) having a specific speed of 20 to 80. It is an impulse turbine based on the patent of Prof. Dohat Banki (1859-1922) of Budapest Technical University.

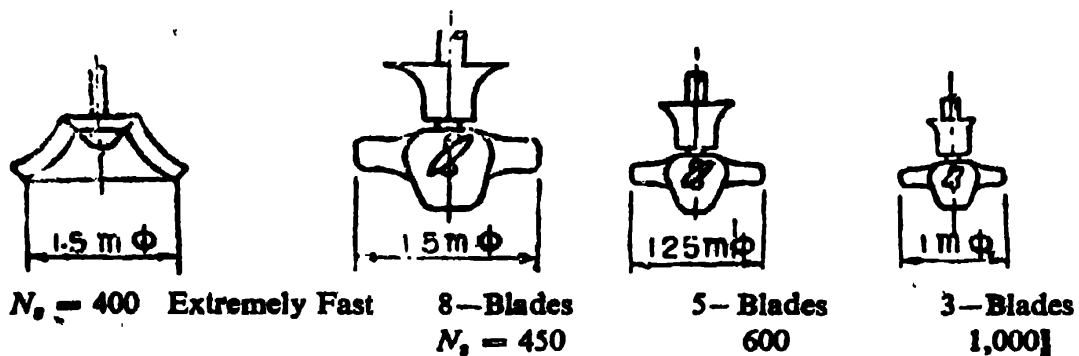
Francis turbines could be designed for a specific speed greater than 300 in which case it will have a special runner as designed by Prof Dubs,  $N_s$  of which ranges from 280 to 520.



(a) PELTON RUNNERS



(b) FRANCIS RUNNERS



(c) DUBS RUNNER

(d) KAPLAN RUNNERS

Different Types of Water Turbine Runners

Comparison of dimensions is made on the basis that each runner develops 100 BHP under 40 m net head, when running at a speed equal to  $10 N_s$ .

Fig 4.10 Classification of Water Turbines According to Specific Speeds  $N_s$

The **bulb or tubular** type of turbine is the latest development for very low head plants. It comprises a Kaplan (or fixed propeller) runner, set axially within a short pressure conduit. The generator is close-coupled to the runner and both are housed in 'bulb' shaped casing which is surrounded by flowing water. The bulb turbine has been useful to harness tidal power.

Special types of runners, such as screw runners, are used for specific speed more than 1,000.

**Problem 4.1** Find the specific speed of a turbine installed at *Khapoli Power House* belonging to *Tata Hydro Electric Co., Bombay*, which develops 17,340 HP under a head of 510 m when running at 300 rpm. Specify the type of turbine employed.

**Solution**

$$P_t = 17,340 \text{ HP} \quad H = 510 \text{ m} \quad N = 300 \text{ rpm}$$

$$\begin{aligned} \therefore \text{Specific speed } N_s &= \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{300 \times \sqrt{17,340}}{510^{\frac{5}{4}}} \\ &= \frac{300 \times 131.3}{510 \times 4.74} = 16.3 \quad \text{Answer} \end{aligned}$$

According to table 4.2 the type of turbine employed is *Pelton*.

**Problem 4.2** The turbine installed at *Ganguwal Power House (Panjab)* develops 34,000 HP under head of 29.9 m. Find the specific speed of this turbine if it runs at 166.7 rpm. Knowing the specific speed, what type of runner would you select for such a turbine? (AMIE)

**Solution**

$$P_t = 34,000 \text{ HP} \quad H = 29.9 \text{ m} \quad N = 166.7 \text{ rpm}$$

$$\begin{aligned} \therefore \text{Specific } N_s &= \frac{N \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{166.7 \times \sqrt{34,000}}{(29.9)^{\frac{5}{4}}} \\ &= \frac{166.7 \times 184}{29.9 \times 2.33} = 440 \quad \text{Answer} \end{aligned}$$

Therefore, according to Table 4.2 *Kaplan* runner is to be selected for the above turbine.

**4.19 Non-Dimensional Factor  $K_s$  for Turbines**—The specific speed  $N_s$  is a dimensional quantity, but a non-dimensional factor  $K_s$  could be deduced for turbines in place of  $N_s$ .

$$K_s = \frac{Q \cdot N^2}{v^3}$$

where

$Q$  = rate of flow in  $\text{m}^3 \cdot \text{s}^{-1}$

$N$  = speed in rpm ;

and  $v$  = velocity of water in  $\text{m} \cdot \text{s}^{-1}$

$$= \sqrt{2gH}$$



To prove  $K_s$  as dimensionless factor

$$K_s = \frac{\frac{m^3}{\text{sec}} \times \left( \frac{1}{\text{sec}} \right)^{\frac{5}{2}}}{\left[ \frac{m}{\text{sec}^2} \right]^{\frac{3}{2}} \times (m)^{\frac{5}{2}}} = \frac{m^3 \times \text{sec}^{\frac{5}{2}}}{m^3 \times \text{sec} \times \text{sec}^{\frac{5}{2}}} = \text{a pure number}$$

Most modern method to classify water turbines would, therefore, be according to the non-dimensional factor  $K_s$ . The values of factor  $K_s$  are given in Table 4.2.

**Problem 4.3** Find the specific speed of a turbine installed at Pykara Power House (South India), which develops 19,265 HP under a head of 855 m. The speed of the turbine is 600 rpm. State the type of turbine that may be required. If this turbine uses 1.9 m<sup>3</sup>/sec of water, what is the value of  $K_s$ ?

**Solution**

$$P_t = 19,265 \text{ HP} \quad H = 855 \text{ m} \quad N = 600 \text{ rpm}$$

$$(a) \text{ Specific Speed } N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{600 \times \sqrt{19,265}}{855^{\frac{5}{4}}} = 18 \text{ Answer}$$

∴ This turbine is Pelton type, using a slow runner.

$$(b) K_s = \frac{Q \cdot N^2}{v^3} \quad \text{where } v = \sqrt{2gH}$$

$$= \frac{1.9 \times 600^2}{(\sqrt{2 \times 9.81 \times 855})^3} = 0.315 \text{ Answer}$$

**4.20 Relation between  $N_s$  and  $K_s$ —**

$$N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{N}{\sqrt{H}} \sqrt{\frac{P_t}{H \cdot \sqrt{H}}}$$

$$\text{where } P_t = \frac{\gamma \cdot Q \cdot H}{75} \cdot \eta_t$$

$$\therefore N_s = \frac{N}{\sqrt{H}} \cdot \sqrt{\frac{\gamma \cdot Q \cdot H \cdot \eta_t}{75 H \cdot \sqrt{H}}}$$

$$= 3.65 \frac{N}{\sqrt{H}} \sqrt{\frac{Q}{\sqrt{H}}} \cdot \sqrt{\eta_t}$$

Dividing both sides by  $\sqrt{2g} \cdot \sqrt{2g}$

$$\frac{N_s}{\sqrt{2g} \cdot \sqrt{2g}} = 3.65 \sqrt{\eta_t} \cdot \frac{N}{\sqrt{2gH}} \sqrt{\frac{Q}{\sqrt{2gH}}}$$

$$\text{But } \sqrt{2gH} = v$$

$$\therefore \frac{N_s}{\sqrt{2g} \cdot \sqrt{2g}} = 3.65 \sqrt{\eta_t} \cdot \frac{N}{v} \cdot \sqrt{\frac{Q}{v}}$$

$$N_s = 34 \sqrt{\eta_t} \cdot \frac{N \sqrt{Q}}{v^{\frac{3}{2}}}$$

$$\begin{aligned} \text{but } K_s &= \frac{Q \cdot N}{v^3} \quad \therefore N_s = 34 \cdot \sqrt{\eta_t} \cdot K_s \\ \text{or } K_s &= \left( \frac{N_s}{34 \cdot \sqrt{\eta_t}} \right)^3 = \left( \frac{N_s}{34} \right)^3 \cdot \frac{1}{\eta_t} \quad \dots(4.2) \end{aligned}$$

Assuming average value of  $\eta_t = 0.9$  and substituting in the above equation

$$K_s = \left( \frac{N_s}{34} \right)^3 \times \frac{1}{0.9} = \left( \frac{N_s}{32.2} \right)^3 = \frac{N_s^3}{1,040} \quad \dots(4.2a)$$

Apparently  $N_s$  also seems to be dimensionless from the above equation, but it must be borne in mind that value 1,040 includes factors such as  $g$  etc.

**Problem 4.4** *It is proposed to develop 2,000 HP at a site where 150 m of head is available. What type of turbine—impulse or a low, medium, or high head reaction turbine would be employed if it had to run at 300 rpm? If the same turbine is now used under a head of 30 m, find the power developed and its rpm.*

**Solution**

$$P_t = 2,000 \text{ HP} \quad H = 150 \text{ m} \quad N = 300 \text{ rpm}$$

(a) Specific speed of the turbine

$$N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{300 \times \sqrt{2,000}}{150^{\frac{5}{4}}} = 25.6$$

The specific speed of a Pelton (impulse) type turbine ranges from 10 to 35, therefore Pelton turbine would be selected for the above project and according to Table 4.2 a normal runner would be employed.

$$(b) \text{ According to Art 3.3, (iii) } \frac{P_{t_1}}{P_{t_2}} = \left( \frac{H_1}{H_2} \right)^{\frac{3}{2}}$$

$$\text{or } P_{t_2} = P_{t_1} \left( \frac{H_2}{H_1} \right)^{\frac{3}{2}}$$

$$\therefore P_{t_2} = 2,000 \times \left( \frac{30}{150} \right)^{\frac{3}{2}} = 180 \text{ HP} \quad \text{Answer}$$

$$(c) \frac{N_2}{N_1} = \left( \frac{H_2}{H_1} \right)^{\frac{1}{2}}$$

$$\text{or } N_2 = N_1 \left( \frac{H_2}{H_1} \right)^{\frac{1}{2}} \quad \dots(\text{refer Eqn 3.2})$$

$$\therefore N_2 = 300 \times \left( \frac{30}{150} \right)^{\frac{1}{2}} = 134.2 \text{ rpm} \quad \text{Answer}$$

**Problem 4.5** *An impulse turbine develops 2,500 HP under a head 70 m. What could be the maximum and minimum speeds of the turbine with a single nozzle? What speed would be the best for coupling to an alternator? How high a speed could a reaction turbine give?*

**Solution**

$$P_t = 2,500 \text{ HP} \quad H = 70 \text{ m}$$

The specific speed of a turbine is  $N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$

The range of the specific speed for the impulse (Pelton) turbine is 10 to 35.

∴ Minimum speed of the turbine is given by :

$$10 = \frac{N \cdot \sqrt{2,500}}{70^{\frac{5}{4}}} \quad \text{or} \quad N = \frac{(70 \times 2.88) \times 10}{50} = 40.2 \text{ rpm} \quad \text{Answer}$$

The maximum speed of impulse turbine :

$$N = \frac{H^{\frac{5}{4}} N_s}{\sqrt{P_t}} = \frac{(70 \times 2.88) \times 35}{50} = 141 \text{ rpm} \quad \text{Answer}$$

The best synchronous speed for 50 cycles frequency can be 125 rpm.

(b) The maximum speed which a reaction turbine of Francis type could give :

$$N = \frac{(70 \times 2.88) \times 300}{50} \quad \dots (\text{as } 300 \text{ is the maximum specific speed})$$

$$= 1,210 \text{ rpm} \quad \text{Answer}$$

The maximum speed which could be obtained by a Kaplan turbine under the same conditions :

$$N = \frac{(70 \times 2.88) \times 1,000}{50} = 4,030 \text{ rpm} \quad \text{Answer}$$

**Problem 4.6** A horizontal Francis turbine has the following specifications—

$$H = 98 \text{ m}$$

$$Q = 10.8 \text{ m}^3/\text{sec}$$

Find the minimum synchronous speed of the turbine, if its efficiency is 87%.

**Solution**

$$\text{Horse Power of the turbine, } P_t = \frac{\gamma \cdot Q \cdot H}{75} \cdot \eta_t$$

$$P_t = \frac{1,000 \times 10.8 \times 98}{75} \times 0.87 = 1,228 \text{ HP}$$

The minimum and maximum specific speeds for a Francis turbine are 60 and 300 respectively.

∴ Minimum speed of Francis turbine—

$$N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} \quad \text{or} \quad 60 = \frac{N \cdot \sqrt{1,228}}{98^{\frac{5}{4}}}$$

$$\text{or} \quad N = \frac{60 \times (98 \times 3.15)}{35} = 529 \text{ rpm}$$

The nearest synchronous speed for frequency of 50 cycles per second is

$$600 \text{ rpm} \quad \text{Answer}$$

**UNSOLVED PROBLEMS**

- 4.1 What are "Multi-Purpose Projects"? Give examples. What percentage of hydro power potential of India has been developed so far?
- 4.2 Define head and discharge. What is the difference between the gross head and net head?
- 4.3 What are the essential components of hydro-electric power plant?
- 4.4 Define "Head Works". For what purpose are they installed?
- 4.5 Name the different types of gates and valves.
- 4.6 What are the trashracks? State their uses. How is the debris removed from the trashracks?
- 4.7 What are waterways? Describe their control works.
- 4.8 What is a forebay? What is the difference between a forebay and a surge tank?
- 4.9 Classify Water Power Plants. Why is it difficult to classify in accordance to heads under which they work?
- 4.10 Describe the basis under which the water power plants can be classified.
- 4.11 Describe with the help of sketch,
  - (a) High head water power plant ;
  - (b) Canal water power plant
  - (c) Low head water power plant ;
  - (d) Medium head water power plant.
- 4.12 Write an essay on the importance of high head and low head hydro-electric power projects. *(Panjab University)*
- 4.13 Illustrate diagrammatically a general layout for a water power scheme comprising a Francis turbine supplied by a penstock and discharging through its draft tube into tail race. Indicate the effective head on turbine, after taking into account the losses upto the turbine intake and the final velocity in the tail race. *(Panjab University)*
- 4.14 What is the difference between run-off river plants and storage plants?
- 4.15 What are pumped storage plants? Where are they used?
- 4.16 What is the difference between base load and peak load plants?
- 4.17 Describe tidal power plants with the help of a sketch. Explain why such plants could not be built so far.
- 4.18 What means are provided to save the penstock from water hammer, when the gates or valves of turbine are quickly closed?
- 4.19 What is impulse turbine and why is it so named?
- 4.20 Name the different water turbines which are used now-a-days. How are these turbines classified?
- 4.21 What is the difference between the impulse and the reaction water turbines?
- 4.22 Are Pelton turbines mostly vertical or horizontal and why?
- 4.23 Is the modern Francis turbine a purely radial flow turbine?
- 4.24 Define specific speed of a turbine. How does this figure help in the design of a turbine?

- 4.25 Justify for the same type of turbine : "If  $N_s$  is less, the size of runner is more."
- 4.26 What is non-dimensional factor  $K_s$  ? How is it related to specific speed ? Why is it preferable to classify the turbines according to  $K_s$  ?
- 4.27 Where are multi-jet Pelton turbines employed ?
- 4.28 Describe Turgo-Impulse turbines. What is their special feature ?
- 4.29 Describe Banki and Dubs turbines. What is the range of specific speed for which each of them is employed ?
- 4.30 What are tubular turbines ? Describe bulb turbines. How do they differ from tubular turbines ?

### Numericals

- 4.31 Determine the specific speed and the type of each of the following water turbines. State the type of turbine runner used in each case :—
- Tilaiya* Power House (D.V.C.) having two turbines, each developing 2,845 BHP, when running at 250 rpm under a maximum head of 23.45 m.
  - Pathri* Hydro Station on the Ganga Canal in U.P. equipped with three Water Turbines, manufactured by Voith (Germany), each developing 9,650 BHP, when running at 125 rpm under a net working head of 9.75 m.
  - Reisseck* (Austria) Power Station having two turbines manufactured by Charmilles (Switzerland), each rated at 31,500 BHP, when running at 750 rpm, under an effective head of 1,770 m, the highest head so far used to develop power anywhere in the world upto date.
  - Aluminium Co* Power Plant (Canada) having two turbines, manufactured by Dominion (Canada), each rated at 142,200 BHP, when running at 327.5 rpm, under a rated head of 758 m.
- [(a) 258, Francis, (b) 712, Kaplan (c) 11.6, Pelton, Slow (d) 30.7, Pelton, fast].
- 4.32 It is proposed to build a turbine to operate at 106 rpm and develop 13,500 HP under a head of 100 m. What type of turbine would you suggest ?  
( $N_s = 39$  ; Multi-nozzle, Pelton Turbine or Turgo Impulse Turbine)
- 4.33 Can a Pelton wheel with single nozzle be obtained to utilize 280 lit/sec of water under a head of 70 m, and run at a speed of 600 rpm ? If not, what may be done ?  
(No, as  $N_s = 45.5$ , therefore either two nozzles with one wheel or two units each with a single nozzle may be used).
- 4.34 The Pelton turbine at Seera de Cubatao (Brazil) is a double overhung, two nozzle, horizontal type and develops, 85,000 HP under a head of 710 m. Determine its specific speed, if it runs at a synchronous speed of 50 cycles per second.

$$\left[ \text{Hint : } P_t \text{ per runner per nozzle} = \frac{85,000}{4} = 21,250 \text{ H} \right]$$

(11.92 with 300 rpm)

## *Pelton Turbine and Other Impulse Turbines*

**5.1 Modern Pelton Turbine**—Among the impulse (or action) water turbines, Pelton turbine is the only type being mostly used these days. It is also called free jet turbine and operates under a high head of water and, therefore, requires a comparatively less quantity of water. The water is conveyed from a reservoir in the mountains to the turbine in the power house through penstocks. The penstock is joined to a branch pipe or lower bend fitted with a nozzle at the end. Water comes out of the nozzle in the form of a free and compact jet. The number of nozzles required for a turbine depends on its specific speed. All the pressure energy of water is converted into velocity head. The water having a high velocity is made to impinge, in air, on buckets fixed round the circumference of a wheel, the latter being mounted on a shaft (refer Fig 5.1). The impact of water on the surface of the bucket produces a force which causes the wheel to rotate, thus, supplying a torque or mechanical power on the shaft. The jet of water strikes the double hemispherical cup shaped buckets at the centre and is deviated on both sides, thus eliminating an end thrust (refer Art 1.15). The water jet after impinging on the buckets is deflected through an angle of about  $165^\circ$ , instead of  $180^\circ$ , so that it may not strike the back of the incoming bucket and retard the motion of the wheel. After performing work on the buckets water is discharged into the tail race. The wheel must be so located that the buckets do not splash into the tail race water when the wheel revolves.

As stated above Pelton turbine is best suited for high heads. It is efficient and reliable when employed under these conditions. It will remain no more efficient for low heads, because for a given power if the head is reduced, the rate of flow has to be increased. Increased flow will require bigger jet diameter, consequently the runner diameter will also be increased. On the other hand the jet velocity and consequently peripheral velocity of runner will be reduced because of low head. These two factors i.e., increase of runner diameter and decrease of runner velocity will make the turbine bulky and slow running for low heads. Reaction turbines described in the next chapters will be used for low heads.

### **Pelton Turbine : Main Components and their Functions**

**5.2 Guide Mechanism**—This mechanism controls the quantity

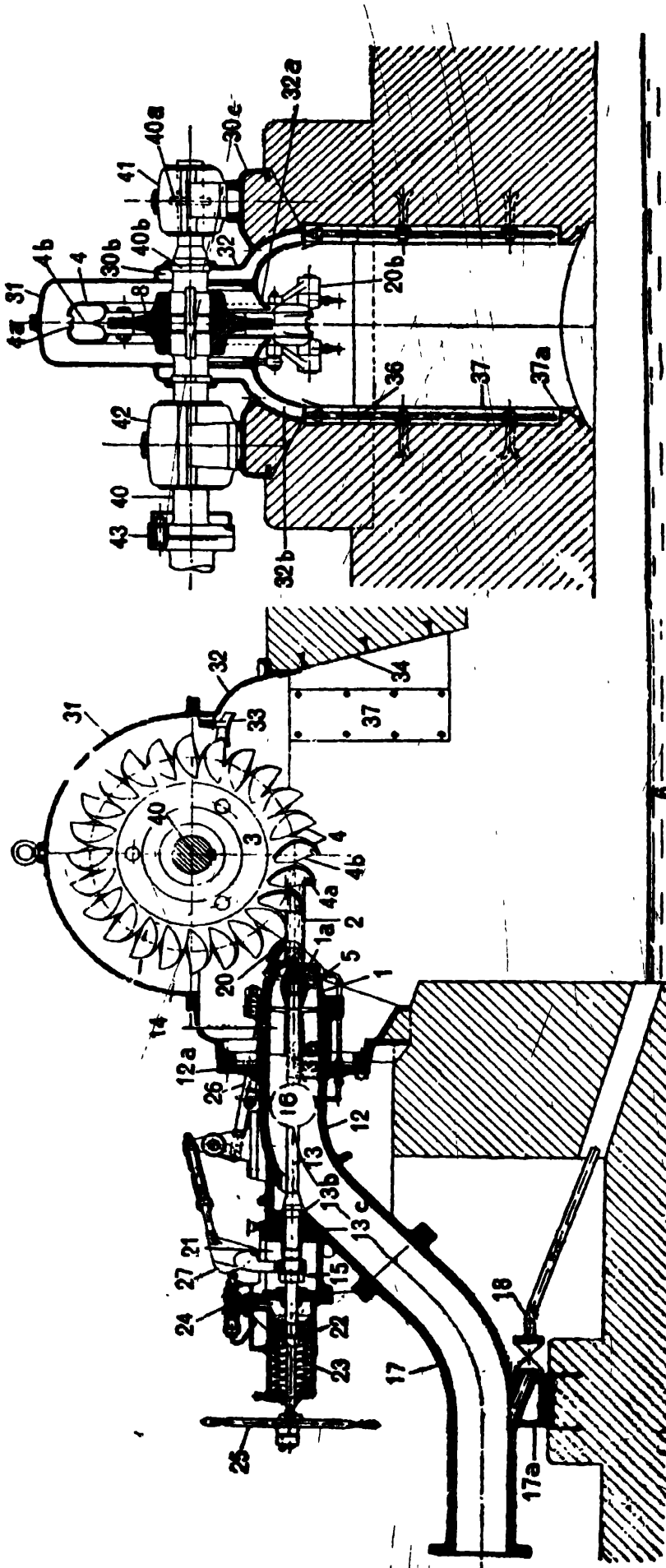


Fig 5.1 Pelton Turbine Manufactured by Messrs Escher Wyss & Co., Ltd. : Zurich, Switzerland

#### List of Parts

1 Nozzle 1a Inset piece 1b Holder 1c Eyes 1d Protecting roof 2 Water jet 3 Runner wheel 4 Runner buckets 4a Shaped-out portion of bucket 4b Centre partition 5 Spear head 5a Spear Point 5c Stud bolt 5d Key 5e Spear body 5f, 5g Cylindrical holes with pins 6 Tail race D<sub>1</sub> Runner diameter d. Jet diameter 7 Lugs of bucket 8 Runner disc 8a Runner boss 9 Fixing bolts with nuts 9a and 9b Bolt head 10 Tensioning bolts 11 Radial keys 12 Inlet bend 12a Flange 13 Spear rod 13a Bronze sleeve 13b Balancing piston 13c Leather packing 14 Guide cross 14a Guide ribs 14b Cylindrical member for foregoing 15 Support for spear rod 16 Cleaning hole 17 Lower bend 17a Anchoring foot 18 Emptying pipe 19 Turbine valve 20 Jet deflector 20a Intercepting piece 20b Levers 21 Relay lever 22 Servomotor piston for spear adjustment 23 Closing spring for spear 24 Distributing valve for nozzle 25 Handwheel for spear adjustment 26 Deflector rod 27 Cam 30 Casing 30a Opening for the inlet bend 30b Lateral chambers 30c Cast iron bearing supports 31 Casing hood 32 Lower part of casing 32a Guide walls 32b Drainage channels for spray water from turbine shaft 33 Spray water catcher 34 Lining 36 Cooling coils 37 Cover for protecting of latter 37a Hole for discharging cooling water 40 Turbine shaft 40a Shaft collar 40b Ring 41 External turbine bearing 42 Internal turbine bearing 43 Shaft coupling 44 Speed governor 45 Flywheel.

of water passing through the nozzle and striking the buckets, thus meeting the variable demand for power. It maintains the speed of the wheel constant even when the head varies. The mechanism essentially consists of a spear fixed to the end of a shaft which is operated by governor (refer Fig 5.1). When the speed of the wheel increases, the spear is pushed into the nozzle thereby reducing the quantity of water striking the buckets. If the speed of the wheel falls, the spear is drawn back allowing a greater quantity of water to pass through the nozzle. Sometimes a sudden decrease in load takes place. Consequently water requirement of the turbine suddenly falls. This necessitates the immediate closure of the nozzle with the spear, which may cause the pipe to burst due to the sudden increase in the pressure of water. In order to avoid such an occurrence, an additional nozzle is used. Water can pass through the nozzle, without striking the buckets. This nozzle is called the *Bypass Nozzle* and is opened when the main nozzle is being closed on a reduction in load taking place.

The modern practice is to provide the guide mechanism with a *deflector* (refer Fig 5.2 and Plate 9). The deflector is a plate connected to the spear rod by means of levers 20b (refer Fig 5.1) the spear rod 5 being operated by the governor. The plate is located between the nozzle and the buckets. When a sudden reduction in load takes place, the governor brings the deflector in front of the buckets thus deflecting the water jet and preventing it from striking the buckets.

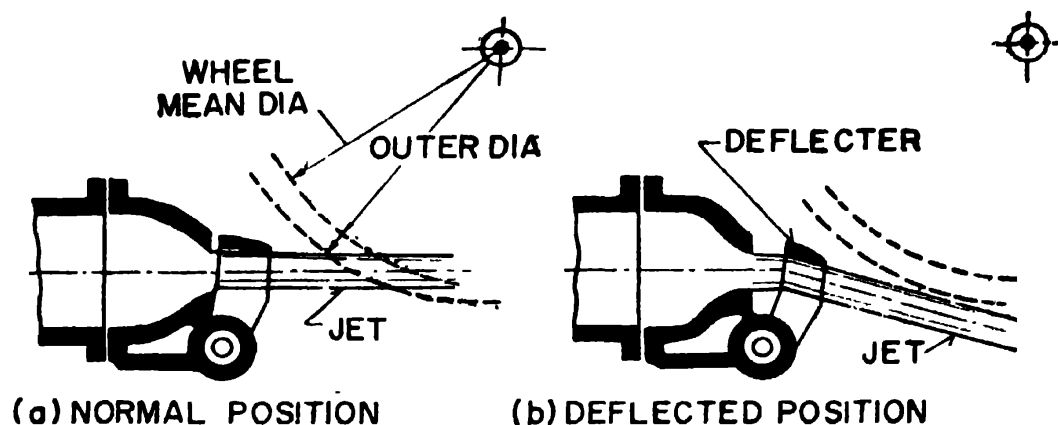


Fig 5.2 Deflector of Pelton Turbine

English Electric Co. have provided some of the Pelton Turbines with *Sewer Jet Diffuser* (refer Plate 10) which disperses or diffuses jet by means of a number of slightly slanting flat blades surrounding the needle and forming a part of spear-head. With the sudden fall in load, the blades instantly go forward into the stream, causing it to whirl and converting the compact jet into a hollow-cone broken into spray, thus destroying the energy by diffusion. For the normal operation, the blades are withdrawn and their ends flushed with the cover, leaving a clear passage for the water to go out through the nozzle.

**5.3 Buckets and Runner**—Each bucket is divided vertically into two parts by a *splitter* which is a sharp edge at the centre, giving the shape of a double hemispherical cup (refer Fig 5.3 and Plate 11). The splitter helps the jet to be divided, without shock, into two parts moving sideways in opposite directions. The rear of the buckets should be shaped so as not to interfere with the passage of water to the bucket preceding in order of rotation. The jet should be deflected backwards when leaving the buckets,



(refer Plate 12), the angle of deflection being about  $160^\circ$ . The buckets form the most important part of the turbine and should be designed to withstand the full force of the jet when the turbine is shut off. It is important to select a suitable material for the buckets, so that they should not crack under the considerable force of the jet. Cast Iron is used for low heads but for higher heads bronze or better still, stainless steel is used. The buckets should be properly polished in order to avoid undue stresses under the action of the jet, which may lead to cracking at sections which are incorrectly shaped.

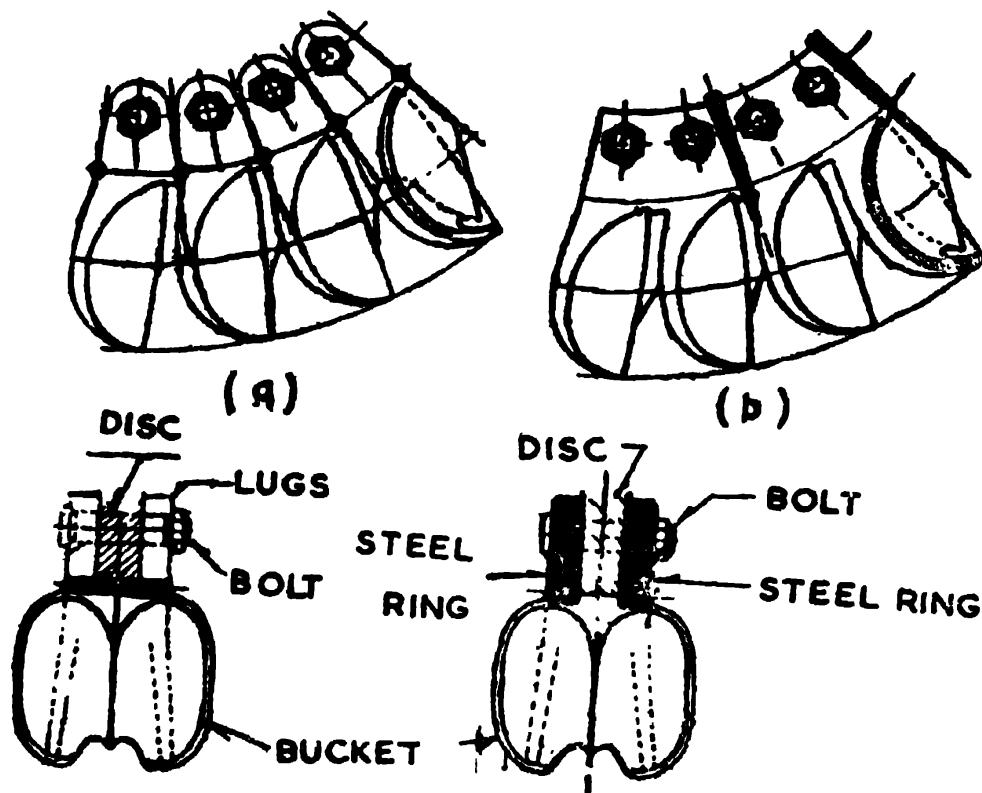


Fig 5.3 Methods of Bucket Attachment (Escher Wyss)

(a) Single Bucket

(b) Pair of Buckets

The buckets are bolted on to the circumference of a round disc forming the runner of the Pelton turbine (refer Fig 5.3). The buckets and the disc can be cast as a single unit which will be more economical. Many turbine manufacturers believe that all the buckets wear equally in a given time, but it is found in practice that buckets may break down one at a time. It is easy to replace a broken bucket unless the runner is cast as a single unit. The runner is made of Cast Iron, Cast Steel or Stainless Steel. Cast Iron is used to reduce cost in turbines designed for low heads.

Sand erosion and corrosion of bucket is shown in Plate 13.

**5.4 Casing**—As already mentioned in Art 4.15 the casing of a Pelton turbine has no hydraulic function to perform. It is necessary only to prevent splashing and to lead the water to the tail race, and also as a safeguard against accidents.

The casing is cast or fabricated. It has to take a force of the jet projecting beyond the runner in the event of overspread. In horizontal shaft units, the casing is split into bed plate and cover so that the runner can be lowered into place or lifted out by a crane. The lowest part of the bed plate is embedded and anchored in the lower house floor. The escape

of water along the shaft is prevented by a seal. Baffles (refer Fig 5.1, No. 33) are arranged to reduce windage loss or to protect the runner and jet from interference by splash.

**5.5 Hydraulic Brake**—After shutting down the inlet valve of turbine, the large capacity runner will go on revolving for a considerable period, due to its inertia. This has necessitated the development of a brake to bring the turbine to a standstill in the shortest possible time. The brake consists of a small nozzle fitted in such a way that on being opened, it directs a jet on the back of the buckets to bring the revolving runner quickly to rest (refer Fig 5.4 a). The least diameter of the brake jet has been found to be equal to 0.6 times\* the least diameter of the main jet.

### 5.6 Different Layouts of Pelton Turbine—

(a) **Arrangements of Jets**—Ordinarily the Pelton turbines have a single jet and a horizontal shaft, as shown in Fig 5.4a. However, the number of jets depend upon the specific speed. It is, therefore, possible that more than one jet may be employed. Fig 5.4 b and c show the arrangements of double jet with horizontal shaft and four jets with vertical shaft respectively. Fig 5.5 and Plate 14 show six jet vertical Pelton turbine arrangement.

(b) **Arrangements of Runner**—The runner of the turbine as well as the rotor of the generator to be driven by the turbine are keyed on the same shaft. The rotor of the generator is generally heavier than the Pelton runner, and is, therefore, supported in two bearings while the turbine runner is keyed on the length of the shaft overhanging beyond one of the bearings. This is known as *single-overhung* unit of runner (refer Fig 5.6 a).

If the turbine is designed for greater power or higher speed, two turbine runners keyed to a single horizontal shaft are installed. They may be arranged together on one side of the generator and each of them having its own bearing as shown in Fig 5.6 b; or one on each of the projecting end of the shaft (refer Fig 5.6 c). The latter arrangement is known as *double-overhung* unit.

(c) **Arrangements of Turbine Shaft**—As already explained Pelton turbines are usually installed with horizontal shaft and are equipped with only one nozzle (refer Fig 5.1 and 5.4 a). The number of runners may be one or two keyed to the single horizontal shaft. Two runners are used to obtain higher speed and more power.

In case the number of jets are two, horizontal shaft arrangement may be employed. For four jets (refer Fig 5.4 c) and six jets (refer Fig 5.5 and Plate 14) vertical shaft arrangement is used.

**5.7 Notable Pelton Turbine Installations of the World**—The largest Pelton turbine of the world till today is at Cimego (Italy) producing 150,000 BHP under a head of 720 m when running at 300 rpm. It is of double overhung type, having one jet per runner.

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\*See "Characteristics of Free Jet Water Turbine in Working and Brake Regions" (in German), by Dr. Jagdish Lal, Published by Springer Verlag, Vienna 1952.

The next largest Pelton turbine, manufactured by Dominion (Canada), is installed at Kemano (Canada) in an underground power station, meant, for aluminium reduction works at Kitimat. This is a vertical-shaft, four jet turbine producing 142,200 BHP at 758 m net head and 327.5 rpm.

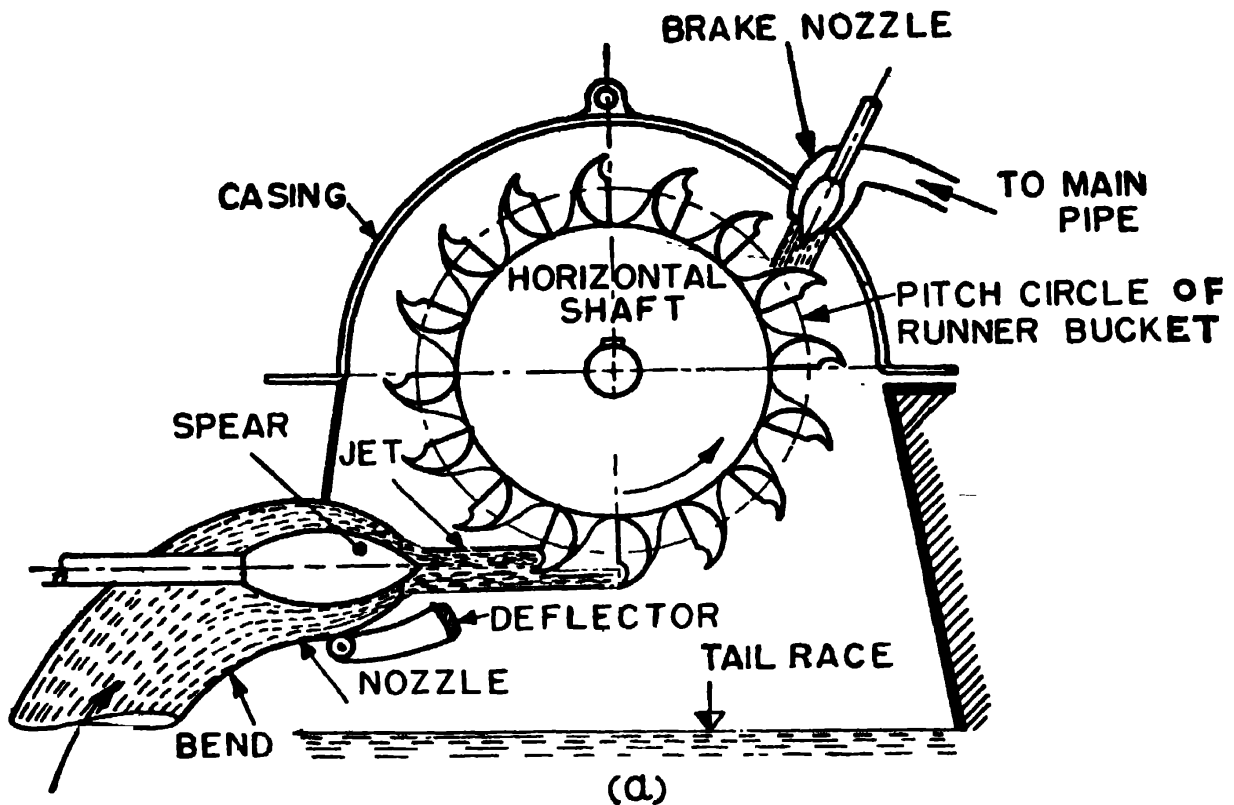


Fig 5.4 (a) Single Jet, Horizontal Shaft Pelton Turbine

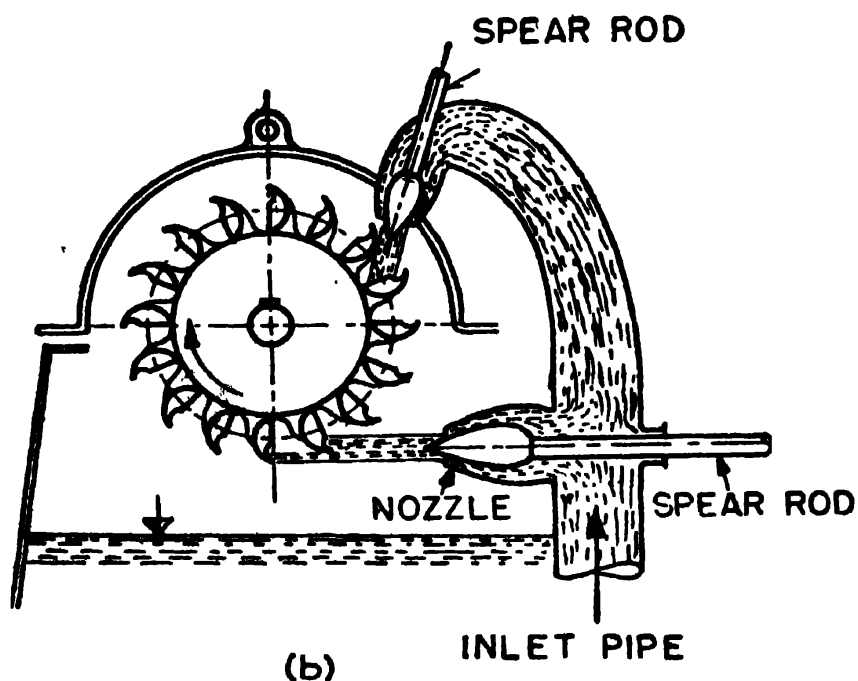


Fig 5.4 (b) Double-Jet, Horizontal Shaft Pelton Turbine

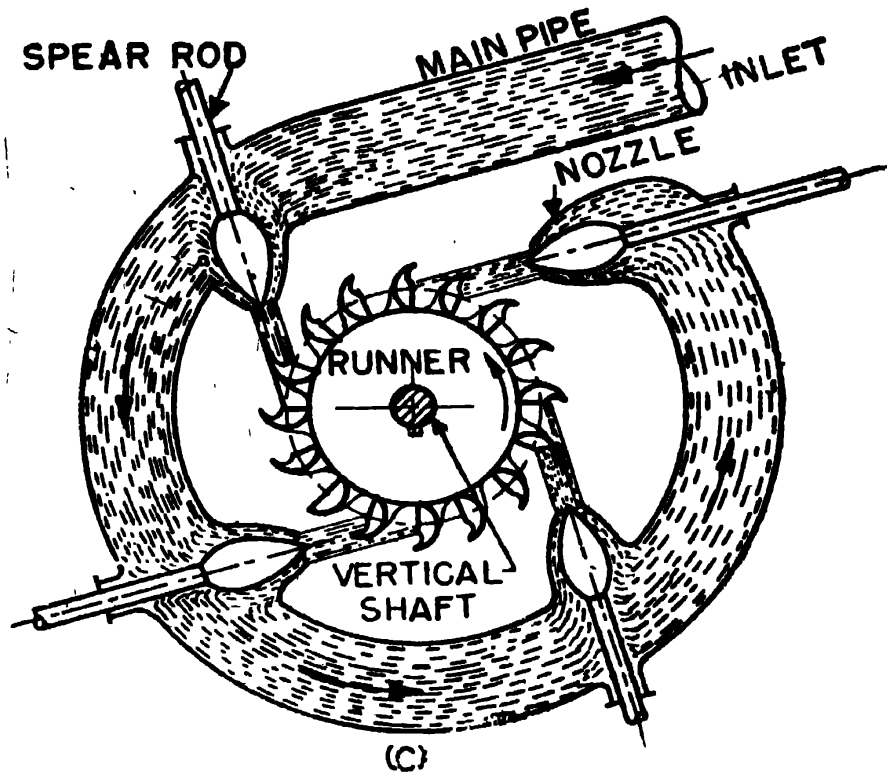


Fig 5.4 (c) Four-Jet Vertical Shaft Pelton Turbine  
Fig 5.4 Arrangements of Jets for Pelton Turbine

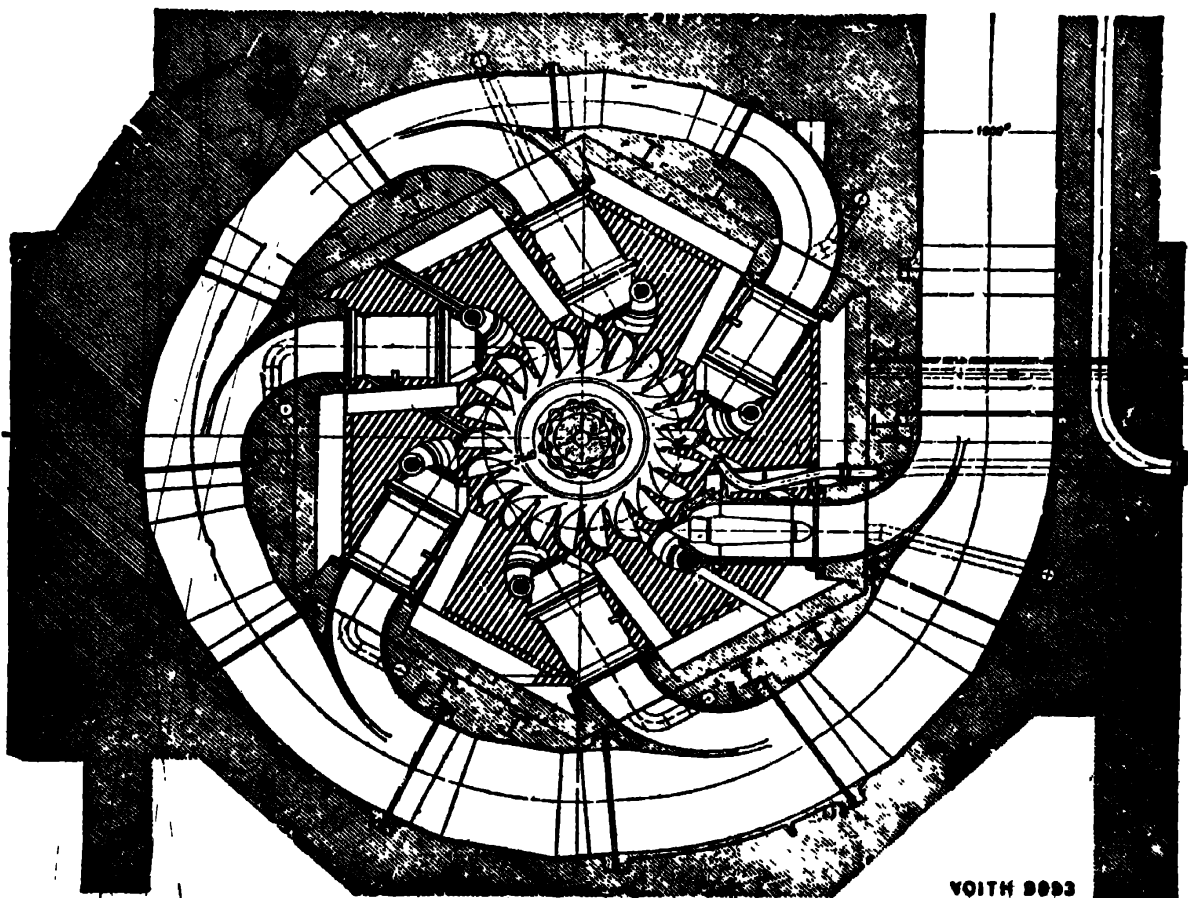


Fig 5.5 Plan of 6-Jet Vertical Pelton Turbine with Inner Governias  
Kurobegawa Power Station.  $H = 540$  m ;  $Q = 18$  m<sup>3</sup>/sec ;  $N = 300$  rpm ;  
 $P_s = 135,000$  HP ;  $N_s = 21$  ;  $D = 2,640$  mm, Inle ; Pipe Dia = 1,865 mm.  
Manufacturers : Voith, Heidenheim (W. Germany)

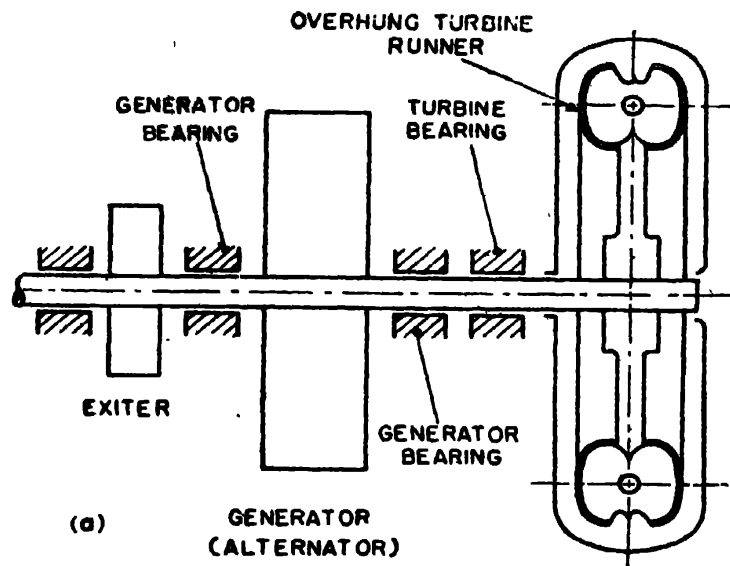


Fig 5.6 (a) Single Overhung Unit

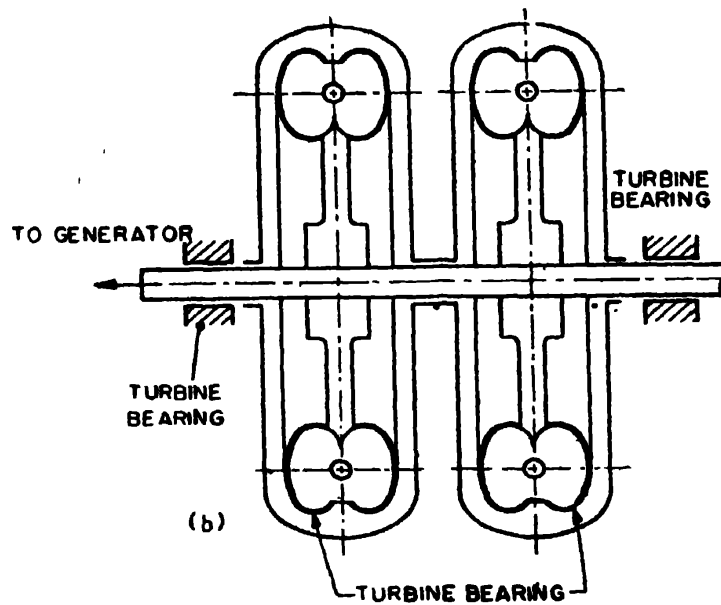


Fig 5.6 (b) Double Runner Arrangement

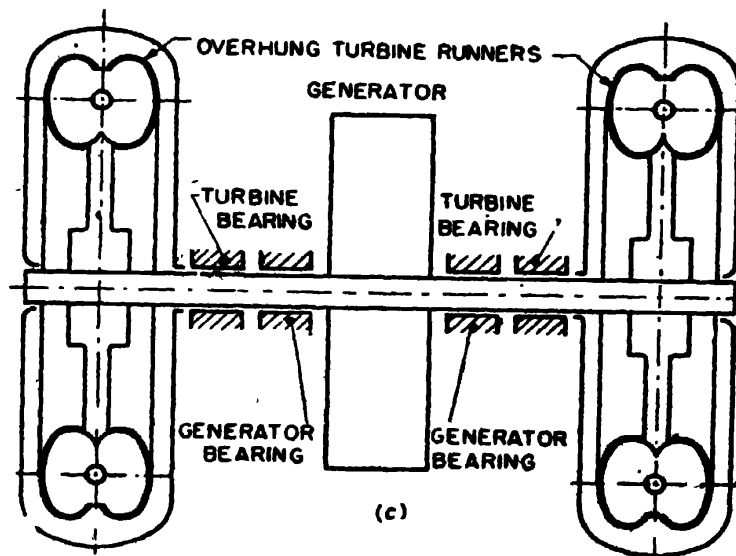


Fig 5.6 (c) Double Overhung Runner Arrangement  
 Fig 5.6 Arrangements of Runners for Pelton Turbines

The highest head used for a Pelton turbine anywhere in the world up-to-date is 1,770 m net. This turbine is installed at Reisseck (Austria) producing 33,800 BHP at 750 rpm. It is manufactured by Charmilles (Switzerland). The next turbine using largest head is installed at Chandoline (Dixence), Switzerland. It delivers 50,000 BHP with two runners (double overhung arrangement) ; when working under 1,748 m gross head and running at 500 rpm. It has discharge of 3.1 to 10.25 m<sup>3</sup>/sec and net head of 1,622 m.

At Seera de Cubatao (Brazil), a double overhung horizontal turbine, developing 85,000 HP (i.e. 42,500 HP per wheel per jet) under a head of 710 m has been installed by Voith Heidenheim (West Germany). The turbine runner has a mean diameter of 3.5 m and has 21 buckets, each bucket weighing 420 kg. Eight such turbines have been installed in the above power house.

Allis-Chalmers double overhung unit in San Francisquito Plant 1 of the city of Los Angeles (USA) has one of the *largest jets* in the world, 35.56 cm in diameter with nozzle diameter of 42.55 cm. Each runner has a diameter of 4.46 m and runs at 171.6 rpm. The maximum head on the turbine is 286.5 m. When tested it delivered 40,800 HP with an efficiency of 81% under 265 m. However, maximum efficiency goes upto 86%.

Fully Plant in Valais (Switzerland) has a jet diameter of 3.81 cm and runner diameter of 3.26 m giving a jet ratio of 93.4—an exception. Four turbines, each delivering 3,000 HP at 500 rpm under a gross head of 1,650 m and net head of 1,470 m have been installed.

At Idikki (India) eight Pelton turbines are being installed each producing 185,000 HP under 630 metres of head and running at 375 rpm.

### 5.8 Pelton Turbine Installations in India—

(1) **The Tata Hydro-Electric Companies Ltd, Bombay**, Under the Tatas Management there are three firms, namely

- (a) The Andhra Valley Hydro-Electric Co. Ltd.,
- (b) The Tata Hydro-Electric Co. Ltd., and
- (c) The Tata Power Co Ltd.

(a) **The Andhra Valley Hydro-Electric Co Ltd** which came into operation in 1921 has got its Power House situated at Bhivpuri. Local trains connect Bombay Victoria Terminus (VT) to Bhivpuri Road (93.5 km). The Power House is 14.5 km from the Bhivpuri Road Railway Station and the only conveyance to reach the destination is the petrol driven trolley run by the Company on the broad gauge.

The Power House is fed by a 22.5 km long  $\times$  1.6 km wide Andhra Lake made up from a Valley by erecting 272 m long dam at Tokarwadi to block Andhra river. The average rainfall in this area is 250 cm per year.

**Penstocks**—Eight penstocks of which six are installed emerge from 5 km tunnel connecting the Andhra Lake. Each penstock has a diameter of 107 cm which reduces to 81.25 cm at the turbine inlet.

**Turbines**—There are six identical Pelton Turbines with governors manufactured by Pelton Water Wheel Co. Ltd., San Francisco, each designed for—Power = 15,200 BHP      Speed = 300 RPM       $\eta$  = 83%

Head = Max 530 m and Min 516 m

Runner Dimensions :  $D_1 = 2.8$  m,  $D_{out} = 3.84$  m,  $d_g = 24.2$  cm ;  
 $z_2 = 18$

**Generators** : There are six identical Alternators manufactured by G.E. (U.S.A.), each developing 15,000 KVA at 5,000 V at 50 cycles frequency. The exciter for each of the alternators is driven by a small separate Pelton turbine.

(b) **The Tata Hydro-Electric Co Ltd**, which came into operation in 1915 has its Power House situated at Khapoli (Bombay-Poona main-Road). This place is also connected by rail through a branch line from Karjat (Bombay-Poona section), 105 km from Bombay.

The Power House is fed from three lakes namely Shirawta, Walwhan and Lonavala. The first two are joined by a tunnel and the water flows from the former towards the latter. From Walwhan Lake, a 7,000 m long duct line carries water to a forebay at Khandalla. Another 435 m long duct joins Lonavala Lake to the main duct. The average rainfall in this area is 414 cm a year.

**Penstocks**—Khandalla Forebay connects the Power House firstly by two pipes each 2,500 m long having a dia of 193.6 cm reducing to 168 cm and then by six pipes each 1,365 m long having a dia of 108.5 cm reducing to 95.5 cm at the turbine inlet.

**Turbines**—The Power House consists of six Pelton Turbines, five of which are manufactured by Escher Wyss & Co Ltd, Zurich and the sixth by the English Electric Co Ltd, Rugby. Five Escher Wyss Turbines are identical and are equipped with governors. Each of these turbines (1954 units) is designed for :

Power = 17,100 HP ; Head = 513 m ;  
 $Q = 2,940$  lit/sec Speed = 300 RPM ;  $\eta_t = 86\%$  ;

Runner Dimensions :  $D_1 = 2,900$  mm ;  $D_{out} = 3,615$  mm ;  
 $D_o = 200$  mm ;  $z_2 = 23$  ;  $s = 158$  mm ;

Each turbine is equipped with a deflector.

The Pelton Turbine manufactured by English Electric Co has the following design data :

Power = 15,200 BHP ; Head = 505 m (net) ; 525 m (max)  
 Speed = 300 RPM ;  $\eta_t = 84\%$  ;  $z_2 = 24$ .

The turbine is fitted with a diffuser (Seewer design) instead of deflector.

**Generators**—There are four Siemens and two G.E. (U.S.A.) make Alternators, each developing 10,000 KVA, 5,000 V at 50 cycles frequency. The exciters for each of the alternators are separately run by small Pelton turbines.

**Tata Power Co Ltd**—which came into operation in 1927 has its Power Station situated at Bhira which is approachable by the Company's trolley from Kolad 30 km away. Kolad is 150 km from Bombay and is connected by bus. Dharamtar which is 40 km from Kolad also connects by bus. One can reach Dharamtar from Bombay by ferry. The nearest Railway Station for Bhira is Lonavala (Bombay-Poona line) about 86 km from Kolad approachable only by Company's car.

The Power Station is fed by Mulshi Lake which is about 45 km from Poona. The catchment area of the lake is 45 sq km. Average rainfall is 250 cm to 625 cm a year at different places of lake.

**Penstocks**—A 13 km long tunnel connects Mulshi lake to two 34 m long pipes of 208 cm diameters. These two pipes are then connected to five pipes each 1,150 m long having a dia of 185 cm reducing to 130 cm towards bottom. Finally ten pipes each 625 m long and of average 91.5 cm diameter connect the five pipes leading to turbine inlets.

**Turbines**—There are six Pelton Turbine sets of double overhung type, consisting of—

(i) Five identical English Electric Pelton turbines (new units erected in 1954), each developing

Power = 25,900 to 31,200 BHP (by two runners) ;

Head = 440 m to 472 m ; Speed = 375 RPM ;  $\eta_t = 86\%$  ;

Runner Dimensions.  $D_1 = 2.28$  m ;  $d_o = 260$  mm ;  $s = 13.5$  cm ;  $z_s = 20$ .

Each turbine is equipped with Seewer type diffuser.

(ii) One Pelton Turbine manufactured by Morgan Smith (USA) having the following data : Power = 36,000 BHP (by two runners) ;

Head = 440 m (net) ; Speed = 375 RPM ;  $\eta_t = 85\%$ .

Runner Dimensions :  $D_1 = 2.21$  m ;  $D_{out} = 2.82$  m ;

$d_o = 23.6$  cm ;  $z_s = 20$  ;  $s = 22.9$  cm.

The turbine is equipped with deflector.

**Generators**—Five English Electric Alternators developing 21,875 KVA, 9,400 to 11,000 V at 50 cycles and one G.E. (USA) make. The exciters are run by smaller size Pelton turbines.

(2) **Koyna Project Dam**—Koyna dam which is 104 m high and 800 m long, impounds a reservoir with a capacity of 2,000 million m<sup>3</sup>. Water is diverted by 1 Km intake channel leading to a 3.75 Km supply tunnel with diameters ranging from 6.4 to 5.3 m which terminates at an underground surge chamber, followed by a valve chamber at the head of 4 pressure shafts leading to underground power station 240 m below the surface. Normal and maximum heads of 415 m and 510 m together with the available discharge led to the adoption of 4 vertical jets, vertical shaft Pelton turbines with normal maximum ratings of 87,000/97,000 HP at 300 rpm for the first stage. These drive generators with rated output of 70 MVA at 0.9 p.f ; rated voltage and 65°C temp. rise in stator and rotor windings. The maximum continuous output of each set is 77 MVA at 0.9 p.f., and line discharge capacity is 70 MVA at 0.9 p.f. Generating voltage 11 KV  $\pm 5\%$  and flywheel effect  $0.404 \times 10^6$  kg-m<sup>2</sup>.

For the second stage dam height has been increased providing a normal head of 488 m and maximum head of 512 m. The corresponding output from the vertical shaft Pelton—6 jet turbines being 106,000 and 110,000 HP. The rated output of alternators are 83.3 MVA at 0.9 p.f. and 80°C temp. rise in stator and rotor winding. A higher speed of 375 rpm has been adopted in view of the fact that these sets had to have the same outside dimensions as the first stage ones.



**SOME MORE PELTON TURBINES**

| <i>Sl. No.</i> | <i>Scheme</i>                                                             | <i>Site of Power House</i>              | <i>Source of Water</i>                         |
|----------------|---------------------------------------------------------------------------|-----------------------------------------|------------------------------------------------|
| 1              | Mandi Hydro-Electric Scheme (refer Fig 5.7) Himachal Pradesh)             | (a) Joginder Nagar (Shanan)<br>(b) Basi | Uhl River (Reservoir at Brot) Tailrace of (a)  |
| 2              | Simla Hydro-Electric Scheme (Simla Municipality)                          | Chamba                                  | Nauti (tributary of Sutlaj River)              |
| 3              | Nainital Hydro Electric Scheme Kumaun Hills, U.P.                         | Nainital                                | Natural lake                                   |
| 4              | Mussoorie Hydro-Electric Ltd. (Mussoorie Municipality)                    | Mussoorie (10 km from city)             | Kiarkuli River                                 |
| 5              | Shillong Hydro-electric Ltd (Assam Govt.)                                 | Shillong                                | Stream nearby                                  |
| 6              | Darjeeling District Hydro-Electric Power Scheme (Darjeeling Municipality) | Sedrapong (6½ km from Darjeeling)       | Mountain Streams                               |
| 7              | Govt. of Mysore, (Electrical Department)                                  | Cauvery                                 | Cauvery Falls                                  |
| 8              | The Satara Hydro-Electric Scheme                                          | Satara                                  | Krishna River                                  |
| 9              | Pykara Hydro-Electric Scheme<br>The Glen Morgan Scheme                    | Pykara                                  | Pykara River                                   |
| 10             | Mahatma Gandhi Hydro-Electric Works (Jog Falls, Mysore)                   | Sharavathi (on Bangalore-Honnar Road)   | Sharavathi River                               |
| 11             | Sharavathi Hydro Project                                                  | Linganamakki                            | Sharavathi                                     |
| 12             | Pallivasal Power Station (Kerala Govt.)                                   | Pallivasal (11 km from Munnar)          | Mudirapuzha River (Tributary of River Periyar) |
| 13             | Mohora Power Station (Kashmir)                                            | Mohora (88 km from Srinagar)            | Jhelum River                                   |
| 14             | Koyna Hydro-Electric Project (Maharashtra)                                | Koyna (240 km from Bombay)              | Koyna River                                    |
| 15             | Kundah Hydro-Electric Project, Madras (Two Stages)                        | Kundah                                  | Kundah River (Nilgiris)                        |

INSTALLATIONS IN INDIA

| Manufacturers or Suppliers          | No. of Turbines | Turbine Specifications |             |                             |           |                             |
|-------------------------------------|-----------------|------------------------|-------------|-----------------------------|-----------|-----------------------------|
|                                     |                 | Power HP Each          | Head m      | Water Quantity lit/s.c each | Speed rpm | Turbine efficiency per cent |
| 1 (a) Boving & Co.                  | 4               | 17,250                 | 510 (av)    | 2,800                       | 428.5     | 88.6                        |
| (b) English Electric HEL            | 3               | 21,000 (4 jets)        | 610 max 335 | 5,570                       | 500       | —                           |
| 2 Boving & Co.                      | 3               | 380                    | 167.5       | 218                         | —         | 88                          |
|                                     | 2               | 600                    | 167.5       | 340                         | —         | —                           |
| 3                                   | 3               | 175                    | 290         | 615                         | —         | 73.5                        |
| 4 Boving & Co.                      | 2               | 685                    | 305         | 240                         | 750       | —                           |
| 5 James Gordon                      | 2               | 380                    | 305         | —                           | 600       | —                           |
| 6 Voith, Germany                    | 2               | 135                    | 168.5       | —                           | 1,000     | 80                          |
| Escher Wyss                         | 2               | 368                    | 173         | 190                         | 1,000     | 85                          |
|                                     | 2               | 294                    | 198         | 139                         | 715       | 80                          |
| 7 Escher Wyss                       | 6               | 1,450                  | 117         | —                           | —         | —                           |
| 8 Voith                             | 2               | 70                     | —           | —                           | 1,000     | —                           |
| Voith                               | 2               | 11,000                 | 940         | —                           | 600       | —                           |
| 9 (a) Escher Wyss                   | 3               | 11,000                 | 850         | —                           | —         | —                           |
| (b) Escher Wyss                     | 2               | 11,000                 | 850         | 1,900                       | 600       | 89.2                        |
| (c) Escher Wyss                     | 2               | 20,000                 | 850         | —                           | —         | —                           |
| 10 Escher Wyss                      | 4               | 17,500                 | 367 m       | —                           | 428.5     | —                           |
|                                     | 4               | 32,500                 | (gross)     | —                           | 428.5     | —                           |
| 11 Neyrpic (France)                 | 8               | 125,000                | 442         | —                           | 300       | —                           |
| 12 (a) Escher Wyss                  | 3               | 6,000                  | 570         | —                           | 750       | —                           |
| (b) Boving & Co.                    | 3               | 10,750                 | 570         | —                           | 600       | —                           |
| Neyrpic (France)                    |                 |                        |             | —                           |           |                             |
| 13 A. Dobbie (USA)                  | 4               | 812                    | 128         | 782                         | 500       | —                           |
| (Two-nozzle, 16 buckets per runner) |                 |                        |             |                             |           |                             |
| 14 Neyrpic (France)                 | 4               | 87,500                 | 475 (net)   | —                           | 300       | —                           |
| Charmiles                           | 4               | 100,000                | —           | —                           | —         | —                           |
|                                     | 2               | 27,000                 | —           | —                           | —         | —                           |
| 15 Dominion Engg (Canada)           | 1               | 29,200 (5 jet)         | 360         | —                           | —         | —                           |
|                                     | 1               | 50,700 (5 jet)         | 690         | —                           | —         | —                           |

(3) **Mahatma Gandhi Hydro-electric Development**—It consists of a storage reservoir at Hirebhasbar, 24 Km upstream of Jog falls, capable of storing 708 million  $\text{m}^3$  of water, a diversion weir at Kargal 4.8 Km upstream of falls, a power channel 5.6 Km long to carry  $33.76 \text{ m}^3/\text{sec}$  and 4 penstocks extending from a forebay to the turbines of the generators

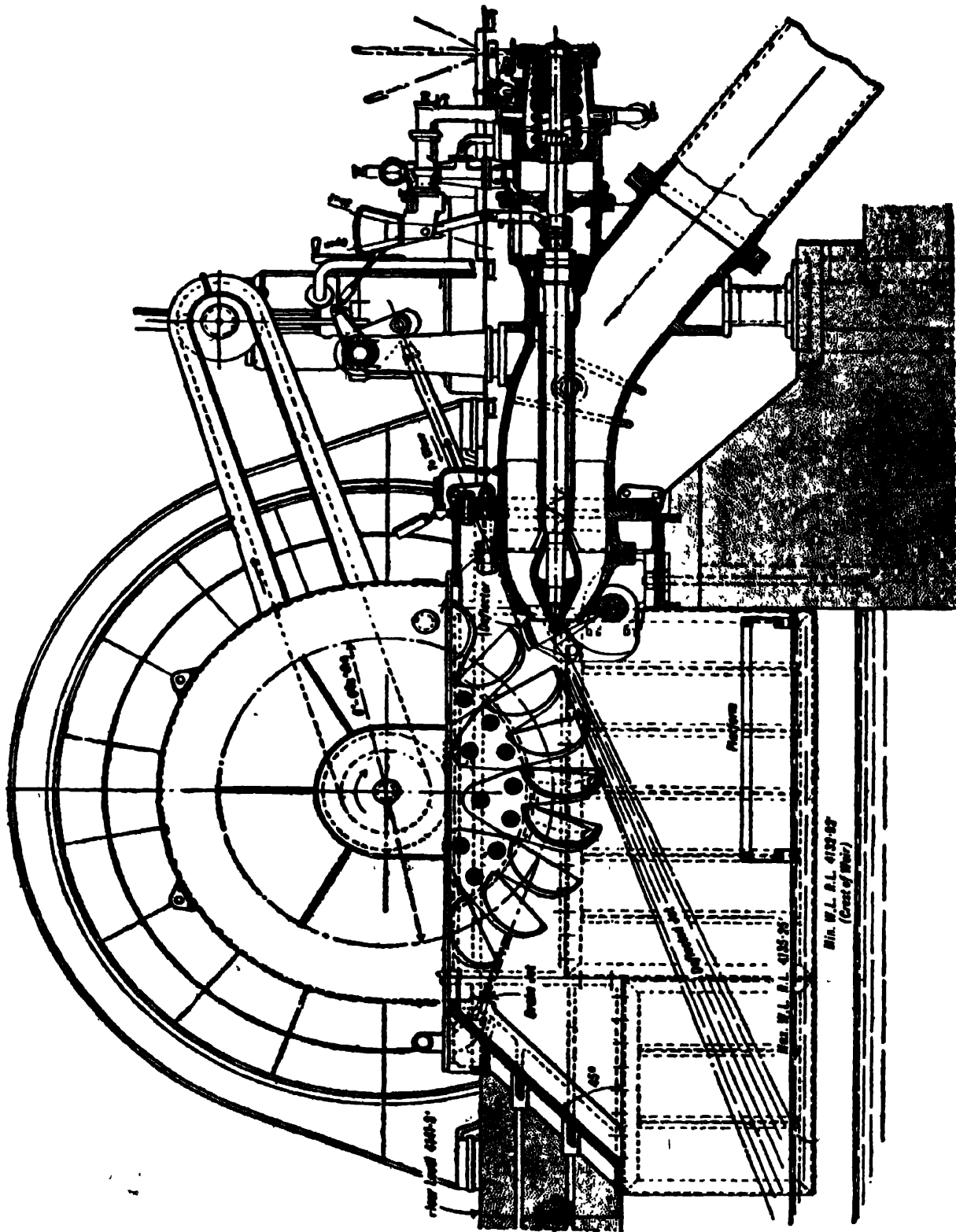


**Fig 5.7 Pelton Turbine (17,250 HP, 510 m head and 428.5 RPM) installed at Joginder Nagar (Mandi Hydro-Electric Scheme). Supplied by Boving & Co. Ltd., London ("The Engineer" dated 8-1-1932). (See unsolved Prob 5.40)**

located in power house in the valley 366 m below the forebay. Total installed capacity is 120 MW.

(4) **Sharavathi Project**—It starts with storage reservoir of 4,419.26 million m<sup>3</sup> at Linganamakki, some 9.65 Km up stream of the falls, from which water is led into reinforced cement concrete conduit along the hill range running on the left bank. After traversing a ridge by a 7.32 m dia water horse shoe tunnel the water enters a subsidiary reservoir created at a dam across the Talakalab stream, a tributary of Sharavathi. The concrete conduit is designed to carry 175.63 m<sup>3</sup>/sec and the Talakalab reservoir has a gross capacity of 141.64 million m<sup>3</sup>. Water from this reservoir is led to the parallel tunnel at Vodenbyle 6.7 m in diameter each designed to carry 147.25 m<sup>3</sup>/sec. The surge tanks, one for each tunnel are 15.25 m diameter and 54.8 m in depth. Five penstocks take from each of the surge tanks, one for each 89.1 MW generator. The total installed capacity is thus 189 MW.

The estimated cost of the project including the first two stages is Rs 124.23 crores.

#### Planned Stages of the Sharavathi Project

|            |                      |           |
|------------|----------------------|-----------|
| Stage I—   | Two units of 89.1 MW | —178.2 MW |
| Stage II—  | Six units of 89.1 MW | —534.6 MW |
| Stage III— | Two units of 89.1 MW | —178.2 MW |

#### Proposed Future Development

Power station at base of storage dam ——— 60.0 MW

Tail race development between Mahatma Gandhi Works and Station under construction ———— 20.0 MW

Tail race development at Gerusoppa dam stream from power station under construction ———— 145.0

Total = 1,160 MW

#### DESIGN OF COMPONENT PARTS OF PELTON TURBINE

**5.9 Turbine Power**—The power available from water can be calculated for a specified net head and rate of flow of water, as follows

$$P_a = \frac{\gamma \cdot Q \cdot H}{75} \quad \dots(5.1)$$

where

$P_a$  = power available in HP ;

$\gamma$  = specific weight of water in kg/m<sup>3</sup> ;

$Q$  = rate of flow of water in m<sup>3</sup>/sec ;

$H$  = net head in m, acting at the turbine inlet.

This formula gives the theoretical available power and the losses in the turbine are not taken into account. To obtain the power supplied on the turbine shaft,  $P_a$  is multiplied by the overall efficiency  $\eta_t$  of the turbine, approximate value of which are given in Table 5.1.

$$\therefore P_t = P_a \cdot \eta_t \quad \dots(5.2)$$

where

$P_t$  = power supplied by the turbine in BHP.

TABLE 5.1  
Practical Data  $\eta_s$

| $\eta_s$ in % | 85  | 88    | 89     | 90      |
|---------------|-----|-------|--------|---------|
| $P_s$ in HP   | 100 | 1,000 | 10,000 | 100,000 |

**5.10 Nozzle and Jet Diameters**—The least diameter of the free jet (refer Fig 5.8) which occurs at the vena contracta is given by the relation

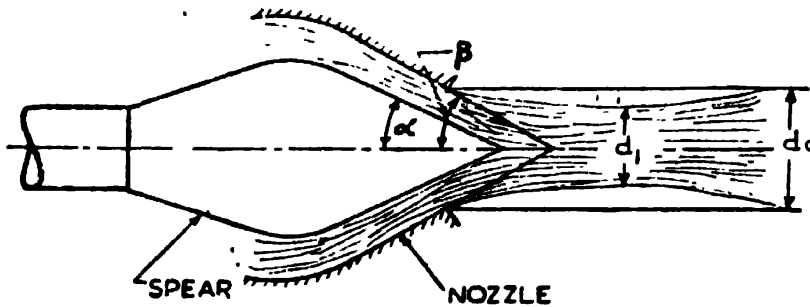


Fig 5.8 Spear, Nozzle and Jet

$$Q = \frac{\pi}{4} \cdot d_1^2 \cdot v_1$$

where  $d_1$  = the least dia of the jet in m ;  
 $v_1$  = velocity of jet in m/sec, measured at vena contracta ;  
 $Q$  = discharge through jet in m<sup>3</sup>/sec ;

Now  $v_1 = K_{v_1} \sqrt{2gH}$

where  $K_{v_1}$  = co-efficient of velocity ;

and  $H$  = net head in m.

Value of  $K_{v_1}$  varies from 0.98 to 0.99

or 
$$d_1 = \sqrt{\frac{4Q}{\pi \cdot K_{v_1} \sqrt{2g} \cdot \sqrt{H}}} \quad \dots(5.3)$$

Substituting average value of  $K_{v_1} = 0.985$ ,

(b) Least diameter of jet

$$\begin{aligned} d_1 &= \sqrt{\frac{4}{\pi \times 0.985 \times \sqrt{2 \times 9.81}}} \cdot \sqrt{\frac{Q}{\sqrt{H}}} \\ &= 0.5415 \cdot \sqrt{Q_1} \end{aligned} \quad \dots(5.3a)$$

Let diameter of the nozzle at the outlet be  $d_o$  (refer Fig 5.8) which is always greater than  $d_1$ . It can be determined as follows—

$$Q = a_o \cdot v_o$$

where  $v_o = K_{v_o} \sqrt{2gH}$

The experimental value of  $K_s$  is 0.81 to 0.83

$$\therefore a_o = \frac{Q}{K_{s_o} \cdot \sqrt{2gH}} \quad \dots(5.4)$$

Value of  $a_o$  is obtained from the following experimental relation :

$$a_o = A \cdot s - B \cdot s^2 \quad \dots(5.5)$$

where  $A$  and  $B$  are constants for a particular nozzle and their values are as follows :

$$A = 2\pi \cdot r_o \cdot \sin \alpha \frac{\beta - \alpha}{\sin(\beta - \alpha)} \quad \dots(5.6)$$

$$B = \frac{\pi \cdot (\beta - \alpha) (\sin \beta - \sin \alpha) \cdot \sin^2 \alpha}{\sin^3(\beta - \alpha)} \quad \dots(5.7)$$

also  $s$  = maximum spear travel ;

$2\alpha$  = angle of spear

and  $2\beta$  = angle of nozzle. (refer Fig 5.8)

The dimensions of the nozzle and the branch pipe are generally given in terms of  $d_1$  and can be determined when  $d_1$  is known.

These values are calculated for

$$\alpha = 25^\circ \quad \beta = 42^\circ.$$

These values of  $\alpha$  and  $\beta$  are mostly used by European Continental Turbine Manufacturers. In UK, however, the values of  $\alpha$  and  $\beta$  are taken as  $22\frac{1}{2}^\circ$  and  $28\frac{1}{2}^\circ$  respectively.

**5.11 Multi-jets**—The specific speed of a Pelton turbine is calculated for a single jet. Thus —

$$N_s = \frac{N \cdot \sqrt{P_s}}{H^{\frac{5}{4}}}$$

Introducing dimensionless quantity  $K_s$  in place of  $N_s$

$$K_s = \frac{Q \cdot N^3}{v^3}$$

$$\text{or } K_s = \left( \frac{N_s}{32.2} \right)^3, \text{ assuming } \eta_t = 90\% \quad (\text{refer Eqn 4.3})$$

Considering  $N_{s_{max}}$  for Pelton Turbine = 32.2 i.e. nearing 35

$$K_s \approx 1 \text{ for a single jet.}$$

If for a single wheel two nozzles i.e., two jets are provided, then total power =  $2 P_s$

$$\text{and specific speed } N_s = \frac{N \cdot \sqrt{2P_s}}{H^{\frac{5}{4}}} = N_s \cdot \sqrt{2}$$

This shows that for specific speed greater than 35 a multi-nozzle Pelton turbine must be employed. Explaining in terms of dimensionless factor, for two nozzles  $K_s$  lies between 1 and 2, for three nozzles  $K_s$  lies between 2 and 3 and so on.

Maximum number of nozzles so far used in a turbine is six in a vertical installation and two in a horizontal installation.

Though multi-jets are essential when the specific speed of a runner is more than 35, but such an arrangement makes the governing of turbine complicated and more expensive.

**5.12 Mean Diameter of Pelton Runner**—Let the mean diameter i.e. pitch circle diameter (PCD) of Pelton runner be  $D_1$  and the speed be  $N$  rpm when the net head is  $H$ .

Circumferential velocity of a bucket,

$$u_1 \propto \sqrt{2gH} \quad \text{or} \quad u_1 = K_{u_1} \cdot \sqrt{2gH}$$

The velocity co-efficient  $K_{u_1}$  is known as *Speed Ratio* i.e., the ratio of the peripheral or linear velocity of the buckets at their mean diameter to the theoretical spouting velocity of water under the head acting on the turbine,

$$\text{or} \quad K_{u_1} = \frac{u_1}{\sqrt{2gH}}$$

Sometimes the speed ratio  $K_{u_1}$  is denoted by  $\phi$ .

Circumferential velocity of wheel can also be given as

$$u_1 = \frac{\pi \cdot D_1 \cdot N}{60} \text{ m/sec}$$

The centre line of the jet is always a tangent to the wheel pitch circle drawn with diameter  $D_1$ , at its lowest point, as shown in Fig 5.9.

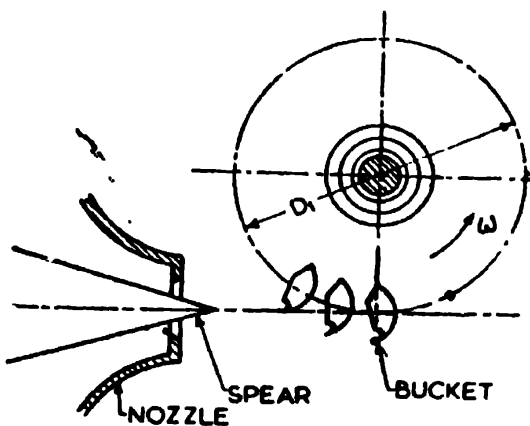


Fig 5.9 Mean Diameter of Pelton Runner

$$\begin{aligned} \text{Now } K_{u_1} \cdot \sqrt{2gH} &= \frac{\pi \cdot D_1 \cdot N}{60} \\ \text{or } D_1 &= \frac{60 \cdot K_{u_1} \cdot \sqrt{2gH}}{\pi \cdot N} \text{ m} \dots (5.8) \\ &= \frac{84.5 K_{u_1} \cdot \sqrt{H}}{N} \text{ m} \\ &\dots (5.8a) \end{aligned}$$

Taking an average value for  $K_{u_1} = 0.45$

$$\begin{aligned} D_1 &= 84.5 \times 0.45 \times \frac{\sqrt{H}}{N} \\ &= 38.6 \frac{\sqrt{H}}{N} \dots (5.8b) \end{aligned}$$

Working speed,  $N$  (RPM), of the turbine depends on the nature of the driven unit. If an electrical generator (alternator) is directly coupled to the turbine,  $N$  will be determined by the relation :

$$f = \frac{p \cdot N}{60} \dots (5.9)$$

where  $f$  is the frequency of the alternator in cycles/sec and  $p$  is the number of pair of poles.

If  $f = 50$  cycles per second, as is usually the case in this country, the speed

$$N = \frac{3,000}{p} \dots (5.9a)$$

**TABLE 5.2**  
**Experimental Values for  $K_{u_1}$  Corresponding to**  
**Maximum Efficiency**

| $N_s$     | 10   | ... | ... | 32   |
|-----------|------|-----|-----|------|
| $K_{u_1}$ | 0.46 | ... | ... | 0.44 |

Cost of a high speed generator being less, it is economical to select a high speed wherever possible. Generally for a Pelton Turbine, the speed is 500, 375, 300 or 250 RPM, depending upon the size of the unit.

After  $N$  and  $K_{u_1}$  have been fixed,  $D_1$  can be easily calculated from Eqn 5.8.

**5.13 Selection of Speed**—For given conditions, Pelton turbines have a wide range of speed. There is a definite speed yielding maximum efficiency as seen from the Table 5.3. If the speed of the turbine is made higher, then

(a) *the specific speed will increase.*

Its advantages are—

- (i) The size of the turbine will become smaller and hence it will be less costly.
- (ii) The jet diameter will decrease, Reduction in jet diameter will raise the jet ratio and enhance the turbine efficiency (refer Table 5.3).

The disadvantage of having higher specific speed is that it necessitates multi-jets with which the governing becomes complicated and more expensive.

(b) *The speed of directly coupled generator will increase.* This means that smaller number of pair of poles are required and hence the generator will also be less costly.

(c) *Material* employed for high-speed machines (turbines and generator) will be costly, as high speed causes great stresses in revolving parts.

All points noted above should be kept in mind and conflicting interest compromised in the selection of a proper speed.

#### 5.14 Jet Ratio of a Pelton Turbine —

Definition : Jet Ratio =  $\frac{\text{Mean diameter of runner}}{\text{Least diameter of jet}}$

Symbolically,  $m = \frac{D_1}{d_1}$

Jet ratio is an important feature of Pelton turbine and influences its characteristics. It is, generally, used instead of specific speed in the selection and design of Pelton Turbines.



Relation between  $m$  and  $N_s$  :

$$N_s = \frac{N \cdot \sqrt{P_1}}{H^{\frac{5}{4}}} \quad \text{where} \quad N = \frac{60 \cdot K_{u1} \cdot \sqrt{2gH}}{\pi \cdot D_1} \text{ RPM}$$

$$\text{Now} \quad P_t = \frac{\gamma \cdot Q \cdot H}{75} \eta_t \quad \text{HP}$$

$$\text{but since} \quad Q = \frac{\pi}{4} d_1^3 \cdot K_{v1} \sqrt{2gH} \quad \text{m}^3/\text{sec}$$

$$P_t = \frac{1,000 \times \frac{\pi}{4} d_1^3 \cdot K_{v1} \sqrt{2gH} \cdot H}{75} \cdot \eta_t$$

$$N_s = \frac{\frac{60 \cdot K_{u1} \cdot \sqrt{2g} \cdot \sqrt{H}}{\pi \cdot D_1} \times \sqrt{\frac{1,000 \times \pi \times \sqrt{2g} \cdot K_{v1}}{4 \times 75} \times d_1^3 \cdot H^{\frac{3}{2}} \eta_t}}{H^{\frac{5}{4}}}$$

$$= \frac{60 \cdot \sqrt{2g} \cdot \sqrt{2g} \cdot \sqrt{1,000}}{\sqrt{\pi} \cdot \sqrt{4 \times 75}} \cdot K_{u1} \sqrt{K_{v1}} \cdot \frac{d_1}{D_1} \cdot \sqrt{\eta_t}$$

$$= 579 \cdot K_{u1} \cdot \sqrt{K_{v1}} \cdot \frac{1}{m} \cdot \sqrt{\eta_t} \quad \dots(5.10)$$

Assuming mean values,

$$K_{u1} = 0.45 \quad \text{and} \quad K_{v1} = 0.985$$

$$\therefore \text{Specific speed } N_s = 260 \times \frac{\sqrt{\eta_t}}{m} \quad \dots(5.10a)$$

In modern turbines the value of jet ratio ranges from 10 to 30 depending on the specific speed and efficiency of the turbine.

TABLE 5.3  
Experimental Values of  $m$

| $m$      | 6.5  | 7.5  | 10   | 20  |
|----------|------|------|------|-----|
| $N_s$    | 35   | 32   | 24   | 10  |
| $\eta_t$ | 0.82 | 0.86 | 0.89 | 0.9 |

For maximum efficiency  $m$  should be from 11 to 14. Highest jet ratio in the world is 110 for the five turbine units of Kt. Glarus Power House in Switzerland. Specifications of each of these Pelton turbines are :

Power = 3,000 HP ;

Speed = 500 RPM ;

$D_1 = 5,360$  mm ;

$d_1 = 48.77$  mm ;

$H_{gross} = 1,650$  m ;

$H_{net} = 1,480$  m ;

The minimum value of jet ratio  $m$  is taken as 10. Small value of  $m$  will mean a smaller diameter of wheel  $D_1$  the jet diameter  $d_1$  remaining constant. This will result in close spacing of buckets with which the turbine efficiency will decrease. In case the value of  $m$  is high, the wheel will become bulky and the efficiency will be less with this arrangement too.

### 5.15 Bucket Dimensions—

Main dimensions of the bucket are given in Fig 5.10.

Length  $L = 2.3$  to  $2.8$  times  $d_1$

Width  $B = 2.8$  to  $3.2$  times  $d_1$

Depth  $T = 0.6$  to  $0.9$  times  $d_1$

Inlet Angle  $\beta_1 \approx 5^\circ$  to  $8^\circ$ ; Outlet angle  $\beta_2 \approx 10^\circ$  to  $20^\circ$  at centre.

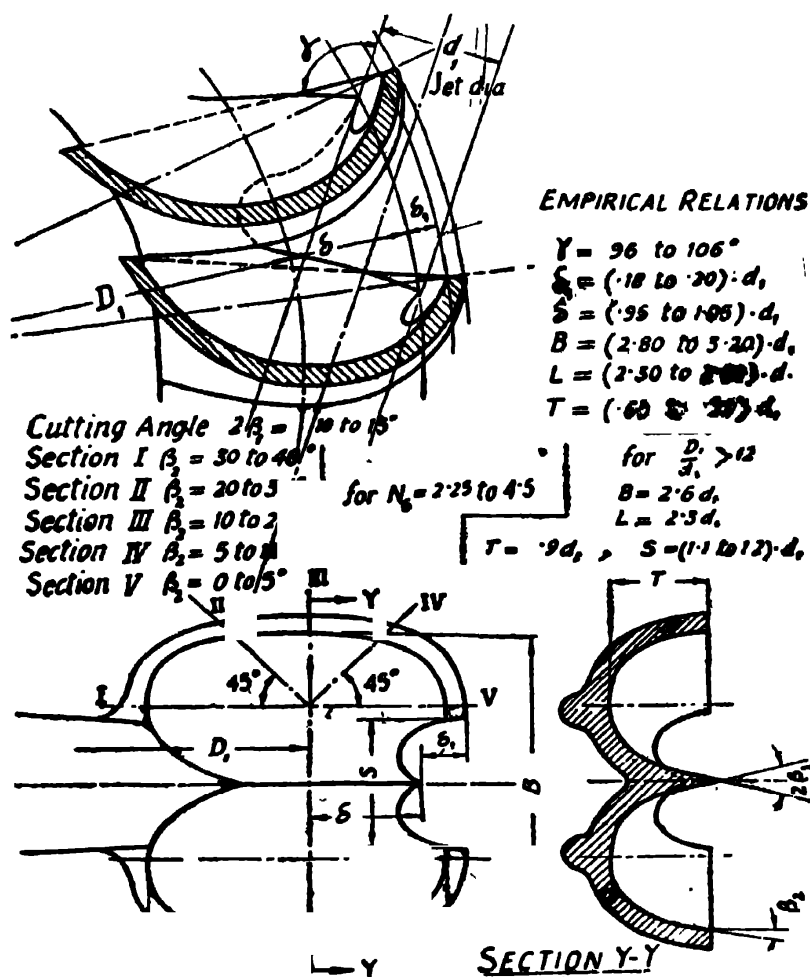


Fig 5.10 Construction of Pelton Runner Blade

The main dimension of the bucket is its width  $B$ . If the bucket width is too small, the water of the jet is not deflected smoothly with the result that there is sufficient dissipation of energy due to turbulence in the bucket. If the width is too large, the surface over which the water flows, becomes also large giving rise to more friction. The optimum values which give maximum efficiency are given above.

**5.16 Minimum Number of Buckets—**Let the number of buckets be  $z$ . Then the angular separation between two buckets i.e., the angular pitch,

$$\theta = \frac{2\pi}{z} \text{ radians} = \left( \frac{360}{z} \right)^\circ$$



$$\therefore \frac{2 (R_1 + k \cdot d_1) \cdot \sin \phi}{v_1} = \frac{\psi}{\frac{u_1}{R_1}}$$

$$\therefore \psi = \frac{2 u_1}{v_1} \left( 1 + \frac{2k}{m} \right) \cdot \sin \phi$$

$$\text{But } u_1 = K_{u_1} \cdot \sqrt{2gH} \quad \text{and} \quad v_1 = K_{v_1} \cdot \sqrt{2gH}$$

$$\therefore \psi = \frac{2 K_{u_1}}{K_{v_1}} \left( 1 + \frac{2k}{m} \right) \cdot \sin \phi$$

$$\text{Further, } \sin \phi = \sqrt{1 - \cos^2 \phi}$$

$$\text{and } \cos \phi = \left[ \frac{1 + \frac{1}{m}}{1 + \frac{2k}{m}} \right]$$

$$\text{then } \psi = \frac{2 K_{u_1}}{K_{v_1}} \cdot \left( 1 + \frac{2k}{m} \right) \sqrt{1 - \left\{ \frac{1 + \frac{1}{m}}{1 + \frac{2k}{m}} \right\}^2}$$

$$\text{or } \psi = \frac{2 K_{u_1}}{K_{v_1}} \cdot \sqrt{\left( 1 + \frac{2k}{m} \right)^2 - \left( 1 + \frac{1}{m} \right)^2} \quad \dots(5.12)$$

both  $\psi$  and  $\phi$  can be calculated.

$$\text{Now } \theta = 2\phi - \psi \quad \dots(5.13)$$

and minimum number of buckets required

$$Z = \frac{2\pi}{\theta_{rad}} = \frac{360}{\theta^\circ} \quad \dots(5.14)$$

From the above deduction it is seen that values of  $\psi$  and  $\phi$  depend on  $K_{u_1}$ ,  $K_{v_1}$ ,  $k$  and  $m$ . Also  $K_{u_1}$  itself depends upon  $m$ ; and  $K_{v_1}$  and  $k$ , depend on  $d_1$  which again is a function of  $m$ . Thus the minimum number of buckets obtained above ultimately depends on  $m$ .

Dr. Taygun\* has suggested the following empirical relation for determination of the number of buckets.

$$Z = 0.5 m + 15 \quad \dots(5.15)$$

This equation holds good for all values of  $m$  from 6 to 35.

If the actual number differs from the value given by the above relation, the efficiency will not be a maximum.

**5.17 Turgo-Impulse Turbine** exclusively manufactured by Messrs Gilbert Gilkes & Gordon Ltd., Kendal (England) is a medium head free jet impulse water turbine. It bridges the gap of specific speed (refer Table 4.2) between the Pelton runner and the Francis runner, which is normally

\*Taygun, H.F. The influence of number of buckets on the efficiency of Pelton runner : Dissertation of Swiss Federal Institute of Technology, Zurich, 1946 (Published in German).

done by employing multi-jets for Pelton runner. The runner is made in one-piece single-discharge buckets (half Francis and half Pelton type),

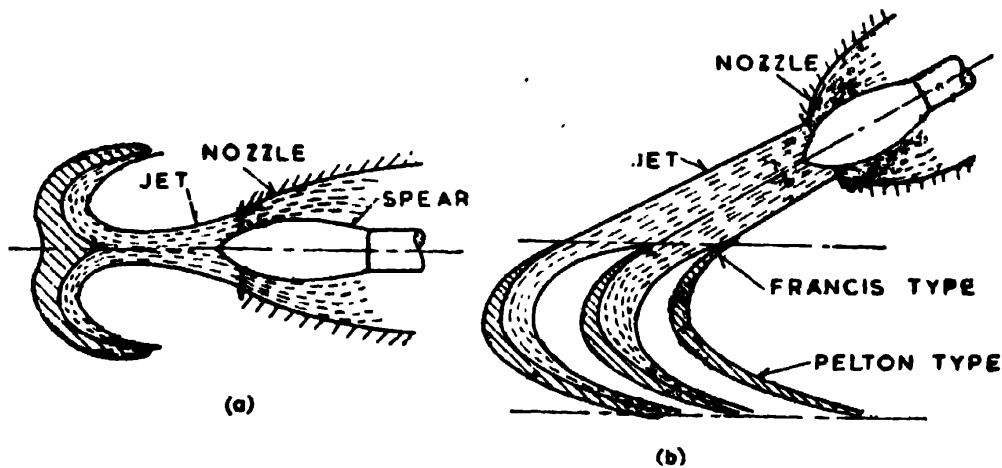


Fig 5.12 Difference between the Water Jet Impinging  
(a) the Pelton Bucket and (b) the Turgo Bucket

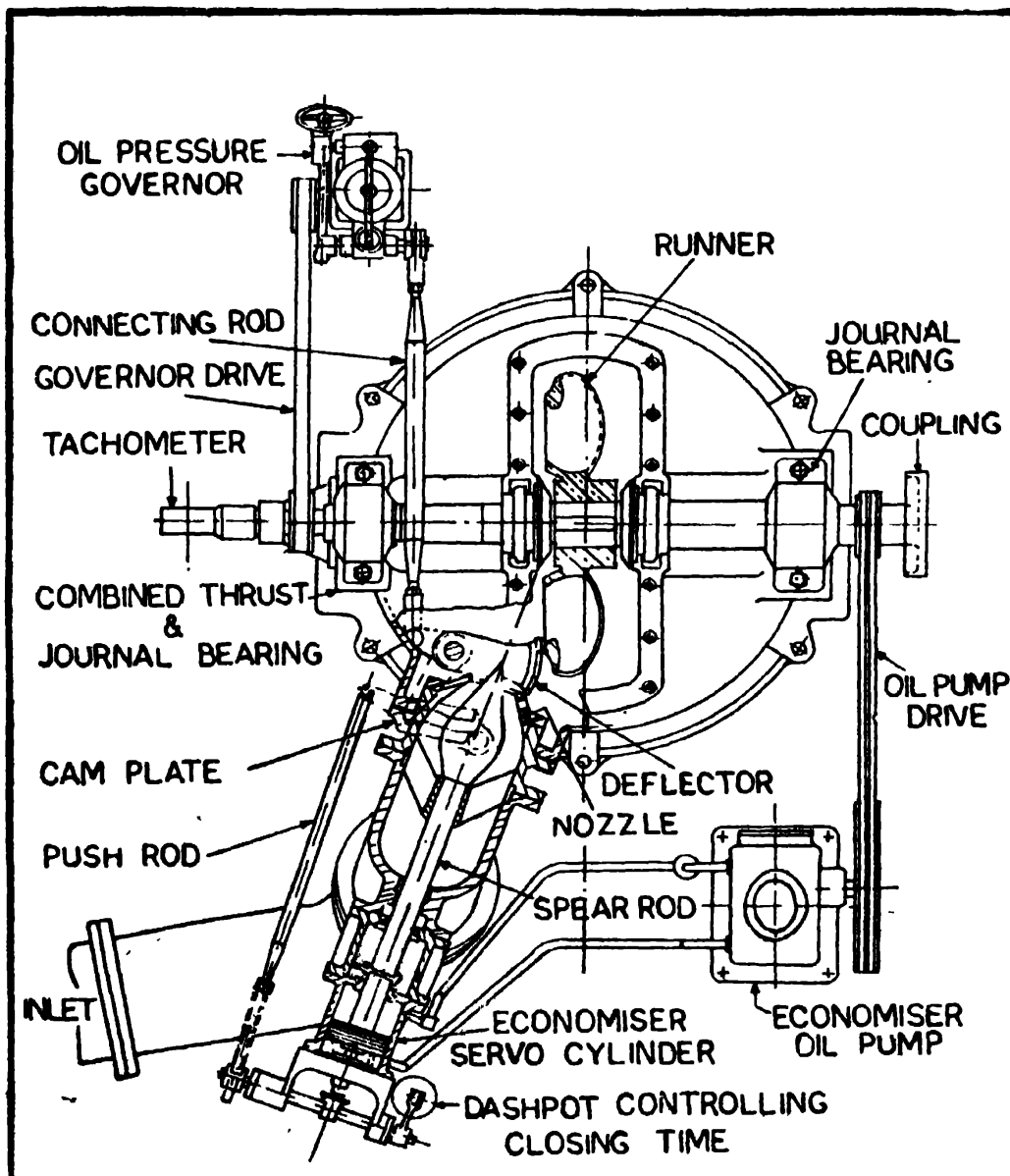


Fig 5.13 Section of a Turgo Impulse Turbine with  
Oil Pressure Governor and Automatic Spear Control

held inwardly by the hub and outwardly by a peripheral band. The jet impinges the buckets obliquely to one side of the runner and discharges at the other. The striking of jet on the buckets is similar to Girard turbine of impulse type (refer Chapter on Introduction Art 3). Fig 5.12 shows the difference between the water jet impinging the Pelton bucket and the Turgo bucket. These turbines have been built upto 10,000 HP ; and for heads upto 300 m. The speed for which these turbines has been designed is as high as 2,000 rpm. Fig 5.13 and Plate 15 show the section through a Turgo-Impulse Turbine with hand regulated spear. Fig 5.13 shows section of such a turbine with oil pressure governor and automatic spear control. The number of jets to the runner is usually one and at most two. The end thrust of the jet is taken by bearings, therefore, the overhung arrangement is not employed for such wheels.

The Power House at Poonch (Jammu and Kashmir State), 320 km from Jammu, fed by Betar River is equipped with two Turgo Impulse turbines, each developing 40 HP at 670 rpm.

**5.18 Banki Turbine** (refer Fig 5.14 and Plate 16)—As already described in Art 4.5, Banki turbine is a free jet, radial flow, impulse turbine

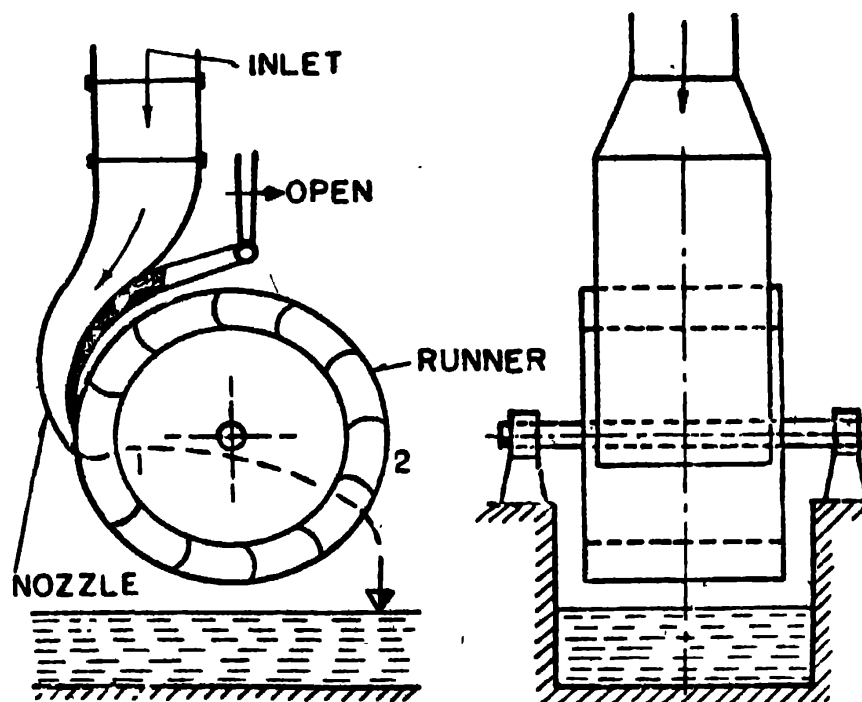


Fig 5.14 Banki Turbine

which like Turgo-impulse turbine, bridges the gap of specific speed (refer Table 4.2) between Pelton and Francis runners. The water jet striking the runner has a rectangular cross section which can be varied lengthwise as well as breadthwise. The water jet strikes the runner blade twice, first at 1 and then at 2 before it leaves the wheel. Thus the velocity of wheel is utilised twice. The runner has a drum-shape, connected to a shaft which is always horizontal. The drum-shaped-runner can be made of any length with which large amount of discharge can be used.

*Practical data :*  $\alpha_1 = 70^\circ$  ;  $u_1 = 0.48 \sqrt{2gH}$

The Banki turbine is generally employed for low head upto about 20 m with an efficiency of 10 to 90%. It can be made of cheaper construction with which it can offer excellent means of exploiting smaller water power resources.

**Problem 5.1** *The Pykara (South India) Power House is equipped with impulse turbines of Pelton type. Each turbine delivers a maximum horsepower of 19,300 when working under a head of 855 m and running at 600 rpm. Find the least diameter of the jet and the mean diameter of the wheel. What would be the approximate diameter of orifice of the nozzle tip? Determine the value of the jet ratio and state if it is within the limits. Specify the number of buckets for the wheel. Take the overall efficiency of the turbine as 89.2%.*

**Solution**

$$P_t = 19,300 \text{ HP}$$

$$H = 855 \text{ m}$$

$$N = 600 \text{ rpm}$$

$$\eta_t = 0.892$$

(a) Least diameter of the jet  $d_1$

$$P_t = \frac{\gamma \cdot Q \cdot H}{75} \cdot \eta_t \quad (\text{refer Eqn 5.3})$$

$$\text{or} \quad 19,300 = \frac{1,000 \times 855 \times Q}{75} \times 0.892$$

$$\therefore \quad Q = \frac{19,300 \times 75}{1,000 \times 855 \times 0.892} = 1.895 \text{ m}^3/\text{sec}$$

$$\text{Further} \quad Q = a_1 \cdot v_1 = \left( \frac{\pi}{4} \cdot d_1^2 \right) (K_{v_1} \cdot \sqrt{2gH})$$

$$\text{Assuming} \quad K_{v_1} = 0.988$$

$$1.895 = \frac{\pi}{4} \times d_1^2 \times 0.988 \times \sqrt{2 \times 9.81 \times 855}$$

$$\text{or} \quad d_1 = \sqrt{\frac{4 \times 1.895}{\pi \times 0.988 \times 4.43 \times 29.2}} = \sqrt{0.189} \\ = 0.1372 \text{ m}$$

$$\text{or} \quad d_1 = 137.2 \text{ mm} \quad \text{Answer}$$

(b) Mean diameter of wheel  $D_1$

$$u_1 = K_{u_1} \sqrt{2gH} = \frac{\pi \cdot D_1 \cdot N}{60}$$

$$\text{Assuming} \quad K_{u_1} = 0.45 \text{ (for maximum efficiency)}$$

$$\therefore \quad D_1 = \frac{60 \times K_{u_1} \cdot \sqrt{2gH}}{\pi \times N} = \frac{60 \times 0.45 \times 4.43 \times 29.2}{\pi \times 600} \\ = 1.85 \text{ m} \quad \text{or} \quad D_1 = 1,850 \text{ mm} \quad \text{Answer}$$

(c) Diameter of nozzle  $d_o = 1.25 \cdot d_1 = 1.25 \times 13.72 = 17.2 \text{ cm}$   
 or  $d_o = 172 \text{ mm}$  *Answer*

(d) Jet ratio  $m = \frac{D_1}{d_1} = \frac{1.85}{0.1372} = 13.5$  *Answer*

As the jet ratio for maximum efficiency is between 11 and 14, this value is, therefore, within the limits.

(e) Number of buckets  $Z = 0.5m + 15$  (refer Eqn 5.15)  
 $= 0.5 \times 13.5 + 15 = 22 \text{ buckets}$  *Answer*

**Problem 5.2** A double overhung Pelton wheel unit is to operate at 30,000 KW generator under an effective head of 300 m at the base of the nozzle. Find the size of jet, mean diameter of runner, synchronous speed and specific speed of each wheel. Assume generator efficiency 93%, Pelton wheel efficiency 85%, co-efficient of nozzle velocity 0.97, speed ratio 0.46 and jet ratio 12.

### Solution

There are two runners keyed on the two ends of the shaft, and the generator lies between them. Each runner is to be taken as one complete turbine. Thus the generator is fed by two Pelton turbines.

$\therefore$  Horsepower developed by each turbine,

$$P_t = \frac{30,000}{2 \times 0.736} \times \frac{1}{0.93} = 22,000 \text{ BHP}$$

Available horsepower of each turbine,

$$P_s = \frac{22,000}{0.85} = 25,900 \text{ HP}$$

$$\text{But } P_s = \frac{\gamma \cdot Q \cdot H}{75} \quad \therefore Q = \frac{P_s \times 75}{\gamma \cdot H} = \frac{25,900 \times 75}{1,000 \times 300} = 6.47 \text{ m}^3/\text{sec}$$

$$\begin{aligned} \text{Velocity of jet } v_1 &= K_{v_1} \cdot \sqrt{2gH} \\ &= 0.97 \times \sqrt{2 \times 9.81 \times 300} = 74.3 \text{ m/sec} \end{aligned}$$

$$\text{Area of jet } a_1 = \frac{Q}{v_1} = \frac{6.47}{74.3} = 0.087 \text{ m}^2$$

$$\therefore \text{Diameter of jet, } d_1 = \sqrt{\frac{0.087 \times 4}{\pi}} = 0.333 \text{ or } 333 \text{ mm} \quad \text{Answer}$$

$$\begin{aligned} \text{Diameter of wheel, } D_1 &= d_1 \times \text{jet ratio} \\ &= 0.333 \times 12 = 4 \text{ m or } 4,000 \text{ mm} \quad \text{Answer} \end{aligned}$$

$$\begin{aligned} \text{Peripheral velocity of the wheel } u_1 &= K_{u_1} \cdot \sqrt{2gH} \\ &= 0.46 \times 74.3 \times \sqrt{300} = 35.3 \text{ m/sec} \end{aligned}$$

$$\text{But } u_1 = \frac{\pi \cdot D_1 \cdot N}{60} \quad \therefore N = \frac{60 \times 35.3}{\pi \times 4} = 168.5 \text{ rpm}$$

Assuming  $f$ , the frequency of generator, as 50 cycles per sec,

$$f = \frac{p \cdot N}{60}, \text{ where } p = \text{number of pair of poles}$$



$$\therefore N_{syn} = \frac{60 \cdot f}{p} = \frac{3,000}{p}$$

$$\text{Assuming } p = 18, \quad N_{syn} = \frac{3,000}{18} = 166.7 \text{ r} \quad \text{Answer}$$

Revised Diameter of the wheel,

$$D_1 = \frac{168.5 \times 4}{166.7} = 4.05 \text{ m}$$

$$= 4,050 \text{ mm} \quad \text{Answer}$$

$$\text{Specific speed } N_s = \frac{N \cdot \sqrt{P_s}}{H^{\frac{5}{4}}} = \frac{166.7 \times \sqrt{22,000}}{300 \times 300^{\frac{1}{4}}}$$

$$= 19.8 \quad \text{Answer}$$

### FORCE, POWER AND EFFICIENCY

**5.19 Velocity Triangles**—In a Pelton wheel the jet strikes a number of buckets simultaneously. It commences to strike the bucket before it has reached a position directly under the centre of the wheel (refer Fig 5.11). The angle which the striking jet makes with the direction of motion of the bucket is denoted by symbol  $\alpha_1$  and in practice it varies from  $8^\circ$  to  $20^\circ$ . As shown earlier in Chapter 1 the force exerted by the jet can be calculated with the help of velocity triangles at inlet and at outlet.

In drawing the typical velocity triangles (refer Fig 5.15) for a Pelton runner the following points should be kept in mind.

$$u_1 = u_2 \dots \dots (\text{since } r_1 \approx r_2)$$

$$w_1 = w_2 \dots \dots (\text{Assuming there is no friction at the blades})$$

$$\alpha_1 < \beta_1, \quad v_1 > v_2, \quad \alpha_2 < \alpha_1$$

$$u_1 < v_1, \quad v_2 < w_2.$$

**5.20 Force Exerted by the Jet**—For the calculation of the force exerted by the jet it is assumed that  $\phi_1 = 0$  i.e., the bucket face is perpendicular to the jet.

$$\text{If } \alpha_1 = 0^\circ, \beta_1 = 180^\circ \text{ (refer Fig 5.16)}$$

$$\text{then } v_{u1} = v_1 \cos \alpha_1 = v_1 = u_1 + w_1$$

$$\text{or } w_1 = v_1 - u_1$$

From velocity triangle at outlet

$$v_{u2} = v_2 \cos \alpha_2 = u_2 - w_2 \cos \beta_2$$

For ideal case,

$$\beta_2 = 0^\circ \text{ i.e., water is deflected back by } 180^\circ$$

$$\therefore v_{u2} = u_2 - w_2 \dots (\cos 0^\circ = 1)$$

$$\text{But } u_1 = u_2 \text{ and } w_1 = w_2 \quad (\text{assuming for ideal case})$$

$$\therefore v_{u2} = u_1 - w_1$$

$$= u_1 - (v_1 - u_1) = 2u_1 - v_1$$

Force exerted by the jet in the direction of  $u_1$

$$F_u = \rho \cdot Q (v_{u1} - v_{u2}) \text{ kg} \quad (\text{refer Eqn 1.21})$$

(assuming that the total quantity  $Q$  strikes the bucket)

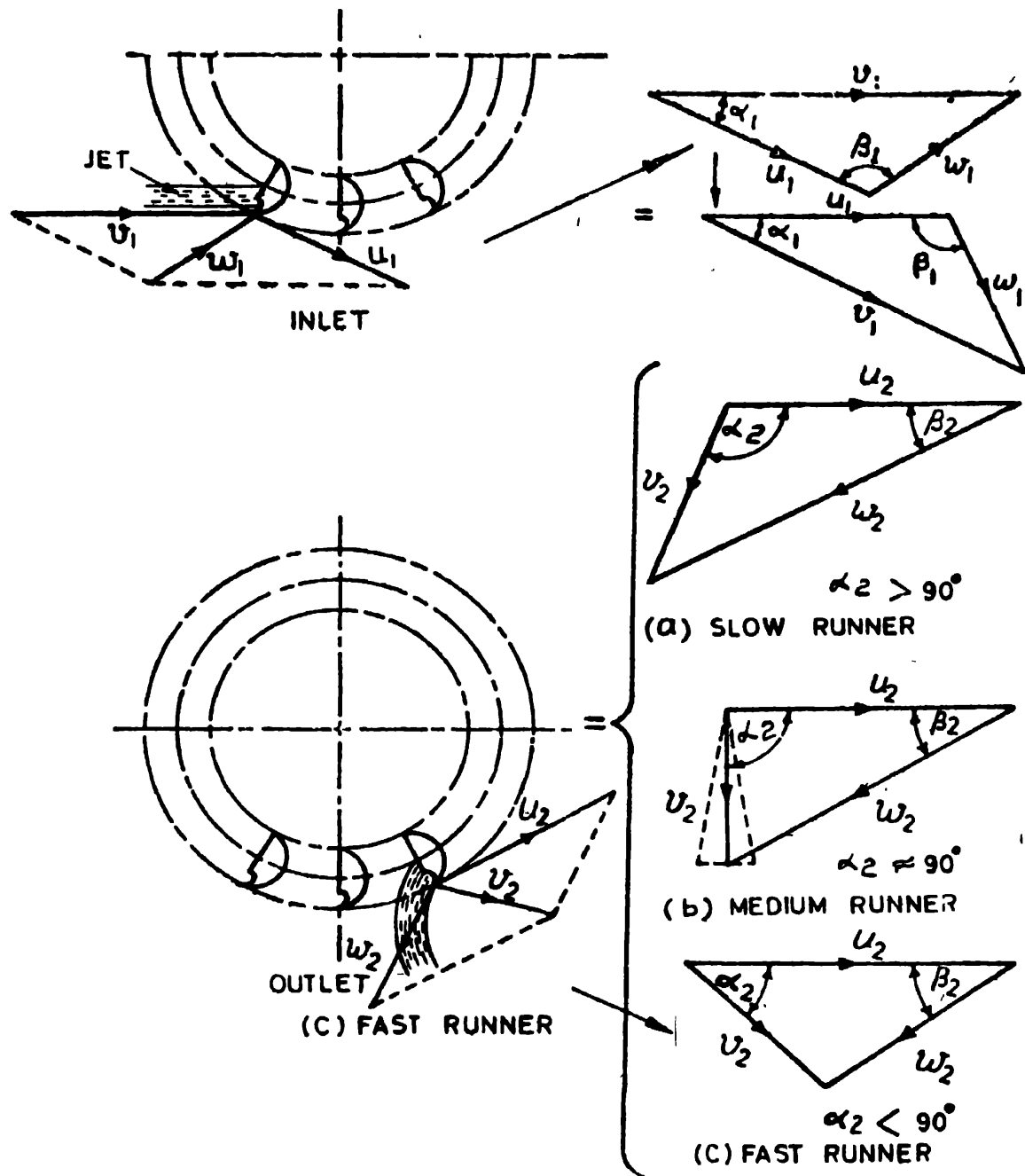


Fig 5.15 Typical Velocity Triangles of Pelton Runner

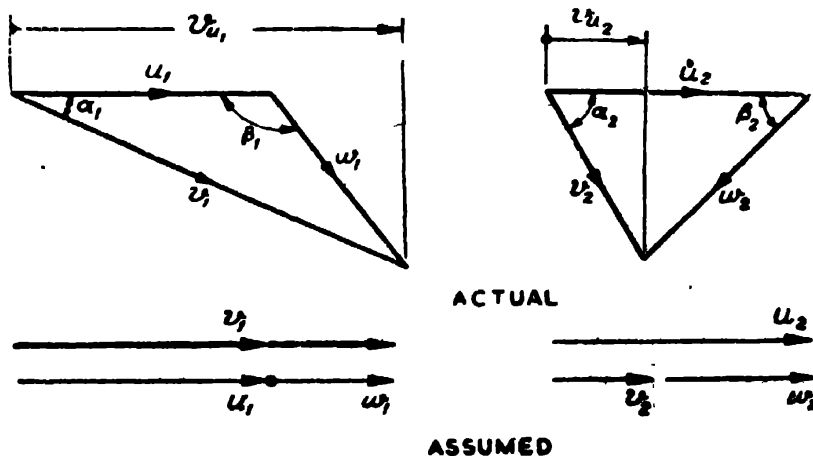


Fig 5.16 Ideal Velocity Triangles

$$\text{or} \quad F_u = \rho Q \{v_1 - (2u_1 - v_1)\} \\ = 2 \rho Q (v_1 - u_1)$$

$$\text{Also, } Q = \frac{\pi}{4} \cdot d_1^2 \cdot K_{v_1} \cdot \sqrt{2gH}$$

$$v_1 = K_{v_1} \cdot \sqrt{2gH}$$

$$\text{and } u_1 = K_{u_1} \cdot \sqrt{2gH}$$

Substituting these values in the expression for  $F_u$

$$F_u = 2 \rho \left( \frac{\pi}{4} \cdot d_1^2 \cdot K_{v_1} \sqrt{2gH} \right) [K_{v_1} - K_{u_1}] \sqrt{2gH} \quad \text{kg} \\ = \rho \cdot \pi \cdot K_{v_1} (K_{v_1} - K_{u_1}) \cdot d_1^2 \cdot g \cdot H \quad \text{kg} \\ = \gamma \cdot \pi \cdot K_{v_1} (K_{v_1} - K_{u_1}) \cdot d_1^2 \cdot H \quad \text{kg} \quad \dots(5.16)$$

Hence, force for unit head and unit diameter,

$$F_{u11} = \frac{F_u}{d_1^2 \cdot H} = \pi \cdot K_{v_1} \cdot (K_{v_1} - K_{u_1}) \cdot \gamma \quad \text{kg} \quad \dots(5.17) \\ \text{per unit head and per unit jet dia.}$$

Force will be maximum when  $K_{u_1} = 0$  i.e., wheel is at rest.

$$(F_{u11})_{\max} = \pi \cdot K_{v_1}^2 \cdot \gamma \\ \text{kg per unit head and per unit jet dia} \quad \dots(5.17a)$$

Substituting average values  $K_{u_1} = 0.985$  and  $\gamma = 1,000 \text{ kg/m}^3$

$$(F_{u11})_{\max} = \pi \cdot (0.985)^2 \times 1,000 \\ = 3,050 \text{ kg per unit head and per unit jet dia.}$$

Under normal working conditions  $K_{u_1} \approx 0.45$ , whence

$$F_{u11} = \pi \times 0.985 \times (0.985 - 0.45) \times 1,000 \\ = 1,657 \text{ kg per unit head and per unit jet dia} \quad \dots(5.17b)$$

For *runaway speed* (i.e., speed at no load or in other words the vanes or wheel running away from the jet with the same velocity as that of the jet,)

$$K_{u_1} = K_{v_1} \quad (\text{refer Fig 5.17})$$

$$\text{then } F_{u11} = 0$$

Theoretically for maximum efficiency,  $\frac{K_{v_1}}{K_{u_1}} = 2$

This can be proved as under :

$$\text{Power } P = F_u \cdot u_1 = 2 \rho Q (v_1 - u_1) \cdot u_1$$

For given  $v_1$ , the power attains maximum value when

$$\frac{dP}{du_1} = 2 \rho Q (v_1 - 2u_1) = 0$$

$$\text{or } v_1 = 2u_1 \quad \text{or} \quad \frac{v_1}{u_1} = 2$$

But in practice, on account of losses,

$$\frac{K_{v1}}{K_{u1}} = 1.8$$

Therefore the runaway speed of a Pelton turbine is 1.8 times the normal working speed.

### 5.21 Work Done and Power Developed by the Jet—

$$\text{Power, } P_H = F_u \cdot u \quad \text{m-kg/sec}$$

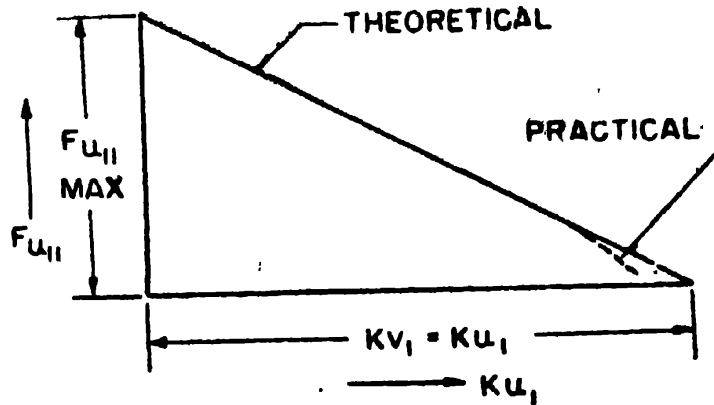


Fig 5.17 Force ( $F_{u11}$ ) vs Speed Ratio ( $K_{v1}$ )

$$\begin{aligned} &= 2 \cdot \frac{\gamma}{g} \cdot \frac{\pi}{4} \cdot d_1^2 \cdot K_{v1} \cdot \sqrt{2gH} \cdot (K_{v1} - K_{u1}) \cdot \sqrt{2gH} \cdot (K_{u1} \sqrt{2gH}) \\ &= \gamma \cdot \pi \cdot d_1^2 \cdot K_{v1} \cdot K_{u1} \cdot (K_{v1} - K_{u1}) \cdot \sqrt{2gH} \cdot H \quad \dots(5.18) \end{aligned}$$

Power developed per unit head and unit jet diameter,

$$P_{H11} = \frac{P_H}{d_1^2 \cdot H \cdot \sqrt{H}} = \gamma \cdot \pi \cdot K_{v1} \cdot K_{u1} \cdot (K_{v1} - K_{u1}) \cdot \sqrt{2g} \quad \dots(5.19)$$

Substituting average values  $K_{v1} = 0.985$  and  $K_{u1} = 0.45$

$$\begin{aligned} P_{H11} &= \pi \times 1,000 \times 0.985 \times 0.45 \times (0.985 - 0.45) \times 4.43 \\ &= 3,333 \text{ kg-m/sec/unit head and unit jet dia.} \end{aligned}$$

$$\text{or } P_{H11} = \frac{3,333}{75} = 44.5 \text{ HP/unit head and jet dia.} \quad \dots(5.19a)$$

### 5.22 Turbine Efficiencies—

(a) **Jet Efficiency or Head Efficiency**—By definition the Jet Efficiency or Head Efficiency,

$$\begin{aligned} \eta_H &= \frac{P_H}{P_a} = \frac{\pi \cdot \gamma \cdot d_1^2 \cdot K_{v1} \cdot K_{u1} \cdot (K_{v1} - K_{u1}) \cdot \sqrt{2gH} \cdot H}{\gamma \cdot \frac{\pi}{4} \cdot d_1^2 \cdot K_{v1} \cdot \sqrt{2gH} \cdot H} \\ &= 4 K_{u1} \cdot (K_{v1} - K_{u1}) \quad \dots(5.20) \end{aligned}$$

For maximum efficiency, assuming  $K_{v1}$  as constant,

$$\frac{d\eta_H}{dK_{u1}} = 0 \quad \text{or} \quad 4(K_{v1} - 2K_{u1}) = 0$$

$$\text{i.e.} \quad K_{u1} = \frac{K_{v1}}{2} \quad \text{or} \quad u_1 = \frac{v_1}{2}$$

$$\begin{aligned} \text{Also } \left( \eta_H \right)_{max} &= 4 \times \frac{K_{v1}}{2} \left( K_{v1} - \frac{K_{v1}}{2} \right) \\ &= K_{v1}^2 \end{aligned}$$

Taking the average value of  $K_{v1} = 0.985$ ,

$$(\eta_H)_{max} = 0.985^2 = 0.97$$

In the ideal case  $(\eta_H)_{max} = 1$  but actually it is within 0.96 to 0.98

**(b) Volumetric Efficiency—**

So far, the force, power and efficiency of the jet have been dealt with. Now the total quantity of water contained in the jet does not strike the bucket and always there is some amount of water that slips and falls in the tail race without doing any useful work. Thus, a new factor called *Volumetric Efficiency* is introduced.

If  $\Delta Q$  be the quantity of water lost on account of slip;

$$\eta_Q = \frac{Q - \Delta Q}{Q} \quad \dots(5.21)$$

Actual value of  $\eta_Q$  is between 0.97 and 0.99

**(c) Hydraulic Efficiency—**Considering the hydraulic losses of the turbine, the hydraulic efficiency can be written as—

$$\eta_h = \left( \frac{H - \Delta H}{H} \right) \left( \frac{Q - \Delta Q}{Q} \right) = \eta_H \cdot \eta_Q \quad \dots(5.22)$$

**(d) Mechanical Efficiency—**There are always some mechanical losses in the transmission of power by the turbine and mechanical efficiency  $\eta_{mech} = 0.97$  to  $0.99$  depending upon—the size and capacity of the unit which may vary from 100 to 1,00,000 HP.

**(e) Final Power Output from Turbines—**

If  $P_s$  be the natural available power produced by jet,

$$P_H = P_s \cdot \eta_H$$

Hydraulic Power generated by turbine,

$$P_h = P_H \cdot \eta_Q = P_s \cdot \eta_H \cdot \eta_Q$$

Net Brake Horse Power developed by the turbine shaft,

$$P_t = P_h \cdot \eta_{mech} = P_s \cdot \eta_H \cdot \eta_Q \cdot \eta_{mech} \quad \dots(5.23)$$

Hence, *Final or Overall Efficiency* of the turbine,

$$\begin{aligned} \eta_t &= \frac{P_t}{P_s} = \frac{P_s \cdot \eta_H \cdot \eta_Q \cdot \eta_{mech}}{P_s} \\ &= \eta_H \cdot \eta_Q \cdot \eta_{mech} \quad \dots(5.24) \end{aligned}$$

Substituting the values for  $\eta_H \cdot \eta_Q$  and  $\eta_{mech}$

$$\begin{aligned} \eta_t &= 0.97 \times 0.98 \times 0.982 \\ &= 0.932 \end{aligned}$$

It must be remembered that in calculating the above values of force, power and efficiencies it was presumed that

$$\beta_1 = 180^\circ, \quad \beta_2 = 0^\circ, \quad w_1 = w_2$$

In practice,

$$\beta_1 = 95^\circ \text{ to } 110^\circ, \quad \beta_2 = 10^\circ \text{ to } 20^\circ \quad \text{and} \quad w_2 = (0.96 \text{ to } 0.98) \cdot w_1$$

More accurate calculations for force, power and efficiencies can be made by taking into account these facts and making the necessary corrections. Some of them are dealt with in the next article.

**5.23 Practical Calculation of Force, Torque, Power and Efficiency—**In practice as the jet impinges on the bucket, the wheel turns through a few degrees such that the direction of motion of the bucket changes. Then the velocity triangles are drawn as shown in Fig 5.18.

The relative velocity of water at inlet,  $w_1$  is given by—

$$w_1 = v_1 - u_1 \quad (\text{Since } v_1 \text{ and } u_1 \text{ are taken as co-linear})$$

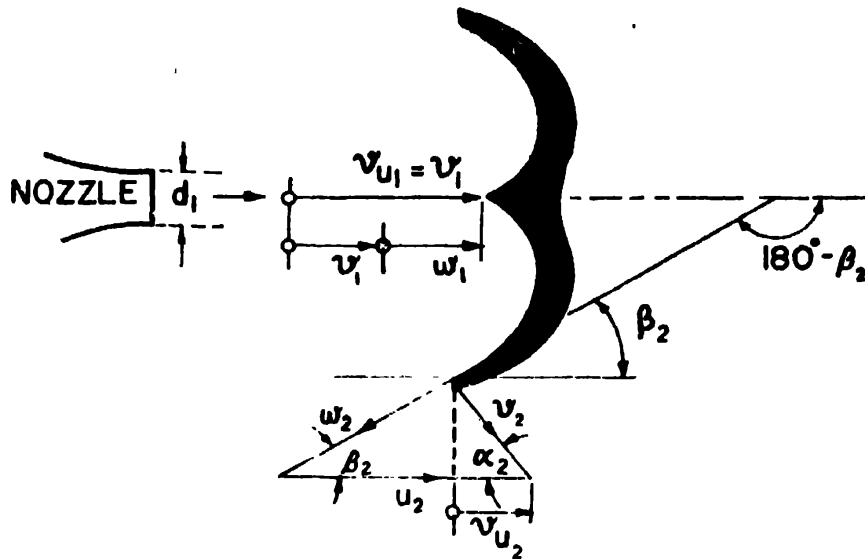


Fig 5.18 Practical Shape of Velocity Triangles Drawn at Bucket Section

Peripheral component of absolute velocity of water is given by

$$\begin{aligned} v_{u1} &= v_1 \cos \alpha_1 = v_1 \quad (\because \beta_1 = 180^\circ \text{ and } \alpha_1 = 0) \\ &= w_1 + u_1 \end{aligned}$$

The relative velocity at outlet,  $w_2$  with which the water leaves the bucket is slightly less than the inlet relative velocity,  $w_1$ . This is due to two reasons—(a) the frictional loss over the inner surface of the bucket (b) the loss due to the jet striking over the splitter of bucket which has always some thickness however sharp it may be.

$\therefore w_2 = k.w_1$  where  $k$  is a co-efficient which takes account of friction loss in flow through the bucket.

Sometimes the relative velocity  $w_2$  is found as follows—

$$\frac{w_1^2}{2g} = \frac{w_2^2}{2g} + k \cdot \frac{w_2^2}{2g}$$

$$\text{or} \quad w_2 = \frac{w_1}{\sqrt{1+k}}$$

Thus the value  $\frac{1}{\sqrt{1+k}}$  is to be substituted for  $k$ .

From outlet velocity triangle—

$$\begin{aligned} v_{u2} &= u_2 - w_2 \cos \beta_2 \\ &= u_2 - k \cdot w_1 \cos \beta_2 \end{aligned}$$

Force of jet on the vane,  $F = \rho Q (v_{u1} - v_{u2})$

$$\text{or} \quad F = \rho Q \{(w_1 + u_1) - (u_2 - k w_1 \cos \beta_2)\}$$

$$\text{Now} \quad u_1 = u_2 = u = \omega r$$

The radius of jet is small as compared to that of wheel, it is assumed that whole of the water impinges on the bucket of radius  $r$ . It is also assumed that whole of the water leaves the bucket at radius  $r$ .

$$\therefore F = \rho Q (w_1 + kw_1 \cos \beta_2) = \rho Q w_1 (1 + k \cos \beta_2)$$

$$\text{or } F = \rho Q (v_1 - u)(1 + k \cos \beta_2) \quad \dots(5.25)$$

$$\left[ \text{or } F = \rho Q (v_1 - u) \left( 1 + \frac{\cos \beta_2}{\sqrt{1+k}} \right) \right] \quad \dots(5.25a)$$

$$\text{Torque on the wheel} = \rho Q (v_1 - u)(1 + k \cos \beta_2) \cdot r \quad \dots(5.26)$$

$$\begin{aligned} \text{Power output of wheel} &= \rho Q (v_1 - u)(1 + k \cos \beta_2) r \cdot \omega \\ &= \rho Q (v_1 - u)(1 + k \cos \beta_2) \cdot u \quad \dots(5.27) \end{aligned}$$

$$\text{Wheel efficiency } \eta_w = \frac{\text{output}}{\text{input}}$$

The input is the kinetic energy of the jet.

$$\begin{aligned} \therefore \eta_w &= \frac{\rho Q (v_1 - u)(1 + k \cos \beta_2) \cdot u}{\frac{1}{2} \rho Q v_1^2} \\ &= \frac{2u (v_1 - u)(1 + k \cos \beta_2)}{v_1^2} \quad \dots(5.28) \end{aligned}$$

The wheel efficiency is maximum when  $\frac{u}{v_1} = \frac{1}{2}$  and assuming  $k$  as constant.

$$\text{or } \eta_w = \frac{2u(v_1 - u) + \left( 1 + \frac{\cos \beta_2}{\sqrt{1+k}} \right)}{v_1^2} \quad \dots(5.28a)$$

The wheel efficiency represents the effectiveness of the wheel in converting the jet energy into mechanical energy. Comparing it with the various efficiencies mentioned above, it will represent the volumetric efficiency and a part of head efficiency (not calculated above). In order to arrive at overall efficiency of the turbine, the wheel efficiency is multiplied by jet efficiency and mechanical efficiency. The jet efficiency takes care of losses in the nozzle and the mechanical efficiency is meant for the bearing friction and windage losses. The overall efficiency for large Pelton turbine is about 85 to 90%.

In addition to the losses explained above, the flow of water is not steady because the buckets move successively in the direction of the jet as the runner revolves. Further the water splashes and scatters as it leaves the buckets. The splashed water after striking the turbine casing forces on to the runner retarding its motion.

**Maximum value of wheel output and wheel efficiency**—For maximum efficiency, differentiate the numerator of Eqn 5.28 with respect to  $u$  and equate it to zero.

$$\frac{d}{du} \left\{ 2u (v_1 - u)(1 + k \cos \beta_2) \right\} = 0$$

$$2v_1 - 4u = 0 \quad \text{or} \quad u = \frac{v_1}{2}$$

Substituting this value of  $u$  is Eqn 5.27 and 5.28 and simplifying :

$$\text{Maximum output of wheel} = \rho Q \cdot \frac{v^3}{4} (1 + k \cos \beta_2) \quad \dots(5.29)$$

$$\text{Maximum wheel efficiency} = \frac{\frac{v_1^3}{2} (1 + k \cos \beta_2)}{v_1^3} = \frac{(1 + k \cos \beta_2)}{2} \quad \dots(5.30)$$

**Problem 5.3** The mean bucket speed of a Pelton turbine is 14 m/sec. The rate of flow of water supplied by the jet under a head of 45 m is 800 litres per second. If the jet is deflected by the buckets at an angle of  $165^\circ$ , find the HP and the efficiency of the turbine.

**Solution**

$$u_1 = u_2 = 14 \text{ m/sec} \quad Q = 800 \text{ lit/sec} = 0.8 \text{ m}^3/\text{sec}$$

$$H = 45 \text{ m} \quad \beta_2 = 180^\circ - 165^\circ = 15^\circ$$

$$\text{Velocity of jet} \quad v_1 = K_{v_1} \cdot \sqrt{2gH} \quad \text{assume } K_{v_1} = 0.985$$

$$v_1 = 0.985 \times \sqrt{19.62 \times 45} = 29.2 \text{ m/sec}$$

$$\text{Assuming} \quad \beta_1 = 180^\circ$$

Draw the inlet velocity triangle (refer Fig 5.19)

$$v_{u_1} = v_1 = 29.2 \text{ m/sec}$$

$$w_1 = v_1 - u_1 = 29.2 - 14 = 15.2 \text{ m/sec}$$

For outlet triangle, assume  $u_1 = u_2$

and  $w_1 = w_2$  (neglecting losses on buckets)

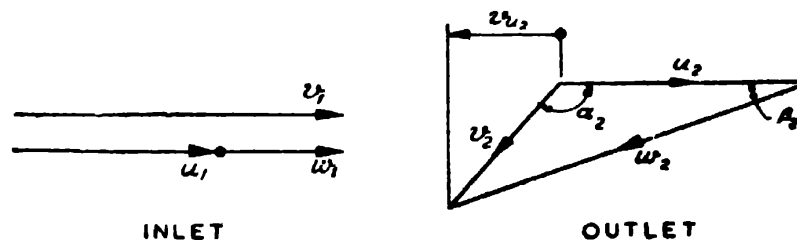


Fig 5.19

$$w_2 = 15.2 \text{ m/sec}$$

$$u_2 = 14 \text{ m/sec}$$

$$v_{u_2} = v_2 \cos \alpha_2 = u_2 - w_2 \cos \beta_2 = 14 - (15.2 \times 0.9659) = 14 - 14.7 = -0.7 \text{ m/sec}$$

$$\text{Work done/sec} = \rho Q (v_{u_1} - v_{u_2}) \cdot u_1 \quad (\text{refer Eqn 1.23})$$

$$= \frac{1,000 \times 0.8}{9.81} \times (29.2 + 0.7) \times 14 \text{ m-kg/sec}$$

$$P_t = \frac{\text{Work done/sec}}{75} = \frac{1,000 \times 0.8 \times 29.9 \times 14}{9.81 \times 75}$$

$$= 445 \text{ HP} \quad \text{Answer}$$

$$\text{Efficiency } \eta_t = \frac{\text{Power developed}}{\text{Available power}} = \frac{445}{\frac{\gamma \cdot Q \cdot H}{75}} = \frac{445 \times 75}{1,000 \times 0.8 \times 45}$$

$$= 0.947 \quad \text{or} \quad 94.7\% \quad \text{Answer}$$



The solution has been found by using the fundamental equation 1.24. However the problem can be solved by applying Eqn 5.27 for wheel output and 5.28 for wheel efficiency.

$$\begin{aligned}
 \text{Wheel output} &= \rho Q (v_1 - u) (1 + k \cos \beta_2) \cdot u \\
 &\quad (\text{where } k = 1, \text{ assuming no losses over the bucket}) \\
 &= \frac{1,000 \times 0.8}{9.81} (29.2 - 14) (1 + 1 \times 0.9659) \times 14 \\
 \text{or } P_w &= \frac{1,000 \times 0.8}{9.81 \times 75} \times 15.2 \times 1.9659 \times 14 = \mathbf{455 \text{ HP Answer}} \\
 \eta_w &= \frac{2 u (v_1 - u) (1 + k \cos \beta_2)}{v_1^3} \\
 &= \frac{2 \times 14 (29.2 - 14) (1 + 1 \times 0.9659)}{29.2^3} = 0.978 \\
 &= \mathbf{97.8\% \quad Answer}
 \end{aligned}$$

The difference between the two efficiencies is due to the fact that  $\eta_t$  is calculated by taking input as available power whereas  $\eta_w$  is determined by the energy input of jet. This means the nozzle losses have been included to find  $\eta_t$  i.e.,

$$\eta_t = \eta_w \cdot k v_1^2 = 0.978 \times 0.985^3 = 0.947$$

**Problem 5.4** A single jet Pelton turbine is required to drive a generator to develop 10,000 KW. The available head at the nozzle is 760 m. Assuming electric generator efficiency 95%. Pelton wheel efficiency 87%, coefficient of velocity for nozzle 0.97, mean bucket velocity 0.46 of jet velocity, outlet angle of the buckets 15 degrees and the relative velocity of the water leaving the buckets 0.85 of that at inlet, find

- (a) the diameter of jet ; (b) the flow in cumecs  
and (c) the force exerted by the jet on the buckets.

If the ratio of the mean bucket circle diameter to the jet diameter is not to be less than 10, find the best synchronous speed for generation at 50 cycles per second and the corresponding mean diameter of the runner.

(AMI Mech E)

### Solution

$$\begin{aligned}
 KW &= 10,000 & H &= 760 \text{ m} & \eta_G &= 0.95 \\
 K_{v_1} &= 0.97 & K_{u_1} &= 0.46 \times 0.97 = 0.446 & \eta_t &= 0.87 \\
 \beta_2 &= 15^\circ & w_2 &= 0.85 w_1, \quad f = 50 \sim & m &\geq 10 \\
 (1 \text{ KW} &= 1.36 \text{ HP} \quad \text{or} \quad \text{HP} = 0.736 \text{ KW})
 \end{aligned}$$

$$\text{Output of the turbine, } P_t = \frac{10,000}{0.736} \times \frac{1}{0.95} = 14,320 \text{ BHP}$$

$$\text{Available horsepower of the turbine, } P_a = \frac{\gamma \cdot Q \cdot H}{75} \text{ HP}$$

$$\text{Also } P_a = \frac{P_t}{\eta_t} = \frac{14,320}{0.87} = 16,460 \text{ HP}$$

$$\therefore Q = \frac{16,460 \times 75}{1,000 \times 760} = 1.62 \text{ m}^3/\text{sec}$$

Also  $Q = \frac{\pi}{4} d_1^2 \cdot K_{v_1} \sqrt{2gH}$

or  $1.62 = \frac{\pi}{4} \times d_1^2 \times 0.97 \times \sqrt{2 \times 9.81 \times 760}$

or  $d_1 = 0.132 \text{ m}$  or **132 mm** *Answer*

Force exerted by jet  $F_u = \rho Q (v_{u_1} - v_{u_2})$  (refer Eqn 1.22)

Assume  $\alpha_1 = 0$  and  $\beta_1 = 180^\circ$

and  $v_1 = K_{v_1} \sqrt{2gH} = 0.97 \times 4.43 \times \sqrt{760} = 118.5 \text{ m/sec}$

$u_1 = K_{u_1} \sqrt{2gH} = 0.446 \times 4.43 \times 27.6 = 54.5 \text{ m/sec}$

From inlet velocity triangle (refer Fig 5.20)

$w_1 = 118.5 - 54.5 = 64 \text{ m/sec}$

$w_2 = 0.85 \times 64 = 54.6 \text{ m/sec}$

$u_2 = u_1 = 54.5 \text{ m/sec}$

$v_{u_1} = v_1 = 118.5 \text{ m/sec}$

From outlet velocity triangle (refer Fig 5.20)

$v_{u_2} = u_2 - w_2 \cos \beta_2$   
 $= 54.5 - 54.6 \times \cos 15^\circ$   
 $= 1.83 \text{ m/sec}$

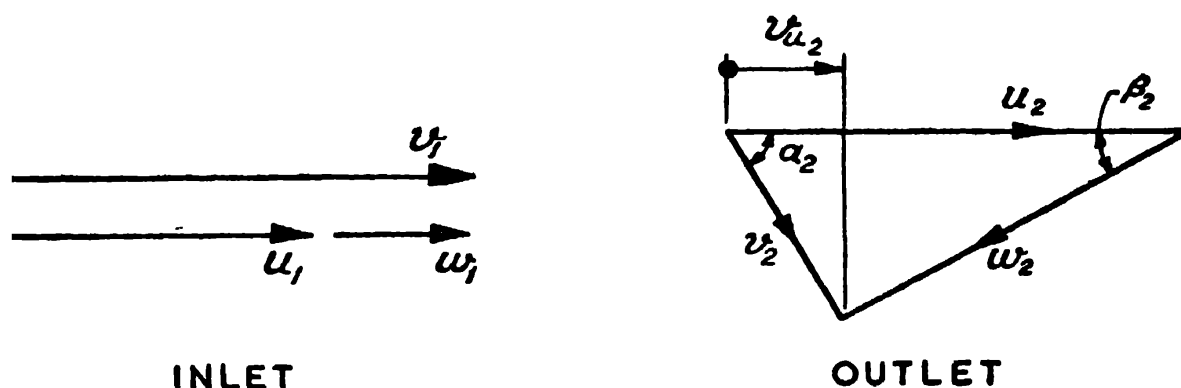


Fig 5.20

$F_u = \rho Q (v_{u_1} - v_{u_2})$   
 $= \frac{1,000 \times 1.62}{9.81} \times (118.5 - 1.83)$   
 $= 19,250 \text{ kg}$  *Answer*

$m = \frac{D_1}{d_1} = 10, \quad \therefore D_1 = 10 \times 132 = 1,320 \text{ mm}$

$u_1 = \frac{\pi \cdot D_1 \cdot N}{60}$  or  $N = \frac{60 \times 54.5}{\pi \times 1.32} = 787 \text{ rpm}$

Frequency generator  $f = \frac{p \cdot N}{60}$ , where  $p$  = No. of pair of poles.

$\therefore$  If  $p = 4$ ,  $N_{syn} = 750 \text{ rpm}$  which is nearest to 787 rpm

Now  $D_{(revtsed)} = \frac{1,320 \times 787}{750} = 1,386 \text{ mm}$  *Answer*

**Problem 5.5** In Pelton wheel installations a small auxiliary nozzle is often fitted, from which a jet of high pressure water can be directed when required on to the reverse side of the buckets, thus acting as a brake and bringing the wheel quickly to rest. Estimate the time required to stop the wheel by the brake nozzle under the following conditions :

Diameter of brake nozzle = 7 cm.

Head at the nozzle = 475 m.

Original velocity of buckets = 45 m/sec.

Corresponding kinetic energy of wheel and all revolving parts = 1,550,300 m-kg.

Assume that the dynamic pressure of the bucket is equivalent to that on flat radial blades.

### Solution

Velocity of jet from brake nozzle,  $v_1 = K_{v_1} \sqrt{2gH}$

Where  $K_{v_1}$  = velocity co efficient  
= 0.985 (assumed)

$$\therefore v_1 = 0.985 \times \sqrt{2 \times 9.81 \times 475} \\ = 95 \text{ m/sec}$$

$$\text{Cross-sectional area of brake nozzle, } a_1 = \frac{\pi}{4} \cdot d_1^2 \\ = \frac{\pi}{4} \times 7^2 = 38.5 \text{ cm}^2$$

$$\therefore \text{ The quantity of water per sec flowing through the brake nozzle, } \\ Q = a_1 \cdot v_1 = 0.00385 \times 95 = 0.366 \text{ m}^3/\text{sec}$$

Initial peripheral velocity of the buckets = 45 m/sec

Final peripheral velocity of the buckets = 0 m/sec

$$\therefore \text{ Average peripheral velocity of the buckets } = 22.5 \text{ m/sec} \\ \text{(assuming uniform retardation)}$$

Then average force acting on buckets

$$F_u = \rho Q \{ v_1 - (-u_1) \}$$

where  $v_1$  = the velocity of jet,

$u_1$  = the peripheral velocity of buckets acting in the opposite direction of  $v_1$

$$\therefore F_u = \frac{1,000 \times 0.366}{9.81} \times (95 + 22.5) = 4,380 \text{ kg}$$

Now, let the mean radius of wheel be  $R_1$ , then

$$\text{Torque or turning moment } M_t = F_u \cdot R_1 = 4,380 \cdot R_1$$

Let  $I$  be the moment of inertia of all moving parts, then

$$\frac{1}{2} I \cdot \omega_1^2 = \text{kinetic energy of all moving parts} \\ = 1,550,000 \text{ m-kg}$$

where  $\omega_1 =$  angular velocity in radian/sec

$$= \frac{u_1}{R_1} = \frac{45}{R_1}$$

$$\therefore I = \frac{1,550,000}{\frac{1}{2} \cdot \omega_1^2} = \frac{1,550,000 \times 2 R_1^2}{45 \times 45}$$

and torque or turning moment  $M_t = I \cdot \alpha$

where  $\alpha =$  the angular retardation in rad/sec<sup>2</sup>

Hence  $F_u \cdot R_1 = I \cdot \alpha$

$$\text{or } 4,380 R_1 = \frac{1,550,000 \times 2 R_1^2}{45 \times 45} \cdot \alpha$$

$$\text{or } \alpha = \frac{4,380 \times 45 \times 45 R_1}{1,550,000 \times 2 R_1^2} = \frac{2.86}{R_1}$$

$$\text{but } \alpha = \frac{\omega}{t} = \frac{\frac{u_1}{R_1}}{t} \quad \therefore \frac{45}{R_1 \cdot t} = \frac{2.86}{R_1}$$

$$\text{or } t = \frac{45}{2.86} = 15.75 \text{ second} \quad \text{Answer}$$

### UNSOLVED PROBLEMS

- 5.1 Write a short note on modern Pelton turbine. Explain why such a turbine is not used for low heads.
- 5.2 Describe with the help of a simple sketch the main components of a Pelton turbine.
- 5.3 Write short notes on : By-pass nozzle ; deflector ; Seewar jet diffuser.
- 5.4 Why is the jet deflected by the buckets between 160° to 165° instead of 180° ?
- 5.5 How are the buckets of Pelton turbine secured to the rim of the wheel ?
- 5.6 Why has a Pelton bucket a notch-like edge ? How does it affect the efficiency of the turbine ?
- 5.7 State the functions of the casing of a Pelton turbine. Why has it no hydraulic function to perform ?
- 5.8 What are the uses of hydraulic brake ? Describe the ratio of brake jet diameter to main jet diameter.
- 5.9 Sketch the layout of single jet, double jet and four jet Pelton turbines.
- 5.10 Sketch the arrangements of single overhung and double overhung Pelton wheels.
- 5.11 What is meant by a single overhung and a double overhung Pelton turbine ? What are the advantages of the latter over the former ?
- 5.12 Under what conditions would you use more runners for a Pelton turbine ?  
(Delhi University)
- 5.13 Under what conditions would you employ more than one nozzle for a Pelton turbine ? What are the disadvantages of such an arrangement ?  
(AMIE)

5.14 On what factors does the number of jets depend in case of Pelton turbines ?

5.15 What importance has the ratio

$$m = \frac{\text{mean diameter of Pelton wheel}}{\text{least diameter of jet}}$$

in designing the Pelton turbine.

(AMIE)

5.16 Evolve a formula for the specific speed of a Pelton wheel in the following form :

$$N_s = K \cdot \sqrt{\eta} \frac{d}{D}$$

where  $N_s$  = specific speed,  $\eta$  = overall efficiency,  
 $d$  = diameter of jet,  $D$  = diameter of bucket circle,  
 $K$  = a constant.

Find out the value of  $K$  when coefficient of discharge for nozzle is 0.96 and the bucket speed = 0.47 of jet speed, (AMIE)

5.17 On what factors do the selection of speed of Pelton turbine depend ?

5.18 How would you determine the minimum number of buckets for a Pelton turbine ?

5.19 Sketch a Pelton wheel bucket and show how it is attached to the wheel. Give its main dimensions.

5.20 Why does the efficiency of Pelton turbine fall if

(a) the value of jet ratio  $m$  is not between 11 and 14,

(b) the value of width of bucket is not optimum.

5.21 Write a short note on Turgo-Impulse turbine. How does it differ from Pelton turbine ?

5.22 Describe Banki turbine and give its uses.

5.23 Prove that the horsepower of a Pelton turbine per unit head and per unit jet diameter is 44.5.

5.24 Differentiate between volumetric, head, mechanical and overall efficiencies of a Pelton turbine.

5.25 Derive an expression giving the relationship between the jet speed and bucket speed for maximum efficiency of a Pelton wheel.

(Madras University)

### NUMERICALS

5.26 A Pelton wheel is to be designed to develop 1,000 HP at 400 rpm. It is to be supplied with water from a reservoir whose level is 250 m above the wheel through a pipe 900 m long. The pipe line losses are to be 5% of gross head. The co-efficient of friction is 0.005. The bucket speed is to be 0.46 of jet speed and efficiency of wheel is 85%. Calculate the pipe line diameter, jet diameter, and wheel diameter. (Pipe = 440 mm, Jet = 84 mm, Wheel = 1.480 m)

(Panjab University)

5.27 The highest head so far used anywhere in the world upto date is for running a Pelton turbine, installed at REISSECK (Austria) which is rated as 31,400 BHP at 750 rpm under an effective head of 1,770 m.

Determine—

- (a) the least diameter of jet ;
- (b) the mean diameter of the Pelton runner ;
- (a) the number of buckets of Pelton runner ; using empirical relation.

$$Z = 0.5 m + 15$$

here  $Z$  = no. of buckets

and,  $m = \frac{\text{mean diameter of Pelton runner}}{\text{least diameter of jet}}$

Assume suitable values of overall turbine efficiency ( $\eta_t$ ), jet velocity co-efficient ( $K_{v_1}$ ) and wheel peripheral velocity co-efficient or speed ratio ( $K_{u_1}$ ) from the following table which gives all the required values against  $N_s$ , the specific speed.

| $N_s$     | 34   | 24    | 10   |
|-----------|------|-------|------|
| $\eta_t$  | 0.82 | 0.89  | 0.9  |
| $K_{v_1}$ | 0.98 | 0.985 | 0.99 |
| $K_{u_1}$ | 0.44 | 0.45  | 0.46 |

(101 mm, 2,180 mm ; 26) (Jadavpur University)

- 5.28 Determine the discharge, least jet diameter, mean runner diameter jet ratio and number of buckets of the following Pelton turbine :

Nainital Power House—

BHP = 172, Head = 300 m Speed = 600 rpm

Assume the following constants :

$$K_{v_1} = 0.98, \quad K_{u_1} = 0.45, \quad \eta_t = 0.75.$$

(57.3 lit/sec ; 31 mm ; 35.6 ; 1,100 mm ; 33) (Panjab University)

- 5.29 The buckets of Pelton wheel deflect the jet impinging on them through  $150^\circ$ . Pressure at the nozzle is  $40 \text{ kg/cm}^2$ . The centre line of the buckets is on a circle of dia 250 cm. Find—

- (a) the best speed of the wheel ;
- (b) the theoretical efficiency ;
- (c) the theoretical HP at a nozzle flow of  $3 \text{ m}^3/\text{sec}$  ;
- (d) the specific speed.

Prove the formula used for specific speed.

(308 rpm ; 90.6% ; 16,000 HP ; 20.8 units) (Panjab University)

- 5.30 A Pelton wheel has a mean bucket speed of 12 m/sec and is supplied with water at the rate of  $0.7 \text{ m}^3/\text{sec}$  under a head of 30 m. If the buckets deflect the jet through an angle of  $160^\circ$ , find the HP and the efficiency of the turbine.  
(264 HP ; 93.4%) (*Madras University & UPSC*)
- 5.31 A Pelton wheel, which may be assumed to have hemispherical buckets, is 60 cm diameter. The available pressure at the nozzle when it is closed is  $15 \text{ kg/cm}^2$  and the supply when the nozzle is open is  $3 \text{ m}^3/\text{min}$ . If the revolutions are 600 per minute, calculate the horsepower of the Pelton wheel and its efficiency.  
(89.5 HP ; 89.5%) (*Madras University*)
- 5.32 A Pelton turbine is required to develop 3,500 HP under a head of 300 m. Its efficiency is 80%. Specific speed is 16 and the ratio of the rim speed to the jet speed is 0.47.  
Assume  $C_v = 0.97$  for the nozzle. Calculate the jet diameter and the rpm of the wheel.  
(13.68 cm ; 338 rpm) (*AICTE*)
- 5.33 A jet of water impinges on a series of curved vanes at an angle of  $30^\circ$  to the direction of motion of the vanes while entering and leaves the vanes horizontally. The head under which the jet issues from the nozzle is 30 m, the co-efficient of velocity for the nozzle is 0.9 and the diameter of the jet after leaving the nozzle is 5 cm. The speed of the vanes is 10 m/sec and the relative velocity of the water at outlet is 0.8 times the relative velocity at inlet. Calculate  
(a) the angle of vane tips at inlet ;  
(d) the horse power developed by the jet ;  
and (c) the efficiency of the system.  
( $50^\circ 48'$  ; 11.8 HP ; 69.2%) (*Rajputana University*)
- 5.34 Show that in an impulse water turbine with equal bucket angles at inlet and outlet and with no frictional and other losses, the maximum efficiency is equal to  $\cos^2 \alpha$ , where  $\alpha$  is the jet angle with the direction of motion of the turbine.  
A Pelton wheel working under a total head of 500 m is supplied with  $0.3 \text{ m}^3/\text{sec}$  of water through a pipe 3.5 km long, co efficient of friction of the pipe material being 0.007. If the wheel develops 800 HP with a wheel efficiency of 85%, determine the necessary diameters of the nozzle and the pipe.  
Neglect losses in nozzles.  
(76 mm ; 302 mm) (*Roorkee University*)
- 5.35 A Pelton turbine is required to develop 5,000 HP at 300 rpm under a head of 360 m. Determine the least diameter of the jet impinging on the buckets of the turbine wheel if the co-efficient of velocity is 0.985. Assume turbine's overall efficiency as 0.9. What would be the mean diameter of the wheel if speed ratio is 0.45. Calculate the size of the nozzle used if its diameter is 25% larger than the least diameter of jet. How many buckets will be used for the wheel for the best efficiency ?  
(133 mm ; 2.4 m ; 183 mm ; 24)
- 5.36 Show that in Pelton wheel, where the buckets deflect the water through an angle of  $180^\circ - \theta$ , the hydraulic efficiency of the wheel is given by

$$\eta_h = \frac{2v(V-v)(1+\cos \theta)}{V^2}$$

where  $V$  is the velocity of jet and  $v$  the velocity of the wheel at the pitch radius.

If the bucket speed is 30 m per sec,  $\theta = 20^\circ$ , and the jet diameter is 12.5 cm., find the jet velocity for maximum efficiency and the corresponding WHP of the turbine.

(60 m/sec ; 1,750 HP as Available Power) (*Bombay University*)

- 5.37 A Pelton wheel is supplied with water in the nozzle box at a pressure head of 50 m. The velocity co-efficient for the nozzle is 0.96. The relative velocity of water while in contact with the buckets is turned through  $150^\circ$ , and is also reduced by 15% owing to friction losses. The required ratio of bucket to jet speed is 0.47 and 0.8 of the energy imparted to the wheel, as determined by the velocity diagrams, is available at the output shaft. The bucket circle diameter is ten times the jet diameter. Find the jet and wheel diameters, and the speed of rotation to develop 200 HP under these conditions.

(62 mm ; 620 mm ; 754 rpm)

- 5.38 The water available for a Pelton wheel is  $4 \text{ m}^3/\text{sec}$  and total head from reservoir to nozzle is 250 m. The turbine has two runners with two jets per runner. All the four jets have same diameters. The pipe line is 3,000 m long. The efficiency of the power transmission through pipe line and nozzle is 91% and efficiency of each runner is 90%. The velocity co-efficient of each nozzle is 0.975 and the co-efficient of friction for the pipe is 0.0045.

Determine—

- (i) the horse power developed by turbine ;
- (ii) the diameter of jet ;
- (iii) the diameter of pipe line.

(10,890 HP ; 14 cm ; 1.26 m) (*London University*)

- 5.39 The following data were obtained from a test on a Pelton turbine—

Area of jet = 75 sq cm ; Discharge = 180 lit/sec ;

Head at nozzle = 30 m ; BHP = 56.0

Horsepower absorbed in friction and windage = 3.0 HP

Determine the energy lost in the nozzle and also the energy absorbed due to losses in the wheel at discharge.

(1.5 HP ; 11.5 HP) (*Roorkee University*)

- 5.40 In Mandi Hydro-Electric Scheme at Jogindar Nagar the water from the high level reservoir at Brot is led through the tunnel (constructed in the mountains) which is connected to four penstocks, each of average diameter of 1,300 mm through a nozzle of 200 mm diameter. The jet of each penstock impinges on the buckets of a Pelton turbine of horizontal type. Thus there are four turbines, each having its own penstock with a nozzle. Each of the penstock is 1.6 km long. The total head available at the centre line of nozzle is 510 m. Calculate the velocity of the water jet issuing out of each nozzle, assuming co-efficient of velocity as 0.98.

The mean diameter of the runner of Pelton wheel is 2,000 mm. The speed ratio is 0.46. If the buckets turn the water to  $165^\circ$ , calculate—

- (a) total water power available at the headworks assuming co-efficient of friction  $f = 0.008$  ;
- (b) power of each jet ;



- (c) hydraulic power of each turbine if 3% of the jet water drops to the tail race without doing any useful work ;  
 (d) BHP of the turbine if 2% of the power calculated under (c) is lost in windage and bearing friction.

[98 m/sec (a) 86,600 HP (b) 20,050 HP (c) 19,200 HP (d) 18,816 BHP]  
 (Delhi University)

5.41 A Pelton turbine has a mean bucket speed of 40 m/sec, and is supplied with water at the rate of 0.55 m<sup>3</sup>/sec, the head of water behind the nozzle being 400 m. If the jet is deflected by the buckets through 170°, find HP developed and the efficiency of the wheel. Assume  $\beta_1 = 0$ . (2,840 HP ; 96%) (AMIE)

5.42 A Pelton wheel is to be designed to the following specifications :—

Power = 16,250 HP ; Head = 381 m  
 Speed = 750 RPM ; Overall Efficiency = 86 per cent

Jet diameter not to exceed  $\frac{1}{8}$  times the wheel diameter

Determine—

- (a) the wheel diameter ; (b) number of jets required ;  
 and (c) the diameter of the jet. (1 m ; 4 ; 118 mm) (Rajasthan University)

5.43 A double overhung Pelton wheel unit is to operate a 30,000 KW generator under the effective head of 305 m at the base of the nozzle. Find the size of jet, mean diameter of runner, synchronous speed and specific speed of each wheel. Assume generator efficiency 93%, Pelton wheel efficiency 85%, co-efficient of nozzle velocity 0.97, speed ratio 0.46 and jet ratio 12.

(330 mm ; 3,960 mm ; 167.67 rpm ; 19.4 units) (Panjab University)

5.44 A twin jet Pelton wheel has a mean bucket circle diameter of 1,680 mm and runs at 500 rpm. When the jets are 152.5 mm diameter, the available head at the nozzle is 488 m. Assuming co-efficient of velocity for the nozzle as 0.98, outlet angle of bucket 15°, relative velocity for water leaving the bucket 0.88 of that of inlet ; and windage and mechanical losses 3% of the water horsepower supplied, find—

- (a) the water horsepower supplied and brake horsepower ;  
 (b) the force of one jet on the bucket ;  
 (c) the overall efficiency.

(22,740 HP ; 19,300 HP ; 17,100 kg ; 84.7%) [AMI Mech E (London)]

5.45 (a) Obtain an expression for the theoretical efficiency of a Pelton wheel when the angle of the bucket at exit makes an angle of  $\theta$  with the direction of jet. Show by a diagram how the efficiency of the wheel will vary as the ratio of velocity of the jet to the velocity of bucket is varied.

- (b) A Pelton wheel has a mean speed of 12.2 m/sec and is supplied with water at the rate of 1,370 litres per second under a head of 30.5 m. If the buckets deflect the jet through an angle of 16° find the horsepower and efficiency of the wheel.

(557 HP ; 95.4%) (Panjab University)

5.46 Design a Pelton turbine with the following data

Head of water = 305 m ; Rate of flow = 240 lit/sec

Speed of turbine = 750 rpm. Assume constants etc.

(Hint : Find HP developed, specific speed, dia of jet and wheel, buckets main dimensions and their number)

(858.7 HP ; N, 16.9 ; 6.34 cm ; 88.7 cm ; Nozzle dia 6.94 cm ; bucket dimensions ; length 16.2 cm, width 19 cm, depth 4.75 cm ; 22 buckets).

# Reaction Turbines I

## FRANCIS AND DERIAZ TURBINES

### (A) FRANCIS TURBINE

**6.1 Modern Francis Turbine** is an inward mixed flow reaction turbine i.e., the water, under pressure, enters the runner from the guide vanes towards the centre in radial direction and discharges out of the runner axially. The Francis turbine operates under medium heads and also requires medium quantity of water. It is employed in the medium head power plants. This type of turbine covers a wide range of heads (30 to 450 m). Water is brought down to the turbine and directed to a number of stationary orifices fixed all around the circumference of the runner (refer Fig 6.1). These stationary orifices are commonly termed as *guide vanes or wicket gates*.

A part of the head acting on the turbine is transformed into kinetic energy and the rest remains as pressure head. There is a difference of pressure between the guide vanes and the runner which is called the *reaction pressure*, and is responsible for the motion of the runner. That is why a Francis turbine is also known as a *reaction turbine*.

In this turbine the pressure at the inlet is more than that at the outlet. This means that the water in the turbine must flow in a closed conduit. Unlike the Pelton type, where the water strikes only a few of the runner buckets at a time, in the Francis turbine the runner is always full of water. The movement of the runner is affected by the change of both the pressure and the kinetic energies of water. After doing its work the water is discharged to the tail race through a closed tube of gradually enlarging section known as the *draft tube*. It does not allow water to fall freely to the tail race level as in the case of a Pelton turbine. The free end of the draft tube is submerged deep in the tail water, thus, making the entire water passage, right from the head race upto the tail race, totally enclosed.

The draft tube converts kinetic head to pressure head. About 70% conversion is possible. By recovering pressure head in a draft tube the pressure at the runner exit is reduced below atmosphere. This makes it possible to install the turbine above tail race level without any loss in available head. This is an important advantage in the use of a reaction turbine over Pelton turbine,

**6.2 Different Types of Francis Turbines**—There are two main types of Francis turbines, namely—

- (i) Closed type and (ii) Open flume type.

**Closed Type Francis Turbine**—In this type the water is led to the turbine through the penstock whose end is connected to the spiral casing

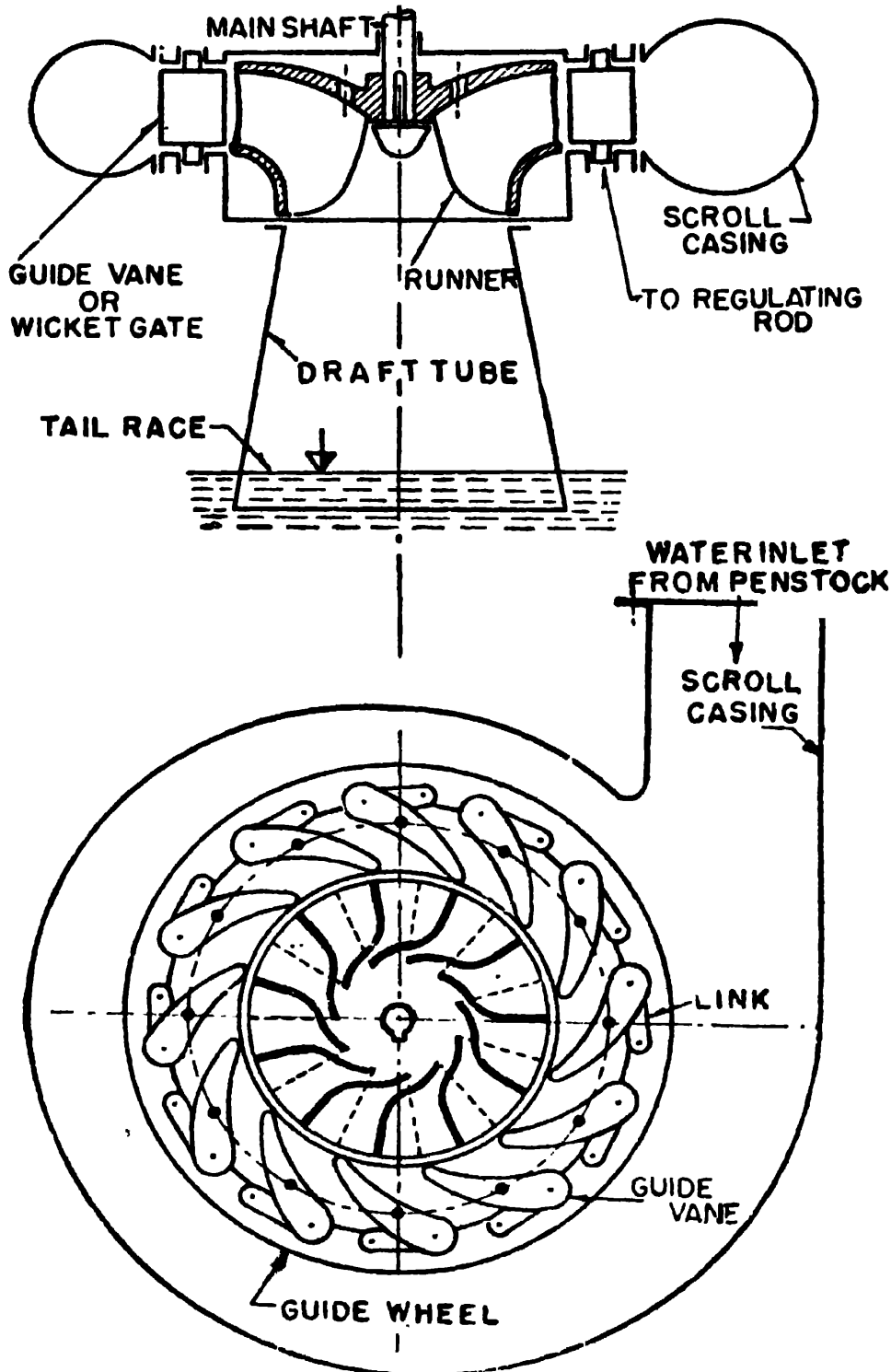


Fig 6.1 Outlines of a Francis Turbine, Vertical Closed Type

of the turbine. This spiral casing directs the water evenly to the guide blades (refer Fig 6.1 and Plate 17). The water then passes through the runner and finally goes to the tail race through the draft tube. The closed

type Francis turbine may be of two types, horizontal and vertical. The horizontal type is used for medium and high heads and the vertical type for medium and low heads. The large units are, however, mostly of the vertical type.

**Open Flume Type Francis Turbine**—In the open flume Francis turbine a concrete chamber replaces the spiral casing (refer Fig 6.2). This type of turbine is used for 5 to 10 m heads. In many power plants axial flow turbine (Kaplan or tubular turbines, refer Chapter 7) is fast replacing this type of turbine.

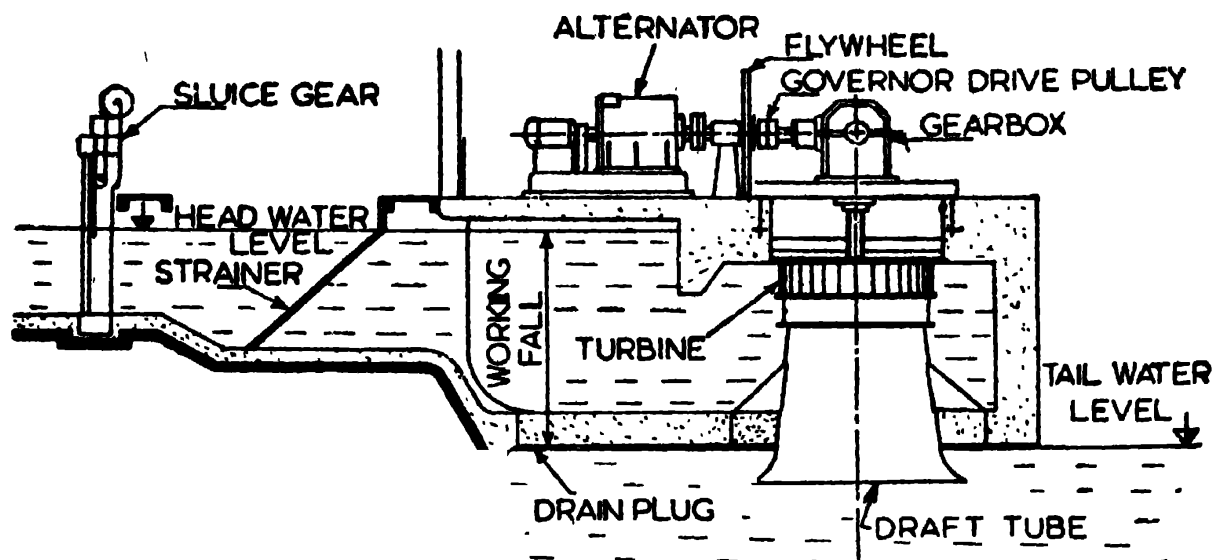


Fig 6.2 Open Flume, Vertical Francis Turbine Layout  
(Manufacturers : Gilbert Gilkes & Gordon Ltd, UK)

These are, however, still employed where the quantity of water is small and the fluctuation in head is more. The turbine is surrounded by water. The guide vanes are adjusted by means of a mechanism, also submerged in water. As the heads are low, the mechanical stresses in the levers and links are also less. The modern tendency is towards equipping the turbine with an external regulating mechanism. In case of a horizontal turbine the shaft is carried through a gland in the chamber wall. The guide vanes, the runner etc. which are submerged in water are on one side of the masonry wall. On the other side are the bearing and electric generator. As the bearing is out of water it is easily accessible. The vertical type open flume turbine is submerged in water through a vertical shaft, and the bearing which is fixed on a masonry structure carries the full turbine load. The generator is, generally, connected horizontally through gears, as it must run at a higher speed than the turbine.

### Main Components of Modern Francis Turbine

**6.3 Penstock**—Penstock is a waterway to carry water from the reservoir to the turbine casing. Trashracks (refer Fig 4.1) are provided at the inlet of penstock in order to obstruct the debris entering in it. Plate 18 shows penstock for Macagua I Hydro Power Station on Rio Caroni (Venezuela) manufactured by Voith, West Germany, having an inside diameter of 7.5 m, made of high quality boiler plates. The penstock sections were manufactured in quarters and welded at the site, due to transport difficulties. Stiffening and anchor-rings and pads were welded to

the penstock at the site. The welding seams were inspected by X-rays and the penstock was tested at twice the operating pressure.

#### 6.4 Spiral Casing or Scroll Casing (refer Fig 6.1 and Plate 19)—

To avoid loss of efficiency, the flow of water from the penstock to the runner should be such that it will not form eddies. In order to distribute the water around the guide ring evenly, the scroll casing is designed with a cross-sectional area reducing uniformly around the circumference, maximum at the entrance and nearly zero at the tip. This gives a spiral shape and hence the casing is named as spiral casing. In the case of big units, the inside circumference of casing has *stay vanes* each directing the water to the guide vanes. The position of the inlet to the spiral casing depends on the direction of water flowing out of the penstock which may vary according to the site. The spiral casings are provided with inspection holes and also with pressure gauge connections. Large spiral casings are made in parts for ease in transport. The spiral casing for vertical reaction turbine is partly or completely grounded in. In such a case the runner is not easily accessible and the dismantling is not so easy as in horizontal type turbine.

The material of scroll casing depends upon the heads which are as follows—

Concrete without steel plate lining—upto about 30 m

Welded rolled steel plate —upto about 100 m

Cast steel —more than 100 m

**6.5 Guide Mechanism** (refer Fig 6.1)—The guide vanes or wicket gates, as they are sometimes called are fixed between two rings in the form of a wheel, known as *guide wheel*. The guide vanes have a cross-section known as aerofoil section. This particular cross-section allows water to pass over them without forming eddies and with minimum friction losses (refer Fig 6.3). Each guide vane can rotate about its pivot centre which is connected to the regulating ring by means of a link and a lever. The ring

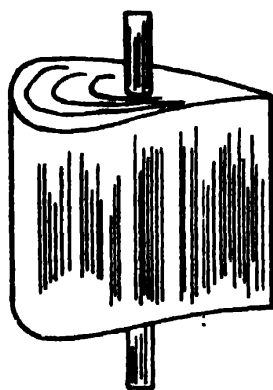


Fig 6.3 Guide Vane

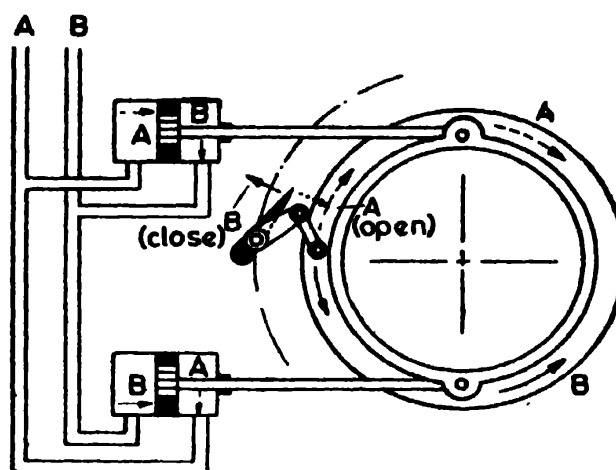


Fig 6.4 Regulation of Guide Vanes

is connected to the regulating shaft by means of regulating rods, generally, two in number (refer Fig 6.4). By rotating the regulating shaft the guide vanes can be closed or opened thus allowing a variable quantity of water according to the needs. The regulating shaft is operated by means of a governor whose function is to keep the speed of the turbine constant at varying loads. With a decrease in load the speed of the turbine always tends to increase. To bring the speed back to the rated value, the governor is used to reduce the guide vane opening thereby allowing less water to strike the runner. The guide vanes are generally made of cast steel.

Plate 20 shows the guide vanes of Francis turbine badly affected by sand erosion.

**6.6 Runner and Turbine Main Shaft** (refer Fig 6.5 and Plate 21)—The flow in the runner of a modern Francis turbine is not purely radial but a combination of radial and axial. The flow is inward, that is, from the periphery towards the centre. The width of the runner depends upon the specific speed. The high specific speed runner is wider than the one which has a low specific speed because the former has to work with a large amount of water. The runners may be classified as (i) slow ; (ii) medium and (iii) fast, depending upon the specific speed (refer Art 4.18). The runner may be cast in one piece or may be made of separate steel plates welded together. The runners are made of cast iron for small output, cast steel for large output and stainless steel or a non-ferrous metal like bronze, when the water is chemically impure and there is danger of corrosion. The runner blades should be carefully finished. Francis runners as large as 7.5 m in diameter (for Krasnoyarsk—USSR) have been manufactured so far.

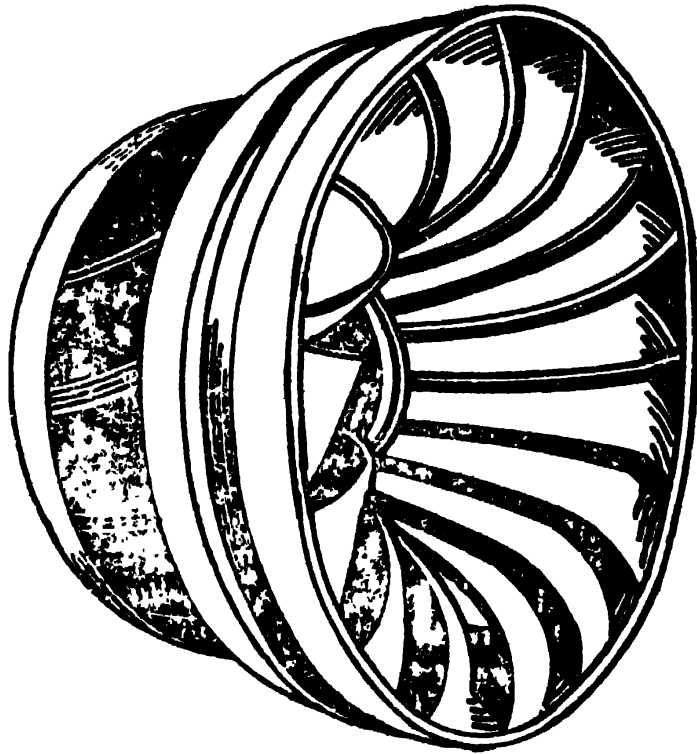


Fig 6.5 Francis Turbine Runner

The runner is keyed to the shaft which may be vertical or horizontal. The turbine is accordingly specified as vertical or horizontal type.

The shaft is generally made of steel and is forged. It is provided with a collar for transmitting the axial thrust to the bearing.

The turbine is generally provided with one bearing (refer plate 17). In vertical type the bearing carries full runner load and acts as thrust-cum-supporting bearing. Right selection of bearing is, therefore, extremely important. The lubrication of the thrust bearing also plays an important part in the running of the turbine. For this purpose an oil pump driven by the main turbine shaft is generally employed.

**6.7 Draft tube** (refer Fig 6.1 and Plate 17)—The water after doing work on the runner passes on to the tail race through a draft tube which is a welded steel plate pipe or a concrete tunnel, its cross-section gradually increasing towards the outlet. The draft tube is a conduit which connects the runner exit to the tail race. The tube should be drowned—approximately one metre below the lowest tail race level. The functions of the draft tube are as follows :—

(i) If the water is discharged freely from the runner, turbine will work under a head equal to the height of the head race water-level above the runner outlet. If an airtight draft tube connects the runner to the tail

race, workable head is increased by an amount equal to the height of the runner outlet above tail race.

The draft tube will, thus, permit a negative (suction) head to be established at the runner outlet thus making it possible to install the turbine above the tail race without loss of head. This can be explained as follows :—

The pressure in the draft tube at the tail race level is atmospheric. If the cross-section of draft tube is kept uniform, the pressure at the runner outlet is equal to the atmospheric pressure minus the height of runner outlet above the tail race level. The available head, measured from head race level to the discharge side of the turbine, is thus the same as if the turbine were erected at tail race level and discharged under atmospheric pressure.

Thus the reaction turbines may be installed in three ways, (a) at the tail race level, (b) above the tail race level and (c) below tail race.

**Example**—Let the difference in level of head race and tail race be 100 m, and for a turbine installed at the tail race level and discharging at atmospheric pressure, the head available for the turbine is 100 m (refer Fig 6.6a).

Now in a case when the turbine is installed 5 m above the tail race level and no draft tube is installed, the head available for the turbine is 95 m (refer Fig 6.6b). With the use of draft tube the water is not discharging at atmospheric pressure but at a negative pressure that is  $-5$  m. Therefore, the available head for the turbine will be  $95 - (-5) = 100$  m. Thus it is possible to install the turbine above tail race level without any loss of available head.

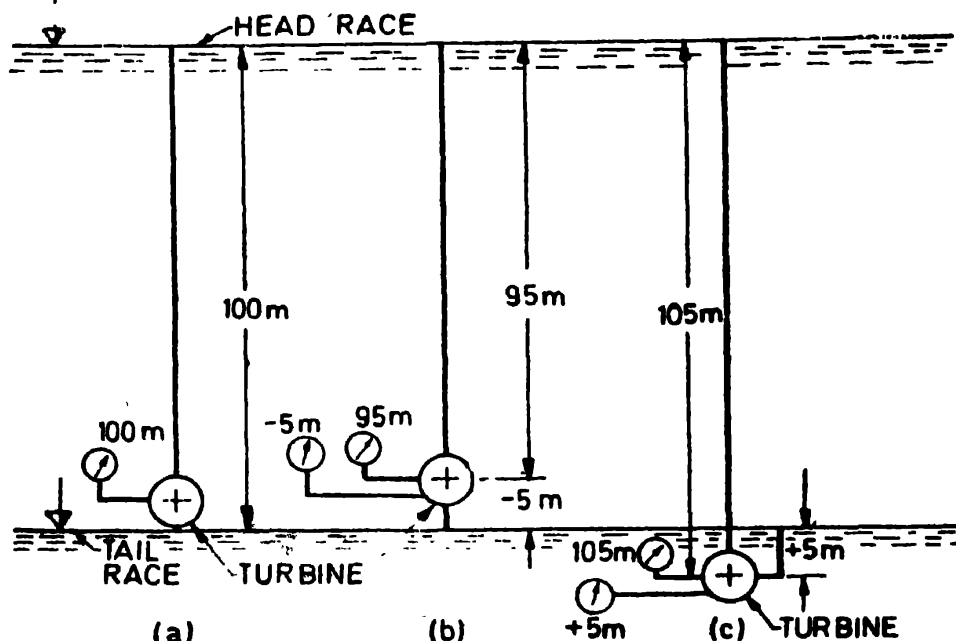


Fig 6.6 Installation of Draft Tube Without Loss of Head

In certain cases the turbine is installed below the tail race level (refer Fig 6.6c). Let the turbine be installed 5 m below the tail race level, then the pressure at the runner exit will be 5 m gauge. The turbine runner exit is 105 m below the head race level. There the available head for the

turbine will be  $105 - 5 = 100$  m. Thus the turbine can be installed below or above the tail race level with the help of draft tube and the available head remains the same.

It may be noted that reaction turbines are seldom installed discharging in atmosphere at the tail race level because the tail race level is variable during the draught and flood periods of the year.

Further the turbine installed below the tail race level will reduce the possibility of cavitation because of the absence of negative head.

(ii) The water leaving the runner still possesses a high velocity and this kinetic energy would be lost if it is discharged freely as in a Pelton turbine. By employing a draft tube of increasing cross-section, the enclosed conduit is extended up to the outlet end of the tube and discharge takes place at a much reduced velocity thus resulting in a gain of pressure head. This increases the negative pressure head at turbine runner exit with which the net working head on the turbine increases. With the increase in net working head on the turbine, output will also increase, thus raising the efficiency of the turbine.

**6.8 Different Types of Draft Tubes**—The draft tube is an integral part of a reaction turbine. The velocity energy of water at runner

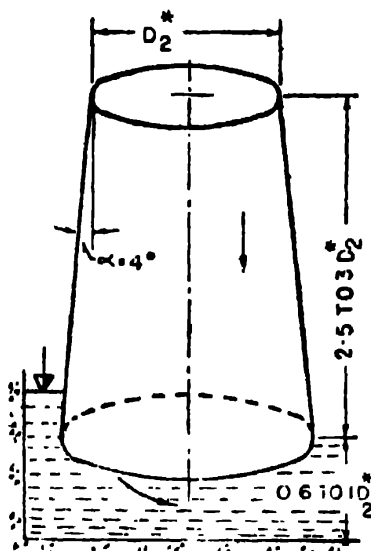


Fig 6.7 (a) Straight Divergent Tube

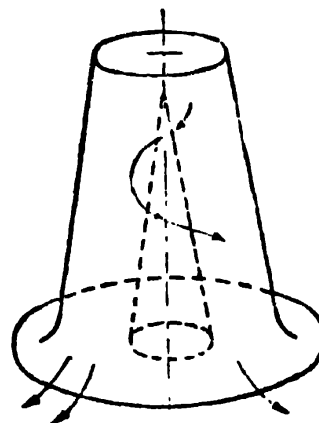


Fig 6.7 (b) Moody's Bell Mouthed Draft Tube

exit is 3 to 15% of the net working head in case of Francis turbines depending upon the specific speed. As the specific speed increases, the value of velocity energy at the runner rises and it will be nearly 45% in the case of a Kaplan turbine. Hence for high specific speed Francis turbines as well as for Kaplan turbines, the second function of draft tube, described in Art 6.7(ii), i.e., recovery of kinetic energy at discharge is more important. Therefore great attention is paid to the shape of draft tube especially meant for high specific speed turbines.

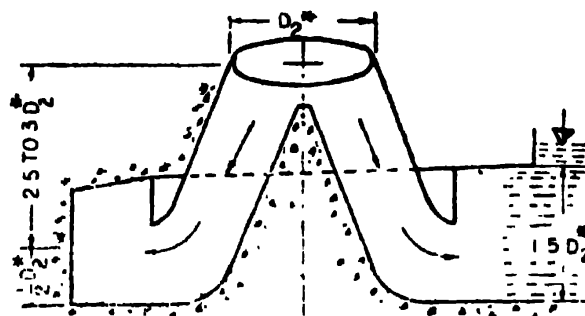


Fig 6.7 (c) Moody's Spreading Type Draft Tube or "Hydracone"



The following are some types of draft tubes, employed in the field—

(a) *Straight Divergent Tube* [refer Fig 6.7 (a)].—The shape of this tube is that of a frustum of a cone. It is employed for low specific speed,

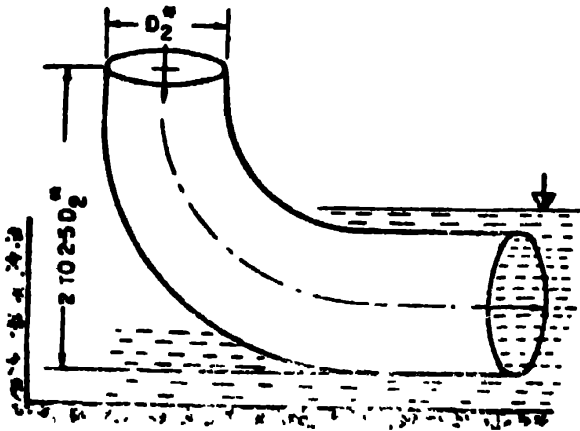


Fig 6.7 (d) Simple Elbow Tube

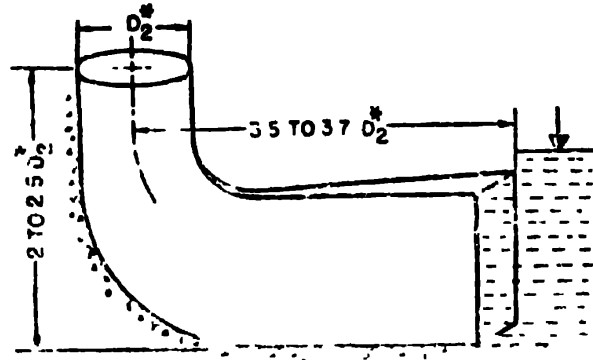


Fig 6.7 (e) Elbow Tube having Circular Cross-section at Inlet but Rectangular at Outlet

Fig 6.7 Different Types of Draft Tube

vertical shaft Francis turbine. The maximum cone angle is  $8^\circ$  (or  $\alpha = 4^\circ$ ). Experiments have shown that if this angle is greater than  $8^\circ$ , the water detaches away from the inner wall of the tube while flowing downwards forming vortices and causing loss of head. The tube must discharge sufficiently low under tail water level. The maximum efficiency which this type of draft tube can yield is 85%, because the kinetic head to be recovered is less. This type of draft tube improves speed regulation on falling load.

(b) *Moody's Spreading Tube or "Hydracone"* (refer Fig 6.7 b and c)—Moody suggested a bell mouthed draft tube (refer Fig 6.7 b) having a solid conical core in the entire central portion of the tube, thus allowing a large exit area without excessive length. When the turbine works at part load or due to high velocity of water at runner exit, the discharging water velocity has a whirl component, it is likely to cause eddy losses. The central cone arrangement is made to reduce the whirl action of discharging water. The efficiency of such a draft tube is about 85%.

(c) *Simple Elbow Tube* (refer Fig 6.7 d)—In order to keep down the cost of excavation, particularly in rock, the vertical length of the draft tube should be minimum. Since the draft tube exit diameter should be as large as possible to recover the kinetic head and at the same time the maximum value of the cone angle is fixed, the draft tube must be bent to keep its definite length. Simple elbow type draft tube will serve such a purpose. Its efficiency is, however, low, about 60%.

(d) *Elbow Tube with a Circular Inlet and a Rectangular Outlet Section* (refer Fig 6.7 e)—This type of draft tube has been designed to turn the water from the vertical to the horizontal direction with a minimum depth of excavation and at the same time having a high efficiency. The transition from a circular section in the vertical leg to a rectangular section in the horizontal leg takes place in the bend. The horizontal portion of the draft tube is generally inclined upwards to lead the water gradually to the level of the tail race and to prevent entry of air from the exit end. The exit end of the tube must be totally immersed in water.

In order to avoid any whirl component of velocity of water at runner exit, one or two piers are constructed in the bend of the draft tube. Such

piers behave similar to the central core of Moody's spreading tube, described above. Fig 6.8 (a) and 6.8 (b) show the single and double pier elbow type draft tubes.

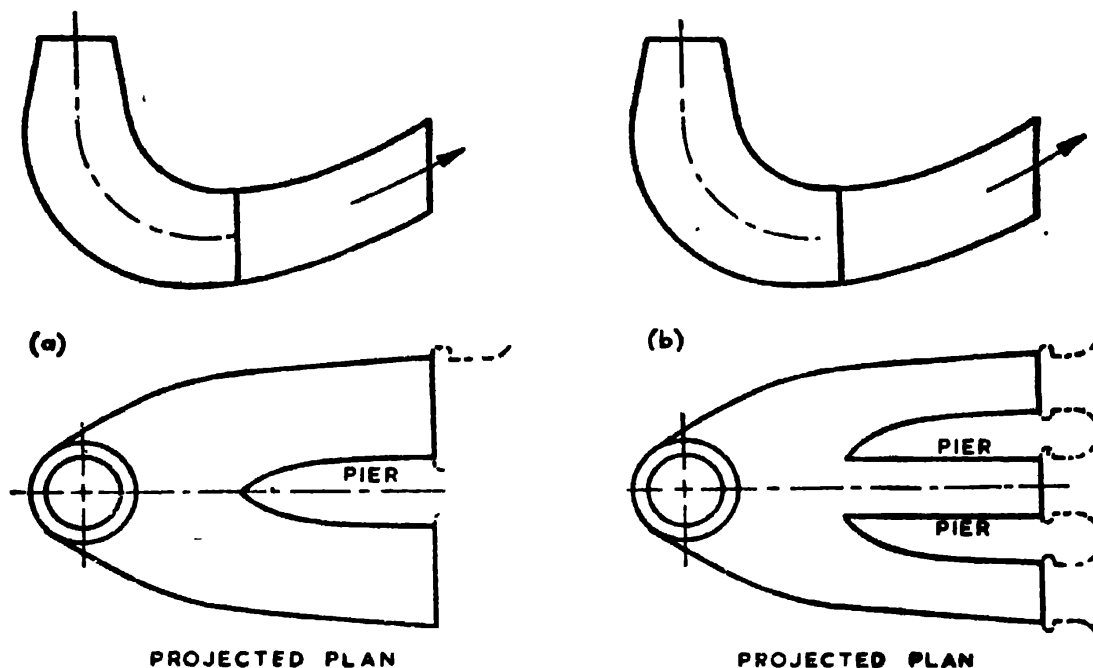


Fig 6.8 Elbow Type Draft Tube (a) Single Pier Type (b) Double Pier Type

**6.11 Power**—Brake horse power and available power of Francis turbine are determined by methods similar to those adopted for Pelton turbine (refer Art 5.9).

**Problem 6.1** *The source of a hydro-electric plant is a stream whose discharge varies with the season. The rainy season lasts for 3 months and gives an average discharge of  $12 \text{ m}^3/\text{sec}$  to the stream. For the rest of the year the discharge is  $3 \text{ m}^3/\text{sec}$ . A reservoir is built such that the whole of the discharge could be utilised at a uniform rate. The mean reservoir level is 200 m above the turbine level. The power house has three turbines whose inlets are connected to the reservoir by means of three pipe lines, each 1,300 m long. The head loss due to friction in each of these pipes is 3 per cent of the gross head. The overall efficiency of each of the turbines is 87%. Assume  $f=0.005$ .*

*Determine (a) the capacity of the reservoir, (b) the diameter of the pipes and (c) the BHP output of the station.*

### Solution

- (a) Average discharge during rainy season for three months  $12 \text{ m}^3/\text{sec}$   
 Average discharge during dry season for nine months  $= 3 \text{ m}^3/\text{sec}$

$$\text{Total discharge for the year} = (12 \times 3) + (3 \times 9) = 63 \text{ m}^3/\text{sec}$$

$$\text{Average discharge for one month} = \frac{63}{12} = 5.25 \text{ m}^3/\text{sec}$$

The reservoir receives the water during the rainy season only and at the same time a discharge of  $5.25 \text{ m}^3/\text{sec}$  is utilised by the power station.

## 6.9 SOME FRANCIS TURBINES

| S. No. | Scheme                                                                                 | Site of Power House                                                  | Source of Water                  |
|--------|----------------------------------------------------------------------------------------|----------------------------------------------------------------------|----------------------------------|
| 1      | Bhakra Dam Project (Punjab)                                                            | Bhakra (14 km from Nangal Rly. Station)                              | Sutlej River (Bhakra Dam)        |
| 2      | Ranbir Canal Scheme (Jammu & Kashmir)                                                  | Satwari (near Jammu where River Tawi & Ranbir Cross)                 | Chenar River                     |
| 3      | Ganderbal Hydro-Station (Jammu & Kashmir)                                              | Ganderbal (21 km from Srinagar)                                      | Sindh River                      |
| 4      | Udhampur (Jammu & Kashmir)                                                             | Udhampur (70 km from Jammu)                                          | Tawi Canal                       |
| 5      | Rajouri Hydro Electric Stn. (Jammu)                                                    | —                                                                    | —                                |
| 6      | Hirakud Dam Project I (Orissa)                                                         | Hirakud (12 km from Sambalpur)                                       | Mahanadi (Hirakud)               |
| 7      | Damodar Valley Corp. (DVC). West Bengal & Bihar                                        | Tilaya (District Hazaribagh, Bihar, Nearest Railway Station Kodarma) | Barakar River (Tilaya Dam)       |
| 8      | Mayurakshi Project (West Bengal & Bihar) (Seasonal Power Project linked with DVC grid) | Messanjar (Dist. Santhal Parganas) Bihar                             | Mayurakshi River                 |
| 9      | Gokak Hydro-Electric Scheme (Bombay)                                                   | Gokak (Dist. Belgaum)                                                | Gokaprabha River                 |
| 10     | Chambal Hydro Electric Scheme (Rajasthan)                                              | Gandhisagar                                                          | Chambal River                    |
| 11     | Tungabhadra Hydro-Electric Scheme (Andhra)                                             | (a) Tungabhadra Dam                                                  | Tungabhadra                      |
| 12     | Machkund Hydro Electric Project (Andhra)                                               | (b) Hampi Machkund (300 km from Vizagapatam)                         | Tungabhadra (Dunduma Dam)        |
| 13     | Cauvery Hydro Electric Scheme (Mysore)                                                 | Sivasamudram (65 km from Mysore)                                     | Cauvery River                    |
| 14     | Cauvery-Mettur Project (Tamil Nadu)                                                    | Mettur (District Salem, Tamil Nadu)                                  | Cauvery River (Mettur Dam)       |
| 15     | Periyar Hydro-Electric Scheme (Tamil Nadu)                                             | Periyar                                                              | Periyar                          |
| 16     | Kerala Govt.                                                                           | Peringal Kutha                                                       | Chalakaudy River                 |
| 17     | Kerala Govt.                                                                           | Neriamangalam                                                        | Tailwaters of Sengulum & Penniar |
| 18     | Upper Sileru Hydro-Electric Project (1st Stage)—Andhra                                 | Upper Sileru                                                         | Sileru River at Guntavada Dam    |
| 19     | Rihand Dam Project (U.P.)                                                              | Rihand, Mirzapur Rly. Stn.                                           | Rihand Dam                       |

## INSTALLATIONS IN INDIA

| Manufacturers<br>or Suppliers | No. of<br>Turbines | Turbine Specifications |               |                                               |              |                              |                                                                                                | Other<br>details |
|-------------------------------|--------------------|------------------------|---------------|-----------------------------------------------|--------------|------------------------------|------------------------------------------------------------------------------------------------|------------------|
|                               |                    | Power<br>HP<br>each    | Head<br>m     | Dis-<br>charge<br>m <sup>3</sup> /sec<br>each | Speed<br>rpm | Vertical<br>or<br>Horizontal |                                                                                                |                  |
| 1. Hitachi (Japan)            | 5                  | 1,50,000               | 120           | 100                                           | 166.7        | Vertical                     | —                                                                                              |                  |
| USSR                          | 5                  | 1,70,000               | —             | —                                             | —            | —                            | —                                                                                              |                  |
| 2. —                          | 4                  | 700                    | 8             | —                                             | 300          | Horizontal                   | —                                                                                              |                  |
| 3. (a) Escher<br>Wyss         | 2                  | 4,365                  | 137           | 2.760                                         | 1,000        | Horizontal                   | —                                                                                              |                  |
| (b) Ganz                      | 2                  | 4,500                  | —             | —                                             | 600          | —                            | —                                                                                              |                  |
| 4. (a) Voith                  | 2                  | 490                    | 58            | 0.765                                         | 1,000        | Horizontal                   | D=0.44 m                                                                                       |                  |
| (b) Voith                     | 2                  | 36,000                 | 31.6          | 94.2                                          | 150          | Vertical                     | D=3.3 m                                                                                        |                  |
| 5. Voith                      | 2                  | 750                    | 7.6           | 8.5                                           | 142          | Vertical<br>with gears       | D=1.4 m                                                                                        |                  |
| 6. Voith                      | 2                  | 32,000                 | 23            | 127.5                                         | —            | Vertical                     | —                                                                                              |                  |
| 7. —                          | 2                  | 2,500                  | 15 to<br>23.5 | —                                             | 250          | Vertical                     | Z <sub>1</sub> =20<br>Z <sub>2</sub> =15<br>N <sub>r</sub> =487<br>rpm                         |                  |
| 8. —                          | 2                  | 2,500                  | —             | —                                             | —            | —                            | —                                                                                              |                  |
| 9. (a) —                      | 3                  | 900                    | 60            | —                                             | —            | —                            | —                                                                                              |                  |
| (b) —                         | 1                  | 1,500                  | —             | —                                             | —            | —                            | —                                                                                              |                  |
| 10. Voith                     | 3                  | 34,000                 | 5.5           | 63.5                                          | 187          | Vertical                     | D=2.68 m                                                                                       |                  |
| 11. (a) Escher<br>Wyss        | 1                  | 680                    | 27            | —                                             | —            | —                            | —                                                                                              |                  |
| (b) Charmilles                | 2                  | 12,500                 | 33.5          | 31                                            | 214          | Vertical                     | —                                                                                              |                  |
| 12. (a) Morgan<br>Smith       | 3                  | 25,000                 | 260           | 7                                             | 600          | Vertical                     | —                                                                                              |                  |
| (b) Voith                     | 3                  | 36,000                 | 255           | 11.75                                         | 600          | Vertical                     | D=1.65 m                                                                                       |                  |
| 13. (a) Boving                | 4                  | 9,000                  | 123           | —                                             | 375          | Horizontal                   | —                                                                                              |                  |
| (b) Boving                    | 5                  | 5,600                  | —             | —                                             | 600          | Horizontal                   | —                                                                                              |                  |
| (c) Escher<br>Wyss            | 1                  | 5,600                  | —             | —                                             | 600          | Horizontal                   | —                                                                                              |                  |
| 14. English<br>Electric       | 4                  | 1,000 to<br>16,000     | 18 to<br>50   | —                                             | 250          | Horizontal                   | D <sub>1</sub> =1.61 m<br>Z <sub>1</sub> =20<br>A <sub>1</sub> =1.11 m <sup>2</sup><br>D=1.7 m |                  |
| 15. Voith                     | 3                  | 150,000                | 374           | 11.38                                         | 750          | Vertical                     | —                                                                                              |                  |
| 16. Charmilles                | 4                  | 13,500                 | 180           | —                                             | 600          | Vertical                     | —                                                                                              |                  |
| 17. Charmilles                | 3                  | 23,000                 | 200           | —                                             | 600          | Vertical                     | —                                                                                              |                  |
| 18. USSR                      | 2                  | 95,000                 | 94            | 20                                            | —            | —                            | —                                                                                              |                  |
| 19. English<br>Electric       | 5                  | 77,000                 | 68.5          | —                                             | 150          | Vertical                     | —                                                                                              |                  |

6.10 Notable Francis Turbine Installations of the World

| S. No. | Plant            | Location              | No. of Units | Unit Capacity (hp) | Head (m) | Speed (rpm) | Other Details                                                                                 |
|--------|------------------|-----------------------|--------------|--------------------|----------|-------------|-----------------------------------------------------------------------------------------------|
| 1.     | Krasnoyarsk      | (Yenisei R.) U.S.S.R. | 10           | 685,000            | 95       | 93.8        | Efficiency=94%; $Q=1140$ lit/sec<br>Runner Dia=7.5 m, Weight=250 tonnes. Mfg. by LMZ (Russia) |
| 2.     | Churchill Falls  | Labrador, Canada      | 10           | 618,000            | 316      | 200         |                                                                                               |
| 3.     | Portage Mt.      | Br. Columbia, Canada  | 10           | 356,500            | 152      | 150         |                                                                                               |
| 4.     | Alcantara        | Spain                 | 4            | 330,000            | 108      | 115.4       |                                                                                               |
| 5.     | Bratsk           | (Angara R.) U.S.S.R.  | 16           | 310,000            | 100      | 150         | $D_2=5.5$ m, Mfg. LMZ (Russia)                                                                |
| 6.     | Seilevare        | Sweden                | 1            | 302,000            | 167      | —           |                                                                                               |
| 7.     | Mossy Rock       | U. S. A.              | 2            | 263,000            | 106      | 128.6       |                                                                                               |
| 8.     | Outardes No. 3   | Quebec, Canada        | 8            | 258,000            | 144      | 163.6       |                                                                                               |
| 9.     | Big Bend No. 2   | Alberta, Canada       | 1            | 250,000            | 118      | 150         |                                                                                               |
| 10.    | Aswan            | Egypt                 | 12           | 246,000            | 58       | 100         |                                                                                               |
| 11.    | Guri             | (Caroni R.) Venezuela | 3            | 242,000            | 92       | 128.6       |                                                                                               |
| 12.    | Furnas           | Brazil                | 6            | 232,000            | 94       | 150         | Mfg : Nohab (Sweden)                                                                          |
| 13.    | Akosombo         | (Volta R.) Ghana      | 4            | 212,000            | 69       | 115.4       |                                                                                               |
| 14.    | Big Bend Unit I  | Alberta, Canada       | 1            | 210,000            | 118      | 163.6       |                                                                                               |
| 15.    | Stornorfors      | Sweden                | 3            | 202,000            | 75       | 125         |                                                                                               |
| 16.    | Lewiston         | New York, U.S.A.      | 13           | 200,000            | 92       | 120         |                                                                                               |
| 17.    | Chute-des-Passes | Quebec, Canada        | 5            | 200,000            | 115      | 200         |                                                                                               |
| 18.    | Messaure         | Sweden                | 2            | 197,000            | 86       | 150         |                                                                                               |
| 19.    | Bersimis II      | Quebec, Canada        | —            | 180,000            | 118      | 163.6       |                                                                                               |
| 20.    | Bersimis I       | Quebec, Canada        | 8            | 175,000            | 266      | 277         |                                                                                               |
| 21.    | Mani Couagan 2   | Quebec, Canada        | 8            | 170,000            | 70       | 120         |                                                                                               |
| 22.    | Grand Coulee     | Washington, U.S.A.    | 18           | 165,000            | 101      | 120         | Mfg : Charmilles (Switzerland)                                                                |
| 23.    | Bortla           | Orgues, France        | —            | 158,000            | 112      | 187.5       | Mfg : Japanese Firm                                                                           |
| 24.    | —                | Suhlo and Julu, Japan | —            | 145,000            | 76       | —           |                                                                                               |
| 25.    | Noxon            | Montana, U.S.A.       | 4            | 140,000            | 49.5     | 100         |                                                                                               |
| 26.    | Kariba           | Rhodesia              | 6            | 140,000            | 86       | 167         |                                                                                               |
| 27.    | Murray I         | Australia             | 10           | 130,000            | 492      | 500         |                                                                                               |
| 28.    | —                | Hemsil (Norway)       | —            | 50,000             | 542      | 750         | Mfg : Kvaerner Brug, Oslo (Norway)                                                            |

∴ Discharge going to the reservoir =  $12 - 5.25 = 6.75 \text{ m}^3/\text{sec}$   
 Hence the capacity of reservoir =  $6.75 \times 3 \times 30\frac{1}{2} \times 24 \times 3,600 \text{ m}^3$   
 (Assuming average number of days in a month =  $30\frac{1}{2}$ )  
 $= 5.31 \times 10^7 \text{ m}^3$  *Answer*

(b) Gross head = 200 m

Head lost due to friction,  $H_f = 200 \times 0.03 = 6 \text{ m}$

but  $H_f = \frac{4f \cdot L}{d} \cdot \frac{v^2}{2g}$  where  $d = \text{dia of pipe};$   
 $L = \text{length of pipe.}$

$$= \frac{4f \cdot L}{2g \cdot d} \left( \frac{Q}{\frac{\pi}{4} d^2} \right)^2 = \frac{4f \cdot L Q^2}{2g \times \left( \frac{\pi}{4} \right)^2 \cdot d^5}$$

where  $Q = \text{the quantity of water through each pipe}$

$$= \frac{5.25}{3} = 1.75 \text{ m}^3/\text{sec}$$

$$\therefore 6 = \frac{4 \times 0.005 \times 1,300 \times 1.75^2}{2 \times 9.81 \times (0.785)^2 \times d^5}$$

$$\text{or } d^5 = 1.091 \text{ m}^5$$

$$\text{or } d = 1.0176 \text{ m} \quad \text{Answer}$$

(c) The BHP output of the station—

$$\text{BHP} = \frac{\gamma \cdot Q \cdot H}{75} \cdot \eta_t = \frac{1,000 \times 5.25 \times (200 - 6)}{75} \times 0.87$$

$$= 10,810 \text{ BHP} \quad \text{Answer}$$

### Design of Component Parts of Francis Turbine

**6.12 Spiral Casting**—The outer curve of the spiral casing is of the form of an archimedean spiral (refer Fig 6.9).

If the inlet diameter and velocity be  $d_i$  and  $v_i$  respectively, quantity of water flowing per second,

$$Q_o = v_i \cdot \frac{\pi}{4} \cdot d_i^2 \quad \dots(6.1)$$

$$\text{also } v_i \propto \sqrt{2gH}$$

$$= K_{v_i} \sqrt{2gH}$$

The maximum value of  $v_i$  is about 10 m/sec.

The inlet diameter  $d_i$  should be less than but may be equal to penstock diameter.

At any angle  $\theta$  (refer Fig 6.9)

$$\text{Radius } R_s = R_i + \frac{\theta}{2\pi} \cdot d_i \quad \dots(6.2)$$

$$\text{Quantity of water } Q = \frac{\theta}{2\pi} \cdot Q_o \quad \dots(6.3)$$

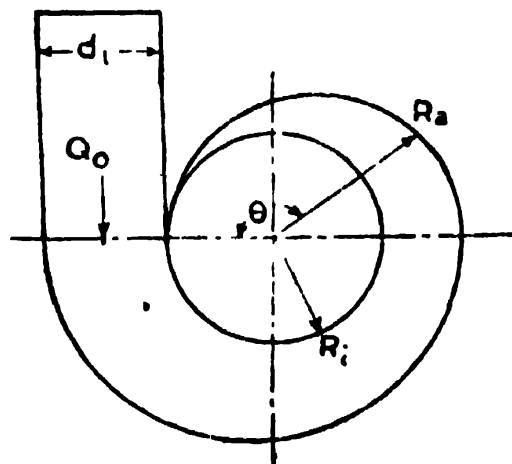


Fig 6.9 Spiral Casing

A full spiral is generally recommended for high head turbine and semi-spiral for low head installation.

TABLE 6.1  
Practical Data for  $K_{v_1}$

| $K_{v_1}$ | 0.2  | ... | ... | ... | 0.15 |
|-----------|------|-----|-----|-----|------|
| $H$ in m  | 25   | ... | ... | ... | 500  |
| $N_s$     | High | ... | ... | ... | Low  |

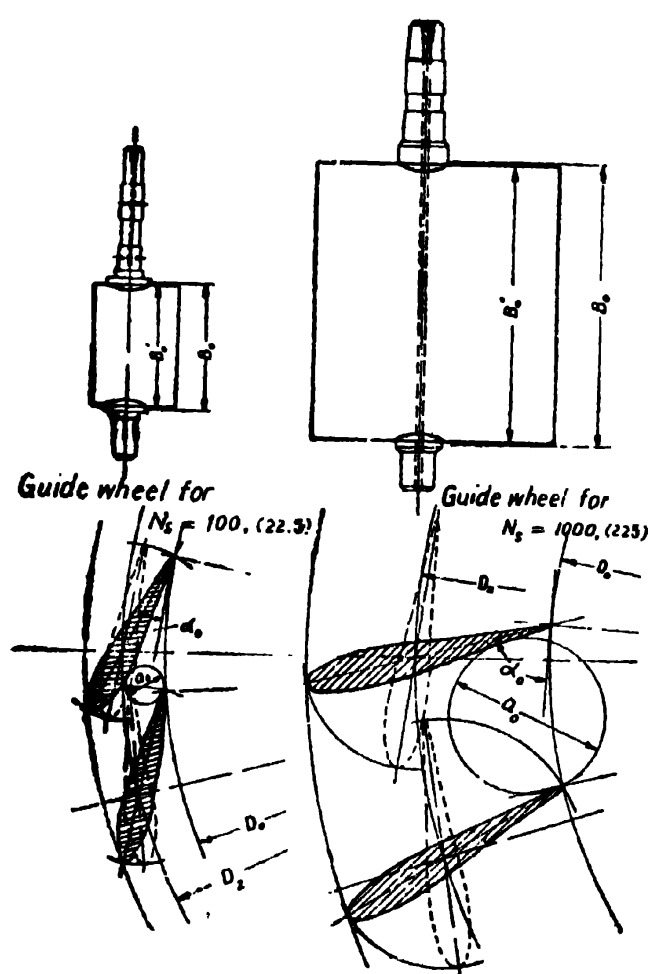


Fig 6.10 Guide Vanes and Guide Wheel

**6.13 Guide Vanes—**Guide Vanes have an aerofoil section to enable stream line flow (refer Fig 6.10).

(a) **Diameter**—Let  $D_o$  be the diameter, measured upto the trailing edge of the guide vane, then tangential velocity,

$$u_o = \frac{\pi \cdot D_o \cdot N}{60}$$

Here  $u_o$  is a hypothetical velocity assumed for the sake of convenience in the calculation of  $D_o$ . Actually guide vanes are not moving.

$$\text{Since } u_o = K_{u_o} \sqrt{2gH}$$

where  $K_{u_o} = 0.7$  to  $1.34$

increasing with specific speed

$$\text{also } u_o = \frac{\pi \cdot D_o \cdot N}{60}$$

$$\therefore D_o = \frac{60 \cdot K_{u_o} \sqrt{2gH}}{\pi \cdot N} \quad \dots(6.4)$$

(b) **Depth or Height**—Let the depth of a guide vane be  $B_o$ . Then if the radial velocity at outlet be  $v_{r_o}$ , neglecting the thickness of trailing edge,

Area across flow  $= \pi \cdot D_o \cdot B_o$

and discharge  $Q = \pi \cdot D_o \cdot B_o \cdot v_{m_o}$

$$= \pi \cdot D_o \cdot B_o \cdot K_{v_{m_o}} \sqrt{2gH}$$

where  $K_{v_{m_o}}$  increases from 0.15 to 0.35 with increase in specific speed (refer Fig 6.32).

$$\text{Finally, } B_o = \frac{Q}{\pi \cdot D_o \cdot K_{v_{m_o}} \cdot \sqrt{2gH}}$$

(c) **Length**—Length of the guide vane,  $L_o \approx 0.3 D_o$  ... (6.5)

(d) **Number of Guide Vanes**—The number of guide vanes  $Z_o$  depends on the diameter  $D_o$  and varies usually from 8 to 24. The guide vanes should be even in number so that the guide wheel can be divided evenly for ease in manufacture and transport when it is very large.

The following table gives the number of guide vanes  $Z_o$  for Francis turbine against different values of  $D_o$ . Two sets have been mentioned, one having a specific speed less than 200 and the other for more than 200.

TABLE 6.2  
Practical Values for  $Z_o$  versus  $D_o$

| $N_s$ | $Z_o = 8$             | 10  | 12  | 14    | 16    | 18    | 20    | 24     |
|-------|-----------------------|-----|-----|-------|-------|-------|-------|--------|
| <200  | Values of $D_o$ in mm |     |     |       |       |       |       |        |
|       | From —                | 250 | 400 | 600   | 800   | 1,000 | 1,250 | >1,700 |
|       | to 250                | 400 | 600 | 800   | 1,000 | 1,250 | 1,700 | —      |
| >200  | From —                | 300 | 450 | 750   | 1,050 | 1,350 | 1,700 | >2,100 |
|       | to 300                | 450 | 750 | 1,050 | 1,350 | 1,700 | 2,100 |        |

**6.14 Francis Runner**—Inlet diameter  $D_1$  is shown in Fig 6.11 up to the end of crown. Sometimes it is measured up to the centre of the entrance edge of runner blade.

In any case,

$$u_1 = \frac{\pi \cdot D_1 \cdot N}{60} \quad \text{and} \quad u_1 = K_{u_1} \cdot \sqrt{2gH}$$

where  $K_{u_1}$  varies from 0.62 to 0.82 increasing with specific speed (refer Fig 6.32)

$$\therefore D_1 = \frac{60 \cdot K_{u_1} \cdot \sqrt{2gH}}{\pi \cdot N} \quad \dots (6.6)$$



Outlet diameter  $D_2$  (refer Fig 6.12)  $\approx \frac{D_1}{2}$

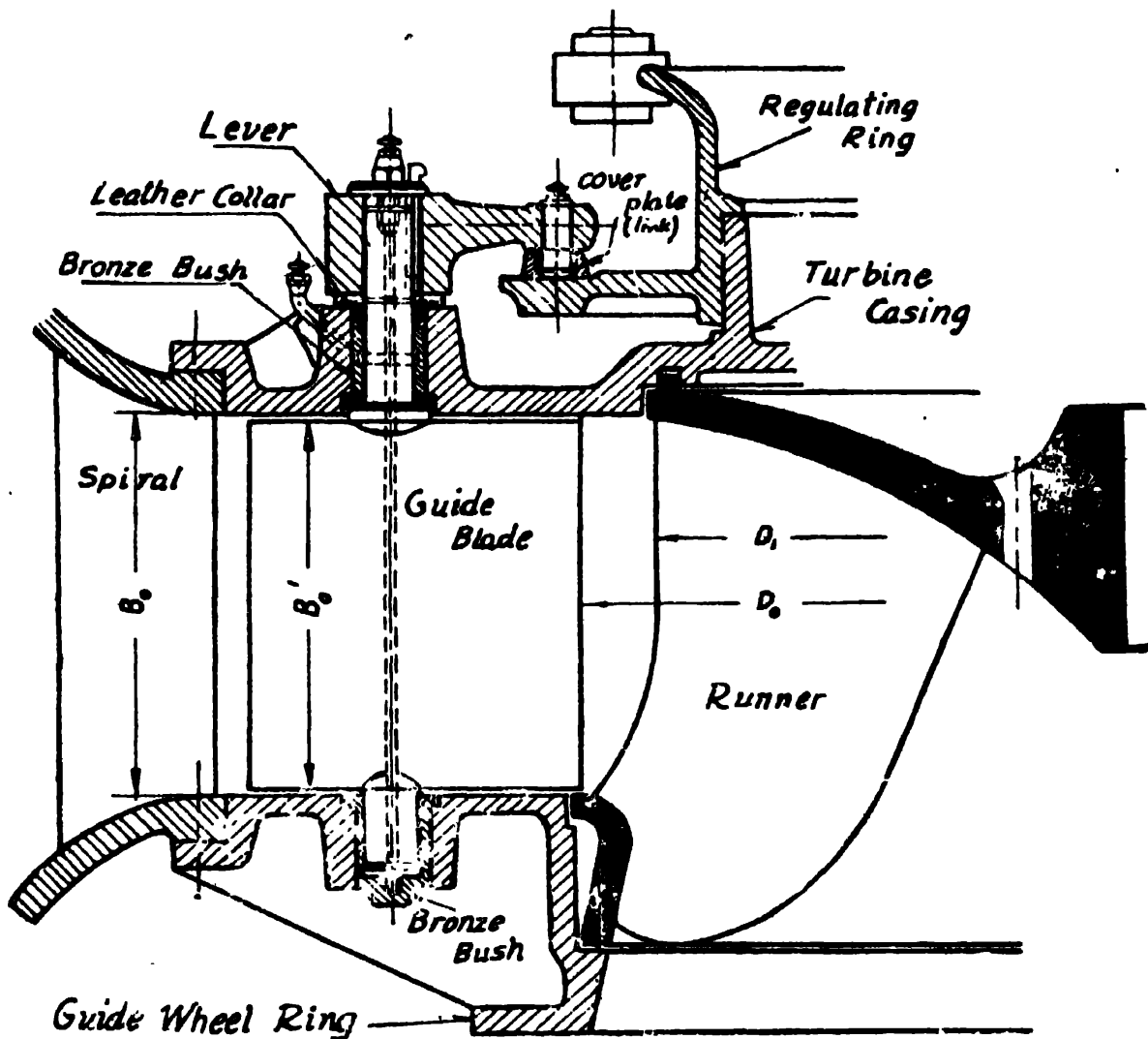


Fig 6.11 Francis Runner with Mechanism for Guide Blade Movement

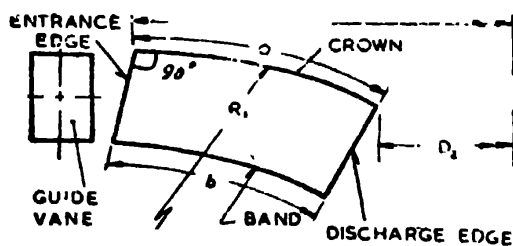


Fig 6.12 Construction of Francis Runner Outlines

Radius of curvature of crown

$$R_1 = \frac{D_1}{2}$$

When  $D_1$ ,  $D_2$  and  $R_1$  are fixed, length of crown can be calculated and length  $b$  of band is 0.7 to 0.75 times  $a$ . The entrance edge is a potential line, and suitable curvature can be given to the discharge edge (refer Fig 6.12).

**Thickness of Runner Blades** varies from 5 to 25 mm depending on  $D_0$ .

**Number of Runner Blades**  $Z_2 = Z_0 \pm 1$  to avoid the synchronous effect.

**6.15 Shapes of Francis Runner and Evolution of Kaplan Runner**—The exact shape of the Francis runner depends on its specific speed. It is obvious from the equation for specific speed (refer Eqn 3.10)



**PLATE 14. Inlet Pipe of Jet Vertical Turbine Manufacturers :  
Voith Heidenheim (W. Germany)**

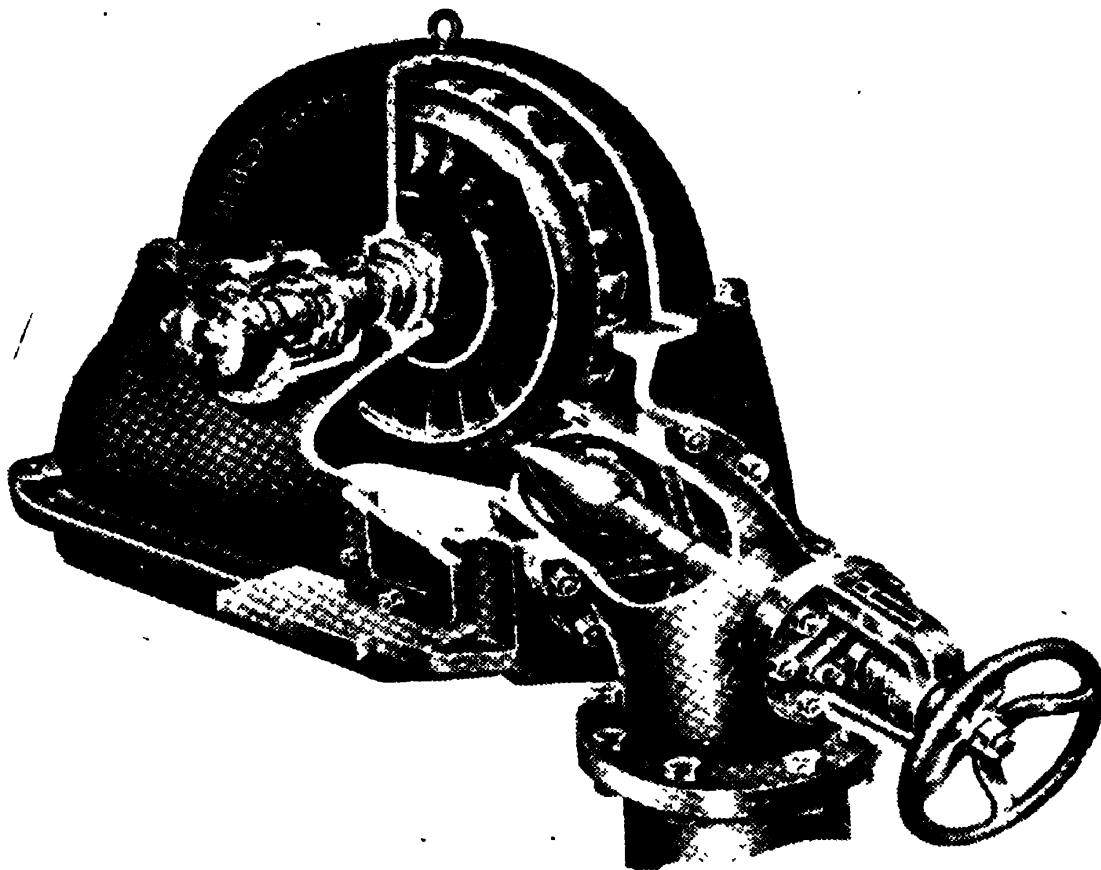


**PLATE 15. Section Through a Turgo-Impulse Turbine with Hand Regulated Spear**



**PLATE 16. Diversion of Penstock for Head 65 m. of Water Dia 4082/3172/2600 mm**



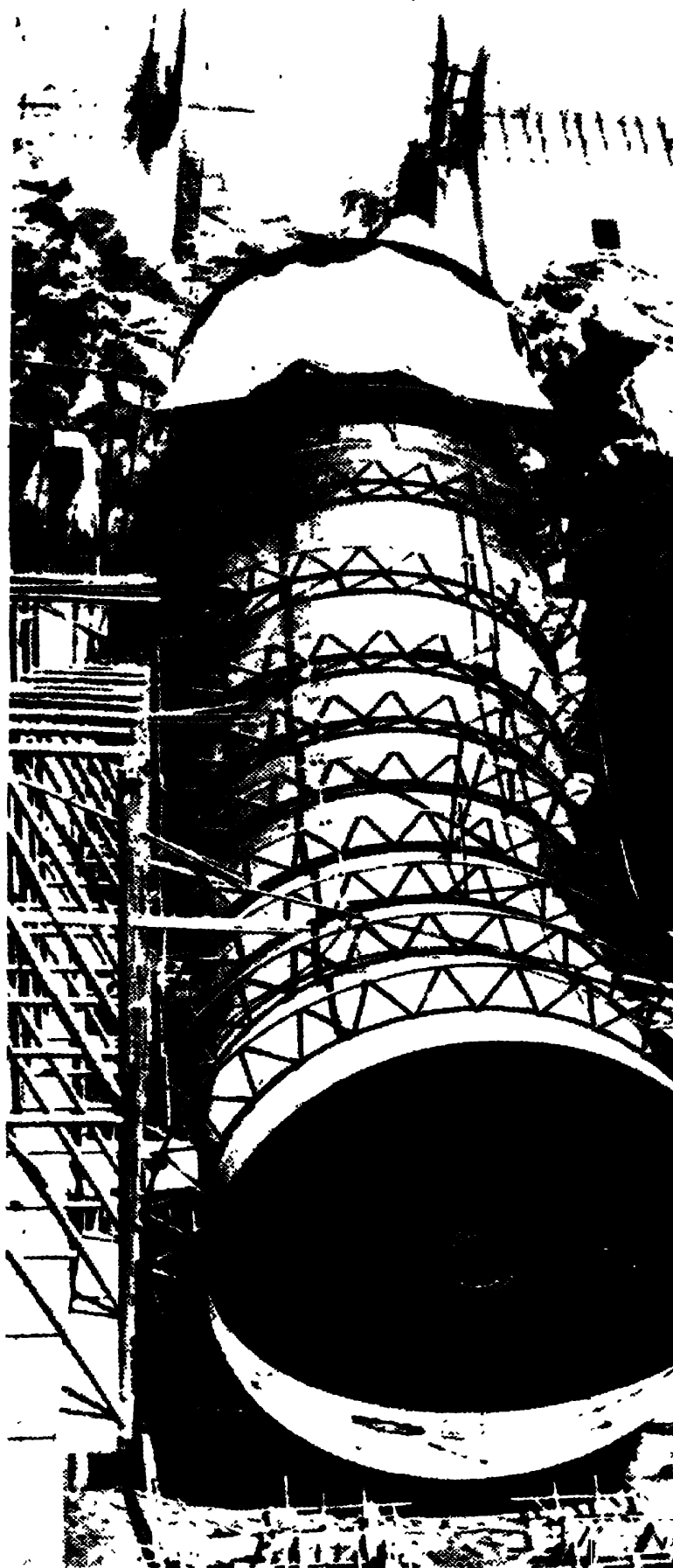


**PLATE 18. Penstock 7.5 m Inside Diameter, 30 m Long, Manufactured by Voith West Germany for Macagua 1 Hydro Power Station on Rio Caroni (Venezuela)**



**PLATE 19.** Spiral or Scroll Casting with the Turbine Runner fitted  
at Escher Wyss work Turbin Output = 55,200 to  
64,800 KW:  $Q = 17,050$  to  $18,200$  lit Sec,  $H = 371$  to  
412 m

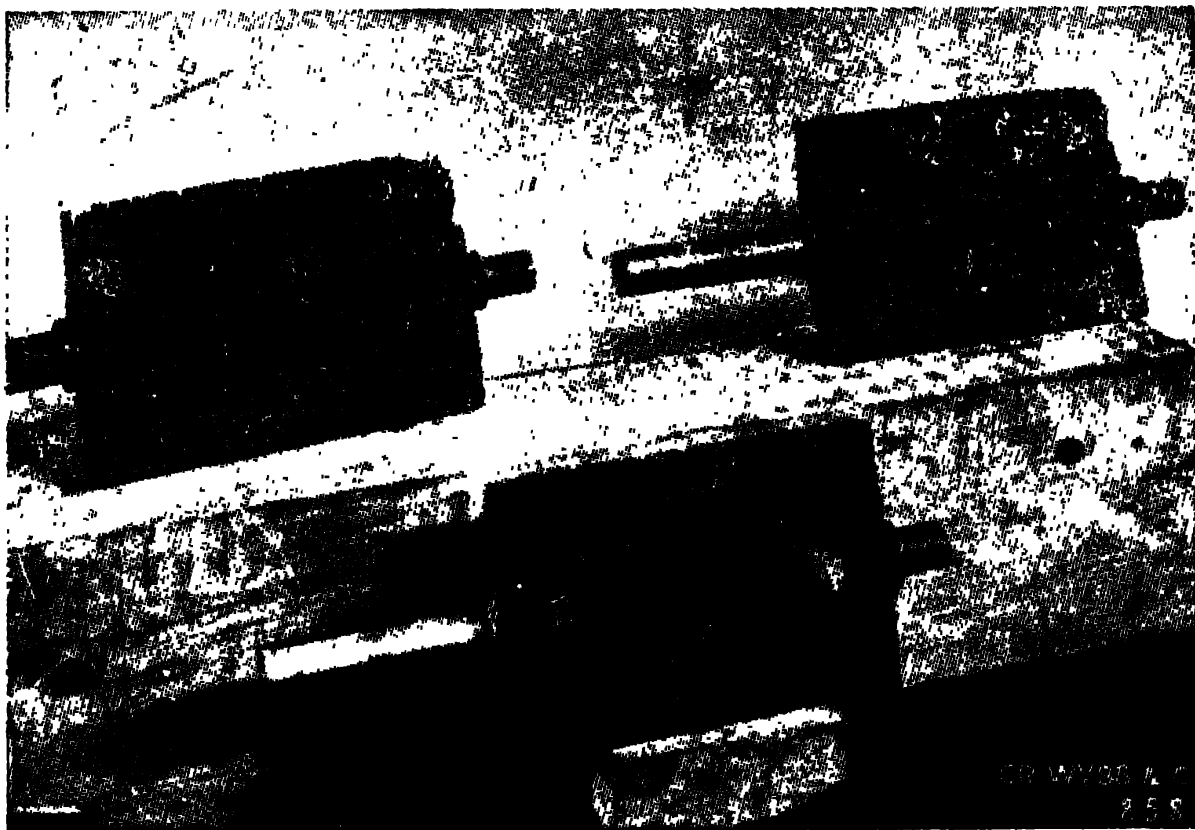


**PLATE 20.** Destruction of Francis Turbine Guide Vanes due to  
Sand Erosion-Escher Wyss (Switzerland)

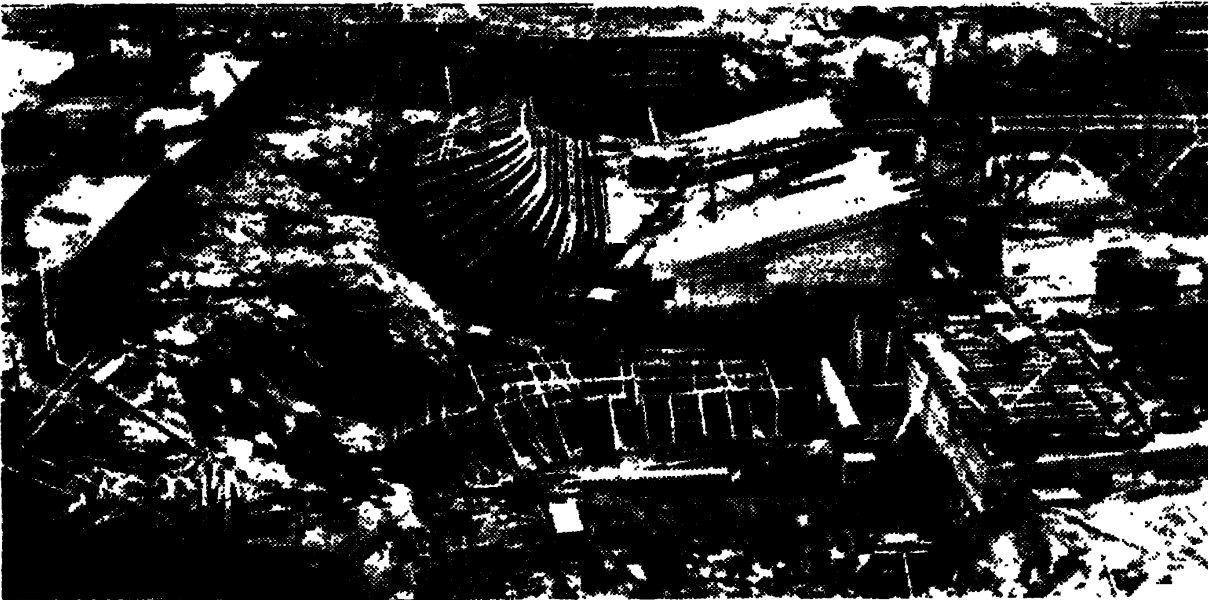


**PLATE 21.** Shuttering of a Draft Tube for Kaplan for Jochensten Power Station at Dunube River, Germany. Manufactured by Voith West Germany

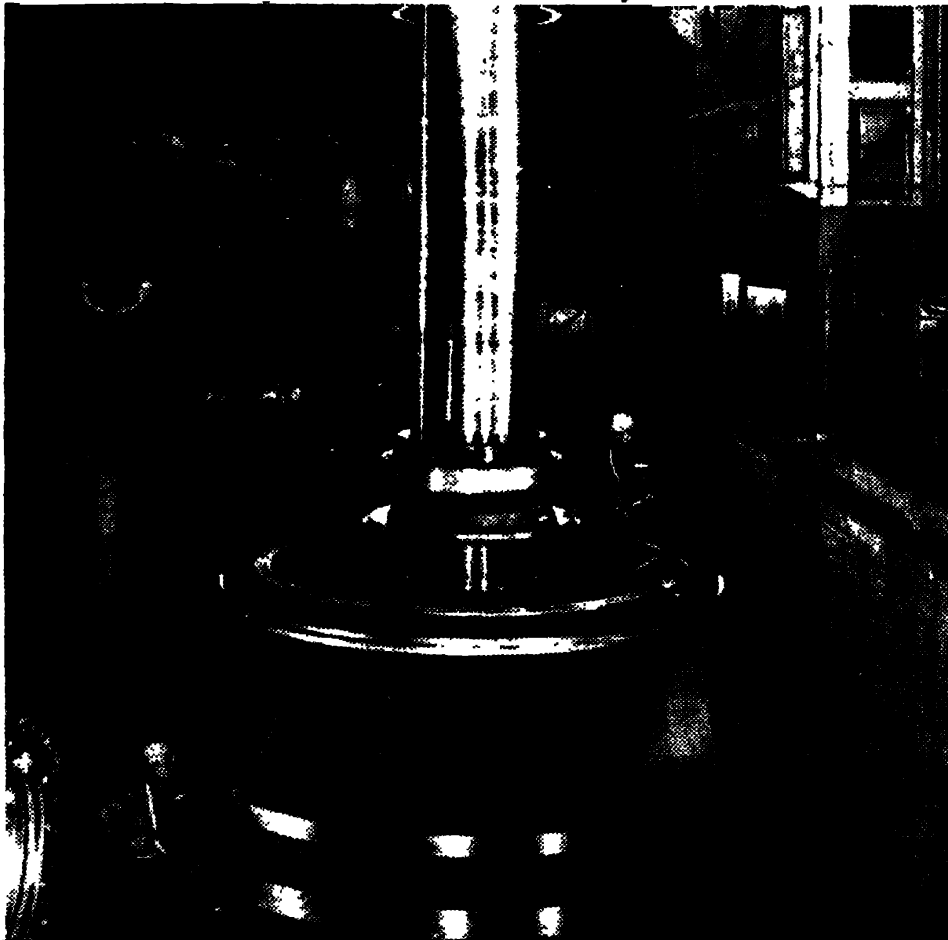


**PLATE 22.** Largest Ally-Steel Francis Runnerj in the whole world 13% Chrome Cast Steel for Macagua 1 Hydro Electric Power Station at Rio Coroni (Venezula) Maximum Runner Dia-7:310 mm Height-2.600 mm, Picture shows Contours of runner blades being grounded and Precision Polished at Voith Heidenheim (West Germany) Work n=

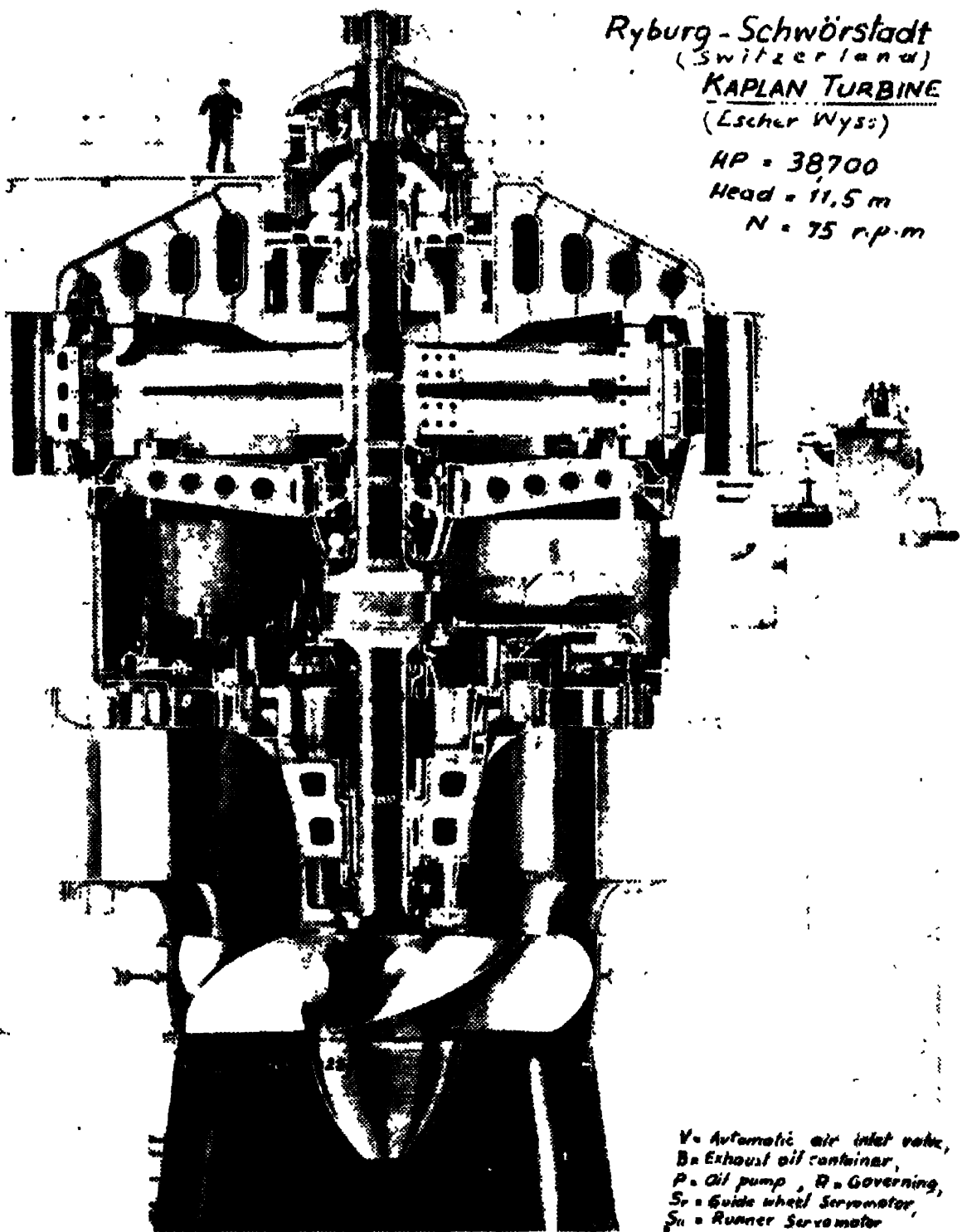


PLATE 23. One of the Largest Kaplan Turbine of Europe at  
 Ryburg Schworastadt Swiss-German (Border)





PLATE 24. Karpan Turbine Runner for Aswan [Egypt] Power Plant Manufactured by Escher Wyss. Zurich

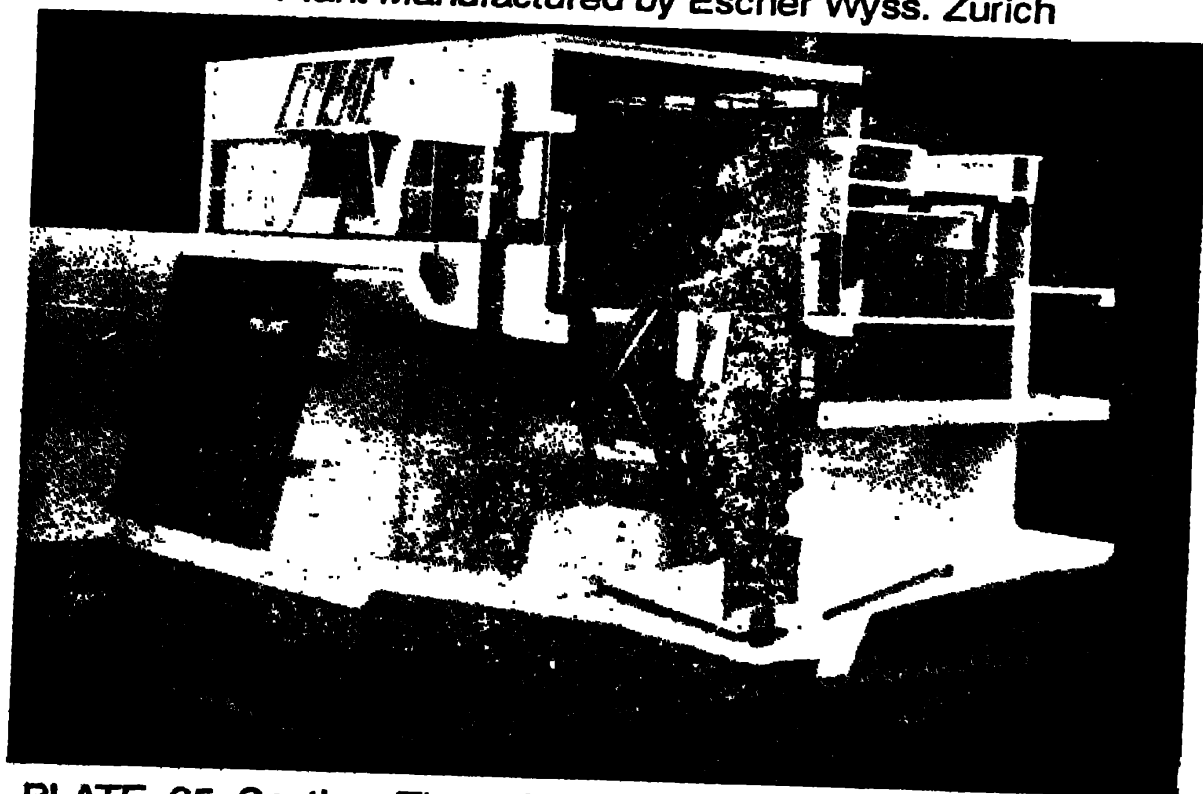


PLATE 25. Section Through a Tubular [Bulb] Turbine Plant Model

that higher specific speed means lower head. This requires that the runner should admit a comparatively large quantity of water. This can be done by increasing either.

(i) Ratio  $\frac{B_o}{D_1}$

or (ii) non-dimensional factor  $K_{v_{m_o}} = \frac{v_{m_o}}{\sqrt{2gH}}$

In the first case the height of guide vanes  $B_o$  and correspondingly the height of runner should increase while in the second case  $v_{m_o}$ , the radial velocity of flow through the guide vanes should increase.

Fig 6.13 shows in stages the changes in the shape of Francis runner with variation of specific speed (refer also Fig 4.10). The first three types have, in order, the slow, the normal and the fast runners. Flow of water through the runner is radial at entrance but more or less axial at exit. As the specific speed increases discharge becomes more and more axial.

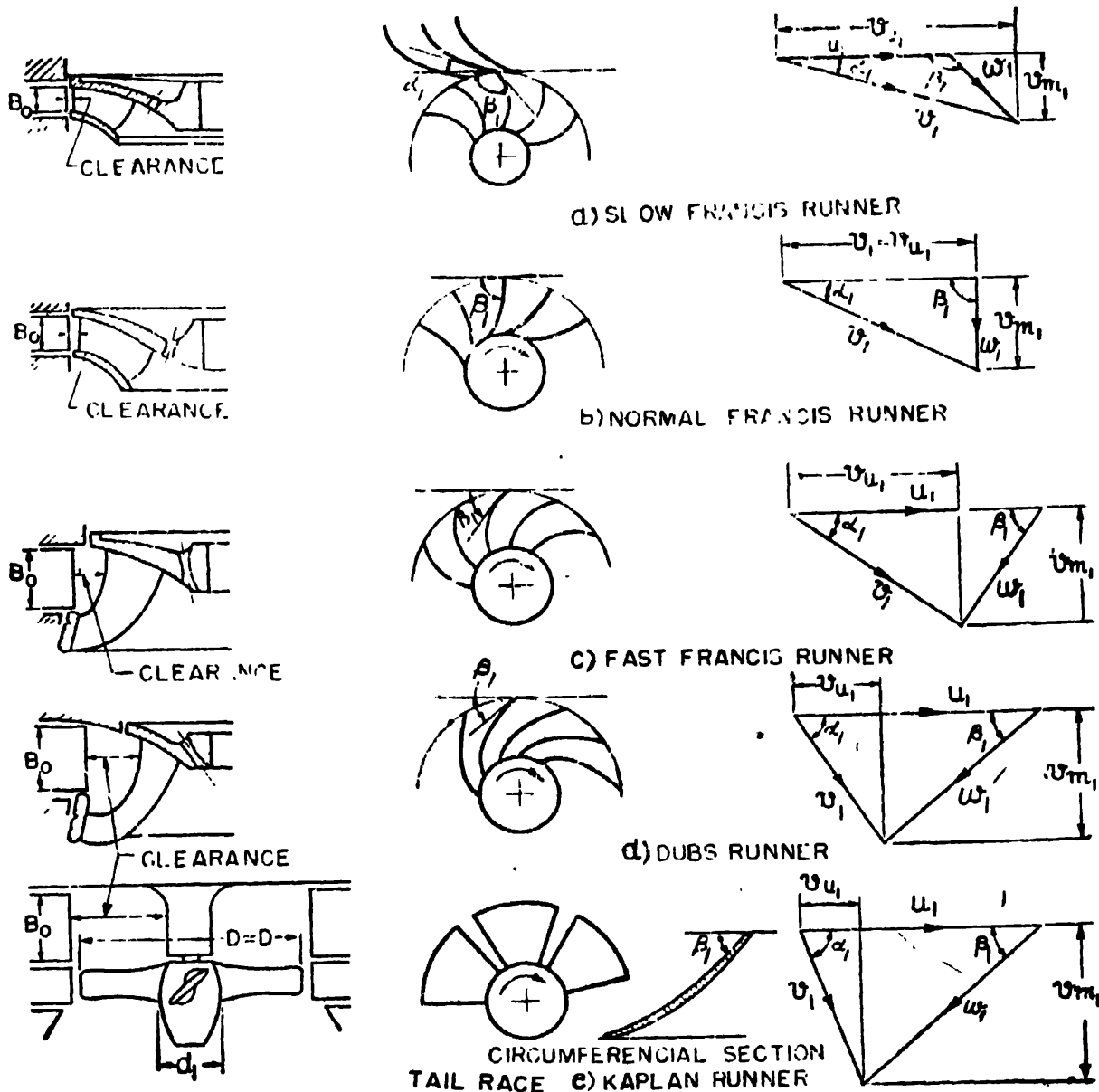


Fig 6.13 Changes in the Shape of Francis Runner and its Inlet Velocity Triangles with the Variation of Specific Speed

Fourth type [refer Fig 6.13 (d)] is the high specific speed Francis runner, better known as Dubs runner. Here the water enters diagonally and the discharge is almost entirely axial. If the band of a Dubs runner is removed, it appears like a propeller type runner. In the strictly propeller type runner, number of blades is less and flow is throughout purely axial.

Fig 6.13 (e) shows such a runner. It is designated Kaplan runner after Kaplan who first designed this type of turbine runner (refer chapter 7).

By drawing the inlet velocity triangle for each of the five runners shown in Fig 6.13 it is seen that decrease in head and increase in specific speed are accompanied with a reduction of inlet velocity  $v_1$  of water. But the radial velocity  $v_{m1}$  increases letting in a large amount of water.

The values of  $N_s$ ,  $K_{u1}$ ,  $\alpha_1$  and  $\beta_1$  for each of the runners are given below.

**(a) Slow Runner :**

$$N_s = 60 \text{ to } 120$$

$$\alpha_1 = 15^\circ \text{ to } 25^\circ$$

$$K_{u1} = 0.62 \text{ to } 0.68$$

$$\beta_1 = 90^\circ \text{ to } 120^\circ$$

$$\frac{B_o}{D_1} = \frac{1}{25} \text{ to } \frac{1}{30}$$

Value of  $\beta_1$  being more than  $90^\circ$  the course of water in the runner is sharply curved and this implies that blades should have a similar curvature

**(b) Normal Runner :**

$$N_s = 120 \text{ to } 180$$

$$\alpha_1 = 25^\circ \text{ to } 32\frac{1}{2}^\circ$$

$$K_{u1} = 0.68 \text{ to } 0.72$$

$$\beta_1 = 90^\circ$$

$$\frac{B_o}{D_o} = \frac{1}{8} \text{ to } \frac{1}{4}$$

**(c) Fast Runner :**

$$N_s = 180 \text{ to } 300$$

$$\alpha_1 = 32\frac{1}{2}^\circ \text{ to } 37\frac{1}{2}^\circ$$

$$K_{u1} = 0.72 \text{ to } 0.76$$

$$\beta_1 = 60^\circ \text{ to } 90^\circ$$

$$\frac{B_o}{D_1} = \frac{1}{4} \text{ to } \frac{1}{2}$$

**(d) Dubs Runner :**

$$N_s = 280 \text{ to } 525$$

$$\alpha_1 = 37\frac{1}{2}^\circ \text{ to } 40^\circ$$

$$K_{u1} = 0.46 \text{ to } 0.70$$

$$\beta_1 = 45^\circ \text{ to } 60^\circ$$

$$\frac{B_o}{D_1} = \frac{1}{2}$$

**(e) Kaplan Runner :**

$$N_s = 300 \text{ to } 1,000$$

$$\alpha_1 < 90^\circ$$

$$K_{u1} = 1.2 \text{ to } 2.4 \text{ (i.e., more than one)}$$

$$\beta_1 < 90^\circ$$

**Outlet Velocity Triangle** for each of these five runners is more or less similar and is shown in Fig 6.14

$\alpha_1 \approx 90^\circ$  for Francis Runners,

$\alpha_1 = 90^\circ$  for Kaplan Runners.

**6.16 Draft Tube Theory**—It has been observed earlier that the steadily increasing cross-sectional area of the draft tube causes a large portion of the kinetic head to be converted to pressure head as the water approaches the tail race.

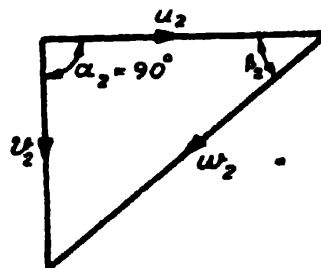


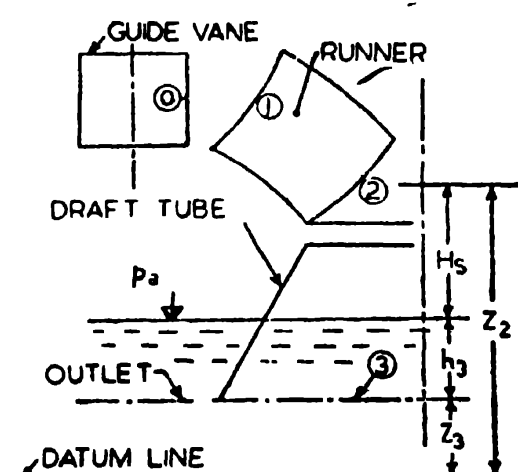
Fig 6.14 Outlet Velocity Triangle for all Reaction (Francis as well as Kaplan) Runners

Thus gain in head is

$$\frac{v_2^2}{2g} - \frac{v_3^2}{2g} = \left\{ 1 - \left( \frac{A_2}{A_3} \right)^2 \right\} \cdot \frac{v_2^2}{2g} \text{ (neglecting frictional loss)}$$

where  $v_2$ ,  $A_2$  and  $v_3$ ,  $A_3$  are the velocities and areas at the points (2) and (3) respectively (refer Fig 6.15).

Had there been no draft tube, the kinetic head  $\frac{v_2^2}{2g}$  would have been



entirely lost. Kinetic head thus saved by the draft tube adds to the effective head of the turbine (refer Eqn 6.7).

The other function of the draft tube is to permit a negative (suction) head to be established at the runner outlet (point 2 in Fig 6.15), which can be proved as follows :

(a) Neglecting frictional loss in the draft tube and applying Bernoulli's Theorem between points (2) and (3)—

Fig 6.15 Draft Tube Theory

$$\frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{v_3^2}{2g} + z_3$$

$$\text{or } \frac{p_2}{\gamma} = \frac{p_3}{\gamma} - (z_2 - z_3) - \frac{v_2^2 - v_3^2}{2g}$$

$$\text{But } \frac{p_3}{\gamma} = \frac{p_a}{\gamma} + h_3$$

$$\therefore \frac{p_2}{\gamma} = \frac{p_a}{\gamma} - (z_2 - z_3 - h_3) - \frac{v_2^2 - v_3^2}{2g}$$

But  $(z_2 - z_3 - h_3) = H_s$  (Height of runner outlet above tail race level)

$$\therefore \frac{p_2}{\gamma} = \frac{p_a}{\gamma} - \left( H_s + \frac{v_2^2 - v_3^2}{2g} \right) \quad \dots (6.7)$$

where  $H_s$  = static suction head

and  $\frac{v_2^2 - v_3^2}{2g}$  = dynamic suction head.

Minimum value of  $\frac{p_a}{\gamma}$  is the vapour pressure at the working temperature. This can be obtained from the following table :

TABLE 6.3

**Vapour Pressure Function of Temperature**

| Temperature °C                          | 0    | 10   | 20   | 30   | 40   | 100  |
|-----------------------------------------|------|------|------|------|------|------|
| Vapour Pressure, $H_v$<br>in m of water | 0.06 | 0.13 | 0.24 | 0.43 | 0.75 | 0.91 |

Pressure  $p_2$  should not be below the vapour pressure, otherwise the vapours will be formed causing cavitation.  $p_a$  and  $p_2$  are thus fixed. If  $v_2$  is more,  $H_s$  must be small. In order to keep the suction head under the cavitation limits, vertical distance between the runner outlet and tail race level must be reduced as the elevation above sea-level increases.  $H_s$  may be negative in which case the runner obviously would be placed below the tail race level.

Considering loss of head  $h_f$  due to friction in the draft tube and applying Bernoulli's Theorem between points (2) and (3).

$$\frac{p_3}{\gamma} + \frac{z_3^2}{2g} + z_3 = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 - h_f$$

$$\text{or } \frac{p_2}{\gamma} = \frac{p_a}{\gamma} - (z_2 - z_3 - h_3) - \frac{v_2^2 - v_3^2}{2g} + h_f$$

Now  $H_s = z_2 - z_3 - h_3$

and  $h_f$  is generally expressed as function of dynamic suction head.

$$\text{or } h_f = k \cdot \frac{v_2^2 - v_3^2}{2g}$$

$\therefore$  Substituting the values of  $H_s$  and  $h_f$

$$\frac{p_2}{\gamma} = \frac{p_a}{\gamma} - \left\{ H_s + (1-k) \cdot \frac{v_2^2 - v_3^2}{2g} \right\} \quad \dots(6.8)$$

Comparing Eqn 6.7 and 6.8, it is seen that with the loss of head  $h_f$ , which always occurs in practice, the value of pressure  $p_2$  increases, thus reducing the danger of cavitation in case of a turbine, but increasing the danger of cavitation in a centrifugal pump.

Let  $1-k = \eta_d$ , which is termed as the draft tube efficiency

$$\text{then } \frac{p_2}{\gamma} = \frac{p_a}{\gamma} - \left( H_s + \eta_d \cdot \frac{v_2^2 - v_3^2}{2g} \right) \quad \dots(6.8a)$$

**Problem 6.2** A Francis turbine is fitted with a vertical conical shaped draft tube. The top and bottom diameters are equal to 60 cm and 90 cm.

respectively. The tube is running full with water flowing downwards, and it has a vertical height of 6 m out of which 1.5 m is drowned in the tail race water. Assume friction loss of head between the top and the bottom points as 0.3 times the kinetic head at draft tube exit. The velocity at exit is 1.5 m per sec. Determine—

(a) the pressure head at the top point of the draft tube in m of water and in kg per sq cm,

(b) the total head at the same point with reference to the tail race as a datum,

(c) the total head at the bottom point with reference to the tail race as a datum,

(d) the power in the water at the top of the tube,

(e) the power in water at the bottom of the tube,

and (f) the efficiency of the draft tube.

### Solution

$$D_2 = 60 \text{ cm}$$

$$D_3 = 90 \text{ cm}$$

$$H = 6 \text{ m}$$

$$H_{L_{2-3}} = 0.3 \cdot \frac{v_3^2}{2g}$$

$$H = 6 - 1.5 = 4.5 \text{ m} \quad v_3 = 1.5 \text{ m/sec}$$

(a) Applying Bernoulli's Theorem between points 2 and 3 (refer Fig 6.15) :

$$\frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 = \frac{p_3}{\gamma} + \frac{v_3^2}{2g} + z_3 - H_{L_{2-3}}$$

$$\begin{aligned} \therefore \frac{p_2}{\gamma} &= \frac{p_3}{\gamma} - (z_2 - z_3) - \left( \frac{v_2^2 - v_3^2}{2g} \right) + 0.3 \cdot \frac{v_3^2}{2g} \\ &= \frac{p_3}{\gamma} - H_s - \left( \frac{v_2^2 - v_3^2}{2g} \right) + 0.3 \cdot \frac{v_3^2}{2g} \end{aligned}$$

$$\text{But} \quad a_2 = \frac{\pi}{4} \times 0.6^2 = 0.282 \text{ m}^2$$

$$a_3 = \frac{\pi}{4} \times 0.9^2 = 0.635 \text{ m}^2$$

$$\therefore v_2 = \frac{a_3 \cdot v_3}{a_2} = \frac{0.635 \times 1.5}{0.282} = 3.38 \text{ m/sec}$$

Substituting the values

$$\begin{aligned} \frac{p_2}{\gamma} &= 10 - 4.5 - \frac{3.38^2 - 1.5^2}{2 \times 9.81} + 0.3 \times \frac{1.5^2}{19.62} \\ &= 10 - 4.5 - 0.469 + 0.0344 \\ &= 10 - 4.9346 \end{aligned}$$

$$\therefore \frac{p_2}{\gamma} = 5.0654 \text{ m of water absolute} \quad \text{Answer.}$$

$$\text{or} \quad \frac{p_2}{\gamma} = -4.9346 \text{ m of water}$$

$$\text{or } \frac{5.0654 \times 1,000}{100 \times 100} = 0.50654 \text{ kg/cm}^2 \text{ absolute}$$

$$\text{or } -\frac{4.9346}{10} = -0.49346 \text{ kg/cm}^2 \text{ Answer}$$

(b) Total head at the top point—

$$\begin{aligned} H_2 &= \frac{p_2}{\gamma} + \frac{v_2^2}{2g} + z_2 \\ &= -4.9346 + \frac{3.38^2}{2 \times 9.81} + 4.5 \\ &= -4.9346 + 0.581 + 4.5 \\ &= 0.1464 \text{ m of water Answer} \end{aligned}$$

(c) Total head at the bottom point—

$$\begin{aligned} H_3 &= \frac{p_3}{\gamma} + \frac{v_3^2}{2g} + z_3 = H_2 - H_{1-3} \\ &= H_2 - 0.3 \cdot \frac{v_3^2}{2g} = 0.1464 - 0.3 \times \frac{1.5^2}{19.62} \\ &= 0.1464 - 0.0344 = 0.112 \text{ m of water Answer} \end{aligned}$$

(d) Power in the water at the top of the tube—

$$\begin{aligned} P_2 &= \frac{\gamma \cdot Q \cdot H_2}{75} = \frac{1,000 \times (0.282 \times 3.38) \times 0.1464}{75} \\ &= 1.86 \text{ HP Answer} \end{aligned}$$

(e) Power in the water at the bottom of the tube—

$$\begin{aligned} P_3 &= \frac{\gamma Q \cdot H_3}{75} = \frac{1,000 \times (0.282 \times 3.38) \times 0.112}{75} \\ &= 1.42 \text{ HP Answer} \end{aligned}$$

(f) Efficiency of draft tube—

$$\begin{aligned} \eta_d &= \frac{\text{actual head converted}}{\text{theoretical head converted}} \\ &= \frac{\frac{v_2^2 - v_3^2}{2g} - 0.3 \times \frac{v_3^2}{2g}}{\frac{v_2^2 - v_3^2}{2g}} = \frac{0.469 - 0.0344}{0.469} \\ &= \frac{0.4346}{0.4690} = 0.925 \text{ or } 92.5\% \text{ Answer} \end{aligned}$$

**6.17 Cavitation**—According to the Bernoulli's equation, if the velocity of flow increases, the pressure will fall. In case of liquid, the pressure cannot fall below vapour pressure which depends upon the temperature and height above mean sea level of the site. Whenever the pressure in any turbine part drops below the evaporation pressure, the liquid boils and a large number of small bubbles of vapour are formed. It may happen that a stream of water cuts short of its path giving rise to eddies and vortices which may contain voids or bubbles. These bubbles mainly formed on account of low pressure, are carried by the stream to higher

pressure zones where the vapours condense and the bubbles suddenly collapse, as the vapours are condensed to liquid again. This results in the formation of a cavity and the surrounding liquid rushes to fill it. The streams of liquid coming from all directions collide at the centre of cavity giving rise to a very high local pressure whose magnitude may be as high as 7,000 atmospheres. Formation of cavity and high pressure are repeated many thousand times a second. This causes pitting on the metallic surface of runner blades or draft tube. The material then fails by fatigue, added *perhaps* by corrosion. Some parts of turbine *e.g.* runner blades may be torn away completely by this process.

The phenomenon which manifests itself in the pitting of the metallic surfaces of turbine parts is known as cavitation because of the formation of cavities. The cavities may be formed on the solid surface or near to it. In case it does not form on the solid surface, the pressure generated in the cavity is propagated by the pressure waves similar to ones occurring in water hammer. The intense pressure is accompanied by a considerable vibration and noise. The sound of the noise is similar to one made by a gravel in a rotating vessel.

'Prof N.S. Govinda Rao suggested a new number called '*The Cavitation Damage*' as a measure of damage due to cavitation.

Turbine parts should be properly designed in order to avoid cavitation because besides damaging the metallic surfaces, cavitation also lowers the efficiency of the turbine. The blade contours are designed for a certain streamline pattern and when the latter alters due to pitting, the resultant torque produced by the flow of water is reduced, thus decreasing the power developed.

From Eqn 6.8, it is seen that the cavitation depends upon—

(a) *Vapour pressure* which is a function of temperature of flowing water (refer Table 6.3).

(b) *Absolute pressure* or barometric pressure due to the location of turbine above mean sea level (refer Table 6.4).

(c) *Suction pressure*  $H_s$  which is the height of runner outlet above tail race level.

(d) *Effective dynamic suction head and absolute velocity of water at runner exit*

$$\text{i.e., } \eta_d \left( \frac{v_2^2 - v_3^2}{2g} \right)$$

and  $v_2$  respectively.

Prof Dietrich Thoma of Munich (Germany) (1881–1943) suggested a cavitation factor  $\sigma$  (sigma) to determine the zone where turbine can work without being affected from cavitation. Its critical value is given by

$$\sigma_{crit} = \frac{H_b - H_s}{H} = \frac{(H_a - H_v) - H_s}{H} \quad \dots (6.9)$$



where

$H_b$  = barometric pressure in metres of water ;  
 $= H_a - H_v$  ;

$H_a$  = average atmospheric pressure in m of water  
 (refer Table 6.4) ;

$H_v$  = vapour pressure in m of water (refer Table  
 6.3) ;

$H_s$  = suction pressure head in m of water (height of  
 runner outlet above tail race) ;

$H$  = working head of turbine in m of water.

Table 6.4 gives the average atmospheric pressure which depends upon the altitude above sea level.

TABLE 6.4

**Altitude vs Average Atmospheric Pressure**

| Altitude            | metre                | 0     | 1,000 | 2,000 | 3,000 | 4,000 |
|---------------------|----------------------|-------|-------|-------|-------|-------|
| Barometric Pressure | <i>mm</i> of mercury | 760   | 676   | 595   | 528   | 463   |
|                     | metre of water       | 10.35 | 9.2   | 8.1   | 7.2   | 6.3   |

Now cavitation factor  $\sigma_{crit}$  depends upon the specific speed of the turbine. Fig 6.16 gives these values for Francis turbine, depending on the experience of McGraw-Hill, New York and ETH, Zurich.

According to Prof Thoma, cavitation can be avoided if the values of  $\sigma$  are not less than the critical value given above. Thus the deciding factors in the selection of reaction turbines are specific speed  $N_s$  and the cavitation factor  $\sigma$ . Prof F. H. Roger\* (USA) suggested the following empirical relation for Francis turbines :

$$\sigma_{crit} = 0.0317 \cdot \left( \frac{N_s}{100} \right)^2 \quad \dots(6.10)$$

The maximum permissible specific speed can thus be calculated :

$$N_s = 100 \cdot \sqrt{\frac{\sigma_{crit}}{0.0317}} \quad \dots(6.11)$$

$$= 562 \cdot \sqrt{\frac{H_b - H_v}{H}} \quad \dots(6.12)$$

Fig 6.17 also shows the effect of cavitation  $\sigma$  on the turbine efficiency  $\eta$ .

\*See Proceedings of the American Society of Civil Engineers Dec. 1937.

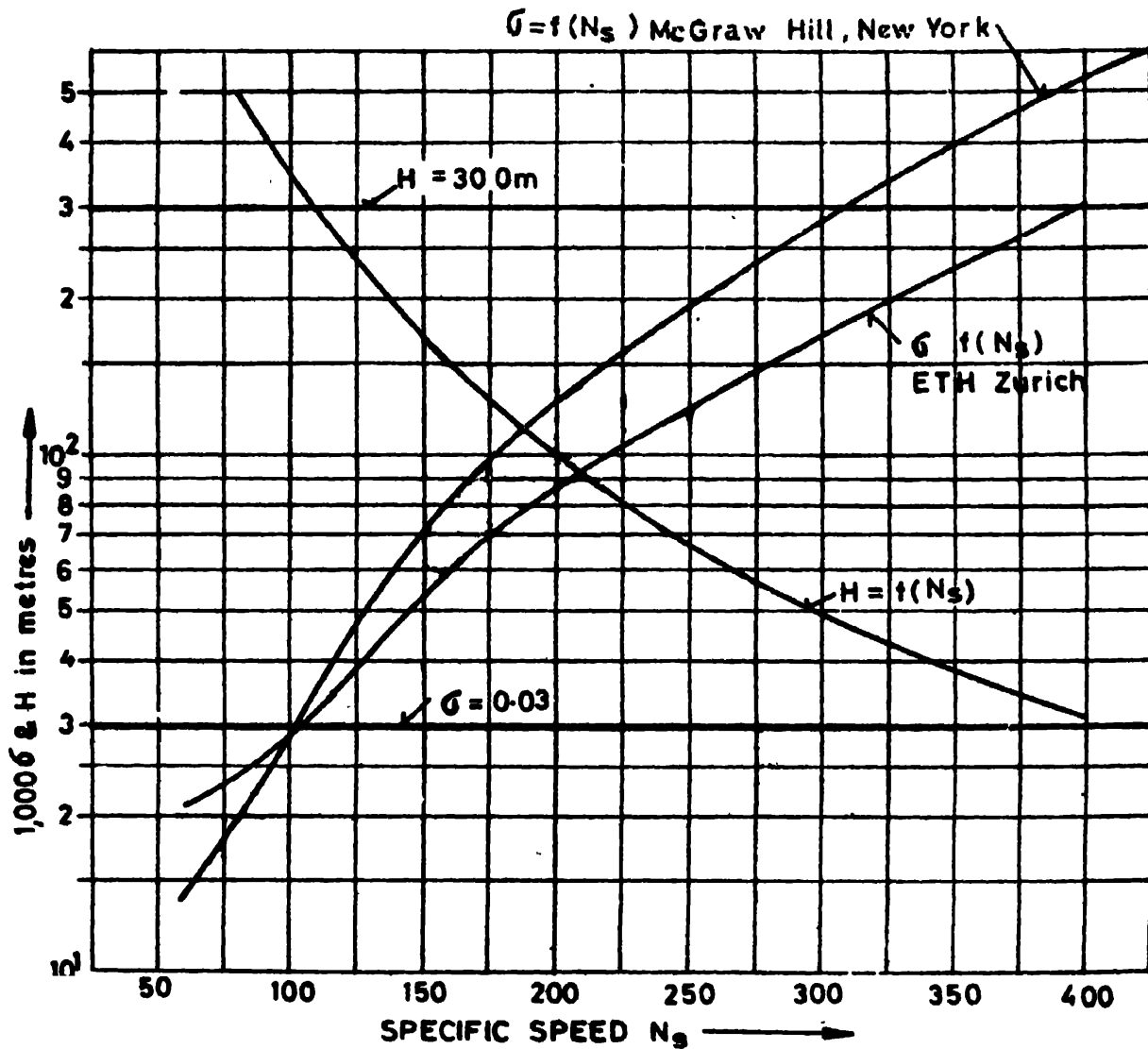


Fig 6.16 Cavitation Factor  $\sigma$  and Head  $H$  versus Specific Speed  $N_s$

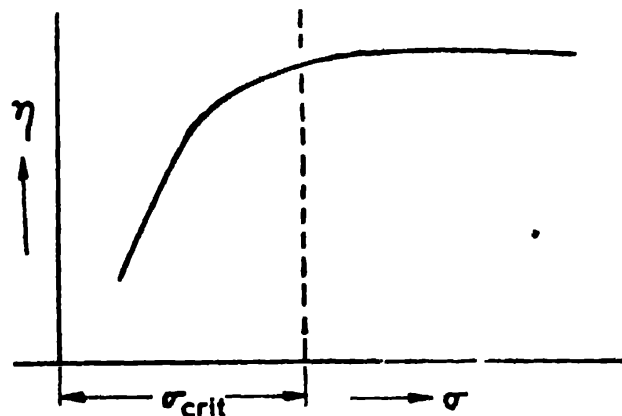


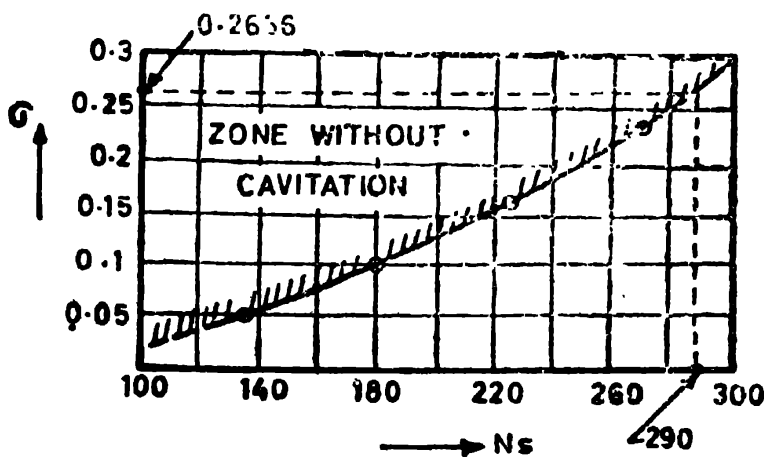
Fig 6.17 Turbine Efficiency  $\eta$  versus Cavitation Factor  $\sigma$

**Problem 6.3** A 16,000 HP reaction water turbine has effective head of 25 m. The runner is 3 m above the tail race. The turbine is installed at an elevation of 300 m above sea level where the barometric head is 9.64 m of water. The temperature of water is 27°C. The following table gives values of the plant sigma  $\sigma$  (Thoma's Cavitation Factor) against specific speed. The

turbine is coupled to a 50 cycles alternator. Calculate the synchronous speed of the turbine.

|          |      |      |      |       |
|----------|------|------|------|-------|
| $\sigma$ | 0.05 | 0.10 | 0.16 | 0.228 |
| $N_s$    | 135  | 180  | 225  | 270   |

(UPSC)

**Solution**Fig 6.18  $\sigma$  versus  $N_s$ 

$$P_t = 16,000 \text{ BHP}$$

$$H = 25 \text{ m}$$

$$H_s = 3 \text{ m}$$

$$H_b = 9.64 \text{ m}$$

(Assume the barometric pressure  $H_b$  at 300 m altitude and at  $27^\circ\text{C} = 9.64 \text{ m}$  of water)

$\therefore$  Thoma's Cavitation factor

$$\sigma = \frac{H_b - H_s}{H}$$

$$\text{or } \sigma = \frac{9.64 - 3}{25} = 0.2656$$

The value of specific speed  $N_s$  corresponding to  $\sigma$  is found from the curve (refer Fig 6.18) drawn with the help of the given table. Thus  $N_s \leq 290$ . It is clear from the curve (refer Fig 6.18) that if  $N_s > 290$ , then cavitation will appear.

$$\text{Now } N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}} \quad \text{or } 290 = \frac{N \times \sqrt{16,000}}{25 \times 2.23}$$

$$\text{or } N = 128 \text{ rpm.}$$

Assuming  $p$ , the number of pair of poles for the alternator as 24, the synchronous speed can be found with the help of Eqn 5.9.

$$\text{Thus } f = \frac{p \cdot N}{60} \quad \dots(\text{refer Eqn 5.9})$$

$$\text{or } 50 \times 60 = p \cdot N$$

$$\text{or } N = \frac{50 \times 60}{24} = 125 \text{ rpm} \quad \text{Answer}$$

which is the nearest synchronous speed to 128 rpm.

**6.18 Methods to Avoid Cavitation—**

(a) **Installation of Turbine Below Tail Race Level**—From Eqs 6.8 and 6.9 it is clear that if the value of  $H_s$  is less, the pressure  $p_s$  and

$\sigma$  will increase, which means the turbine may work in a safe zone. In case of high specific speed turbines such as Kaplan turbine, runner exit velocity  $v_2$  is very high which increases the value of the expression  $\frac{v_2^2 - v_1^2}{2g}$  in Eqn 6.8 and lowers the value of pressure  $p_2$ , thus bringing the cavitation zone nearer. In order to keep  $p_2$  within cavitation limits, the value of  $H_s$  is made negative which means the runner is installed below the tail race level. For such installations the turbine remains always under water. Therefore, when any repair or inspection of turbines is carried out the water has to be pumped out by closing the passages. This is a disadvantage because it is quite expensive to pump the water out and also it requires considerable time. It is therefore, advisable to avoid installation of the turbine below tail race level.

**(b) Cavitation-Free Runner of Reaction Turbine**—The above method being very expensive, modern trend is to develop a cavitation-free runner. It can be made after thorough research.

The experiments are carried out on a model turbine runner in a laboratory, because theoretical calculations merely do not help. A few suitable blade angles, blade lengths etc. are tried for a particular runner and finally the best values are adopted. Photographs are taken by means of stroboscope while the experiments on turbine models are carried out. The parts of runner, where cavitation appears, is clearly visible from the photographs taken. The most modern method of ascertaining cavitation phenomenon is by knowing the accurate pressure distribution along the runner blade section. This is also accomplished by carrying out tests on models driven by air in a wind tunnel, and running it exactly in the same working conditions as an actual-size machine. By this method the points of low pressure are found out. The velocities of these points should not be more than the critical values. If these values fall below the critical range, the velocities are to be altered to avoid danger of cavitation.

It has been found by the author while conducting research experiments on Kaplan turbine runner blades that if the blade profile is made thicker, in order to make it mechanically strong, the pressure at the thicker places, near the boss of runner, may fall below the evaporation pressure (critical pressure), thus bringing the possibility of cavitation. In such cases it has been felt to make the blades flatter. The cross section of such a blade is aero-foil shape and the pressure on both sides, especially on suction side is carefully tested.

**(c) Use of Material**—*Cast iron* is less resistive against cavitation, because it contains more carbon contents. *Cast steel* is better than cast iron. In case cast steel parts are affected by cavitation, it is possible to repair them by welding. Welded places can resist the cavitation more than ordinary surfaces. *Stainless steel* is the best material to avoid cavitation, however, it is expensive. Therefore, the turbine runners are made of cast steel and then coated with stainless steel. Where water is chemically impure, *bronze* is preferred which also resists the cavitation better than cast steel. The pitting in case of bronze is filled by soldering.

The latest research carried out on materials resistance to cavitation shows that pressed plates appear to behave better than castings and whilst aluminium, bronze and zinc alloys are promising, there is a preference for chrome nickel stainless steels.

(d) **Use of Machining**—The turbine runner blades are highly polished, as it is seen in practice that with rough surfaces, the likelihood of cavitation occurrence is more, due to interruption of smooth and stream line flow, causing vortices.

**6.19 Selection of Speed**—The empirical relations mentioned in Art 6.17 made it possible to predetermine  $N_s$  for given  $H_b$ ,  $H_s$  and  $H$ . Also using equations 6.10 and 6.12, approximate values of specific speed  $N_s$  can be estimated. Further from  $N_s$ , the power output  $P_i$  of the turbine can be worked out from specific speed Eqn 3.10. Because the turbine is almost always directly coupled to an alternator, a synchronous speed (refer Eqn 5.9 a) must be chosen.

**6.20 Runaway Speed**—The runaway speed of a turbine is the maximum speed attained by the runner under maximum head at full gate opening, when the external load (i.e. generator) is disconnected from the system and the governor ceases to function. All rotating parts must be designed to withstand the runaway speed which varies among the manufacturers with the design of turbine and generator. The range of runaway speed in terms of normal working speed for various types of turbines is generally as follows—

TABLE 6.5  
Runaway Speed ( $N_r$ ) in terms of Working Speed ( $N$ )

| Turbine Type    | Runaway Speed  |
|-----------------|----------------|
| Pelton turbine  | 1.8 to 1.9 $N$ |
| Francis turbine | 2 to 2.2 $N$   |
| Kaplan turbine  | 2.5 to 3 $N$   |

The exact value of runaway speed of any turbine can be predicted from the model tests in the laboratory. R.E. Krueger in the work published by Bureau of Reclamation, USA, has given the following equation to predict the runaway speed

$$N_r = K_n \cdot N \cdot \left( \frac{H_{max}}{H} \right)^{\frac{1}{2}} \quad \dots(6.13)$$

where

$$K_n = 0.00676 N_s + 1.7$$

$N_r$  = runaway speed ;

$H_{max}$  = maximum head not including water hammer in m ;

$H$  = working head in m.

Laboratory experiments have shown that with the appearance of cavitation the runaway speed of reaction turbine increases, probably because of reduced fluid friction. In case of Kaplan turbine this effect is significant. The runaway speed of reaction turbine may also vary with the gate opening.

**(B) DERIAZ TURBINE**

**6.21 Deriaz Runner (Mixed-Flow Variable Pitch Pump-Turbine Runner)**—Francis turbine has a fixed blade runner which gives maximum efficiency at a particular load condition. At part-load and over-load conditions the turbine shows a much lower efficiency. So in a case where load is variable, Francis turbine due to overall low efficiency is not economical. Further experience has shown that fixed blade runner turbine of Francis type generally suffers from an unstable hydraulic condition in the draft tube at gate openings between approximately 30 to 60%.

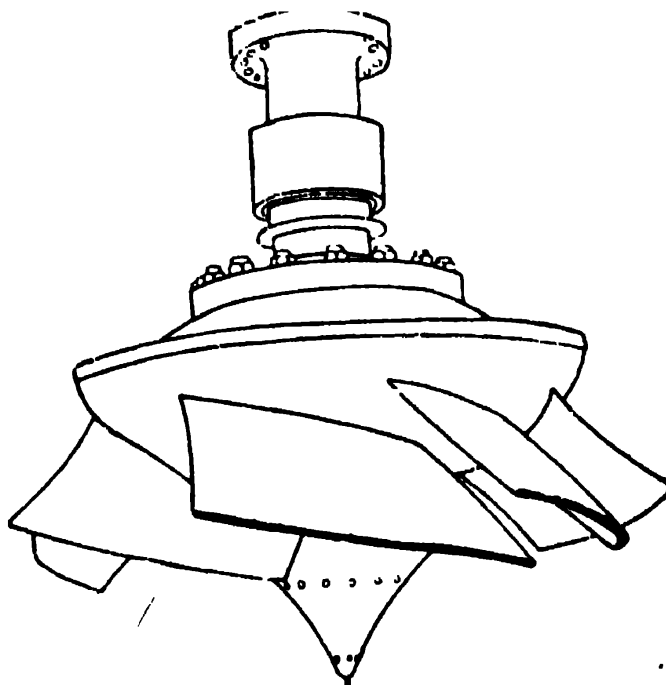


Fig 6.19 Assembly of the Deriaz Reversible Runner (Pump-Turbine Runner) for Sir Adam Beck Pumping-Generating Station in Canada

Paul Deriaz\* Dip. Ing. (ETH Zurich) worked in this field to overcome this difficulty. He utilized the idea of Kaplan turbine (refer chapter 7) in which maximum efficiency is attained at variable load conditions, by the use of movable blade runner. The Kaplan turbine is an axial flow turbine and is used for low heads. So, P. Deriaz designed a variable pitch mixed flow runner to be used for medium heads approximately upto 200 m. The Deriaz runner is surrounded by a conventional guide and gate mechanism of reaction turbine.

**6.22 The Influence of Variable Pitch on Hydraulic Performance**—The implications of variable pitch operation may be demonstrated by comparing hydraulic runner with a multi-start variable pitch screw. The pitch of a propeller can be defined in precisely the same way as the pitch of a screw thread *i.e.*, as the axial displacement of the screw for 360° angular rotation. From Fig 6.20, the relation between pitch and thread slope at the periphery is given by :

$$\tan \beta = \frac{t}{\pi D} \quad \dots(1)$$

where

$t$  = pitch ;

$D$  = outside diameter of screw ;

$\beta$  = thread slope at periphery.

The four bladed propeller of Fig 6.20 is equivalent to a “four start” screw, the pitch of which has the same significance as the forward movement of the propeller when rotating in a stationary medium. Conversely, when the plane of the propeller is maintained stationary in space, the

resultant volume displacement (neglecting the space occupied by the hub) is given by :

$$Q = \frac{\pi D^3}{4} \times \frac{t N}{60}, \quad \dots(2)$$

where  $N$  = rational speed of the propeller. Combining (1) and (2),

$$Q = \frac{\pi^3 D^3 N}{240} \tan \beta \quad \dots(6.14)$$

$$\text{or} \quad \tan \beta = \frac{240}{\pi^3} \cdot \frac{Q}{D^3 N} \quad \dots(6.14a)$$

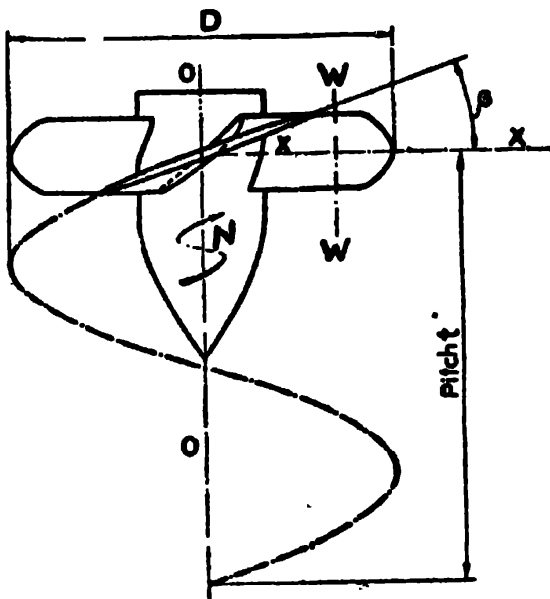


Fig 6.20 Comparison of Multi-start Variable Pitch Screw with Turbine Runner

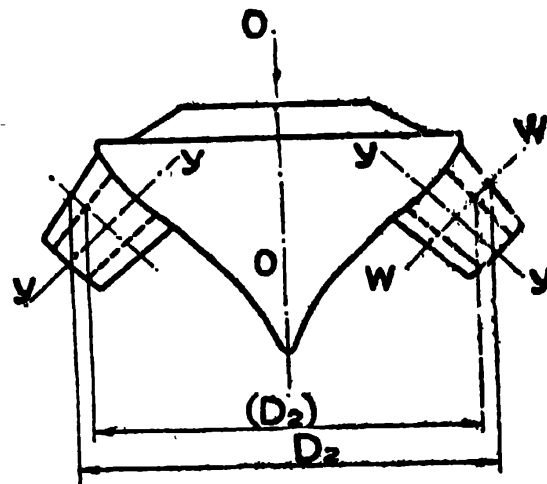


Fig 6.21 Blade Adjustment of Deriaz Runner

Thus, the volume displacement of the propeller for constant rational speed is directly proportional to  $\tan \beta$ . Control of the rate of flow is obtained by "feathering" or varying the propeller pitch.

The slope given by Eqn 6.14a is applicable much more generally to all turbomachines and in particular, to non-axial flow propellers, such as centrifugal pumps, and mixed flow runners of Francis turbines. In all these the ratio  $\frac{Q}{D^3 N}$ , which is the equivalent of pitch, can be varied provided the design allows for variation of the blade angle  $\beta$ .

For purely axial flow machines, such as marine and aircraft propellers and Kaplan turbines and pumps, the variation of blade angle is obtained by rotation of the blades on an axis  $X-X$  perpendicular to the axis  $O-O$  of runner rotation, as shown in Fig 6.20.

For non-axial flow machines having a blade profile similar to that of the mixed flow Francis turbine, the blade angle adjustment cannot be made on the perpendicular axis of Fig 6.20, but must be made as in Fig 6.21—i.e., on an axis  $Y-Y$  inclined to the rotational axis of the machine. The geometry so illustrated is characteristic of the mixed flow Deriaz runner. The axis  $Y-Y$  is determined by the inclination found most

desirable hydraulically and mechanically and may even be parallel to the axis of runner rotation.

It is now important to note that, whereas in Fig 6.20 the inlet and outlet diameters for a given flow line  $W-W$  are equal, these diameters differ in the mixed flow runner of Fig 6.21.

**6.23 Geometry of Deriaz Runner**—Since concept of Kaplan turbine (refer chapter 7) has been applied to Francis runner for the design of Deriaz runner, the reader is expected to go through the next chapter before understanding the geometry of Deriaz runner given below :

The blades of Kaplan turbine are fitted to the hub (refer Fig 6.22). The runner blades are rotated by a mechanism housed in the hub, which

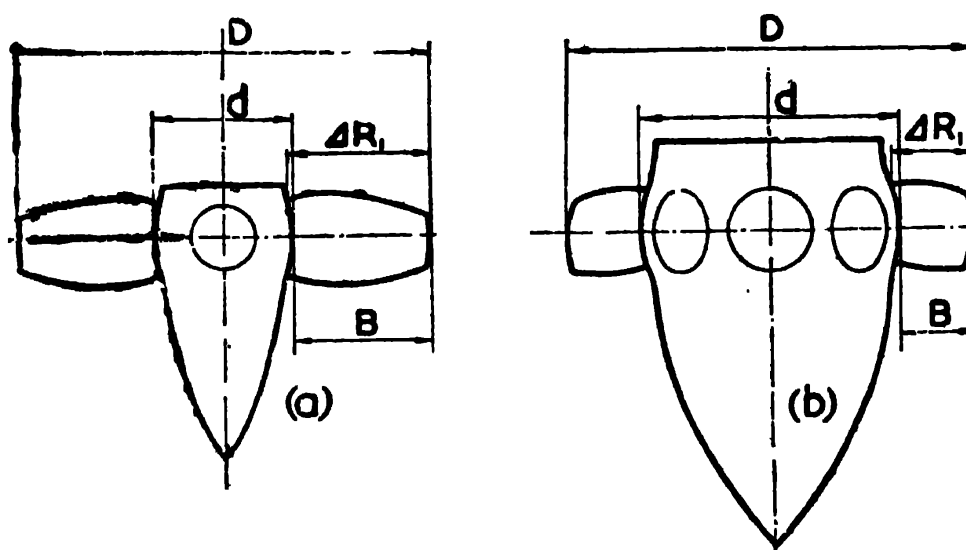


Fig 6.22 Kaplan Turbine Runner  
(a) For Low Head (b) For High Head

is responsible for varying to pitch of Kaplan runner. The hub diameter  $d$  increases with increase of head, thus decreasing the blade length  $\Delta R_1$ . This results in relatively higher water frictional losses over the hub surface with which the turbine efficiency decreases. It has been experienced that with a head of 40 metres the ratio of hub diameter to runner diameter  $\frac{d}{D_1}$  will be 0.52 and with 80 metres head it increases to about 0.65.

The runner vanes or  $\Delta R_1$  (refer Fig 6.23) become narrow which decreases the efficiency. Deriaz\* suggested to revert to mixed flow arrangement of the Francis turbine in such cases. In a Deriaz runner the blades instead of being at right angles to the hub will be inclined at an angle of  $45^\circ$ . It is stated that a runner of this type can be used for a head upto 200 m.

The runner is surrounded by a conventional gate mechanism, which in conjunction with the rotatable blades, gives a flat efficiency curve and overload capacity designed for the same specifications. The principal features of Deriaz Runner are—



1. The axis obliquity allows easier location of operating gear without excessive increase in the hub-diameter ratio  $\frac{d}{D}$  (refer Fig 6.24). The ratio  $\frac{B}{D}$  is also more favourable to good efficiencies.

The radial extent of the inlet edge  $\Delta R$ , decreases from Fig 6.24 (a) to 6.24 (b) for increasing heads.

2. The oblique position of the blade trunnions  $Y-Y$  gives a significant reduction in the loading of the outer trunnion journal because of the centrifugal force on the blade, acting partly against hydraulic loading.

3. The spherical surfaces  $B-B$  of the hub and of the skirt permit rotation of the vanes while maintaining close clearances.

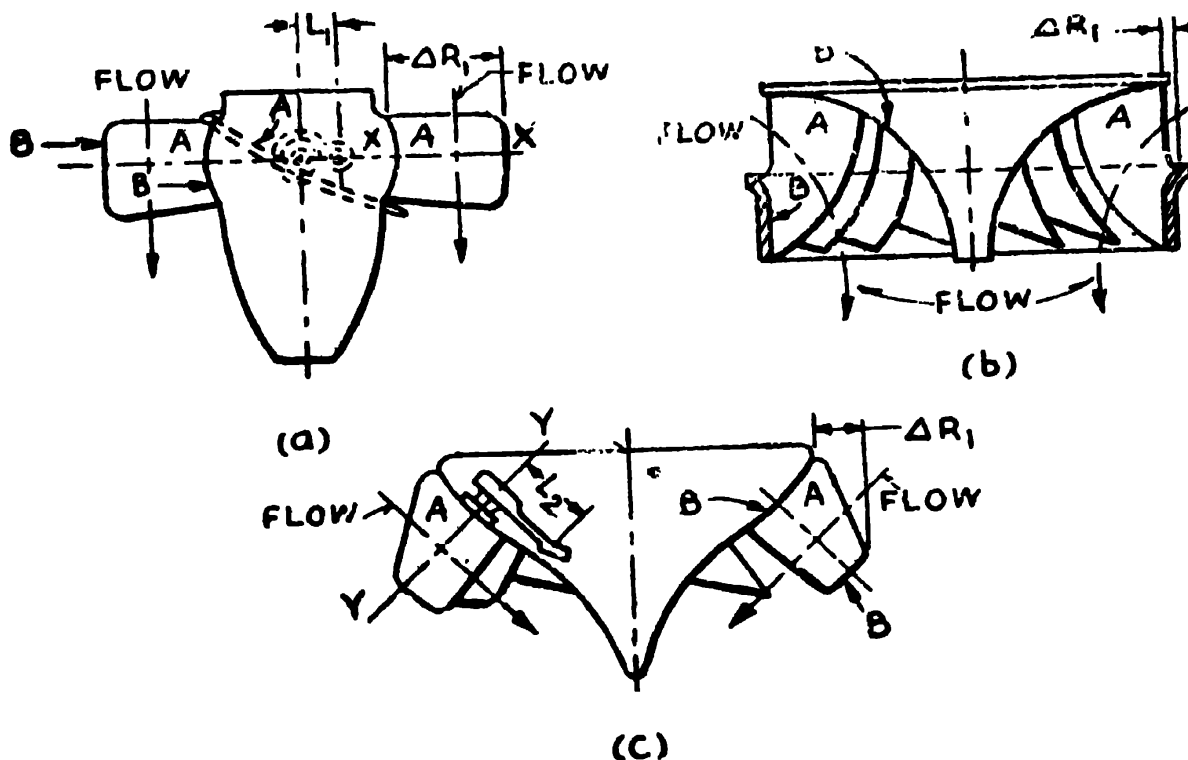


Fig 6.23 Comparison of Construction of Francis and Deriaz Runners

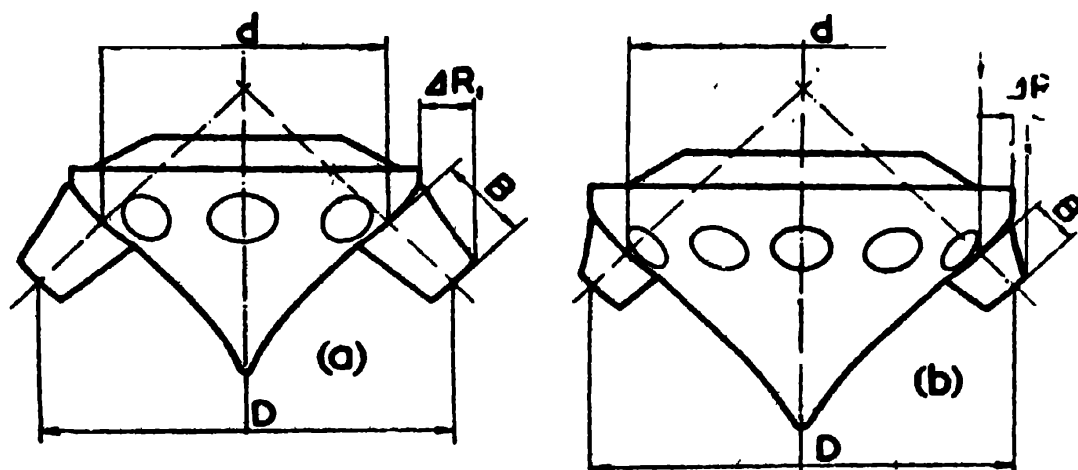


Fig 6.24 Deriaz Runners : (a) For Low Head (b) For High Head

4. Securing advantage of longer levers  $L_2$  (refer Fig 6.23c) and less crowded construction in the boss.

5. Simpler runner blades than those of Kaplan. If desirable the blades can be made to recover the whole cross-sectional area, in the closed position. This is an essential difference between Deriaz and Kaplan designs. In the closed position of Kaplan runner the vanes come in contact with each other at the periphery, but a large gap remains between the vanes near the hub.

#### 6.24 Advantages of Runner Pitch Variation—

(1) The “flattening” of the efficiency-load characteristics obtained with the mixed flow variable pitch runner is illustrated in Fig 6.25 where the characteristics of this machine is compared with that of the fixed

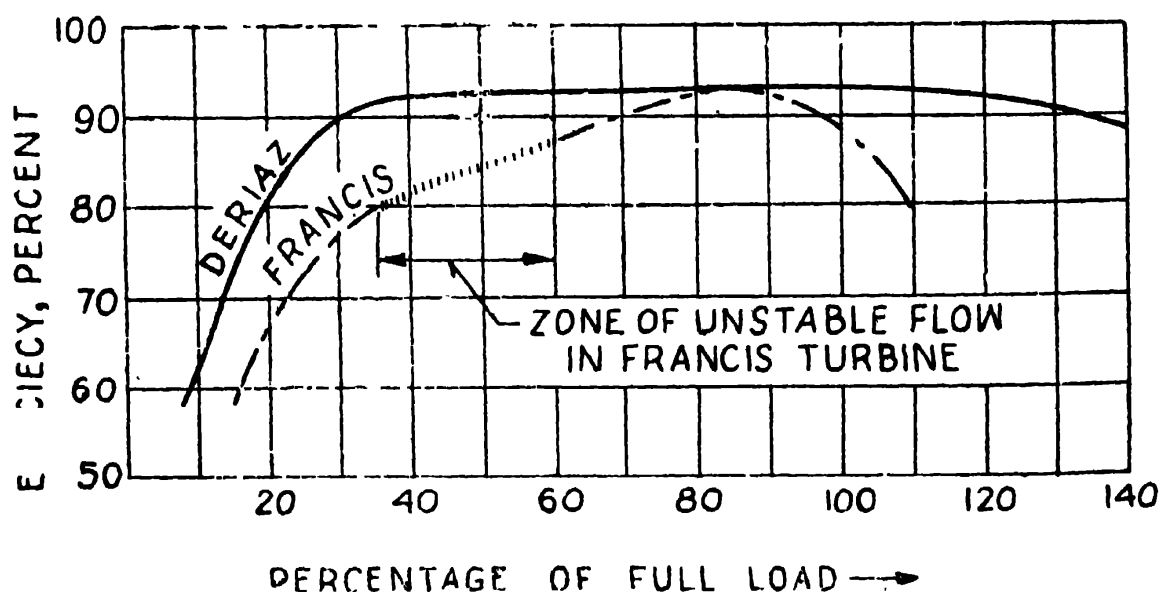


Fig 6.25 Comparison of Efficiencies of Francis (Fixed Blade) and Deriaz Turbines

pitch Francis type. The extension of the flat characteristic in the overload range should be particularly noted. Thus Deriaz runner can be employed for peak and part (even at 30%) conditions without loss of efficiency.

(2) The Deriaz turbine is particularly suitable when it is used as one machine pump-turbine in pumped storage plants without lowering the efficiency of either turbine or pump. A pumped storage hydroelectric plant pumps water at off peak hours of electric demand by means of surplus power into a high level natural or artificial reservoir, in order to utilise the stored energy (*i.e.* water in the reservoir) at periods when it is most needed (refer chapter 10).

It is economical to use Deriaz turbine working as a turbine or a pump in pumped storage plants.

Figs 6.26 and 6.27 show typical discharge and efficiency characteristics, in relation to head, of the Deriaz variable pitch runner when operating as a pump. In Fig 6.26, the discharge and efficiency characteristics for varying head are plotted for three different blade settings corresponding to 75, 100 and 120% blade openings. As would be expected each

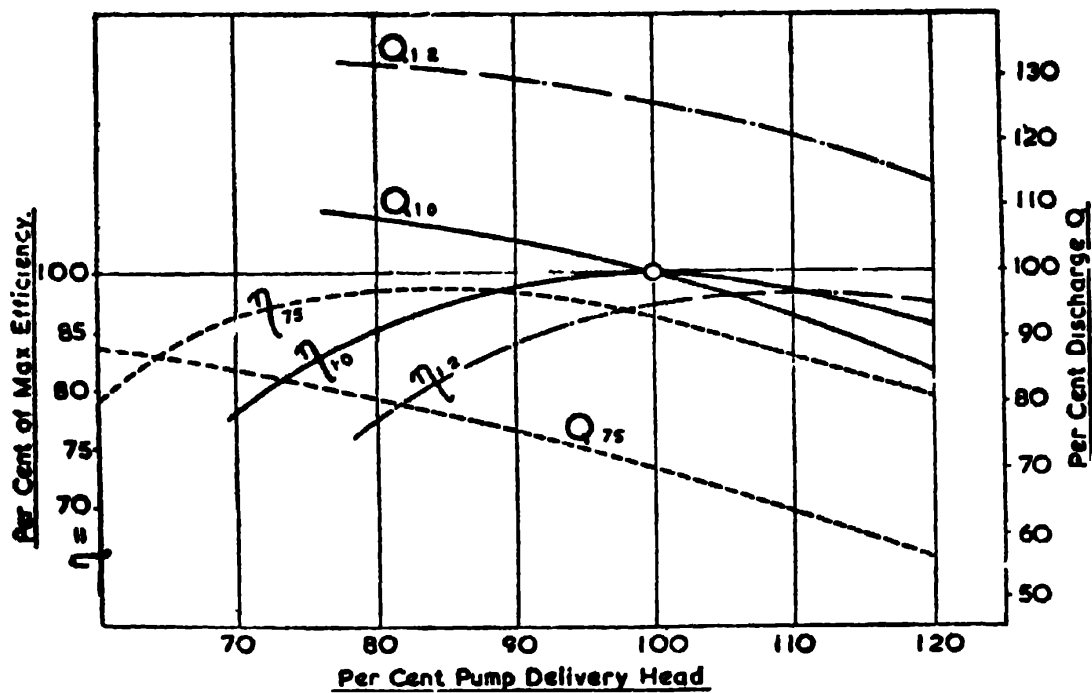


Fig 6.26 Typical Discharge-Efficiency Characteristics of Deriaz Runner. When Operating at Pump with Varying Head For Three Blade settings

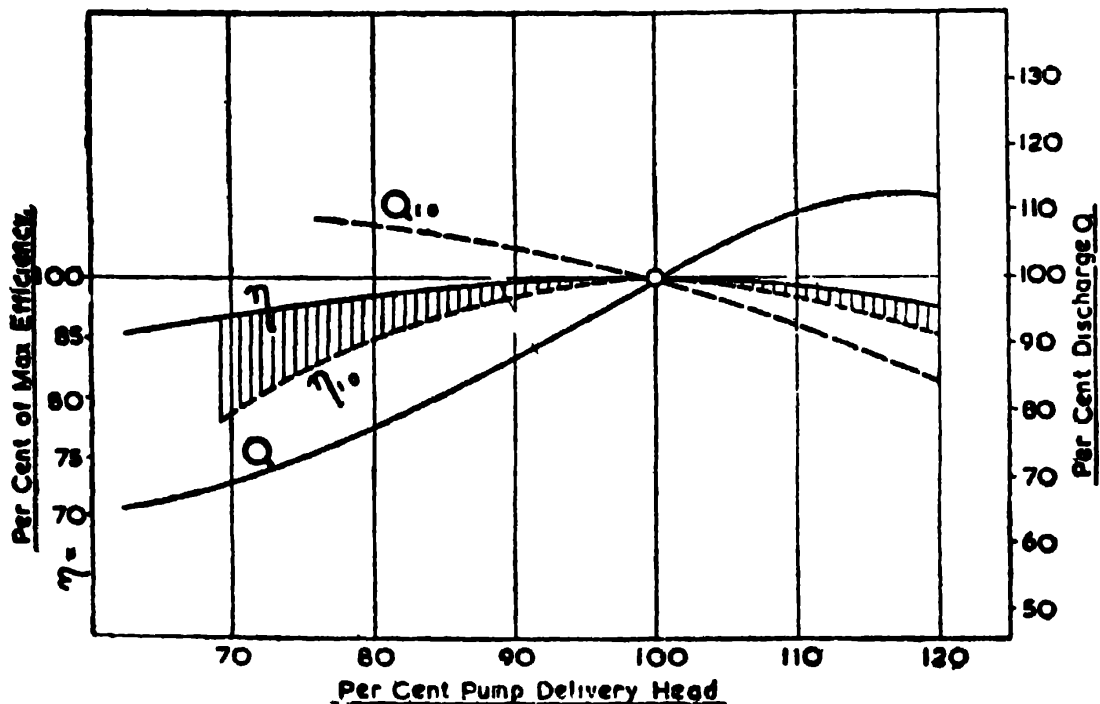


Fig 6.27 Discharge-Efficiency Characteristics of Deriaz Runner For Varying Head Showing Formation of the Efficiency curve

of these characteristics corresponds, in general form, to the performance of a fixed pitch Francis runner. The reduction in discharge, as the head increases, will be noted for each blade opening. However, in Fig 6.27, the full line curves show the result of choosing the blade angle for optimum efficiency at each head. This demonstrates how a flat efficiency curve is obtained over a wide head range. The shaded area of the figure represents

the efficiency gain over the head range resulting from the variable pitch system.

It will be noted how this choice of optimum blade angle now results in rising discharge characteristic with increasing head. This is the opposite of that obtaining with the fixed pitch runner and this characteristic of increasing discharge with increasing head can be further controlled in the design by suitable location of the exit edge of the runner blade. This rising discharge characteristic is of major importance in pumped storage projects where considerable variation in the storage reservoir area with head can occur. If the setting of the pump turbine is correctly selected, high overloads at high efficiency are achieved over that range of head where the maximum water volume has to be pumped.

*Runner speed considerations*—The high efficiency obtained over a wide head range permits a variable pitch reversible pump-turbine to operate with the advantages of peak efficiency at the same speed for pumping and generating duty. Fixed blade reversible pump-turbines require a higher speed for pumping operation than when generating so as to perform at optimum efficiency for both duties at a given head. In some cases this has led to the use of different running speeds for pump and turbine operations, with the attendant complications of synchronous machine pole changing techniques. Such different speeds are unnecessary with Deriaz runner as the effective runner diameter is altered by feathering and the zones of peak efficiency operation are brought within reach for both duties.

*Inlet Guide Vane and Blade Operating Mechanism Considerations*—For optimum conditions when operating as a turbine, the angle  $\alpha$ , at which water is admitted to the runner must be suitably controlled. For reduced losses at the turbine runner exit, the inlet vortex should be formed to maintain the condition given by ;

$$\frac{v_{u1} \cdot u_1}{g} = H \cdot \eta \quad \dots(\text{refer Eqn 1.52})$$

Since the machine operates at constant angular velocity this condition must be fulfilled by suitably guiding the incoming water according to head and discharge. The usual guide mechanism of Francis turbine is suitable for Deriaz runner, but when the machine has to operate as a pump, it is desirable to reduce the number of guide vanes, provided that these are not used to shut off the flow completely. This condition holds when the runner blades themselves permit complete closure.

The guide vane and blade control arrangements to meet these conditions are illustrated in Fig 6.28 which shows the cross-section of the reversible pump turbines at the Niagara Station of Hydro-Electric Power Commission of Ontario. The guide vane requirements are met by the provision of one adjustable flap 12 on each of the fixed stay vanes 10. The axis on which the flaps pivot is  $45^\circ$  to the vertical, each flap being controlled by an individual oil pressure servomotor 11. The flaps control the flow angle at inlet to the runner and are deflected to a maximum at the lower heads on turbine duty. They do not, however, provide for closure.

Stabilisation of the flow at the runner exit when pumping, is achieved by setting the flaps in line with stay vanes.

In Fig 6.28, the blade lever 2 is keyed to the trunnion of the obliquely positioned blade 1, and engages with the operating spider 4 which

is keyed to the vertical rotor shaft of the servo motor 5. This shaft is concentric with the runner axis. The essential feature of the assembly is the

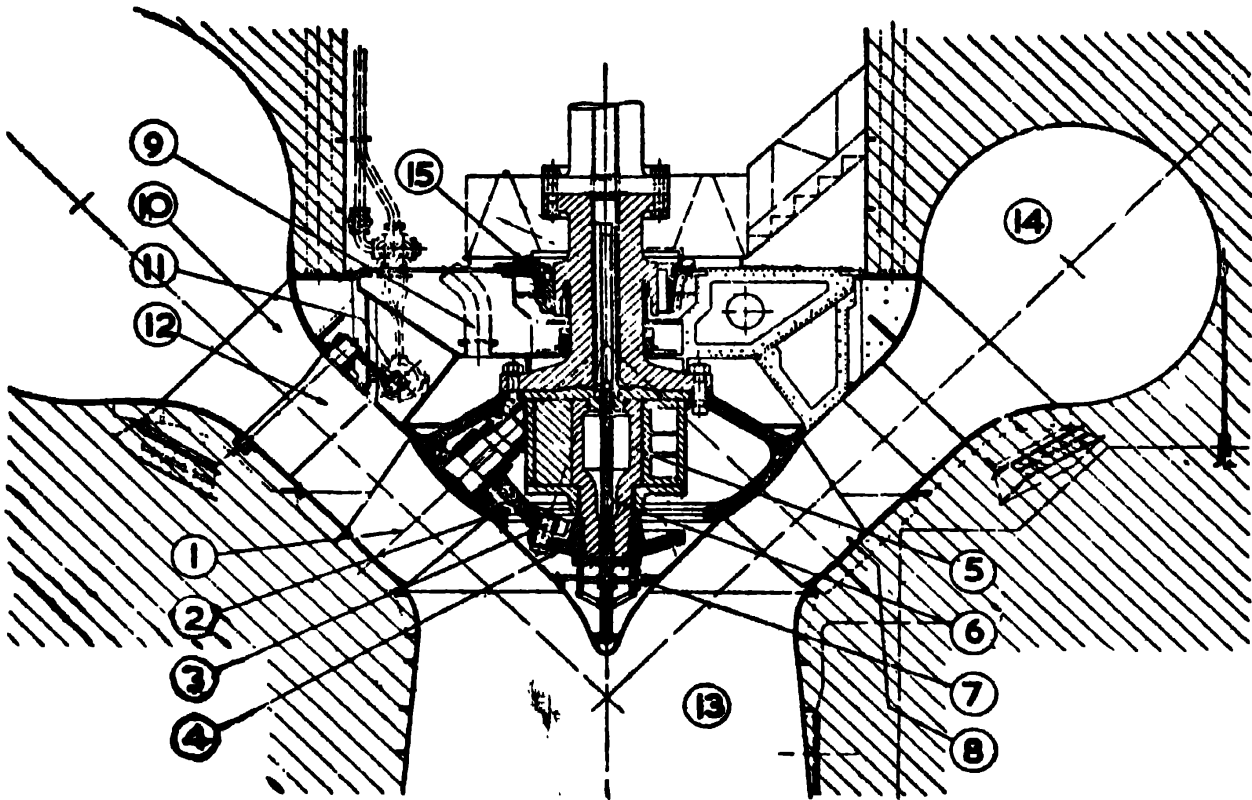


**Fig 6.28 Sectional Elevation Through Deriaz Reversible Pump-Turbine at Sir Adam Beck Niagara Pumping Generating Station Ontario, Canada**

common intersection on the servomotor centre line of the blade trunnion and spider pivot axes. The rotation of the servomotor rotor relative to the runner hub is translated into an axial displacement of the central tube 6 by the cam 7 to operate the governor restoring mechanism. The feature of complete closure leads to the following operating advantages :

(a) The machine can be shut down without closure of guide or gate apparatus or inlet valve. It is then unnecessary for the gate apparatus to control the discharge at closure, thus permitting considerable simplification and reduction of the overall size of the machine and consequent reduction in cost of the power house structure.

(b) The torque required to rotate the runner with shut blades at full operating speed is much reduced, thus permitting easy starting as a pump by throwing the synchronous machine directly on the line even at full operating head.

(c) Operation of the electrical machine as a synchronous condenser does not involve tail water depression by compressed air.

(d) The change from pump to turbine operation involves reversal of the direction of rotation. However, as far as the hydraulic side is concerned, it is only necessary to feather the blades, stop and then restart. This operation requires a few minutes at the most, since the inlet valve, head or guide vane apparatus remains fully open. The turbine output is then adjusted as occasion demands by simply controlling the blade angle.

(e) Starting for turbine operation is done most conveniently in the same way as for pumping, the electrical machine acting as a starting motor. No synchronisation by governor control is required as the machine is thrown straight on the line. This results in considerable simplification in operation, particularly for automatic plant.

(f) It is possible to provide complete closure of the machine at shut down after pumping cycle is complete. Simple and convenient achievement of this condition is possible and no emptying of the penstock is required. The obliquity of the vane is employed to advantage by arranging a reduction in tip clearance on shut down by simply displacing the runner axially downwards in relation to the stationary skirt envelope.

**6.25 Deriaz Turbine Installations**—The first Deriaz turbines were installed at the Sir Adam Beck Niagara Pumping Generating station in Canada. Each is rated at 46,000 HP as a turbine under 25 m head (refer Fig 6.19 and 6.28) and discharges as a pump 110 to 140 m<sup>3</sup>/sec under heads from 18 to 27 m. Six such pump-turbines for the above Power Station of Hydro-Electric Power Commission of Ontario have been supplied by English Electric, Canada. These machines balance the Niagara river flow. These have the added advantage of low starting torque, over-load capacity and assurance of stability of operation during starting.

TABLE 6.6

**World Installations of Deriaz Turbines and Pump Turbines  
(Mixed Flow Adjustable Blade)**

| <i>Plant and Location</i> | <i>No. of<br/>units</i> | <i>Capacity<br/>each HP</i> | <i>Head<br/>m</i> | <i>rpm</i> |
|---------------------------|-------------------------|-----------------------------|-------------------|------------|
| Ajaure, Sweden            | 1                       | 124,000                     | 58<br>(40 to 70)  | 166·7      |
| *Valdecanas, Spain        | 3                       | 110,000                     | (60 to 76)        | 150        |
| Bukhtarma, USSR           | —                       | 106,000                     | 61                | 150        |
| Amagase, Japan            | —                       | 70,000                      | 57                | 180        |
| Yanase (Shikhu), Japan    | —                       | 60,500                      | 97<br>(57 to 97)  | 257        |
| *Niagara, Canada          | 6                       | 46,000                      | 25                | 92·3       |
| *Kuromatagawa, Japan      | —                       | 24,500                      | 73<br>(39 to 78)  | 300        |

**TORQUE, POWER & EFFICIENCIES**

**6.26 Force and Torque**—The resultant dynamic force exerted by the water on the runner vanes in the direction of rotation (refer Eqn 1.22).

$$F_u = \rho Q (v_{u_1} - v_{u_2})$$

Force equivalent of motion at inlet  $F_{u_1} = \rho Q \cdot v_{u_1}$

Force equivalent of motion at outlet  $F_{u_2} = \rho Q \cdot v_{u_2}$

\*Pump-Turbines.

The action of the stream on the vanes of a radial flow runner can be determined by finding the total torque produced by all the elementary forces over the vanes. The runner is considered to be divided into a number of parts of equal area, each constituting what may be called a *fractional turbine*.

Let  $dM_H$  be the turning moment of a fractional turbine and  $dQ$  the quantity of water flowing through it.

Equivalent turning moment of fluid motion at inlet

$$dM_{H_1} = dF_{u_1} \cdot R_1 = \rho dQ \cdot v_{u_1} \cdot R_1$$

Similarly equivalent turning moment at outlet

$$dM_{H_2} = dF_{u_2} \cdot R_2 = \rho dQ \cdot v_{u_2} \cdot R_2$$

Resultant torque  $dM_H = dM_{H_1} - dM_{H_2} = \rho dQ \cdot (v_{u_1} \cdot R_1 - v_{u_2} \cdot R_2)$

$$\text{Hence, } M_H = \int dM_H = \rho \int_0^Q (v_{u_1} \cdot R_1 - v_{u_2} \cdot R_2) \cdot dQ \quad \dots(6.15)$$

Exact value of  $M_H$  can be obtained by graphical integration (refer Fig 6.29)

$$\text{Roughly } M_H = \rho Q (v_{u_1} \cdot R_1 - v_{u_2} \cdot R_2) \quad \dots(6.15a)$$

**6.27 Power**—Let  $P_H$  be the power developed by the turbine.

Then the power of a fractional turbine

$$\begin{aligned} dP_H &= dM_H \cdot \omega \\ &= \frac{\gamma}{g} \cdot dQ \cdot (v_{u_1} \cdot R_1 \cdot \omega - v_{u_2} \cdot R_2 \cdot \omega) \\ &= \frac{\gamma}{g} \cdot dQ \cdot (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2) \end{aligned}$$

$$\therefore P_H = \int dP_H = \frac{\gamma}{g} \int_0^Q (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2) \cdot dQ \quad \dots(6.16)$$

Thus the power  $P_H$  can also be determined by graphical integration (refer Fig 6.30).

$$\text{Approximately, } P_H = \rho Q (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2) \quad \dots(6.16a)$$

(refer Art 1.18)

This is the expression for total hydraulic power developed by the turbine, considering the loss of head including that due to outlet velocity etc., but neglecting the volumetric losses.

## 6.28 Efficiency—

(a) **Head Efficiency**—Let the total head loss in turbine be  $\Delta H$  and the net operating head  $H$ .

Then efficiency  $\eta_H = \frac{H - \Delta H}{H} = 1 - \frac{\Delta H}{H}$

This is known as the “Head efficiency”.

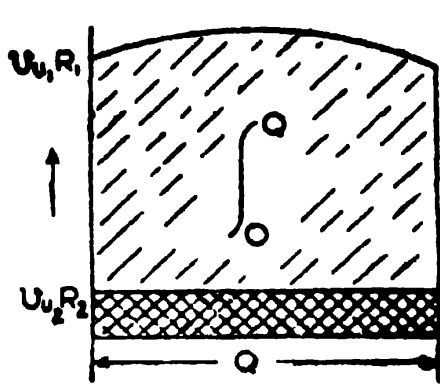


Fig 6.29  $v_u \cdot R$  vs  $Q$

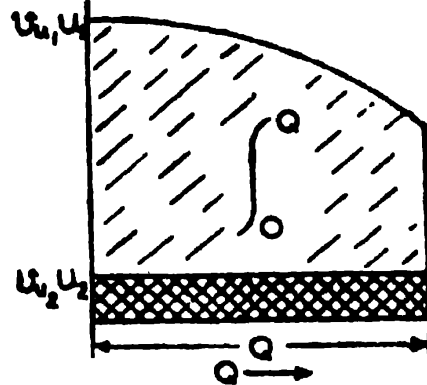


Fig 6.30  $v_u \cdot u$  vs  $Q$

But  $P_H = P_s \cdot \eta_H$   
 $= \gamma \cdot Q \cdot H \cdot \eta_H$

$$\therefore \frac{\gamma \cdot Q}{g} (v_{u1} \cdot u_1 - v_{u2} \cdot u_2) = \gamma \cdot Q \cdot H \cdot \eta_H$$

or  $\eta_H = \frac{v_{u1} \cdot u_1 - v_{u2} \cdot u_2}{gH}$  ... (6.17)

$$= \frac{2(v_{u1} \cdot u_1 - v_{u2} \cdot u_2)}{2gH}$$

Substituting :  $v_{u1} = K_{v_{u1}} \cdot \sqrt{2gH}$ , and  $v_{u2} = K_{v_{u2}} \cdot \sqrt{2gH}$

$$u_1 = K_{u1} \cdot \sqrt{2gH} \quad \text{and} \quad u_2 = K_{u2} \cdot \sqrt{2gH}$$

$$\therefore \eta_H = 2(K_{v_{u1}} \cdot K_{u1} - K_{v_{u2}} \cdot K_{u2})$$

But  $u_2 = \frac{R_2}{R_1} \cdot u_1$  (Since  $\omega = \frac{u_1}{R_1} = \frac{u_2}{R_2}$ )

$$K_{u2} = \frac{R_2}{R_1} \cdot K_{u1}$$

$$\therefore \eta_H = 2K_{u1} \left\{ K_{v_{u1}} - K_{v_{u2}} \cdot \frac{K_{u2}}{K_{u1}} \right\}$$

$$= 2K_{u1} \left( K_{v_{u1}} - K_{v_{u2}} \cdot \frac{R_2}{R_1} \right) \quad \dots (6.18)$$

If the discharge is radial, i.e.,  $\alpha_2 = \frac{\pi}{2}$

then  $\cos \alpha_2 = 0$ , and  $K_{v_{u2}} = 0$

$$\therefore \eta_H = 2K_{u1} \cdot K_{v_{u1}} \quad \dots (6.18a)$$



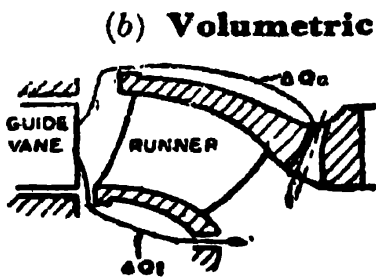


Fig 6.31 Water Quantity Loss Escaping Through the Clearance Between the Guide Wheel and the Runner

(b) **Volumetric Efficiency**—Let  $\Delta Q$  be the amount of water that passes over to the tail race through some passage (refer Fig 6.31) other than the runner and doing no useful work.

$$\text{Then } \Delta Q = \Delta Q_u + \Delta Q_l$$

where  $\Delta Q_u$  = upper clearance loss ;

$\Delta Q_l$  = lower clearance loss

(refer Fig 6.31)

$$\text{Volumetric efficiency, } \eta_Q = \frac{Q - \Delta Q}{Q} = 1 - \frac{\Delta Q}{Q} \quad \dots(6.19)$$

TABLE 6.7

Practical Data of  $\eta_H$  and  $\eta_Q$  for Francis Turbine

| $N_s$ | $K_s$ | $\eta_H$ | $\eta_Q$ |
|-------|-------|----------|----------|
| 65    | 5     | 0.87     | 0.955    |
| 300   | 96    | 0.91     | 0.965    |

(c) **Hydraulic Efficiency**—Total hydraulic loss in a turbine is made up of total head loss and volumetric loss. The actual hydraulic power of the turbine is obtained by considering the total hydraulic loss.

$$\text{Thus } P_h = \gamma (Q - \Delta Q)(H - \Delta H)$$

$$\text{and } P_a = \gamma \cdot Q \cdot H$$

$\therefore$  Hydraulic efficiency

$$\begin{aligned} \eta_h &= \frac{P_h}{P_a} = \frac{\gamma \cdot (Q - \Delta Q)(H - \Delta H)}{\gamma \cdot Q \cdot H} \\ &= \left(1 - \frac{\Delta Q}{Q}\right) \left(1 - \frac{\Delta H}{H}\right) \\ &= \eta_Q \cdot \eta_H \end{aligned} \quad \dots(6.20)$$

(d) **Mechanical Efficiency**—Brake horse power of a turbine is the hydraulic power minus the mechanical losses

$$P_b = P_h - \Delta P_{mech}$$

Mechanical loss may be due to bearing friction and windage,

Denoting the two by  $\Delta P_{mech_1}$  and  $\Delta P_{mech_2}$  respectively,

$$\text{then } \Delta P_{mech_1} = K_1 \cdot G^{\frac{1}{2}} \cdot N^{\frac{1}{2}}$$

where  $G$  = the total load on bearing,

$N$  = the speed in rpm,

$K_1$  = a constant,

$$\Delta P_{mech_2} = K_2 \cdot N^3$$

These formulae have been derived from practical results. Now mechanical efficiency

$$\eta_{mech} = \frac{P_h - \Delta P_{mech}}{P_h} = 1 - \frac{\Delta P_{mech}}{P_h} \quad \dots(6.21)$$

$\therefore$  Brake horse power,

$$P_t = \eta_h \cdot \eta_{mech}$$

(e) **Overall Efficiency**—Let  $\eta_t$  be the overall efficiency of the turbine.

$$\begin{aligned} \text{Then } \eta_t &= \frac{P_t}{P_s} \\ &= \frac{P_h \cdot \eta_{mech}}{P_s} = \frac{P_s \cdot \eta_Q \cdot \eta_H \cdot \eta_{mech}}{P_s} \\ &= \eta_Q \cdot \eta_H \cdot \eta_{mech} \quad \dots(6.22) \end{aligned}$$

**6.29 Discharge Through Francis Turbine**—Let quantity of water flowing per sec be  $Q$

then  $Q = \text{area across flow} \times \text{velocity of flow.}$

Area across radial flow at inlet  $= (\pi D_1 - z_2 \cdot t) \cdot B_o$

where  $D_1$  = the inlet diameter of runner ;

$z_2$  = the number of blades in runner ;

$t$  = the thickness of blades ;

and  $B_o$  = the height of runner  $\approx$  height of guide vanes.

Radial velocity of flow at inlet,

$$v_{m_1} = v_1 \cdot \sin \alpha_1$$

$$\therefore Q = (\pi D_1 - z_2 t) B_o \cdot v_{m_1} \quad \dots(6.23)$$

Now the area occupied by blade edges is usually 5 to 10% of  $\pi D_1$

In general,  $(\pi D_1 - z_2 t) = K \cdot \pi D_1$

where  $K$  = percentage of net flow area  
i.e. 0.90 to 0.95

Also  $B_o$  is proportional to  $D_1$

$$\text{Let } \frac{B_o}{D_1} = K_B \quad \therefore B_o = K_B \cdot D_1$$

$$\text{and } v_{m_1} = K_{v_{m_1}} \sqrt{2gH}$$

$$\therefore Q = K \cdot \pi \cdot K_B \cdot K_{v_{m_1}} \sqrt{2g} \cdot D_1^2 \cdot \sqrt{H} \quad \dots(6.23a)$$

The factor  $(K \cdot \pi \cdot K_B \cdot K_{v_{m_1}} \cdot \sqrt{2g})$  which is constant for geometrically similar turbines is called specific flow and is denoted by  $Q_{11}$

$$\text{Then } Q = Q_{11} \cdot D_1^2 \cdot \sqrt{H} \quad \dots(\text{refer Eqn 3.6})$$

$Q_{11}$  is therefore the quantity of water required by the turbine when working under unit head and with unit runner inlet diameter ( $D_1$ ).

The values of various constants for Francis Turbines are given in Fig 6.32.

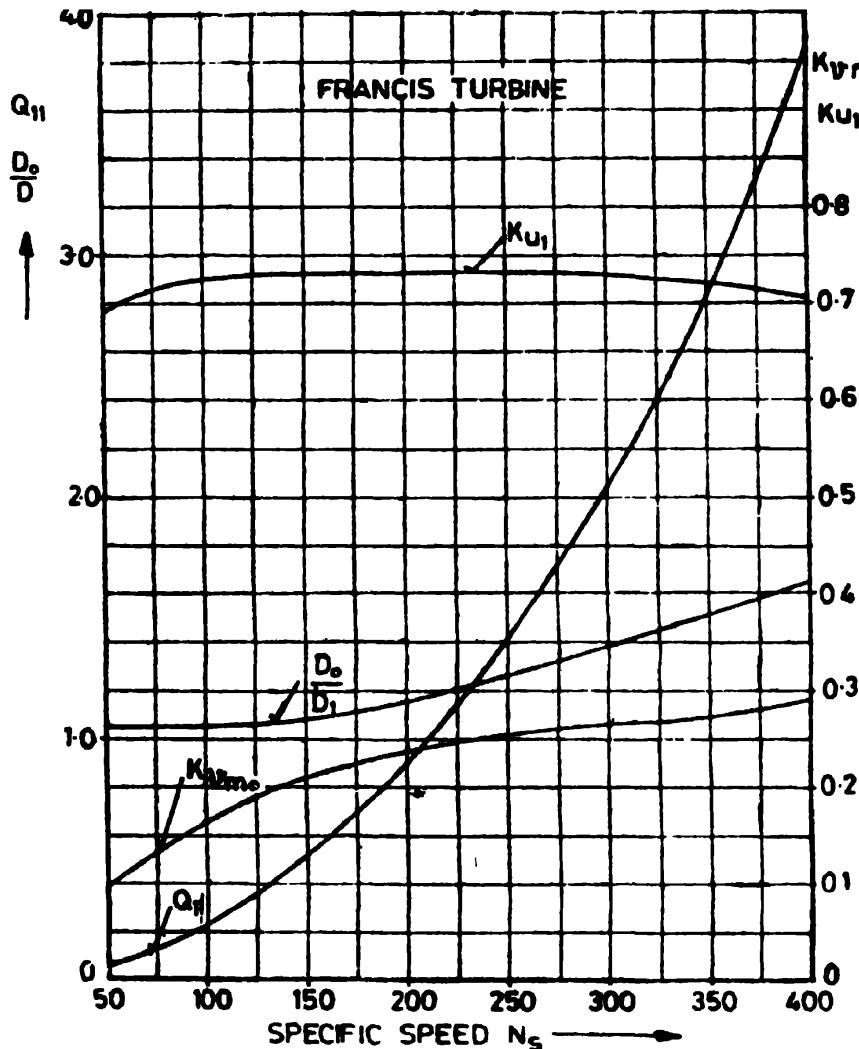


Fig 6.32 Francis Turbine Constants vs Specific Speed

**Problem 6.4** An inward flow reaction turbine has outer and inner diameters of the wheel as 1 metre and 0.5 metre respectively. The vanes are radial at inlet and the discharge is radial at outlet and the water enters the vanes at an angle of  $10^\circ$ . Assuming the velocity of flow as constant and equal to 3 metres/sec, find the speed of the wheel and the vane angle at outlet. (Calcutta University)

**Solution**

$$\begin{aligned} D_1 &= 1 \text{ m} & D_2 &= 0.5 \text{ m} & \beta_1 &= 90^\circ & (\text{vanes radial}) \\ \alpha_2 &= 90^\circ (\text{discharge radial}) & \alpha_1 &= 10^\circ & v_{m1} &= v_{m2} = 3 \text{ m/sec} \end{aligned}$$

Drawing the velocity triangles at inlet and outlet—

The peripheral velocity of wheel at inlet is calculated from the inlet velocity triangle as follows :

$$u_1 = \frac{v_{m1}}{\tan \alpha_1} = \frac{3}{\tan 10^\circ} = 0.1763 = 17 \text{ m/sec}$$

$$\text{Now } u_1 = \frac{\pi D_1 N}{60} \quad \text{or} \quad N = \frac{60 u_1}{\pi D_1} = \frac{60 \times 17}{\pi \times 1} \\ = 325 \text{ rpm} \quad \text{Answer}$$

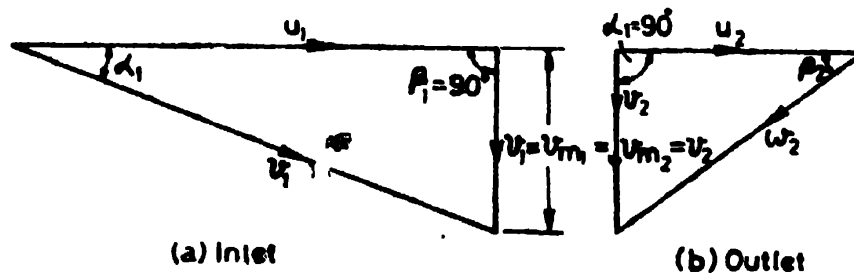


Fig 6.22 Velocity Triangles

Considering velocity triangle at outlet—

Peripheral velocity at outlet

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 325}{60} = 8.5 \text{ m/sec}$$

$$\text{Further } \tan \beta_2 = \frac{v_{m_2}}{u_2} = \frac{3}{8.5} = 0.353$$

$$\therefore \beta_2 = 19^\circ 27' \quad \text{Answer.}$$

**Problem 6.5** A inward flow reaction turbine with a supply of 550 lit/sec under a head of 15 m develops 100 HP at 375 rpm. The inner and the outer diameters of the runner are 50 cm and 75 cm respectively. The velocity of water at exit is 3 m/sec. Assuming that the discharge is radial and that the width of the wheel is constant, find the actual and the theoretical hydraulic efficiencies of the turbine and the inlet angles of the guide and wheel vanes. (AMIE)

### Solution

$$Q = 0.55 \text{ m}^3/\text{sec} \quad D_1 = 75 \text{ cm} \quad H = 15 \text{ m} \quad D_2 = 50 \text{ cm}$$

$$P_s = 100 \text{ HP} \quad v_{m_2} = 3 \text{ m/sec}$$

$$N = 375 \text{ rpm} \quad \alpha_2 = 90^\circ \text{ (radial discharge)} \quad B_1 = B_2$$

(a) Peripheral velocity of wheel at inlet

$$u_1 = \frac{\pi D_1 \cdot N}{60} = \frac{\pi \times 0.75 \times 375}{60} = 14.75 \text{ m/sec}$$

Velocity of flow at exit,  $v_{m_2} = 3 \text{ m/sec}$

As  $\alpha_2 = 90^\circ$ ,  $v_{m_2} = v_2$  (refer Fig 1.29 c)

$$\text{Work done/sec by the turbine per kg of water} = \frac{v_{u_1} \cdot u_1}{g} \quad (\text{refer Eqn 1.52})$$

but this is equal to the head utilised by the turbine, i.e.,

$$\frac{v_{u_1} \cdot u_1}{g} = H - \frac{v_2^2}{2g}$$

(Assuming there is no loss of pressure head at outlet)

$$\text{or } \frac{v_{u_1} \times 14.75}{9.81} = 15 - \frac{3^2}{19.62}$$

$$= 15 - 0.458 = 14.542 \text{ m}$$

$$\text{or } v_{u_1} = \frac{14.542 \times 9.81}{14.75} = 9.7 \text{ m/sec}$$

$$\text{Work done/sec by the turbine wheel} = \frac{\gamma \cdot Q}{g} (v_{u_1} \cdot u_1)$$

...(refer Eqn 1.52)

$$\text{or Horsepower developed by the wheel} = \frac{\gamma \cdot Q}{75} \cdot \frac{(v_{u_1} \cdot u_1)}{g}$$

$$= \frac{1,000 \times 0.55}{75} \times \frac{9.7 \times 14.75}{9.81}$$

$$= 106.5 \text{ HP}$$

Available horsepower or water horsepower

$$= \frac{\gamma \cdot Q \cdot H}{75} = \frac{1,000 \times 0.55 \times 15}{75} = 110 \text{ HP}$$

Brake horsepower = 100 HP

$$\text{Overall turbine efficiency } \eta_t = \frac{100}{110} \times 100 = 90.9\% \quad \text{Answer}$$

(This is the actual hydraulic efficiency as required in this problem)

$$\text{and hydraulic efficiency } \eta_h = \frac{106.5}{110} \times 100 = 96.82\% \quad \text{Answer}$$

(This is the theoretical hydraulic efficiency as required in this problem).

$$(b) \quad Q = (\pi \cdot D_1 \cdot B_1) v_{m_1} = \pi \cdot D_2 \cdot B_2 \cdot v_{m_2}$$

(refer Eqn 2.1 and 6.23) (neglecting blade thickness)

$$\therefore v_{m_1} = v_{m_2} \cdot \frac{D_2}{D_1} \quad (\because \text{width } B \text{ is constant})$$

$$= 3 \times \frac{50}{75} = 2 \text{ m/sec}$$

Drawing inlet velocity triangle (refer Fig 1.30) with the help of  $u_1$ ,  $v_{m_1}$  and  $v_{u_1}$ ,

$$\tan \beta_1 = \frac{v_{m_1}}{u_1 - v_{u_1}} = \frac{2}{14.75 - 9.7} = \frac{2}{5.05} = 0.396$$

$$\text{or Inlet angle of wheel vane } \beta_1 = \tan^{-1} 0.396$$

$$= 21^\circ - 36' \quad \text{Answer}$$

$$v_1 = \sqrt{v_{u_1}^2 + v_{m_1}^2} = \sqrt{9.7^2 + 2^2} = 9.9 \text{ m/sec}$$

$$\text{and } v_{u1} = v_1 \cos \alpha_1$$

$$\therefore \cos \alpha_1 = \frac{v_{u1}}{v_1} = \frac{9.7}{9.9} = 0.982$$

$$\therefore \text{Inlet angle of guide vane } \alpha_1 = \cos^{-1} 0.982 = 10^\circ - 53' \text{ Answer}$$

**Problem 6.6** An experimental inward flow reaction turbine rotates at 370 rpm. The wheel vanes are radial at inlet and the inner diameter of the wheel is half the outer diameter. The constant velocity of flow in the wheel is 2 m/sec. Water enters the wheel at an angle of  $10^\circ - 4'$  to the tangent to the wheel at inlet. The breadth of the wheel at inlet is 75 mm and the area of flow blocked by the vanes is 5% of the gross area of flow at inlet. Find —

- (a) the net available head at the wheel ;  
 (b) the wheel vane angle at outlet ;  
 (c) the outer and inner diameters of the wheel ;  
 and (d) the theoretical water horsepower developed by the wheel. (AMIE)

**Solution**

$$\begin{aligned} N &= 370 \text{ rpm} & D_2 &= 2 D_1 & v_{m1} &= v_{m2} = 2 \text{ m/sec} \\ \alpha_1 &= 10^\circ - 4' & B_1 &= 75 \text{ mm} & k &= 1 - 0.05 = 0.95 \\ \beta_1 &= 90^\circ & (\because \text{ vanes are radial at inlet}) \end{aligned}$$

$$(c) \ v_{m1} = v_1 \sin \alpha_1$$

$$\begin{aligned} \text{or } v_1 &= \frac{v_{m1}}{\sin 10^\circ - 4'} = \frac{2}{0.1748} \\ &= 11.43 \text{ m/sec} \end{aligned}$$

$\therefore$  Velocity of whirl at inlet

$$\begin{aligned} v_{u1} &= v_1 \cos \alpha_1 = 11.43 \times \cos 10^\circ - 4' \\ &= 11.25 \text{ m/sec} \end{aligned}$$

$$\text{As } \beta_1 = 90^\circ, v_{u1} = u_1 \text{ (refer Fig 6.33a)}$$

$$\text{But } u_1 = \frac{\pi \cdot D_1 \cdot N}{60}$$

$$\therefore 11.25 = \frac{\pi D_1 \times 370}{60}$$

$$\text{or } D_1 = \frac{11.25 \times 60}{\pi \times 370} = 0.58 \text{ m or } 580 \text{ mm Answer}$$

$$D_2 = \frac{580}{2} = 290 \text{ mm Answer}$$

(a) Net available head at the wheel

= Work done/sec by the wheel per kg of water

$$= \frac{v_{u1} \cdot u_1 - v_{u2} \cdot u_2}{g}$$

$$= \frac{v_{u_1} \cdot u_1}{g} \quad (\text{assuming radial discharge at outlet})$$

$$= \frac{11.25 \times 11.25}{9.81}$$

$$= 12.9 \text{ m} \quad \text{Answer}$$

$$(b) \quad u_2 = \frac{\pi \times 0.290 \times 370}{60} = 5.63 \text{ m/sec}$$

$$v_{m_2} = 2 \text{ m/sec}$$

$$\tan \beta_2 = \frac{v_{m_2}}{u_2} = \frac{2}{5.63} = 0.355$$

$$\therefore \beta_2 \text{ the wheel vane angle at outlet} = \tan^{-1} 0.355 = 19\frac{1}{2}^\circ \quad \text{Answer}$$

(d) Quantity of water flowing through the wheel

$$= \text{area of flow} \times \text{velocity of flow}$$

$$\text{or} \quad Q = (k \cdot \pi \cdot D_1 \cdot B_1) \cdot v_{m_1} = 0.95 \times \pi \times 0.58 \times 0.075 \times 2$$

$$= 0.26 \text{ m}^3/\text{sec}$$

$$\text{Water Horsepower } P_s = \frac{\gamma \cdot Q \cdot H}{75} = \frac{1,000 \times 0.26 \times 12.9}{75}$$

$$= 44.72 \text{ HP} \quad \text{Answer}$$

**Problem 6.7** An inward flow Francis turbine is required to develop 5,000 BHP when operating under a net head of 30 m and the specific speed to be about 270.

Assuming guide vane angle at full gate opening  $30^\circ$ , hydraulic efficiency 90%, overall efficiency 87%, radial velocity of flow at inlet  $0.3 \sqrt{2gH}$ , blade thickness coefficient 5%, draw the inlet velocity diagram and find—

(a) the nearest synchronous speed to drive an alternator to give a frequency of 50 cycles per sec ;

(b) the diameter and the width of the runner at inlet ;

and (c) the theoretical inlet angle of the runner vanes.

[AMI Mech E (London)]

### Solution

|                            |                             |                    |
|----------------------------|-----------------------------|--------------------|
| $P_t = 5,000 \text{ BHP}$  | $\alpha_1 = 30^\circ$       | $H = 30 \text{ m}$ |
| $v_{m_1} = 0.3 \sqrt{2gH}$ | $N_s = 270$                 | $k = 0.95$         |
| $\eta_h = 0.9$             | $f = 50 \text{ cycles/sec}$ | $\eta_o = 0.87$    |

$$(a) \text{ Specific speed of the turbine } N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$\text{or} \quad N = \frac{N_s \cdot H^{\frac{5}{4}}}{\sqrt{P_t}} = \frac{270 \times 30 \times 2.33}{\sqrt{5,000}} = 268 \text{ rpm}$$

$$\therefore \text{ Nearest synchronous speed} = \frac{3,000}{p} = \frac{3,000}{11} = 272.8 \text{ rpm}$$

(Assuming  $p$ , the number of pair of poles = 11) Answer

$$(b) \text{ Velocity of flow } v_{m_1} = 0.3 \sqrt{2gH}$$

$$= 0.3 \times 4.43 \times \sqrt{30} = 7.26 \text{ m/sec}$$

$$\text{But } v_{m_1} = v_1 \cdot \sin \alpha_1$$

$$\text{or } 7.26 = v_1 \sin 30^\circ$$

$$\text{or } v_1 = \frac{7.26}{0.5} = 14.52 \text{ m/sec}$$

$$v_{u_1} = v_1 \cos 30^\circ = 14.52 \times 0.866 = 12.6 \text{ m/sec}$$

Work done/sec by the turbine runner per kg of water

$$= \frac{v_{u_1} \cdot u_1}{g} = \eta_h \cdot H$$

$$= 0.9 \times 30 = 27 \text{ m}\cdot\text{kg/sec}$$

$$\text{Peripheral velocity } u_1 = \frac{27 \times 9.81}{12.6} = 21 \text{ m/sec}$$

$$\text{But } u_1 = \frac{\pi \cdot D_1 \cdot N}{60}$$

$$\text{or } D_1 = \frac{60 \cdot u_1}{\pi \cdot N} = \frac{60 \times 21}{\pi \times 272.8} = 1.47 \text{ m} \quad \text{Answer}$$

$$\text{Now } P_t = \frac{\gamma \cdot Q \cdot H}{75} \cdot \eta_h$$

$$\text{or } 5,000 = \frac{1,000 \times Q \times 30}{75} \times 0.87$$

$$\text{or } Q = 14.4 \text{ m}^3/\text{sec}$$

Further quantity of water flowing through the runner

$$\text{or } Q = (k \cdot \pi \cdot D_1 \cdot B_1) \cdot v_{m_1}$$

$$\text{or } B_1 = \frac{Q}{k \cdot \pi \cdot D_1 \cdot v_{m_1}}$$

$$\therefore B_1 = \frac{14.4}{0.95 \times \pi \times 1.47 \times 7.26} = 0.453 \text{ m} \quad \text{Answer}$$

(c) Draw the inlet velocity triangle (refer Fig 1.29)

$$\tan \beta_1 = \frac{v_{m_1}}{u_1 - v_{u_1}} = \frac{7.26}{21 - 12.6} = \frac{7.26}{8.4} = 0.865$$

or the inlet angle of the runner vanes

$$\beta_1 = \tan^{-1} 0.865 = 40^\circ - 48' \quad \text{Answer}$$

**Problem 6.8** In an inward flow reaction turbine the peripheral velocity of the wheel at inlet is given by  $v = \phi \sqrt{2gH}$  and the radial velocity of flow  $v_r = \psi \sqrt{2gH}$ .

The breadth of the wheel at inlet is 'n' times the diameter of the wheel at inlet. If the turbine efficiency is 85 percent and the area taken up by vanes



at inlet is 5 percent of the peripheral area at inlet, prove that the specific speed of the turbine is equal to  $150 \phi \sqrt{\psi \cdot n}$ .

A runner of the above type having a specific speed of 220 rpm is required to develop 9,000 horsepower under a head of 85 m. Taking  $\psi$  as 0.18 and  $n = 0.2$ , calculate the rpm and the diameter of the runner. (UPSC)

### Solution

$$\begin{aligned} K_{u_1} &= \phi & K_{v_{m_1}} &= \psi & \therefore K_{v_{m_1}} &= 0.18 \\ v_{m_1} &= v_f & B_1 &= n D_1 & \therefore B_1 &= 0.2 D_1 \\ \eta_t &= 0.85 & P_t &= 9,000 \text{ BHP} & u_1 &= v \\ N_s &= 220 & H &= 85 \text{ m} & k &= 1 - 0.05 = 0.95 \end{aligned}$$

(a) The quantity of water flowing through the turbine = area of flow  $\times$  velocity of turbine

$$\begin{aligned} \therefore Q &= 0.95 \times \pi \times D_1 \times B_1 \times v_{m_1} \\ &= 0.95 \times \pi \times n \times D_1^2 \times \psi \sqrt{2gH} \end{aligned}$$

$$\text{or } Q_{11} = 0.95 \times \pi \times 4.43 \times n \cdot \psi$$

$$\begin{aligned} \text{Now } u_1 &= \phi \sqrt{2gH} \\ &= \frac{\pi D_1 \cdot N}{60} \end{aligned}$$

$$N = \frac{60 \cdot u_1}{\pi \cdot D_1} = \frac{60 \phi \cdot \sqrt{2gH}}{\pi \cdot D_1}$$

$$\text{Now } N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$= \frac{\frac{60 \phi \sqrt{2gH}}{\pi \cdot D_1} \times \sqrt{\frac{1,000 \times 0.95 \times \pi \times 4.43 \times n \times \psi \times D_1^2 \times H^{\frac{3}{2}} \times H \times 0.85}{75}}}{H^{\frac{5}{4}}}$$

$$\begin{aligned} \text{or } N_s &= \phi \times \frac{60 \times 4.43}{\pi} \times \frac{\sqrt{H}}{D_1} \\ &\quad \times \sqrt{\frac{1,000 \times 0.95 \times \pi \times 4.43 \times 0.85}{75}} \times D_1 \times H^{\frac{3}{4}} \sqrt{n \cdot \psi} \\ &\quad H^{\frac{5}{4}} \end{aligned}$$

$$\text{or } N_s = 150 \cdot \phi \cdot \sqrt{n \cdot \psi}$$

Q E D

$$\text{or } N_s = 150 \cdot K_{u_1} \cdot \sqrt{K_B \cdot K_{v_{m_1}}}$$

$$(b) \quad N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$\text{or } 220 = \frac{N \times \sqrt{9,000}}{85^{\frac{5}{4}}} = \frac{N \times 94.9}{95 \times 3.03}$$

$$\text{or } N = \frac{220 \times 85 \times 3.03}{94.9} = 600 \text{ rpm } \text{Answer}$$

$$\begin{aligned} Q_{11} &= 0.95 \times \pi \times 4.43 \times n \cdot \psi \\ &= 0.95 \times \pi \times 4.43 \times 0.2 \times 0.18 \\ &= 0.475 \end{aligned}$$

$$\text{But } Q = \frac{75 \times P_i}{\gamma \cdot H \cdot \eta_i} = \frac{75 \times 9,000}{1,000 \times 85 \times 0.85} = 9.35 \text{ m}^3/\text{sec}$$

$$\text{Now } Q = Q_{11} \cdot D_1^3 \cdot H^{\frac{1}{2}}$$

$$\begin{aligned} \therefore D_1 &= \sqrt[3]{\frac{Q}{Q_{11} \cdot \sqrt{H}}} = \sqrt[3]{\frac{9.35}{0.475 \times \sqrt{85}}} \\ &= 1.46 \text{ m } \text{Answer} \end{aligned}$$

### UNSOLVED PROBLEMS

- 6.1 Describe briefly an inward flow turbine. (AMIE)
- 6.2 Sketch a Francis turbine showing its different parts.
- 6.3 What is the maximum head for which a Francis turbine has been employed?
- 6.4 Where are open flume reaction water turbines employed? Describe such a type of turbine briefly.
- 6.5 Write short notes on—  
Scroll casing, wicket gates, runner of reaction turbines.
- 6.6 What is the necessity of stay vanes in a spiral casing?
- 6.7 What are the functions of a draft tube?
- 6.8 Prove that a draft tube prevents for the loss of head of a reaction turbine.
- 6.9 What are the uses of a draft tube? Sketch and name the different types of draft tubes and state which one of them gives the maximum efficiency. Name the modern turbines which require the use of draft tubes. Why does a Pelton turbine not possess any draft tube? (AMIE)
- 6.10 What is Hydracone? What are its advantages?
- 6.11 How was the elbow type draft tube developed?
- 6.12 Why is it necessary to choose the number of Francis runner blades as odd and the number of guide vanes as even?
- 6.13 Show by the help of diagrams and velocity triangles how the runner of a Francis turbine can be developed to that of a Kaplan type. (AMIE)
- 6.14 Explain the changes in guide blade angle and runner blade angle at inlet with increase of speed ratio?
- 6.15 Draw typical velocity triangles at inlet and at outlet for low, medium and high specific speed Francis turbines as well as Kaplan turbines.
- 6.16 How do the losses in the draft tube effect the pressure at runner exit?

- 6.17 What is meant by "cavitation"? How and where does it occur in Water Power Plants? Derive an equation for the lowest head required by a reaction turbine to place it below the tail race level.
- 6.18 Why does it become necessary to install a water turbine below the tail race level?
- 6.19 What is Thoma's factor of cavitation and what is its significance for water turbines?
- 6.20 On what factors does the cavitation in water turbines depend?
- 6.21 Describe some method to avoid cavitation in water turbines.
- 6.22 Write short notes on the selection of working speed and runaway speed of reaction turbines.
- 6.23 Describe Deriaz turbine with the help of a sketch.
- 6.24 Describe how Deriaz runner can be employed for over and part loads with the maximum efficiency.
- 6.25 Explain how Deriaz developed mixed flow variable pitch turbine runner. Can this runner be used as an impeller of a pump?
- 6.26 State the main advantages of a Deriaz runner.
- 6.27 How Deriaz runner can be used as a turbine or pump without loss of efficiency?
- 6.28 Explain the geometry of a Deriaz runner.
- 6.29 What are the hydraulic losses in a reaction water turbine? Define hydraulic efficiency.
- 6.30 How will you determine the overall efficiency, if the volumetric, head and mechanical efficiencies are given?
- 6.31 What do you understand by—  
 (i) Setting of a turbine; (ii) Cavitation index, and (iii) Speed ratio of a turbine. (AMIE)

### Numericals

- 6.32 Talaiya Power House (DVC) is equipped with two water turbines each developing 2,840 HP when running at 250 rpm under a maximum head of 23.4 m.  
 (a) Calculate the specific speed of the turbine.  
 (b) State the suitable type of the turbine and its runner.  
 (c) Determine approximately the inlet diameter of the runner if the speed ratio is 0.8.  
 (d) Draw the typical inlet and outlet triangles of velocities.  
 (26, Francis, fast runner, 1.3 m) (Jadavpur University)
- 6.33 An inward flow turbine, having an overall efficiency of 75% is required to give 175 HP. The head is 6 m, velocity of the periphery of wheel is  $0.8 \sqrt{2gH}$ , and the radial velocity of flow is  $0.5 \sqrt{2gH}$ . The wheel is to make 250 rpm and the hydraulic losses in the turbine are 20% of the available energy. Determine  
 (a) the angle of the guide blade at inlet;  
 (b) the wheel vane angle at inlet;

- (c) the diameter of the wheel ;  
 (d) the width of the wheel at inlet.

Assume the turbine as reaction and radial discharge.

(34°—48' ; 49°—36' ; 662 mm ; 371 mm) (Panjab University)

- 6.34 An inward flow reaction turbine runner is required to operate under a head of 10 m at a speed of 175 rpm and to develop 200 HP. Find the diameter of the runner at inlet and outlet, the discharge, the guide vane angle and the runner vane angle at inlet and outlet, assuming the following data—

Outlet diameter =  $0.66 \times$  inlet diameter

Peripheral speed of inlet =  $0.75\sqrt{2gH}$

Velocity of flow =  $0.16\sqrt{2gH}$  = constant

Discharge radial

Hydraulic efficiency = 86%

Overall efficiency = 81%

(1.148 m ; 0.75 m ; 1.885 m<sup>3</sup>/sec ; 42°—45' ; 18°—36')

- 6.35 A Francis turbine gives the following performance

$H = 5$  m,  $Q = 1.5$  m<sup>3</sup>/sec, BHP = 90, rpm = 200,

Part gate opening = 0.9, Dia = 75 cm

- (a) Compute the efficiency and speed ratio ;  
 (b) Assuming the same wheel to operate at same speed and same gate opening under a head of 15 m, what will be the new rpm,  $Q$  and HP ? Check the results by recomputing from the efficiency and speed ratio.

[(a) 90%, 0.795 ; (b) 346.4 rpm, 2.598 m<sup>3</sup>/sec, 467.64 HP]

- 6.36 A Francis turbine has a wheel diameter of 1 m at the entrance and 0.5 m at the exit. The vane angle at the entrance is 90° and the guide vane angle is 15°. The water at the exit leaves the vanes without any tangential velocity. The head is 30 m and the radial component of flow is constant. What would be the speed of the wheel in rpm and vane angle at exit ? State whether the speed calculated is synchronous one or not. If not, what speed would you recommend to couple the turbine with an alternator of 50 cycles ? (324 rpm ; 28°—12' ; 300 rpm) (UPSC)

- 6.37 An inward flow turbine wheel works under a head of 20 m and makes 380 rpm. The diameter of the outer circumference of the wheel is 60 cm and of the inner circumference 30 cm. The velocity of water entering the wheel is 13 m/sec and the angle it makes with the tangent to the wheel is 10°. Assuming the radial velocity of flow through the wheel to be constant, and that the water leaves the wheel in a radial direction, determine the direction of the tangent to the vane of the wheel at the inlet and outlet. Determine also the hydraulic efficiency of the turbine.

(110°—30' ; 21° tangent to the periphery ; 77.5%)

- 6.38 Find the leading dimensions of the runner of an inward flow reaction water turbine to develop 850 HP at 1,000 rpm under a head of 100 m assuming a guide vane angle of  $16^\circ$  : axial length of blade at inlet 0.1 times the outer diameter, inner radius 0.6 time the outer radius ; radial velocity of flow constant ; final discharge radial ; hydraulic efficiency 0.88 ; overall efficiency 0.86 ; allowance for blade thickness 5%.

$$(D_1 = 0.527 \text{ m} ; D_2 = 0.316 \text{ m} ; B_1 = 5.27 \text{ cm} ; \beta_1 = 112\frac{1}{2}^\circ ; \beta_2 = 31^\circ - 37')$$

(Jadavpur University)

- 6.39 The following data refers to an inward flow reaction turbine :

Supply =  $1 \text{ m}^3/\text{sec}$  at 25 m head,

Wheel diameters = 75 cm at outlet and 50 cm at inlet,

Radial exit velocity =  $2.5 \text{ m/sec}$ ,

Inlet vane angle =  $35^\circ$

Calculate the horsepower and rpm of the turbine. Assume the width of the wheel as constant, and turbine efficiency 80%.

(266.7 HP ; 326 rpm)

(AICTE)

- 6.40 An inward flow turbine works under a total head of 30 m. The velocity of wheel periphery at inlet is  $15 \text{ m/sec}$ . The outlet pipe of the turbine is 30 cm in diameter and the turbine is supplied with 250 litres of water per second. The radial velocity of flow through the wheel is the same as the velocity in the outlet pipe. Neglecting friction, determine—

(a) the vane angle at inlet,

(b) the guide blade angle,

(c) HP of the turbine.

( $10^\circ - 37'$  ;  $40^\circ - 12'$  ; 100 HP)

(Madras University)

- 6.41 Discuss the general theory of the inward flow reaction turbine and deduce an expression for the maximum hydraulic efficiency. State the reasons that lead to the adoption of comparatively smaller inlet angles of runner blades for higher heads and larger ones for lower heads. Show also that when the inlet angle is  $90^\circ$  for the medium heads, the velocity of flow being constant, the hydraulic efficiency is expressed by  $\frac{2}{2 + \tan \alpha}$ , where  $\alpha$  is the guide blade angle.

(Bombay University)

- 6.42 233 litres of water per second are supplied to an inward flow reaction turbine. The head available is 11 m. The wheel vanes are radial at inlet and the inlet diameter is twice the outlet diameter. The velocity of flow is constant and equal to  $1.83 \text{ m/sec}$ . The wheel makes 370 rpm. Find—

(a) guide vane angle ;

(b) wheel vane angle at inlet ;

(c) inlet and outlet diameters of the wheel ; and

- (d) the width of the wheel at inlet and outlet. Neglect the thickness of the vanes. Assume that the discharge is radial and that there are no losses in the wheel

( $10^\circ - 4'$  ;  $90^\circ$  ; 534 mm ; 267 mm ; 76.25 mm ; 152.5 mm)

(Osmania University)

- 6.43 Two inward flow reaction turbines working under the same head and same hydraulic efficiency have also runners of the same diameters viz., 550 mm and the velocity of flow for both is 5.5 m/sec. One of the runner A has an inlet blade angle of  $65^\circ$  and a speed of 520 rpm, while the other B has an inlet angle of  $115^\circ$ . What is the speed of the runner B ? (432 rpm) (Bombay University)

- 6.44 As an element of water passes through the runner of an inward-flow turbine, its total energy and its total energy undergo considerable changes. Show this by sketching graphs roughly to scale between "radial distance of element from the axis" (abscissae) and "pressure energy" and "total energy" of element (ordinates) for an idealised vertical shaft turbine as follows-

Outer diameter = 1.25 m ; inner diameter = 0.7625 m ; total head above wheel = 186 m ; radial component of flow velocity of water = 6.23 m/sec ; inlet runner blade angle =  $90^\circ$ . Assume that 5% of the total head is lost in friction and eddying in the guide apparatus, before the water reaches the wheel, and that 5% is lost in the runner itself. The water escapes from the runner into the tail-race at atmospheric pressure. Make any other reasonable assumptions. What would be the pressure head of the water in the clearance space between guide blades and runner blades ?

[AMI Mech E (Lond)]

- 6.45 A reaction water turbine is equipped with a straight flaring draft tube having top and bottom diameters of 0.5 m and 0.75 m respectively. The water velocity at the top is 3 m/sec, where the elevation is 5 m above the level of the tail water. Assuming a loss in the draft tube equal to half the velocity head at exit, compute

- (a) the pressure head at the top ;  
(b) the total head at the top with reference to the tail race as a datum ;

- (c) the total head at the exit ;  
(d) the power in water at the top and at exit ;

- and (e) the power loss in the tube due to friction.

( $-5.323$  m ;  $0.135$  m ;  $0.09$  m ; 1.06 HP ; 0.707 HP ; 0.353 HP)

- 6.46 A turbine runner has an exit velocity of 10 m/sec. The loss of head due to friction and other causes in the draft tube should not exceed 1.5 m. What maximum height of setting will you recommend for the turbine if the cavitation is to be avoided ?

Assume

- (i) the velocity of water at the outlet of draft tube as 2.5 m/sec,

- and (ii) the cavitation commences when the pressure is 2.5 m of water absolute.

(4.73 m)

(Delhi University)

- 6.47 A plant is located at an elevation of 100 m MSL. The average water temperature is 15°C. The turbine develops 50,000 HP at a rated speed of 112.5 rpm and under a head of 40 m. Calculate the maximum allowable suction head for the draft tube. Given—

|          |     |     |     |     |     |      |      |       |
|----------|-----|-----|-----|-----|-----|------|------|-------|
| $\sigma$ | 1   | 0.5 | 0.4 | 0.2 | 0.1 | 0.05 | 0.03 | 0.016 |
| $N_s$    | 890 | 515 | 400 | 267 | 182 | 129  | 98   | 71    |

and at 100 m MSL.

|            |     |      |     |     |      |    |
|------------|-----|------|-----|-----|------|----|
| °C         | 75  | 70   | 55  | 45  | 30   | 15 |
| $H_b$ in m | 6.5 | 7.25 | 8.5 | 9.0 | 9.75 | 10 |

$H_b$  = Height of barometric column in m. (3.16 m)

- 6.48 A Francis turbine develops 550 BHP at an overall efficiency of 82% when working under a static head of 5 metres ; the draft tube being cylindrical and of diameter 2.3 metres. What increase in power and efficiency of turbine would you expect ; head, speed and discharge remaining the same ; if a tapered draft tube having an outlet diameter of 3.2 metres and efficiency of conversion of 90% were substituted for the cylindrical one ?

(21.5 HP ; 3.9%)

(*Madras University*)

## Reaction Turbines II

### [PROPELLER, KAPLAN AND TUBULAR (OR BULB) TURBINES]

#### (A) Propeller and Kaplan Turbines

**7.1 Propeller and Kaplan Turbines**—The propeller-shaped runner evolved from the Francis mixed flow runner (refer Art. 6.15) fulfills the

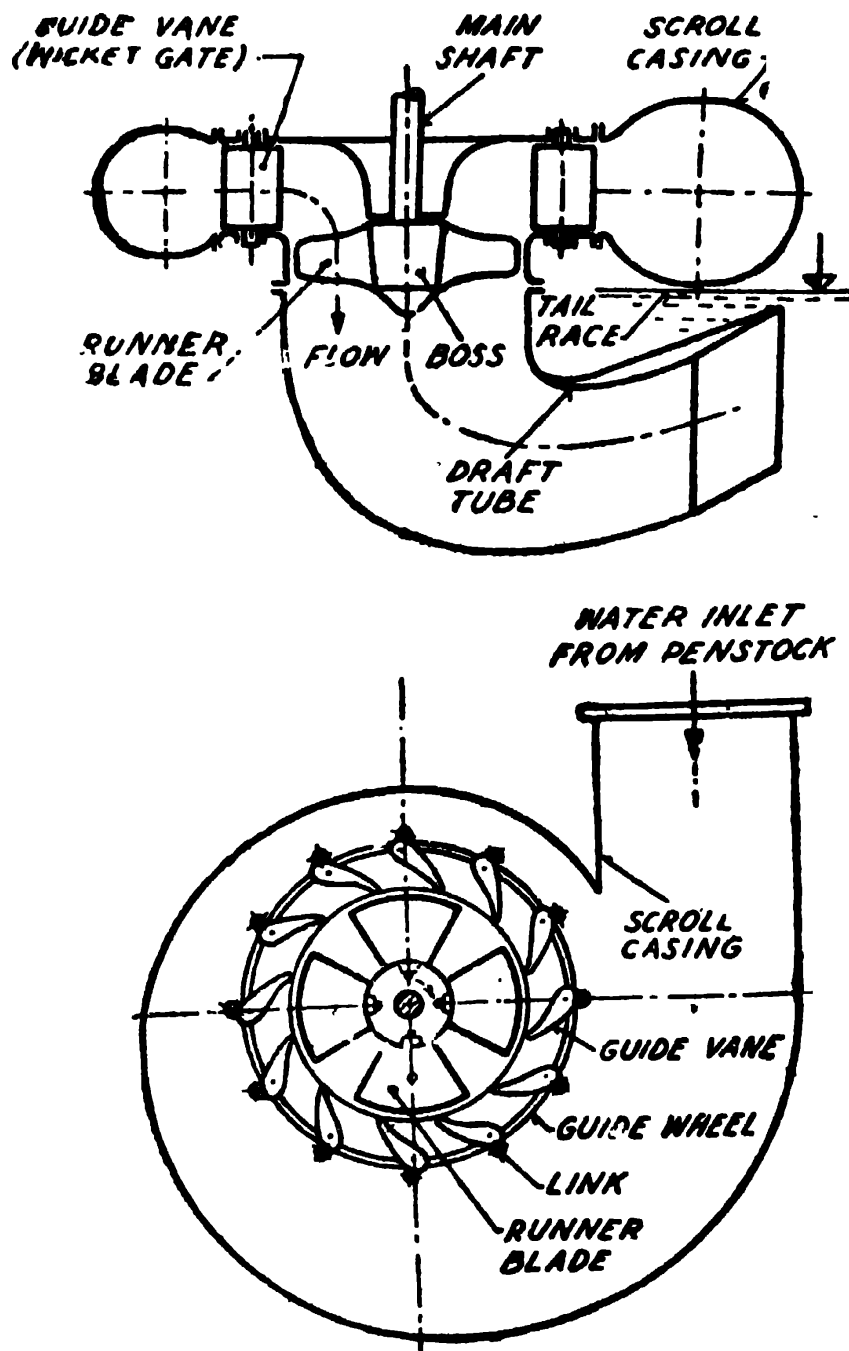


Fig 7.1 Outlines of a Kaplan Turbine



need for a faster unit using a large quantity of flow with low head. The turbine having a propeller-shaped runner is known as *propeller turbine*, which is an axially flow reaction turbine (refer Fig 7.1). Similar to Francis turbine it also operates in an entirely closed conduit from inlet to tail race. The flow passages of propeller turbine operate with very high velocities, therefore, cavitation is likely to occur. It is due to cavitation that the maximum permissible head of such a turbine is restricted.

Propeller turbine has fixed runner blades as in case of Francis type. The power-load efficiency curve (refer Fig 7.2) for propeller turbine is peaked indicating a poor performance at part and overloads. It was Kaplan

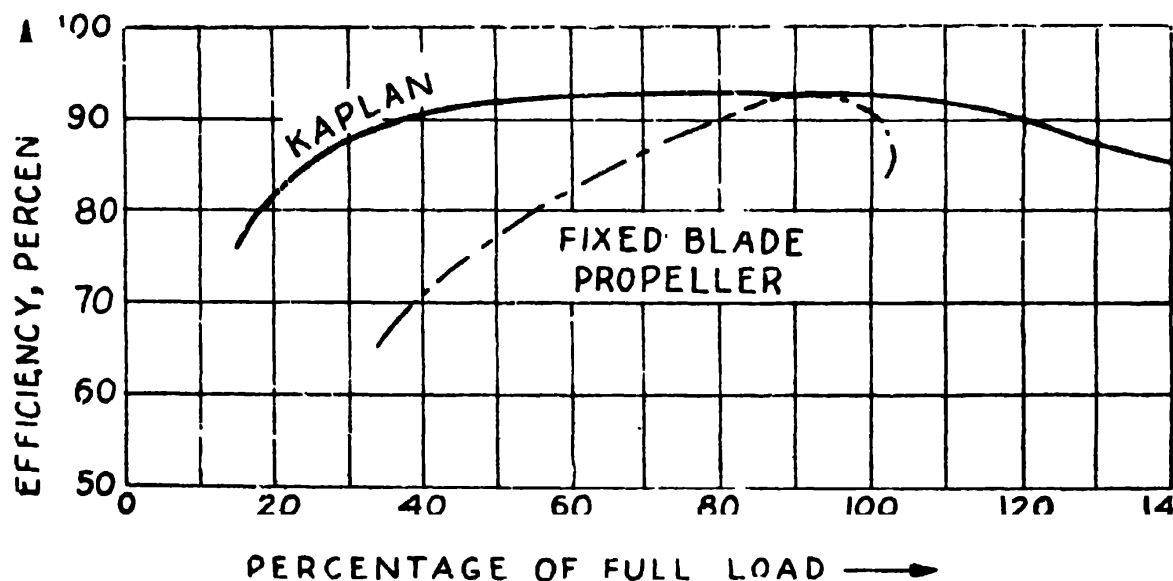


Fig 7.2 Comparison of Efficiencies of Propeller (Fixed-Blade) and Kaplan Turbines

who developed in early 1920's a turbine in which the pitch of the propeller blades was automatically adjusted by the governor, while the turbine is in motion at *constant speed*, to suit the particular load, thus maintaining the optimum efficiency over a wide range of loading and giving a "flat efficiency" curve (refer Fig 7.2) by improving its performance at part and overloads. The runner blade actuating mechanism is accommodated in the boss (or hub) and in the main turbine shaft which is made hollow. The Kaplan turbine is more costly than propeller turbine. The evolution of Deriaz runner (refer Chapter 6 B) was based on the Kaplan design.

Thus Kaplan turbine is just a propeller turbine in which the runner blades are made adjustable. The propeller turbine can be employed economically when it has to work constantly under full load, otherwise Kaplan turbine will be preferred. As already explained Propeller and Kaplan turbine are used where comparatively low head and large quantity of water are available.

As seen from Fig 7.1, there is a considerable space between the ends of guide vanes and the leading edge of runner. The direction of flow changes from radial to axial in this space. Simultaneously in this space, the flow pattern changes approximately to free spiral vortex (i.e.,  $v_u \cdot r = \text{constant}$ ). The guide vanes impart the whirl component to the flow and the runner blades remove the whirl making the discharge purely axial and transforming the whirl into useful power.

All parts such as spiral casing, guide mechanism and draft tube of the propeller and Kaplan turbines except the runner are similar (refer Fig 7.1)

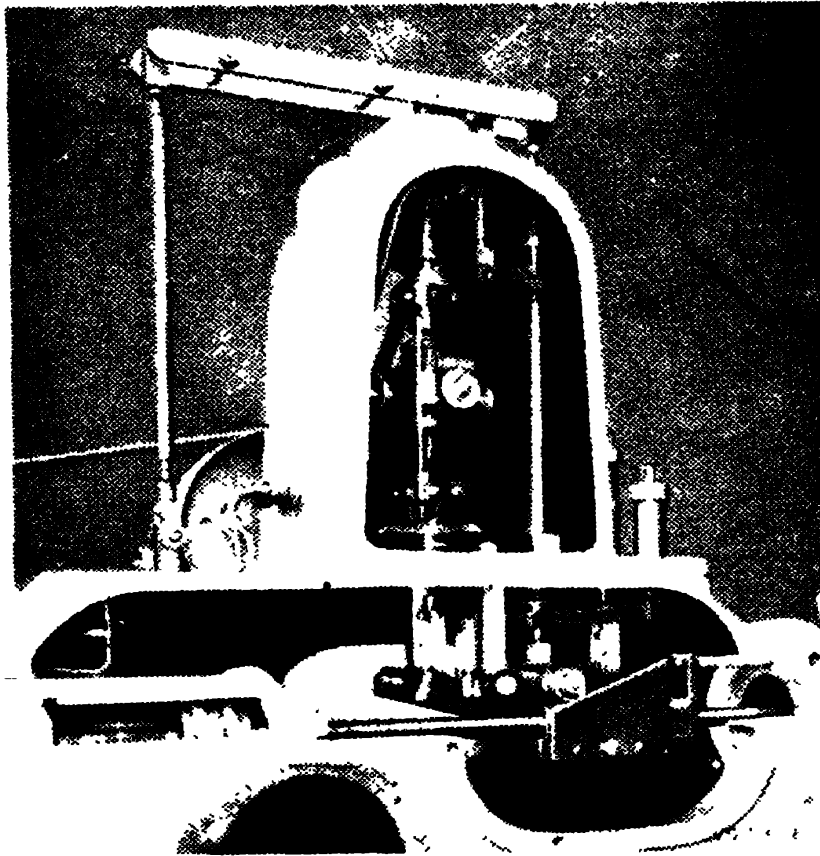


PLATE 26.(a) Turbine Speed Governor whose Operation Diagram is given in Fig. 8.2 Manufacturers : Voith West Germany

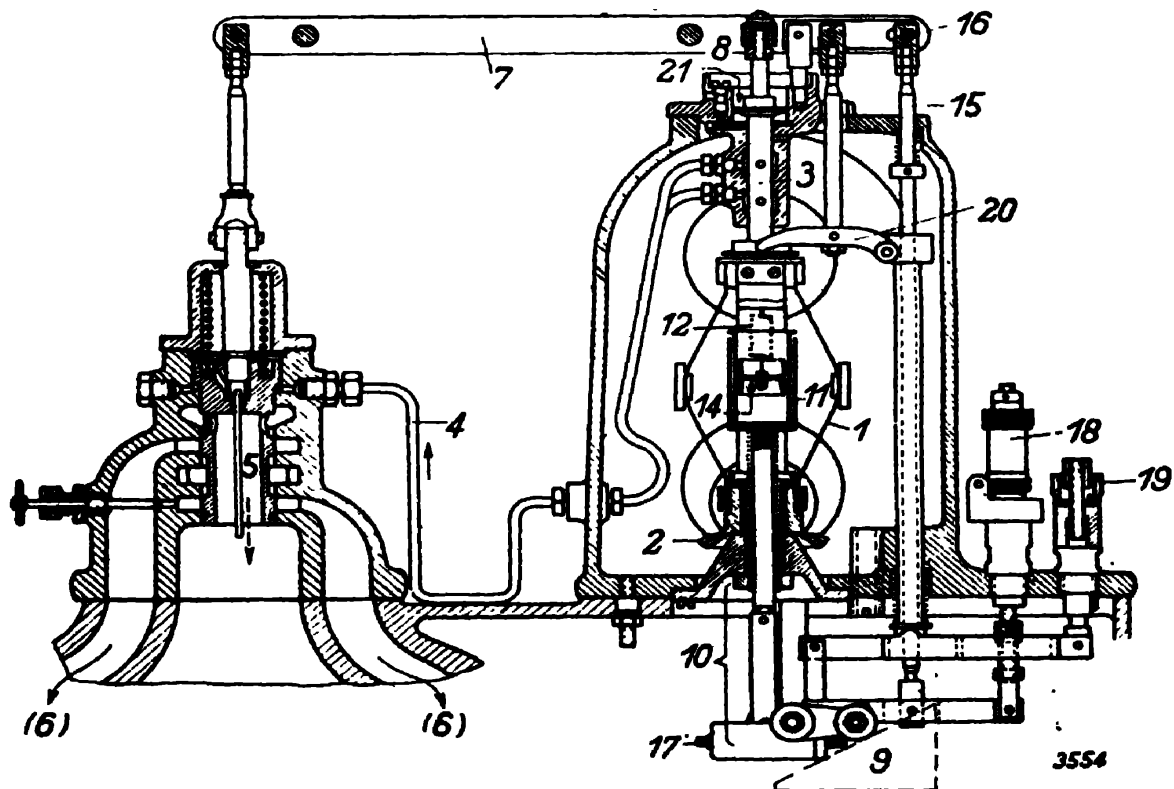
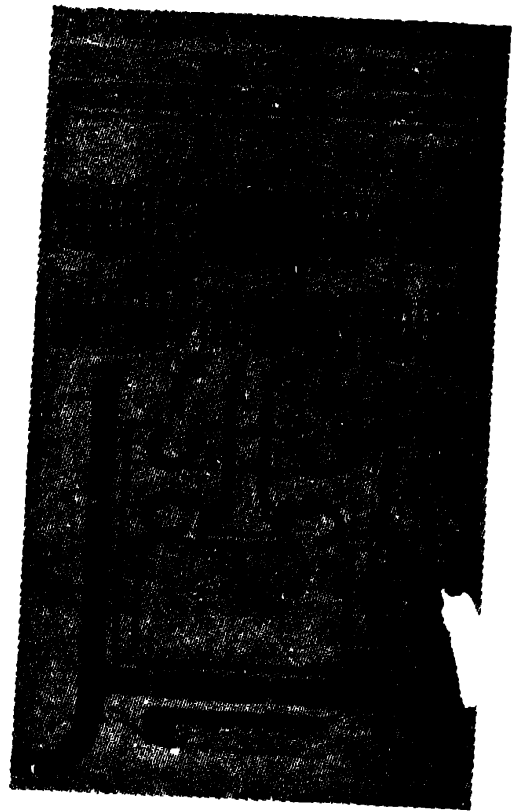


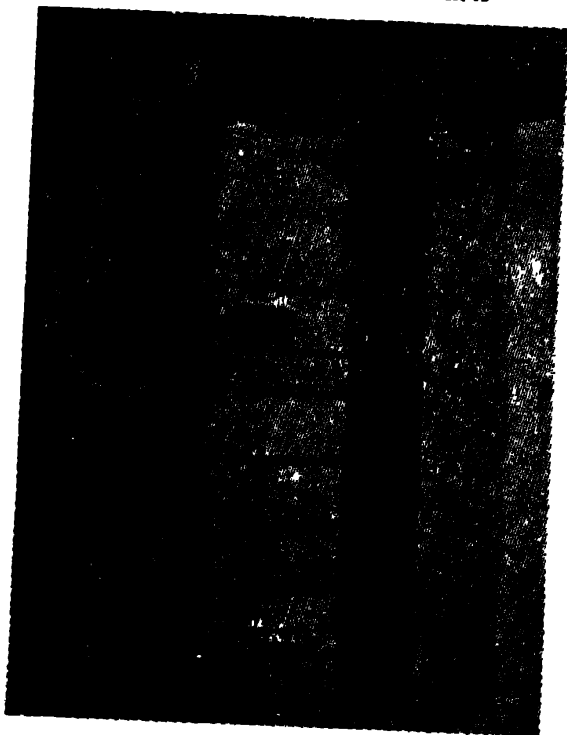
PLATE 26.(b) Sectional Drawing of Turbine Speed Governor whose Pictorial View is given above. Manufacturers : Voith West Germany



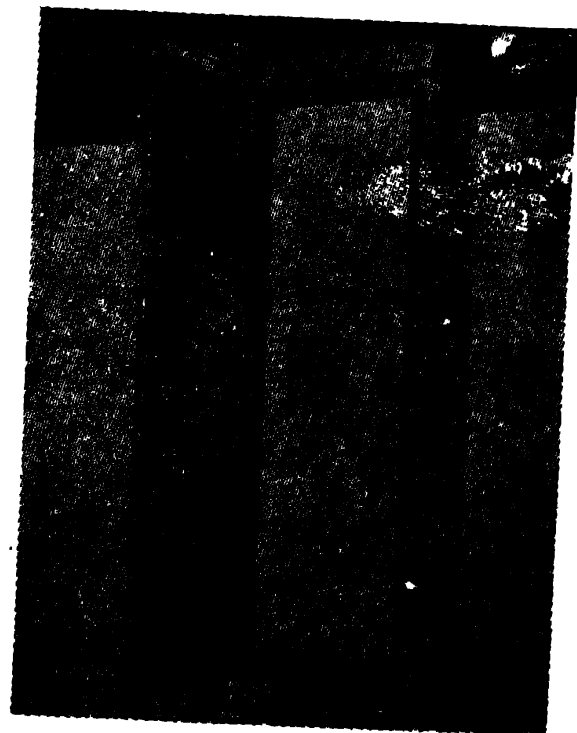
PLATE 27 [a] Front of  
Cabinet Ac-  
tuator Govern-  
ing Equipment  
for Haspranget  
Power Plant



[b] Back Cover Removed :  
showing Electric Con-  
nections and Wiring



[a]



[b]

PLATE 28. Regulator Cubicles Containing Electrical Part of  
Governor Manufactures : Asea (Sweden)

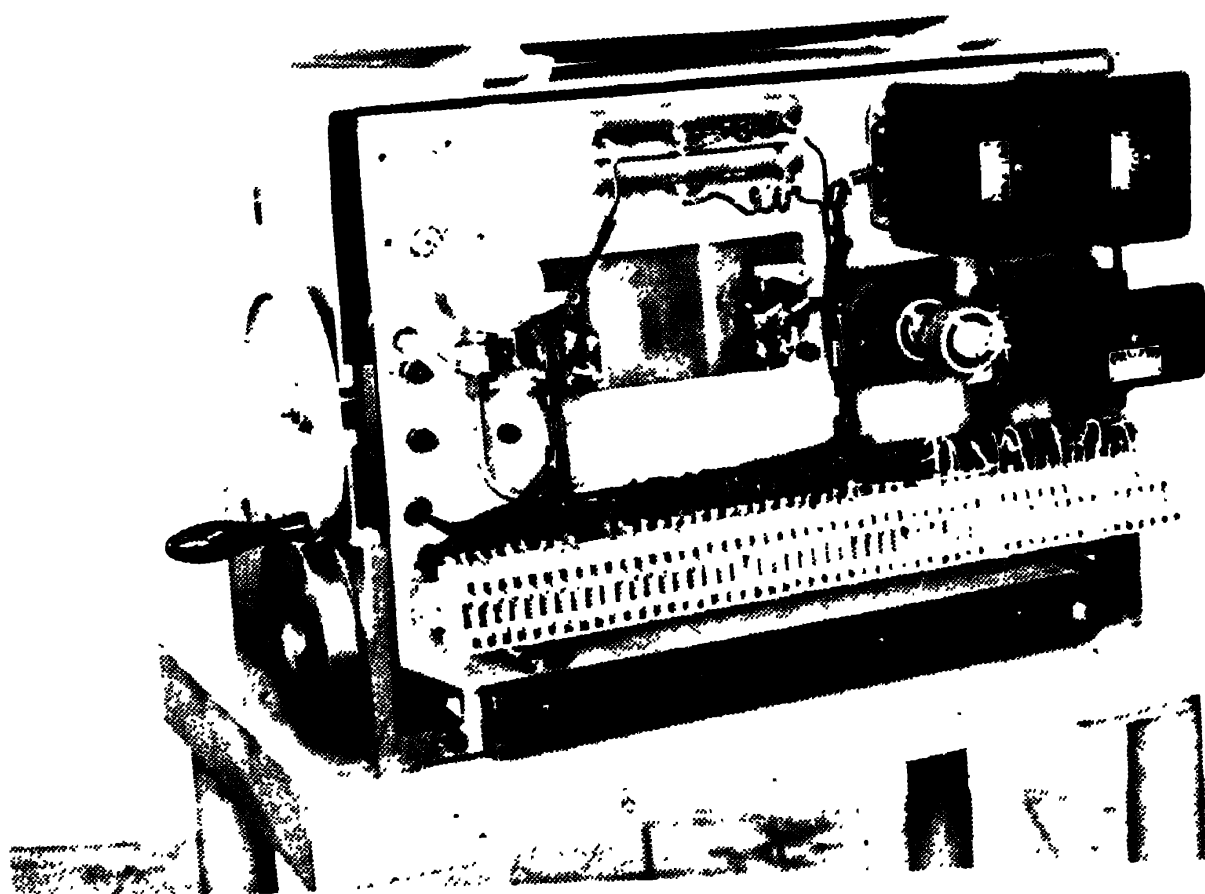
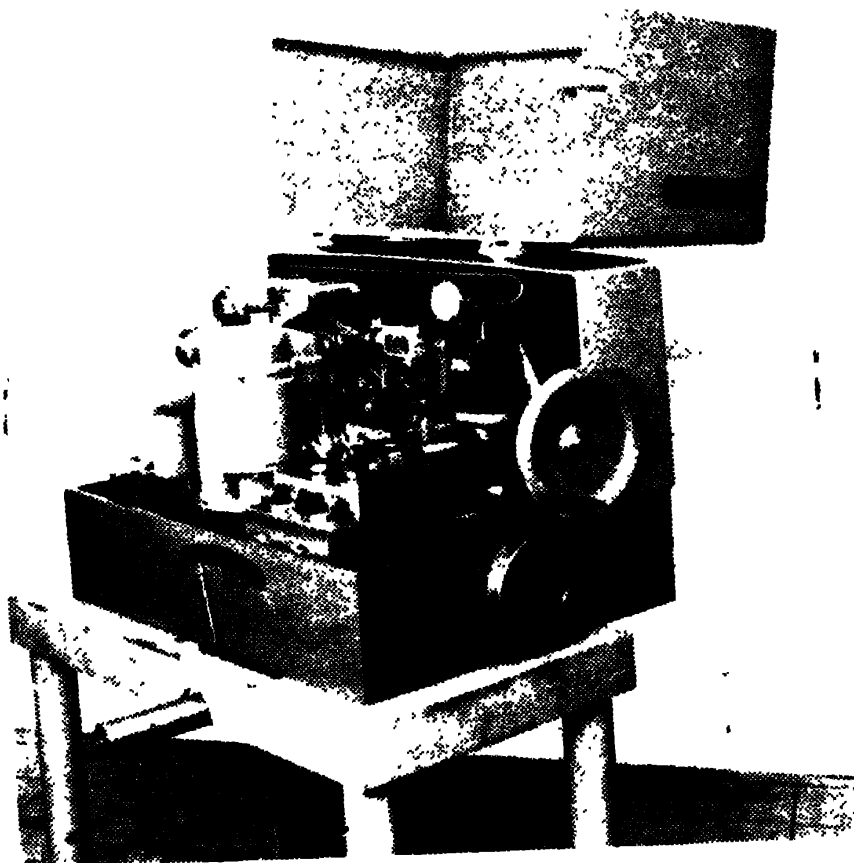


PLATE 29. New Actuator Cabinet for Electro-Hydraulic Governor Manufactures : Asea (Sweden)

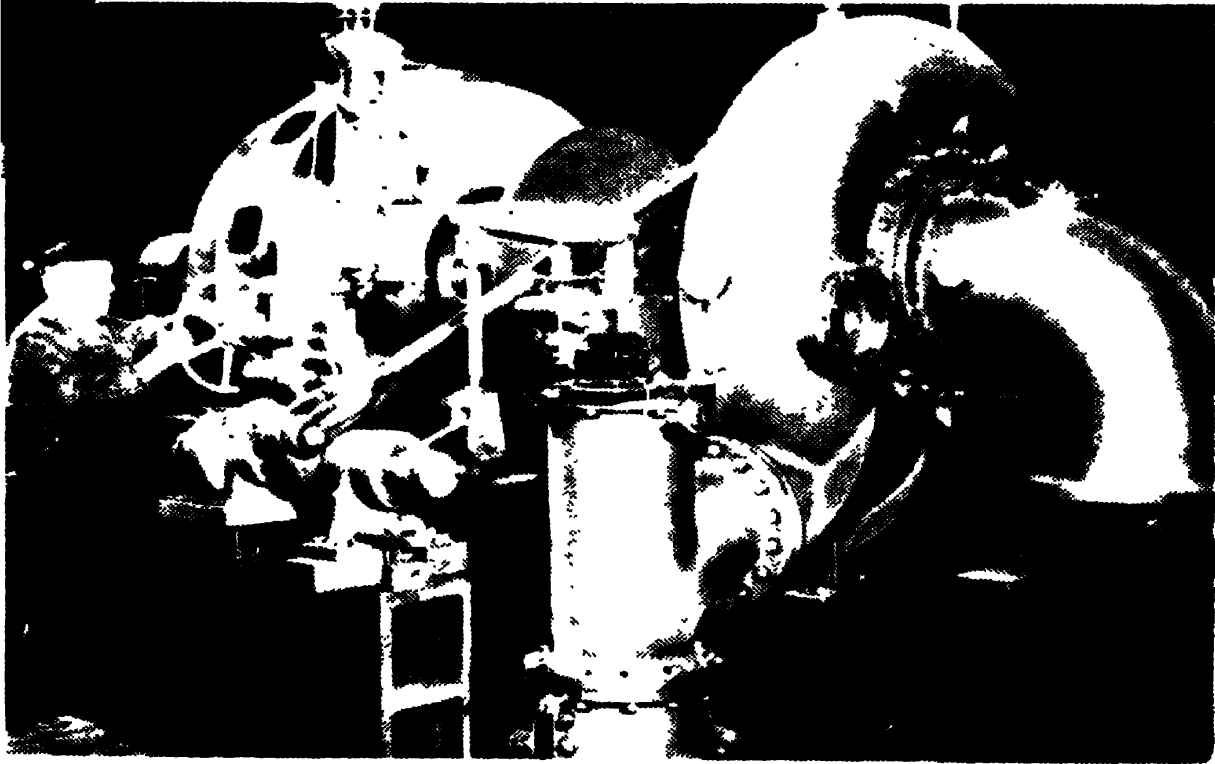


**PLATE 30. Pressure Regulator with Reaction Turbine**

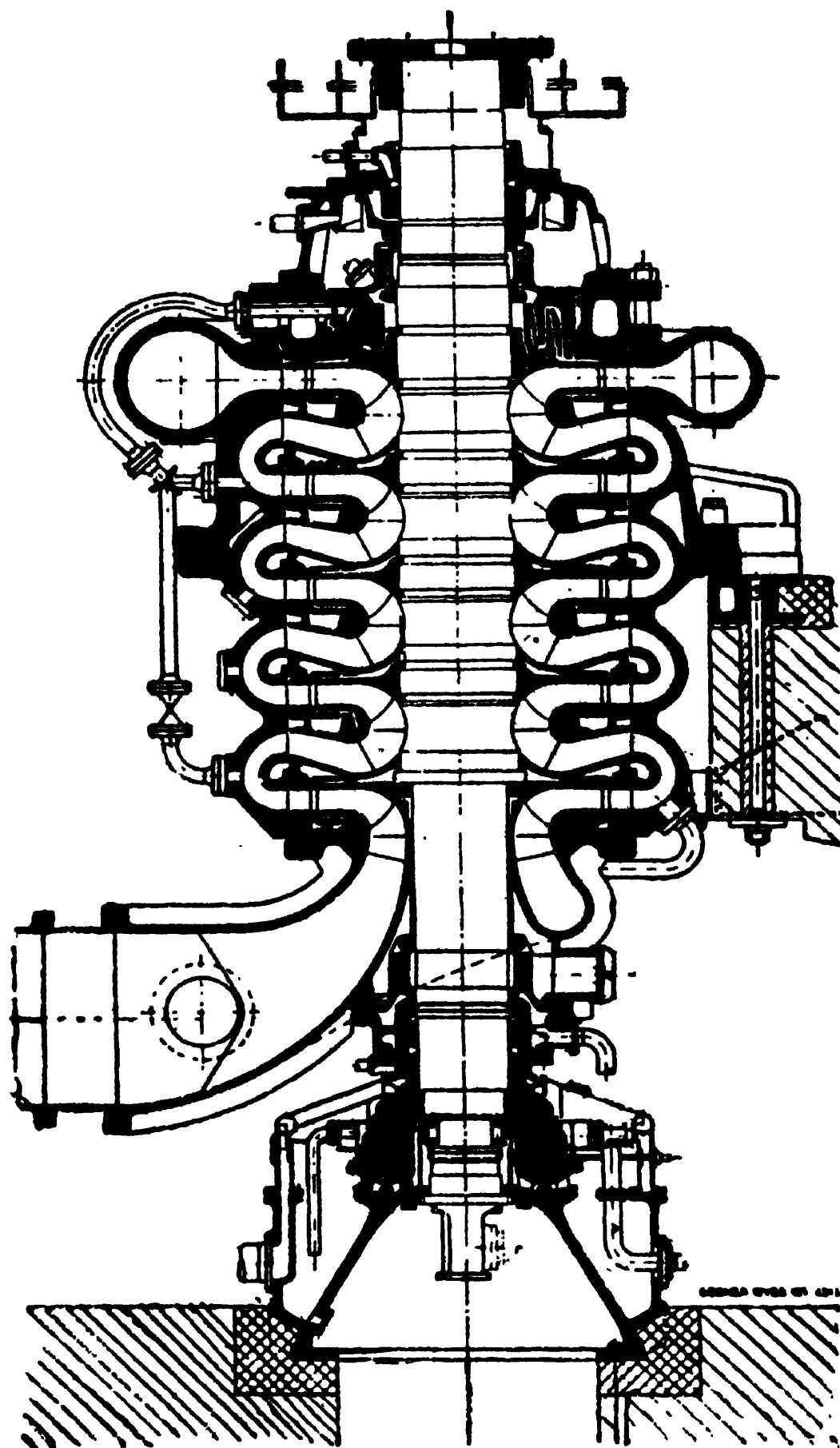


PLATE 31. Sectional View of Storage Pump at Luenersee  
9Austria Manufactures : Escher Wyss

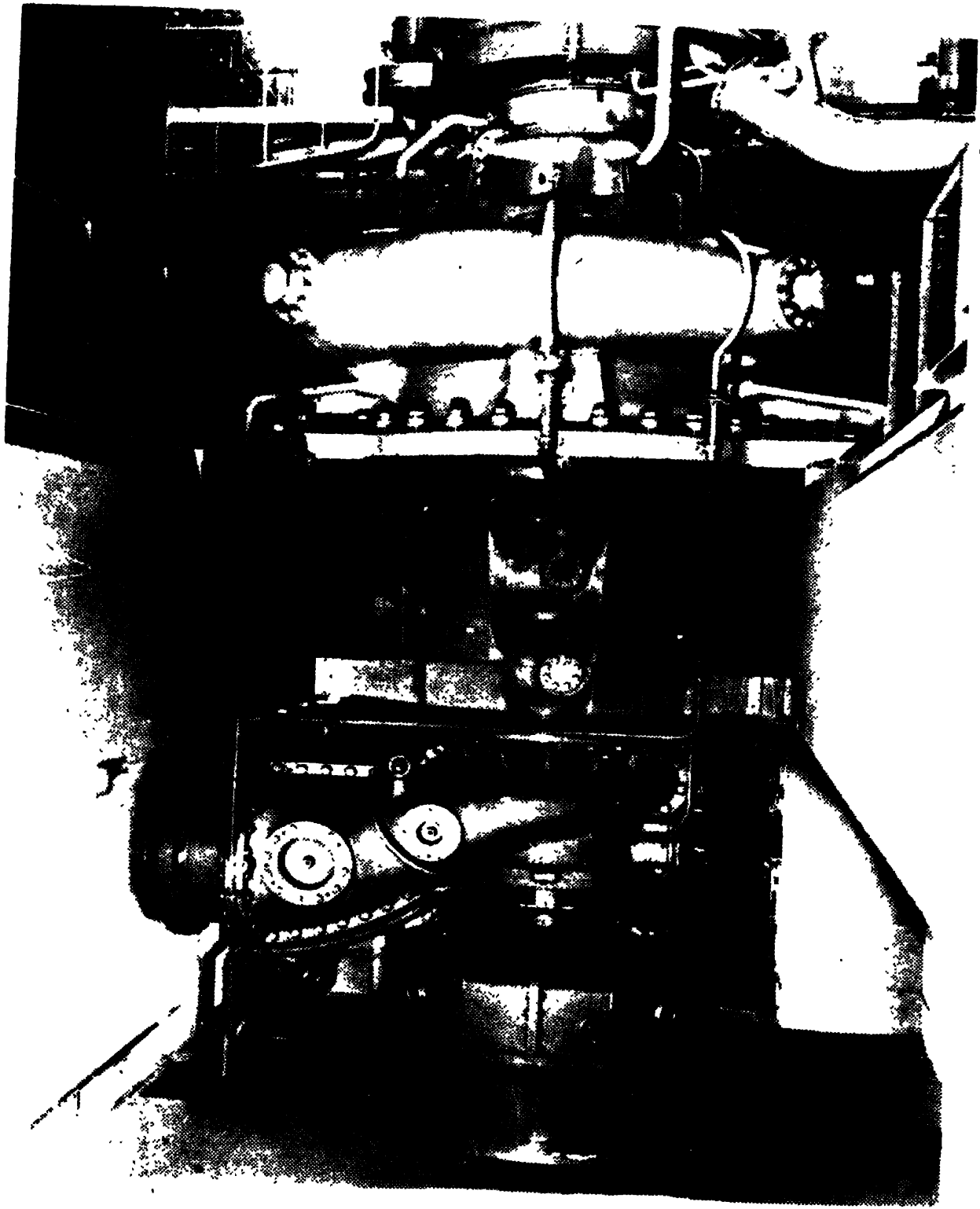


PLATE 32. Escher Wyss Luenersee (Austria) Storage Pump,  
Pictorial View





Runaway speed of a Kaplan turbine is 2.5 to 3 times its normal working speed whereas that of Francis turbine is 2 to 2.2 times (refer Table 6.5). The revolving parts of the turbo set are designed to withstand runaway speed of 2.5 to 3.5 times the working speeds. Emergency oil-pressure systems and runner braking vanes have been devised to prevent the maximum runaway speed being attained particularly where intake gates are omitted.

Plate 23 and Fig 7.3 give pictorial and sectional views of one of the largest Kaplan turbines of Europe. Fig 7.4 shows section through another Kaplan turbine on famous Danube river of Europe.

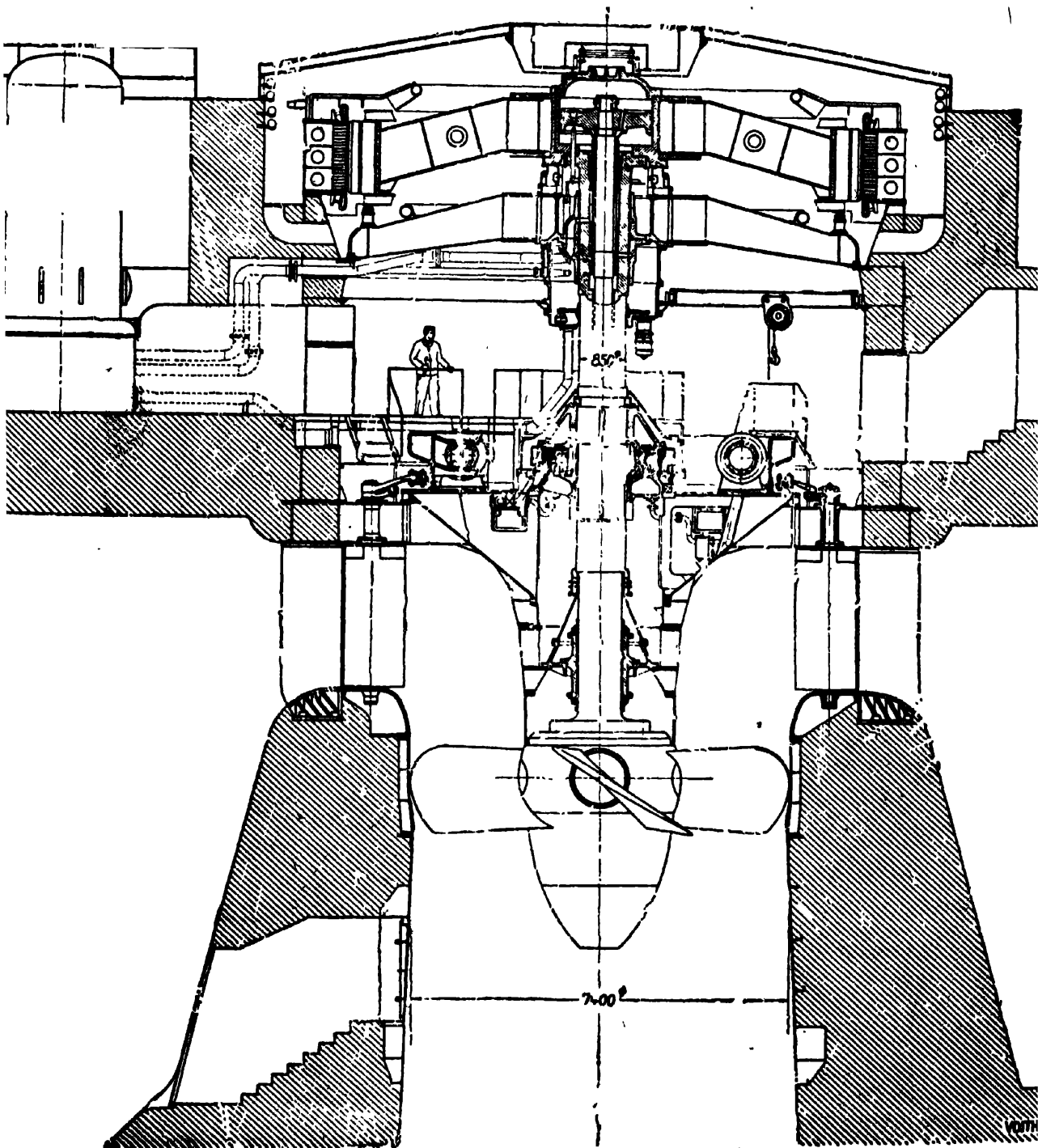


Fig 7.4 Section through Kaplan Turbine Manufactured by Voith, W. Germany for Jochenstein Run-of-River Power Plant on Danube River  $P_t=44,000$  HP;  $Q=350$  m<sup>3</sup>/sec;  $H=9.6$  m;  $N=65.2$  rpm,  $Z=5$  blades. Size of turbine is compared with the size of a man.

**TABLE 7.1**  
**Some Notable Fixed Blade Propeller Turbine Installations of World**

| No. | Plant               | Location        | No. of Units | Unit Capacity HP | Head m | Speed rpm |
|-----|---------------------|-----------------|--------------|------------------|--------|-----------|
| 1   | Harmon              | Ontario, Canada | 2            | 95,400           | 31     | 100       |
| 2   | Kipling             | Ontario, Canada | 2            | 95,400           | 31     | 100       |
| 3   | Big Bend            | S. Dakota, USA  | 8            | 91,500           | 20.4   | 81.8      |
| 4   | Conowigo (II Stage) | Maryland, USA   | 4            | 86,200           | 26.1   | 120       |
| 5   | Little Long         | Ontario, Canada | 2            | 85,200           | 27.4   | 100       |
| 6   | Robert Mosses       | New York, USA   | 8            | 80,100           | 24.6   | 94.7      |
|     |                     |                 | 8            | 78,500           | 24.6   | 94.7      |
| 7   | Robert Saunders     | Ontario, Canada | 16           | 76,000           | 24.6   | 94.7      |
| 8   | Wilson              | Alabama, USA    | 3            | 75,600           | 26.1   | 105.9     |
| 9   | Beauharnois No. 3   | Quebec, Canada  | 10           | 74,700           | 23.7   | 94.7      |

**TABLE 7.2**  
**Some Notable Kaplan Turbine Installations of World**

| No. | Plant         | Location                     | No. of Units | Unit Capacity HP | Head m          | Speed rpm | Other Details                                       |
|-----|---------------|------------------------------|--------------|------------------|-----------------|-----------|-----------------------------------------------------|
| 1   | John Day      | USA                          | 20           | 215,000          | 28.6            | 90.0      |                                                     |
| 2   | Volgograd     | Volgograd USSR (Volga River) | 22           | 171,400          | 22.3            | 62.2      | Mfg : LMZ (Russia),<br>D = 9.3 m                    |
| 3   | Grand Rapids  | Manitoba, Canada             | 3            | 152,000          | 36.5            | 112.5     |                                                     |
| 4   | Kuibishev     | USSR                         | 20           | 146,000          | 19              | 62.2      |                                                     |
| 5   | Ice Harbour   | USA                          | 3            | 145,000          | 27.1            | 90.0      |                                                     |
| 6   | Rocky Reach   | Washington, USA              | 7            | 142,000          | 28.0            | 90.0      |                                                     |
| 7   | Jactaquac     | New Brunswick<br>Canada      | 2            | 142,000          | 33.7            | 112.5     |                                                     |
| 8   | Jupia         | Brazil                       | —            | 140,000          | 14—24.3         | 78.3      |                                                     |
| 9   | Cowan's Ford  | N. Carolina, USA             | 3            | 123,000          | 28.0            | 105.9     |                                                     |
| 10  | Wanapam       | Washington, USA              | 10           | 122,000          | 24.3            | 85.7      |                                                     |
| 11  | Porsi         | Sweden                       | 2            | 121,800          | 33              | 115.0     |                                                     |
| 12  | Priest Rapids | Washington, USA              | 8            | 133,000          | 25.6            | —         | Mfg : English Electric,<br>D <sub>1</sub> = 7.2 m   |
|     |               |                              | —            | 116,000          | 23.7            | 85.7      |                                                     |
| 13  | Aschach       | Danube River,<br>Austria     | 4            | 100,000          | —               | —         | Mfg : Voith (W. Germany),<br>D <sub>1</sub> = 8.4 m |
|     |               |                              | —            | 9,220            | 15              | 68.7      |                                                     |
| 14  | St. Martin    | Austria                      | —            | 15,000           | 77.5<br>Highest | 600       | Mfg : Voith, 7 blades                               |

**TABLE 7.3**  
**Some Kaplan Turbine Installations in India**

| <i>Sl. No.</i> | <i>Scheme</i>                                  | <i>Turbine Details</i>                                                                                                                                                                                                                                                                                     | <i>Site of Power House</i>                                                                                 | <i>Source of Water</i> |
|----------------|------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------|------------------------|
| 1              | Nangal-Bhakra Project (Punjab)                 | Two Vertical Kaplan Turbines each having $P_t=34,000$ HP, $H=29.8$ m, $N=166.7$ rpm. Runaway speed=458 rpm $Z_1=20$ , $Z_2=6$ and one Hitachi (Japan) Kaplan Turbine                                                                                                                                       | Two similar Power Houses at Ganguwal and at Kotla (10 and 24 km) from Nangal Railway Station respectively) | Nangal Hydel Channel   |
| 2              | Ganga Hydro-Electric Scheme (U.P.)             | Ten stations having Kaplan Turbines with power varying from 400 to 4,000 HP ( <i>Pathri</i> —3 Voith vertical turbines, each of 9,650 HP under 9.62 m; $Q=84.4$ m <sup>3</sup> /sec; $N=125$ rpm; $D=3.89$ m and <i>Mohammedpur</i> —3 English Electric turbines, each of 4,300 HP, 5.3 m head at 125 rpm) | Ranipur, Pathri, Bahadrad, Salawa, Chitaura, Nirganj, Mohammedpur, Sumera, Palra and Bhola                 | Ganga Canal            |
| 3              | Sarda Hydel Project (U.P.)                     | Three vertical Kaplan turbines each of 19,200 HP (English Electric)                                                                                                                                                                                                                                        | Khatima                                                                                                    | Sarda Canal            |
| 4              | Hirakud Dam Project II (Orissa)                | Four vertical Kaplan Turbines, each of 52,000 HP and two Voith vertical turbines, each of 36,700 HP under a head of 2.7 m; 150 rpm; $Q=127.5$ m <sup>3</sup> /sec; $D=4.49$ m                                                                                                                              | Hirakud (10 km from Sambalpur)                                                                             | Mahanad (Hirakud Dam)  |
| 5              | Radhanagri Hydro-Electric Scheme (Bombay)      | Four vertical Kaplan Turbines each having $P_t = 1,720$ HP, $H = 14$ to 36 m (29 m average), $N = 600$ rpm                                                                                                                                                                                                 | Radhanagari (Kolhapur)                                                                                     | Bhogavati River        |
| 6              | Nizamsagar Project (Andhra)                    | Three vertical Kaplan turbines each having 7,050 HP                                                                                                                                                                                                                                                        | Nizamsagar                                                                                                 | Nanjira River          |
| 7              | Tungabhadra Hydro-Electric Scheme (Tamil Nadu) | Two vertical Kaplan Turbines from Hitachi (Japan) each having $P_t = 13,800$ HP, $N = 214$ rpm, $H = 20$ m                                                                                                                                                                                                 | Tungabhadra Dam Power Station                                                                              | Tungabhadra Dam        |
| 8              | Panchat Hill Power Station (DVC)               | One vertical Kaplan turbine (Swedish make) $P_t=56,000$ HP at $H = 24.6$ m; and 90,000 HP at 34.5 m, $D_1 = 5.37$ m; $N = 125$ rpm $N_r = 370$ rpm; $Z_1 = 24$ $Z_2 = 5$ ; Runner weight = 89 tonne, Turbine shaft dia 2.9 m (bore 0.342 m) Shaft weight = 30 tonne.                                       | Panchat Hill                                                                                               | Panchat Hill           |

### 7.2 Notable Propeller and Kaplan Turbine Installations of the World and India—The following tables show various installations :

(a) Table 7.1 shows some notable fixed blade propeller turbine installations of the world.

(b) Table 7.2 shows some notable Kaplan turbine installations of the world.

(c) Table 7.3 shows some Kaplan turbine installations in India.

From Table 7.2, it will be seen that most powerful Kaplan turbine of the world to date belongs to John Day Plant of USA. The highest head Kaplan turbine of the world is installed in St. Martin Power Plant in Austria producing 15,000 HP, under 77.5 m head at 603 rpm with 7 runner blades, manufactured by Voith (West Germany).

**7.3 Adjustment of Kaplan Blades**—Kaplan runner blades unlike the Francis can be adjusted to vary passage area between the two blades. This can be done while the turbine is in operation, by means of a servomotor mechanism operating inside the hollow coupling of turbine and generator shaft. Servomotor mechanism consists of a cylinder with a piston working under oil pressure on either side. The piston is connected to the upper end of a regulating rod or blade operating rod, the up and down movement of which turns the blades (refer Fig 7.5, 7.6). Regulating rod passes through the turbine mainshaft which is made hollow for this purpose. Movement of this rod is controlled by a governor (cf. Chapter 8). Motion of the regulating rod is transmitted to the blades through suitable link mechanism enclosed in the runner hub. Fig 7.6 shows details of Kaplan runner blades rotating mechanism. The oil head to supply the pressurised oil to the servomotor through the hollow generator shaft is mounted on the exciter.

The Englesson's Patent, first used in 1922, to house the servomotor in the hub itself instead of in the coupling of turbine and generator shaft,

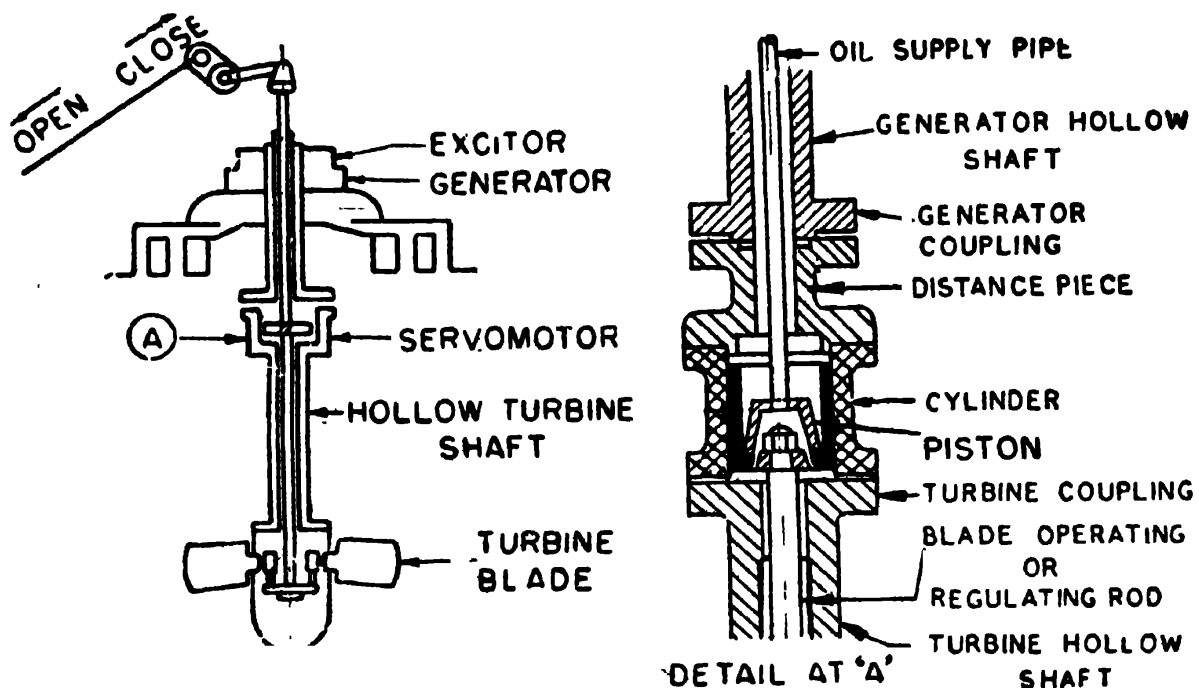
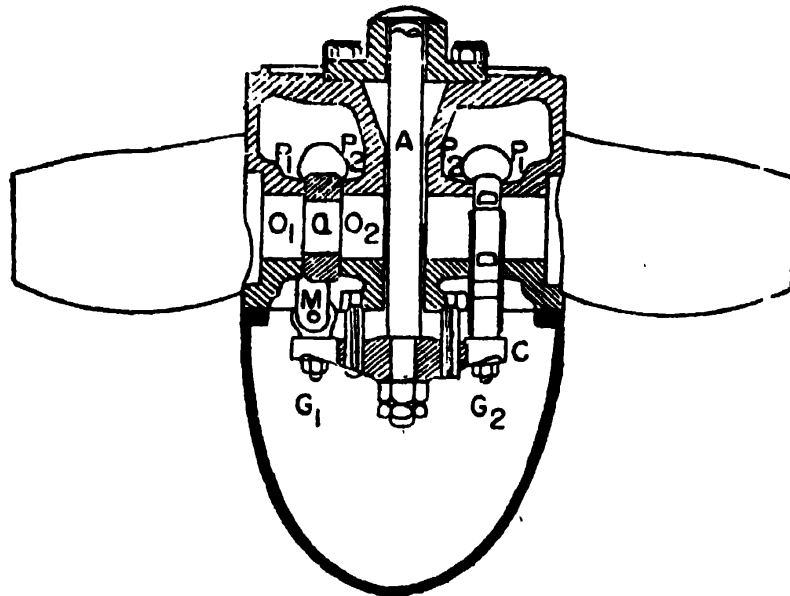
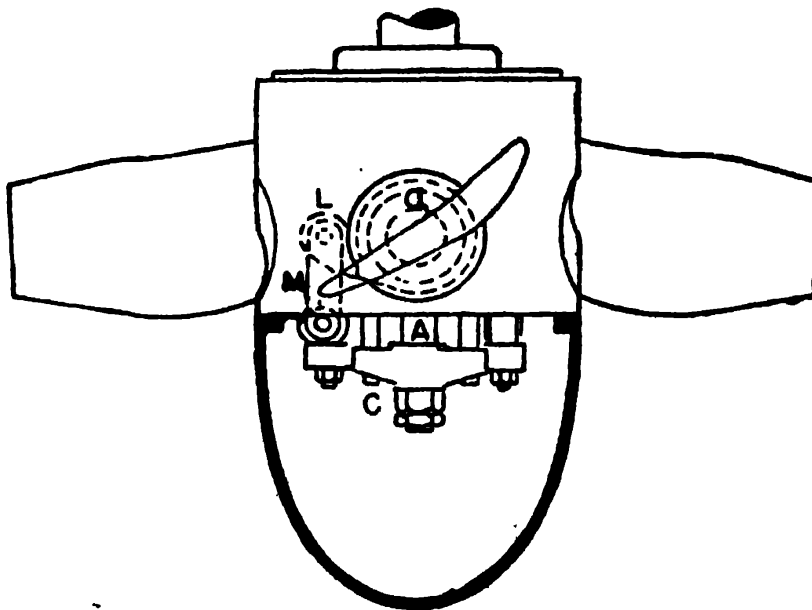


Fig 7.5 Mechanism for Controlling the Blade Movement of Adjustable Blade Runner

has recently been modified by English Electric Co. The servomotor consists of a fixed piston surrounded by a mobile cylinder of small diameter



(a) Front Sectional Elevation



(b) Side View

Fig 7.6 Kaplan Turbine Runner Blades Rotating Mechanism and long stroke. The usual complicated connecting mechanism between the runner blade shafts and their operating levers is eliminated by making each runner blade and operating lever as one solid casting. All operating levers are directly and simultaneously operated by the mobile servomotor cylinder. This arrangement offers easy accessibility to the servomotor without resorting to major dismantling.

**7.4 Guide Vanes for Propeller or Kaplan Turbine**—The number of guide vanes  $Z_0$  can be had from Table 7.4, which are given for various guide wheel diameters  $D_0$  in mm. The value of  $D_0$  can be calculated from the following equation :

$$D_0 = \frac{60 \cdot K_{u_0} \sqrt{2gH}}{\pi \cdot N} \quad (7.1)$$

where  $K_{u_0}$  can be had from the curve  $K_{u_0}$  versus  $N$ , given in Fig 7.11.

**TABLE 7.4**  
**Runner Blades versus Guide Wheel Diameter**

| $Z_0$ |      | 8   | 10  | 12  | 14    | 16    | 18    | 20    | 24     |
|-------|------|-----|-----|-----|-------|-------|-------|-------|--------|
| $D_0$ | From | —   | 300 | 450 | 750   | 1,200 | 1,600 | 2,200 | >4,000 |
| mm    | To   | 300 | 450 | 750 | 1,200 | 1,600 | 2,200 | 4,000 | —      |

**7.5 Outlines of Propeller or Kaplan Runner**—(refer Fig 7.7, 7.8 and Plate 24)—The flow of water at inlet and at outlet is axial. The space between guide-vanes outlet and runner inlet is known as the “Whirl Chamber”. The outer curve of the this chamber is an ellipse (refer Fig 7.8),

Semi-major axis of ellipse,  $a = 0.13 D_1$

Semi-minor axis of ellipse,  $b = 0.16 D_1$  to  $0.2 D_1$

Diameter of boss or hub  $d_1 = 0.35 D_1$  to  $0.6 D_1$

Section of the blades is an aerofoil. Open and closed positions are clearly shown in Section X-X, Fig 7.8

Pitch  $t = \frac{\pi \cdot D_2}{Z_2}$  where  $Z_2 =$  number of runner blades.

Ratio  $l \approx 0.9$  to  $1.05$ .

For velocity triangles at inlet and outlet refer to Art 6.13 and Fig 6.14.

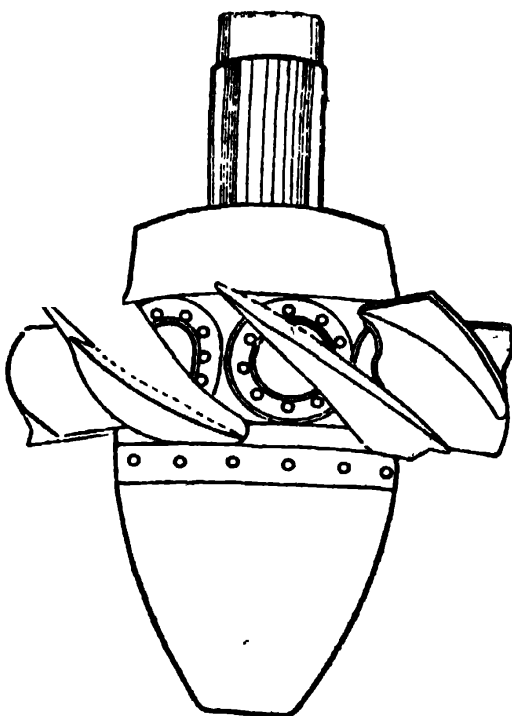


**Fig 7.7 Kaplan Turbine Runner**

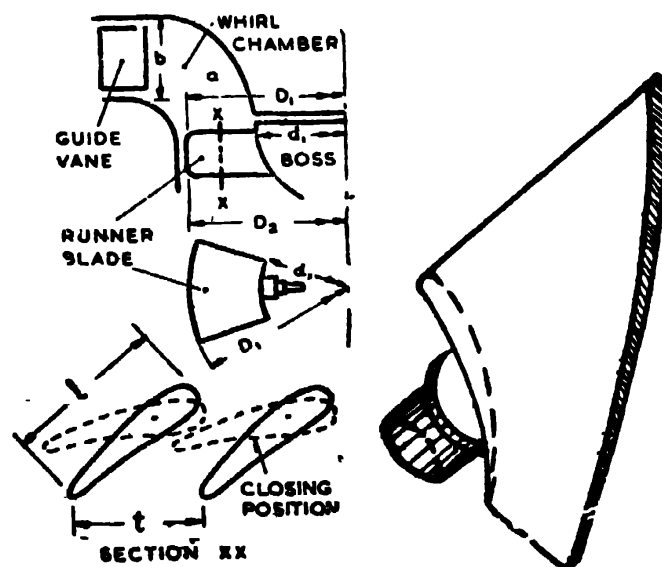


**Fig 7.8 Outlines of Kaplan or Propeller Runner with One Blade**

**7.6 Cavitation in Propeller and Kaplan Turbines**—The phenomenon of cavitation and methods to avoid it have been explained in Chapter 6 on Francis Turbines (refer Art 6.17 and 6.18). This holds good for propeller and Kaplan turbines also. The runner of a propeller or Kaplan turbine is effected by cavitation at the following places (refer Fig 7.9) :

(I) Outer edge of the runner blade, where it is known as *cleavage cavitation* because it takes place at the clearance of the runner blades and the cylindrical opening in which the runner revolves.

(II) Base of runner blade where it is called *throat cavitation* because it acts as a throat for the flow of fluid.

(III) Surface of runner blade and there are two different surfaces :

(a) Half of the blade surface as shown and the cavitation is called as *surface cavitation* ;

and (b) Inlet blade surface where the cavitation is known as *inlet cavitation*.

The cavitation factor or plant sigma  $\sigma$  given for Francis turbine by Eqn 6.9 also holds good for propeller or Kaplan turbine for the installation of the unit. The cavitation factor  $\sigma_{crit}$  depends upon the specific speed of the turbine and its values are given in Table 7.5. These values differ for

TABLE 7.5

**Cavitation Factor  $\sigma_{crit}$  versus Specific Speed  $N_s$  for Kaplan Turbine**

| $N_s$           | 300 to 450  | 450 to 550  | 550 to 650   | 650 to 700 | 700 to 800 |
|-----------------|-------------|-------------|--------------|------------|------------|
| $\sigma_{crit}$ | 0.35 to 0.4 | 0.4 to 0.45 | 0.45 to 0.55 | 0.85       | 1.05       |

various sources and Fig 7.10 shows the curves giving such values of  $\sigma$  versus  $N_s$ . Further following relation\* was suggested to find  $\sigma_{crit}$  for propeller turbine,

$$\sigma_{crit} = 0.3 + 0.0024 \left( \frac{N_s}{100} \right)^{2.73} \quad (7.2)$$

For Kaplan turbine  $\sigma_{crit}$  is 10% higher than for a similar propeller turbine.

\*Refer Proceedings of American Society of Civil Engineers, Dec. 1917.



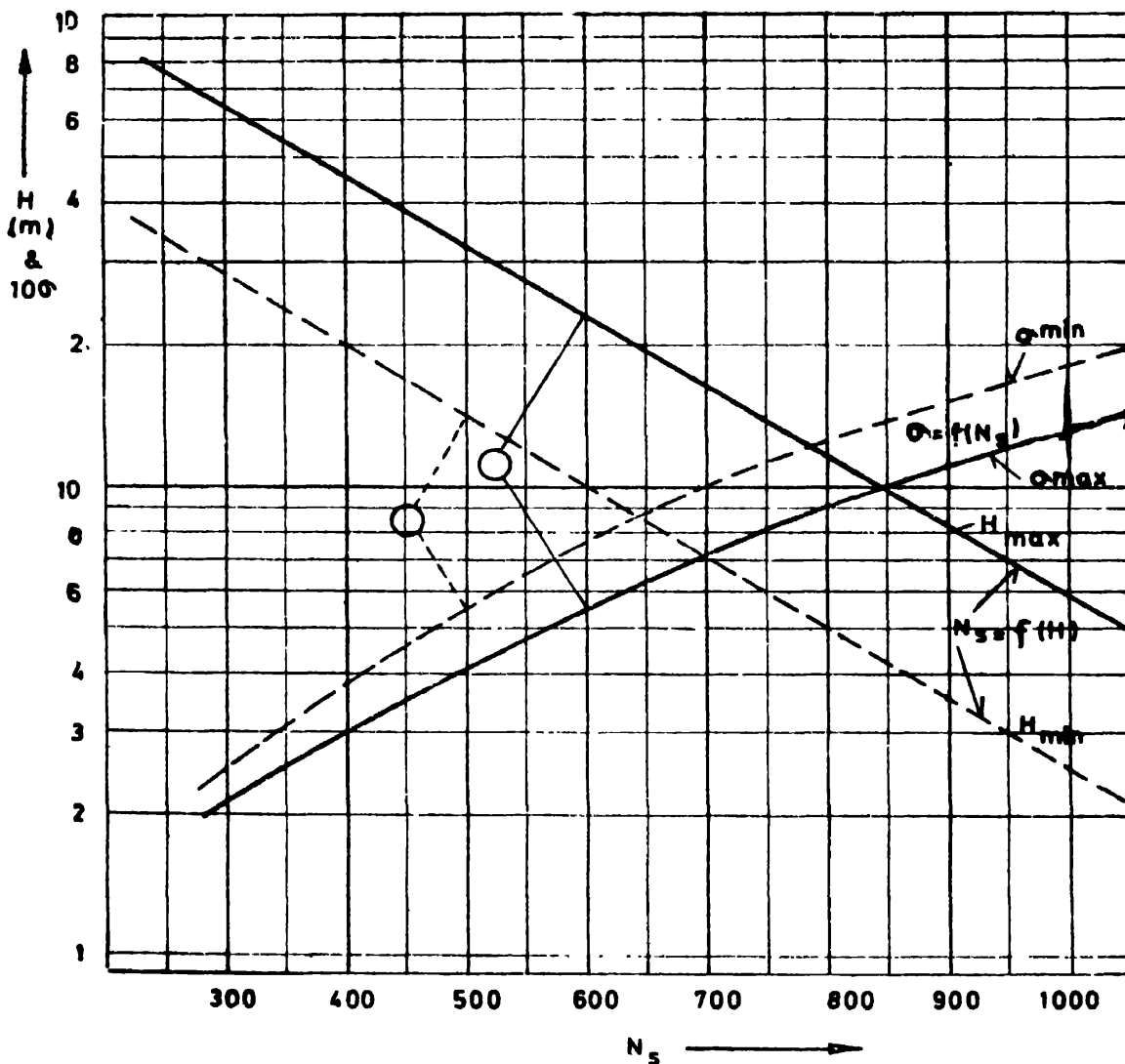


Fig 7.10 Cavitation Factor versus Specific Speed  $N_s$  for Kaplan Turbine

**7.7 Force, Torque, Power and Efficiencies**—For the calculations of force, torque, power and various efficiencies the formulae derived in Art 6.26 to 6.28 for Francis turbine, will hold good for propeller or Kaplan turbine also.

TABLE 7.6

**Practical Values of  $\eta_H$  and  $\eta_Q$  for Kaplan Turbine**

| $N_s$ | $K_s$ | $\eta_H$ | $\eta_Q$ |
|-------|-------|----------|----------|
| 300   | 96    | 0.91     | 0.965    |
| 1,000 | 980   | 0.95     | 0.985    |

**7.8 Rate of Flow Through Propeller or Kaplan Runner**—Let quantity of water flowing per second be  $Q$ , then

$Q = \text{area across flow} \times \text{velocity of flow.}$

Area across flow  $= K \frac{\pi}{4} (D_1^2 - d_1^2)$ , where  $K =$  percentage of net flow area obtained after deducting area occupied by the blades.

$D_1 = D_2 =$  the external diameter of the runner ;

$d_1 =$  the diameter of the runner boss or hub.

Velocity of flow  $v_{m_1} = K_{v_{m_1}} \cdot \sqrt{2gH}$

$$\therefore Q = \text{area} \times \text{velocity} = \frac{\pi}{4} K \cdot K_{v_{m_1}} \cdot \sqrt{2g} (D_1^2 - d_1^2) \sqrt{H} \dots (7.3)$$

$$\text{or } Q = Q_{11} \cdot (D_1^2 - d_1^2) \cdot \sqrt{H}$$

$$\text{where } Q_{11} = \frac{\pi}{4} K \cdot K_{v_{m_1}} \sqrt{2g} \quad (7.4)$$

$Q_{11}$ , is a factor, constant for geometrically similar turbines and is

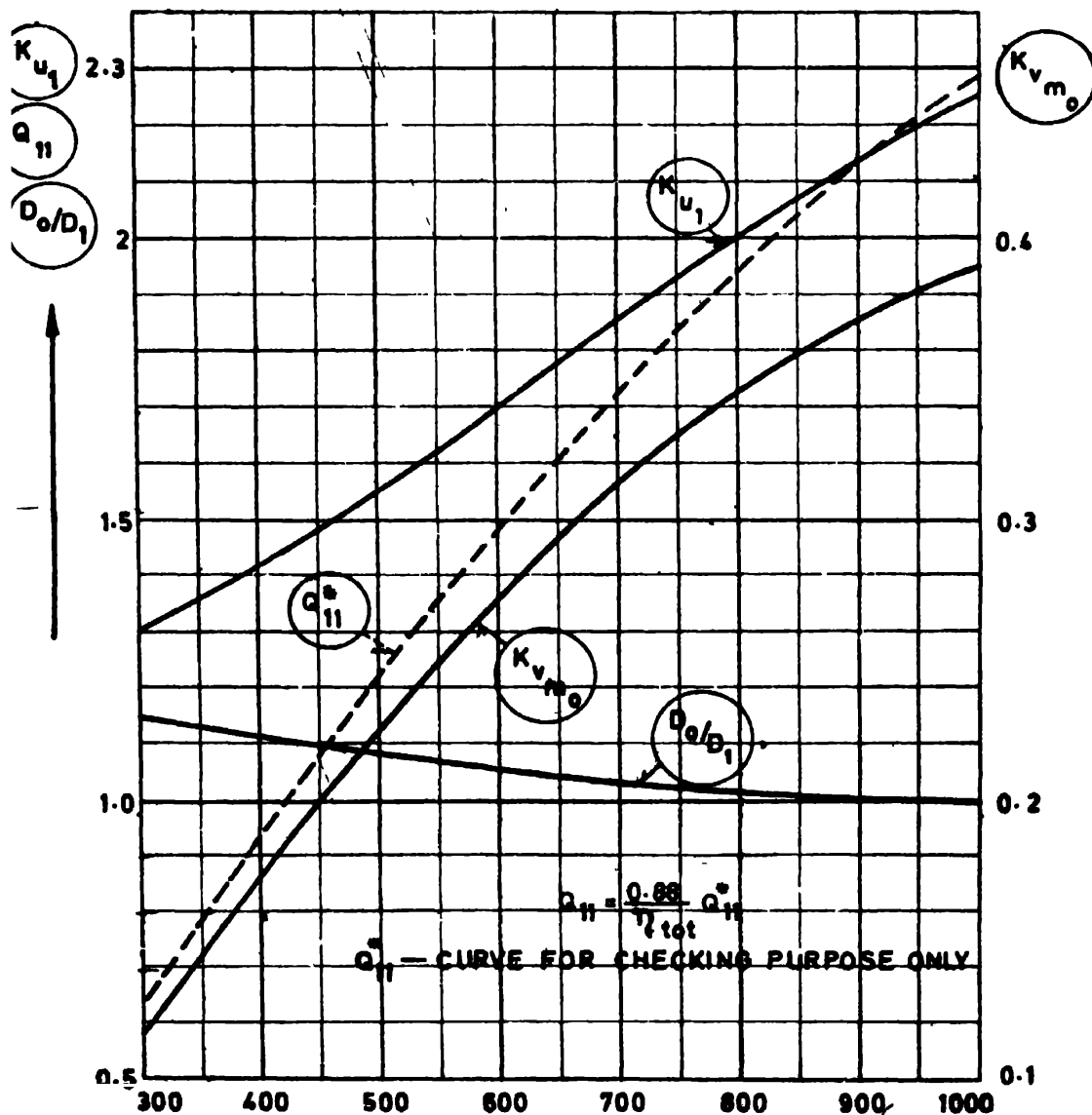


Fig 7.11 Kaplan Turbine Constants *versus* Specific Speed

called *specific flow*. Its value varies from 0.6 to 2.175 depending upon  $N_s$  (refer Fig 7.11).

The ratio of the hub diameter to runner external diameter  $\frac{d_1}{D_1} = v$  depends upon working head and it is determined from the relation,

$$v = 0.38 + \frac{H}{220} \quad \dots(7.5)$$

Specific flow  $Q_{11}$  as well as non-dimensional factors  $K_{u_1}$ ,  $K_{v_{m_0}}$ ,  $\frac{D_n}{D_1}$  etc., can be plotted against specific speed  $N_s$  (refer Fig 7.11).

Here  $D_0$  is the diameter of guide wheel.

**Problem 7.1** A Kaplan turbine develops 8,000 HP under an effective head of 5 m. Its speed ratio is 2 and flow ratio is 0.6 and the diameter of the boss = 0.35 times the external diameter of the runner. Mechanical efficiency of the turbine is 90 percent. Calculate the diameter of the runner, speed of the runner and also its specific speed.

**Solution**

$$\begin{aligned} P_t &= 1,000 \text{ HP}; & H &= 5 \text{ m}; & \eta_t &= 90\%; \\ K_{u_1} &= 2; & K_{v_{m_1}} &= 0.6; & d &= 0.35 D_1 \end{aligned}$$

(Here mechanical efficiency of turbine means the overall turbine efficiency)

$$\eta_t = \frac{\text{Brake Horsepower}}{\text{Available Horsepower}} = \frac{P_t}{P_a}$$

$$\text{or} \quad 0.9 = \frac{8,000}{\frac{\gamma \cdot Q \cdot H}{75}}$$

$$\therefore Q = \frac{8,000}{0.9} \times \frac{75}{1,000 \times 5} = 133.3 \text{ m}^3/\text{sec}$$

$$\text{Flow ratio } K_{v_{m_1}} = 0.6$$

$$\begin{aligned} \frac{v_{m_1}}{\sqrt{2gH}} &= \frac{v_{m_1}}{4.43 \times 2.23} \end{aligned}$$

$$\text{or Velocity of flow, } v_{m_1} = 0.6 \times 4.43 \times 2.23 = 5.92 \text{ m/sec}$$

Discharge = velocity of flow  $\times$  area of flow

$$\begin{aligned} \therefore \text{Area of flow} &= \frac{\text{discharge}}{\text{velocity of flow}} = \frac{Q}{v_{m_1}} = \frac{133.3}{5.92} \\ &= 22.5 \text{ m}^2 \end{aligned}$$

$$\text{But area of flow} = \frac{\pi}{4} D_1^2 - \frac{\pi}{4} d^2$$

$$\therefore 22.5 = \frac{\pi}{4} D_1^2 (1 - 0.35^2) = 0.689 D_1^2$$

$$\begin{aligned} D_1 &= \sqrt{\frac{22.5}{0.689}} = \sqrt{32.7} \\ &= 5.7 \text{ m or } 5,700 \text{ mm} \quad \text{Answer} \end{aligned}$$

$$\text{Speed ratio } K_{u_1} = 2 = \frac{u_1}{\sqrt{2gH}} = \frac{\pi \cdot D_1 \cdot N}{60 \cdot \sqrt{2gH}}$$

$$\therefore N = \frac{K_{u_1} \cdot \sqrt{2gH} \times 60}{\pi \times D_1} = \frac{2 \times 4.43 \times 2.23 \times 60}{\pi \times 5.7} = 66 \text{ rpm} \quad \text{Answer}$$

Best synchronous speed of the turbine would be 75 rpm ; when coupled to a 50 cycles/sec generator having 40 pairs of poles.

Specific speed of turbine

$$N_s = \frac{N \sqrt{P_t}}{H^{\frac{5}{4}}} = \frac{66 \times \sqrt{8,000}}{5^{\frac{5}{4}}} = 792 \text{ Answer}$$

**Problem 7.2** A propeller turbine runner has an outer diameter of 4.5 m and an inner diameter of 2 m. It develops 28,000 HP when working under a head of 20 m. The turbine is directly coupled to an alternator having 22 pairs of poles. The hydraulic efficiency is 94% and overall efficiency is 88%. Find the discharge through the turbine. Find the runner vane angles at inlet and outlet, at the hub and at the edge of the blade.

**Solution**

$$P_t = 28,000 \text{ HP}; \quad H = 20 \text{ m}; \quad \eta_h = 0.94$$

$$\eta_i = 0.88; \quad D_1 = 4.5 \text{ m}; \quad d_1 = 2 \text{ m}$$

Here the outer diameter of the runner corresponds to the edge of the blade. In other words this is the diameter of the runner.

The inner diameter of the runner means the hub diameter.

Assuming  $f = 50$  cycles

$$\text{Speed of turbine } N = \frac{60f}{P} = \frac{60 \times 50}{22} = 136.4 \text{ rpm}$$

Consider the velocity triangles at the outer end of blade (i.e., at radius  $R_1 = 2.25 \text{ m}$ ). Let the conditions at inlet be represented by suffix 1 and those at outlet by suffix 2, then  $D_1 = D_2$  (refer Fig 7.12a).

Generally in a propeller turbine, the discharge is radial i.e.,  $v_{u_2} = 0$

$$\text{Now } u_1 = u_2 = \frac{\pi D_1 N}{60} = \frac{\pi \times 4.5 \times 136.4}{60} = 32 \text{ m/sec}$$

$$P_t = \frac{\gamma Q H}{75} \eta_i \quad \therefore Q = \frac{75 P_t}{\gamma H \cdot \eta_i}$$

$$\text{or } Q = \frac{75 \times 28,000}{1,000 \times 20 \times 0.88} = 119.25 \text{ m}^3/\text{sec} \quad \text{Answer}$$

$$\text{Further } \eta_h = \frac{v_{u_1} u_1}{gH} \quad (\because v_{u_2} = 0, \text{ refer Eqn 1.52})$$

$$\text{or } v_{u_1} = \frac{gH \cdot \eta_h}{u_1} = \frac{9.81 \times 20 \times 0.94}{32} = 5.76 \text{ m/sec}$$

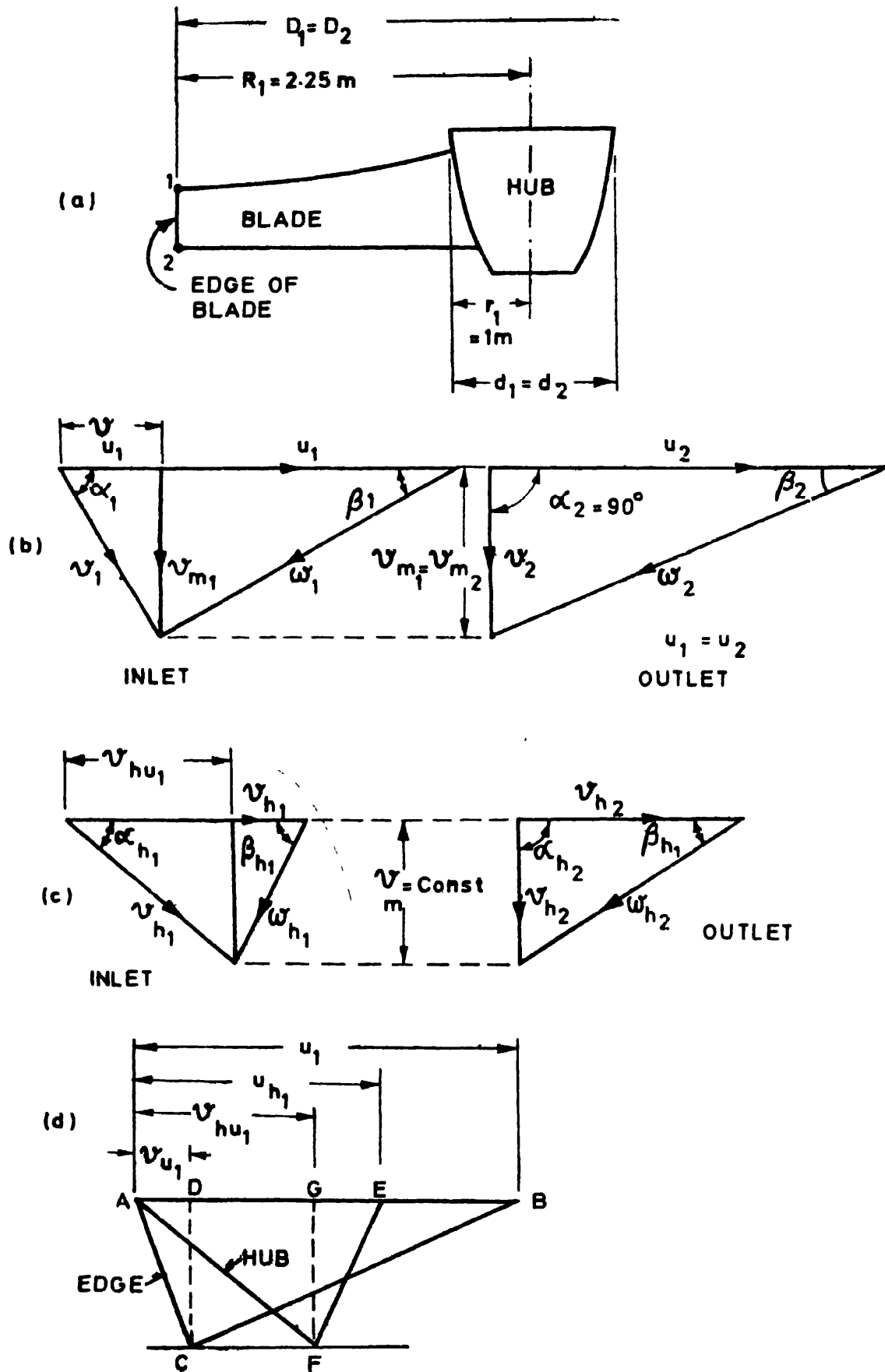


Fig 7.12 Propeller or Kaplan Runner Velocity Triangles at Edge and Hub of Blade

(a) Outlines of Runner (b) Inlet and Outlet Velocity Triangles at Edge (c) Inlet and Outlet Velocity Triangles Hub (d) Inlet Velocity Triangles at both at Edge and Hub.

$$\begin{aligned}\text{velocity of flow } v_{m_1} = v_{m_2} &= \frac{Q}{\text{area of flow}} \\ &= \frac{119.25}{\frac{\pi}{4}(4.5^2 - 2^2)} = 9.35 \text{ m/sec}\end{aligned}$$

Then inlet velocity triangle will become as shown in Fig 7.12b. Drawing the outlet velocity triangle at the edge of the blade, both of the diagrams are drawn at the same radius  $R_1 = R_2 = 2.25 \text{ m}$ .

$$\alpha_1 = \tan^{-1} \frac{v_{m_1}}{v_{u_1}} = \tan^{-1} \frac{9.35}{5.76} = \tan^{-1} 1.625 = 58^\circ - 20'$$

Vane angles at the edge of runner blade—

$$\beta_1 = \tan^{-1} \frac{9.35}{32 - 5.76} = \tan^{-1} \frac{9.35}{26.24} = 19^\circ - 36'$$

$$\beta_2 = \tan^{-1} \frac{9.35}{32} = 16^\circ - 15' \quad \text{Answer}$$

At the hub :  $d_1 = d_2 = 2 \text{ m}$

For free vortex distribution along the inlet edge of the blade,

$$v_{u_1} \cdot R_1 = v_{h_{u_1}} \cdot r_1$$

or velocity of whirl at inlet of hub,

$$v_{h_{u_1}} = \frac{v_{u_1} \cdot R_1}{r_1} = \frac{5.76 \times 2.25}{1} = 13 \text{ m/sec}$$

Blade peripheral velocity of hub

$$v_{h_1} = v_{h_2} = \frac{\pi d_1 N}{60} = \frac{\pi \times 2 \times 136.4}{60} = 14.2 \text{ m/sec}$$

Assume, velocity of flow  $v_m$  as constant i.e., it will be same at the hub and at the edge of runner i.e., 9.35 m/sec

$$\alpha_{h_1} = \tan^{-1} \frac{9.35}{13} = 35^\circ - 20'$$

Vane angles at the hub of the runner

$$\beta_{h_1} = \tan^{-1} \frac{9.35}{1.2} = 83^\circ - 42'$$

$$\beta_{h_2} = \tan^{-1} \frac{9.35}{14.2} = 33^\circ - 24'$$

The inlet and outlet velocity triangles at the hub are shown in Fig 7.12(c). Combining the inlet velocity triangles at the edge of the blade and that at the hub, as drawn in Fig 7.12(d), it will be seen that the whirl component will be decreasing from the hub to the edge of the runner blade i.e.,  $ABC$  is inlet triangle at edge of blade and  $AEF$  is inlet triangle at the hub, and from these two triangles whirl component  $AD < AG$ .

**7.9 Application of Aerofoil Theory to Axial Flow Turbo-machinery : Aerofoil Theory**—Let a solid body, in the form of a flat plate be totally immersed in a fluid stream having a velocity  $w$  relative to

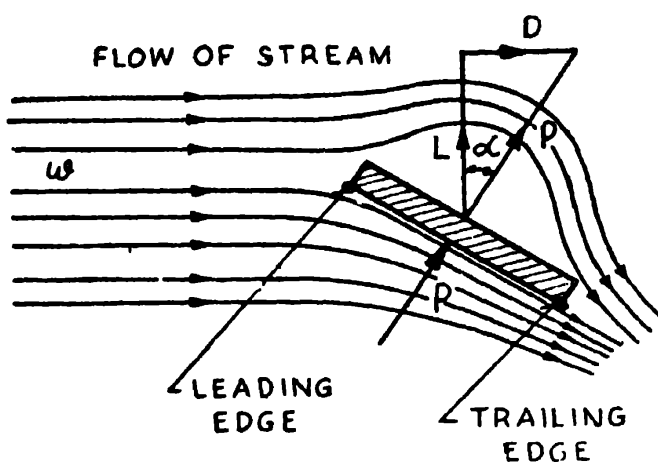


Fig 7.13 Aerofoil Theory

plate motion (refer Fig 7.13). Let the plate be inclined at angle  $\alpha$  to the direction of flow. The plate will then be subjected to dynamic or kinetic pressure resulting from momentum changes induced in the liquid. Let the pressure be represented by  $P$  acting below the plate and normal to its surface as shown in Fig 7.13. Resolve  $P$  into two components vertical and horizontal. The vertical component  $P \cos \alpha$  is called *Lift* and is denoted by  $L$ . The horizontal component  $P \sin \alpha$  is called *Drag* and is denoted by  $D$ .

Lift is induced due to difference of pressure on the two sides of the plate. The stream lines strike the leading edge of the plate. Some are diverted towards the bottom of the plate and the rest are raised above the upper surface and then drop, joining again at the trailing end of the plate. Vortices are produced on the upper surface of the plate which make the pressure negative at that place. Then the resultant pressure  $P$  is the sum of the positive pressure acting below the plate and the negative pressure acting on the upper surface of the plate.

The resultant pressure  $P$  is proportional to the projected area  $A$  of the body normal to the flow and the square of the kinetic energy of stream, thus

$$\begin{aligned}
 P &\propto \frac{\gamma \cdot w^2}{2g} \cdot A \\
 \text{or} \quad P &= C \cdot \frac{\gamma \cdot w^2}{2g} \cdot A \\
 &= C \cdot \rho \cdot \frac{w^2}{2} \cdot A \quad \quad \quad .(7.6)
 \end{aligned}$$

where

$\gamma$  = the specific weight of fluid ;  
 $w$  = the relative velocity of stream ;  
 $A$  = the area of plate ;  
 $C$  = the coefficient of pressure ;  
 $\rho$  = the density of fluid  
 $= \frac{\gamma}{g}$  ;

The lift will thus be equal to :

$$L = C_L \cdot \rho \cdot \frac{w^2}{2} \cdot A \quad \quad \quad .(7.7)$$

where  $C_L$  = the lift coefficient.

and Drag will be written as

$$D = C_D \cdot \rho \cdot \frac{w^2}{2} \cdot A \quad \dots(7.8)$$

where  $C_D$  = the drag coefficient.

The coefficients  $C_L$  and  $C_D$  depend upon the shape of body (or blade), roughness of surface. Reynolds' number, aspect ratio and angle of attack  $\alpha$  (refer Fig 7.14).

Lift is the fluid-force component on the body at right angles to the relative approach velocity. Drag is the fluid-force component parallel to the relative approach velocity exerted on the body by the moving fluid.

**Aerofoil Section**—The plate immersed in fluid, as shown in Fig 7.13 has too much drag. A typical section known as aerofoil section is used in practice which has the least drag. Fig 7.14 shows such a section. The front nose of the section is rounded off smoothly and the tail is made a sharp edge. The top surface is curved smoothly. The bottom surface is not very important.

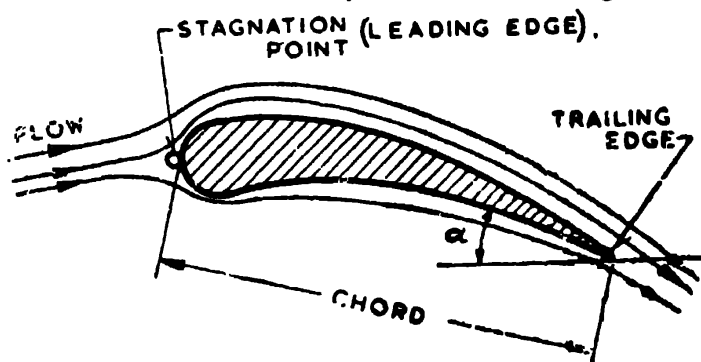


Fig 7.14 Aerofoil Section

Some definitions concerning aerofoil section are given below :

**Stagnation Point** is the leading edge where the stream strikes first and is bifurcated smoothly. The two parts of stream meet again at the trailing edge without formation of eddies.

**Angle of Attack**—If a horizontal line is drawn from the trailing edge, the angle  $\alpha$  (refer Fig 7.14) it makes with the bottom surface of section, is known as angle of attack.

**Aspect Ratio** is the ratio of the lengths of the two sides of the plate and is equal to  $\frac{h}{l}$  (refer Fig 7.15).

**Problem 7.3** An aeroplane is flying at a speed of 560 kmph. The chord area of each of the two wings is 8 sq m. The air resistance of all the parts other than the two wings is 30% of the total wing resistance. The propulsion efficiency of propeller is 80%. The lift and drag coefficients  $C_L$  and  $C_D$  at the angle of incidence of the flights are 0.36 and 0.02 respectively. The atmospheric pressure and temperature at the altitude of the flight are 0.7 kg/cm<sup>2</sup> and 240° K respectively. Determine the lift and the power required to drive the aeroplane. Assume  $pV = WRT$ , where  $R = 29.27$ .

**Solution**

$$w = 560 \text{ kmph} = \frac{560 \times 1,000}{3,600} = 155 \text{ m/sec ;}$$

$$p = 0.7 \times 100^2 \text{ kg/m}^2$$



Total area of wing  $A = 8 \times 2 = 16 \text{ m}^2$

$$C_L = 0.36 \text{ and } C_D = 0.02$$

$$pV = WRT \quad \text{or } p = \frac{W}{V}RT$$

$$\text{or } p = \gamma RT \quad \text{or } p = \rho g R T$$

$$\therefore \rho = \frac{p}{gRT} = \frac{0.7 \times 100^2}{9.81 \times 29.27 \times 240} = 0.1015 \text{ kg sec}^2 \text{ m}^{-4}$$

$$\begin{aligned} \text{Lift } L &= C_L \cdot \rho \cdot \frac{w^2}{2} \cdot A \\ &= 0.36 \times 0.1015 \times \frac{155^2}{2} \times 16 \times \frac{1}{1,000} \text{ tonne} \\ &= 7 \text{ tonne } \text{ Answer} \end{aligned}$$

$$\begin{aligned} \text{Drag } D &= C_D \cdot \rho \cdot \frac{w^2}{2} \cdot A \\ &= 0.02 \times 0.1015 \times \frac{155^2}{2} \times 16 \text{ kg} \\ &= 390 \text{ kg} \end{aligned}$$

$$\therefore \text{ Total drag } D_{tot} = 1.3 \times 390 = 506 \text{ kg}$$

$$\text{Total power absorbed} = \frac{D_{tot} w}{75} = \frac{506 \times 155}{75} \text{ HP}$$

$$\begin{aligned} \text{Power required to drive the plane} &= \frac{506 \times 155}{75 \times 0.8} \\ &= 1,310 \text{ HP } \text{ Answer} \end{aligned}$$

**Application of Aerofoil Theory to Axial Flow Turbo-machinery**—Aerofoil section is stream-lined and causes the least resistance to flow. Hence the guide blades of all reaction water turbines and runner blades of propeller and Kaplan turbines have an aerofoil section. Fluid mechanics draws no distinction between two cases giving rise to the relative motion whether the body (or blade) moves through the stationary fluid or fluid passes by the stationary body. Therefore, it is possible to test the still aeroplane models in the wind tunnel and the towing torpedo model in the still water tank, for the determination of the characteristics of prototype machines. Model experiments of Kaplan turbine bladings are also made in the wind tunnel to find the best and efficient blade section. Spacing between the two blades or the blade pitch should be such that the fluid may pass without interference and that it does the maximum amount of work.

Fig 7.15 shows a section of Kaplan or propeller turbine blade.

Let  $l$  = the length of blade chord ;

$t$  = the circumferential pitch of blade.

The blade length  $b$  is divided into a number of blade elements. The lift and drag of each blade element are respectively given by—

$$\Delta L = C_L \cdot \Delta b \cdot l \cdot \rho \cdot \frac{w^2}{2}$$

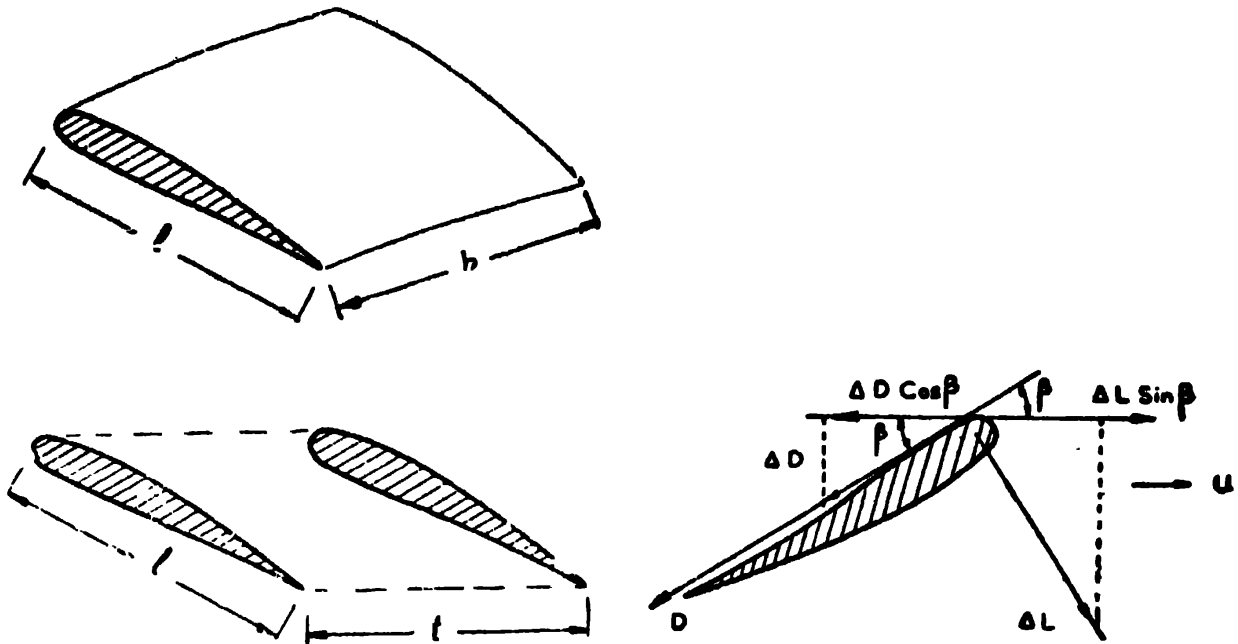


Fig 7.15 Kaplan and Propeller Turbine Blade Sections

$$\text{Drag } \Delta D = C_D \cdot \Delta b \cdot l \cdot \rho \cdot \frac{w^3}{2}$$

Force on blade element (refer Fig 7.15)—

$$\Delta F_u = \Delta L \cdot \sin \beta - \Delta D \cdot \cos \beta$$

$$\begin{aligned} \Delta F_u &= l \cdot \Delta b \cdot \rho \cdot \frac{w^3}{2} (C_L \cdot \sin \beta - C_D \cdot \cos \beta) \\ &= l \cdot \Delta b \cdot \rho \cdot \frac{w \cdot w \cdot \sin \beta}{2} (C_L - C_D \cot \beta) \end{aligned}$$

but  $w \sin \beta = v_m =$  radial velocity or velocity of flow

$$\therefore \Delta F_u = l \cdot \Delta b \cdot \rho \cdot \frac{w \cdot v_m}{2} (C_L - C_D \cot \beta) \quad \dots(1)$$

On the other hand—

$$\Delta F_u = \frac{\gamma \cdot \Delta q}{g} \cdot \Delta v_u$$

where  $\gamma =$  the specific weight of fluid ;

$\Delta v_u =$  the peripheral velocity of blade element

and  $\Delta q = \Delta b \cdot t \cdot v_m$

$$\therefore \Delta F_u = \frac{\gamma \cdot \Delta b \cdot t \cdot v_m}{g} \cdot \Delta v_u \quad \dots(2)$$

Equating (1) and (2)

$$\begin{aligned} \frac{\gamma \cdot \Delta b \cdot t \cdot v_m \cdot \Delta v_u}{g} &= l \cdot \Delta b \cdot \rho \cdot \frac{w \cdot v_m}{2} (C_L - C_D \cot \beta) \\ &= \frac{l \cdot \Delta b \cdot \gamma \cdot w \cdot v_m}{2g} (C_L - C_D \cot \beta) \end{aligned}$$

$$\therefore \Delta v_u = \frac{l}{t} \cdot \frac{w}{2} (C_L - C_D \cot \beta)$$

Denoting  $\frac{l}{t}$  by  $\lambda$

$$\Delta v_u = \lambda \cdot \frac{w}{2} (C_L - C_D \cot \beta) \quad \dots(3)$$

**Practical Data—**

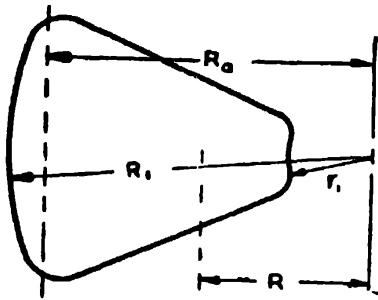


Fig 7.16 Outline of Propeller Blade

$$\lambda_s = \frac{l_s}{t_s} \quad \text{and} \quad \lambda_s = N_s^{\frac{2}{3}} = 80$$

where  $N_s$  = the specific speed ;

$$R_o = 0.95 R_1 \text{ (refer Fig 7.16)}$$

$$\lambda = \lambda_s + v \left( 1 - \frac{R}{R_1} \right)$$

$$\text{where } v = \frac{r_1}{R_1} \quad \text{and } t = \frac{\pi \cdot D}{z_s}$$

Power developed  $\Delta P_H$  by Blade Element—

$$\Delta P_H = \Delta F_u \cdot u$$

$$\begin{aligned} \text{or } \Delta P_H &= l \cdot \Delta b \cdot \rho \cdot \frac{w \cdot v_m}{2} (C_L - C_D \cot \beta) \cdot u \\ &= l \cdot \Delta b \cdot \rho \cdot \gamma \cdot \frac{w \cdot v_m}{2g} (C_L - C_D \cot \beta) \cdot u \end{aligned} \quad \dots(7.9)$$

Head Efficiency  $\eta_H$  of Blade Element—

$$\begin{aligned} \eta_H &= \frac{\Delta P_H}{\Delta P_s} \quad \text{where } \Delta P_s = \gamma \cdot \Delta q \cdot H \\ &= \gamma \cdot \Delta b \cdot t \cdot v_m \cdot H \end{aligned}$$

$$\therefore \eta_H = \frac{l \cdot \Delta b \cdot \frac{\gamma \cdot w \cdot v_m}{2g} (C_L - C_D \cot \beta) \cdot u}{\gamma \cdot \Delta b \cdot t \cdot v_m \cdot H} \quad \dots(7.10)$$

$$\begin{aligned} &= \frac{l}{t} \cdot \frac{w}{\sqrt{2gH}} \cdot \frac{u}{\sqrt{2gH}} (C_L - C_D \cot \beta) \\ &= \lambda \cdot K_w \cdot K_u \cdot (C_L - C_D \cot \beta) \end{aligned} \quad \dots(7.11)$$

**Problem 7.4** A Kaplan turbine is fitted with four aerofoil blades and has a speed of 120 rpm. The mean radius of blade circle is 1.5 m and the blade length in a radial direction is 0.5 m. The chord of the aerofoil blade is inclined at  $25^\circ$  to the direction of motion. The chord length is 2.5 m. The values of  $C_L$  and  $C_D$  for the angle of incidence, used are 0.7 and 0.04 respectively. The turbine is supplied with water under a head of 8 m. Neglecting the area occupied by the blade thickness and assuming a velocity of flow at 5 m/sec, calculate the horsepower developed and the theoretical efficiency of the turbine.

**Solution**

$$z_s = 4$$

$$b = 0.5 \text{ m}$$

$$N = 120 \text{ rpm}$$

$$\beta_1 = 25^\circ$$

$$R_1 = 1.5 \text{ m}$$

$$l = 2.5 \text{ m}$$

$$H = 8 \text{ m} \quad v_{m_1} = 5 \text{ m/sec}$$

From velocity triangle (refer Fig 1.14a)

$$v_{m_1} = w_1 \sin \beta_1 = 5 \text{ m/sec}$$

$$\text{or} \quad w_1 = \frac{5}{\sin 25^\circ} = \frac{5}{0.4226} = 12 \text{ m/sec}$$

$$\begin{aligned} \text{From } F &= \rho \cdot A \frac{w_1^2}{2} (C_L \sin \beta_1 - C_D \cos \beta_1) \\ &= \frac{1,000}{9.81} \times (0.5 \times 2.5) \times \frac{12^2}{2} (0.7 \times 0.4226 - 0.04 \times 0.9063) \\ &= \frac{1,250}{9.81} \times 72 \times 0.25957 = 2,340 \text{ kg} \end{aligned}$$

Power developed by the turbine

$$\begin{aligned} P_H &= \frac{F \cdot u \cdot z_1}{75} = \frac{2,340}{75} \times \left( 2\pi \times 1.5 \times \frac{120}{60} \right) \times 4 \\ &= 2,350 \text{ HP} \quad \text{Answer} \end{aligned}$$

Efficiency of the turbine

$$\eta_H = \frac{F \cdot u \cdot z_1}{w \cdot Q \cdot H}$$

$$\text{where } Q = (k \cdot 2\pi \cdot R_1 \cdot b) v_{m_1} \quad \text{and } k = 1$$

$$= (1 \times 2\pi \times 1.5 \times 0.5) \times 5 \quad \text{m}^3/\text{sec}$$

$$\begin{aligned} \therefore \eta_H &= \frac{2,350 \times 75}{1,000 \times (2\pi \times 1.5 \times 0.5 \times 5) \times 8} \\ &= 93.5\% \quad \text{Answer} \end{aligned}$$

### (B) TUBULAR (OR BULB) TURBINES

**7.10 Tubular or Bulb Turbine**—Kaplan turbine when employed for very low head has to be installed below the tail race level, thus requiring a deep excavation. Further for the Kaplan turbine installation there are a number of bends at inlet casing and the draft tube of elbow type through which the water flows describing 'Z' path giving rise to continuous losses at the bends. Whenever the turbine is repaired or dismantled, the generator has to be removed first.

The cost of turbine and that of civil engineering works using conventional Kaplan turbine with deep excavation is very high. The efficiency of such plants working under low head is less due to excessive losses at the bends. Therefore, efforts have been made to reduce the overall cost and improve the efficiency of the power plant keeping these two things in view.

Arno Fischer in 1937 developed in Germany a modified axial flow turbine which was known as, *tubular turbine*. The turbo-generator set using tubular turbine has an outer casing having the shape of a bulb. Such a set is now termed as *bulb set* and the turbine used for the set is called a *bulb turbine*. The bulb unit is a water tight assembly of turbine and

generator with horizontal axis, submerged in a stream of water. Such a revolutionary concept has led to complete modification of the usual arrangement of the various elements that constitute a low head unit. The axis of rotation coincides with the axis of the passage of water, which is generally straight.

The economical harnessing of fairly low heads on major rivers is now possible with high-output bulb turbines. The idea of such axial-flow, rectilinear draft tube machines is quite old, but its application could not be developed until the electrical engineer succeeded in producing a highly compact and sealed alternator and hydraulics expert took full advantage of bulb turbine forms to increase output and specific speed.

### 7.11 Comparison of Water Power Station Using Bulb Turbines with those Employing Conventional Kaplan Turbines—A

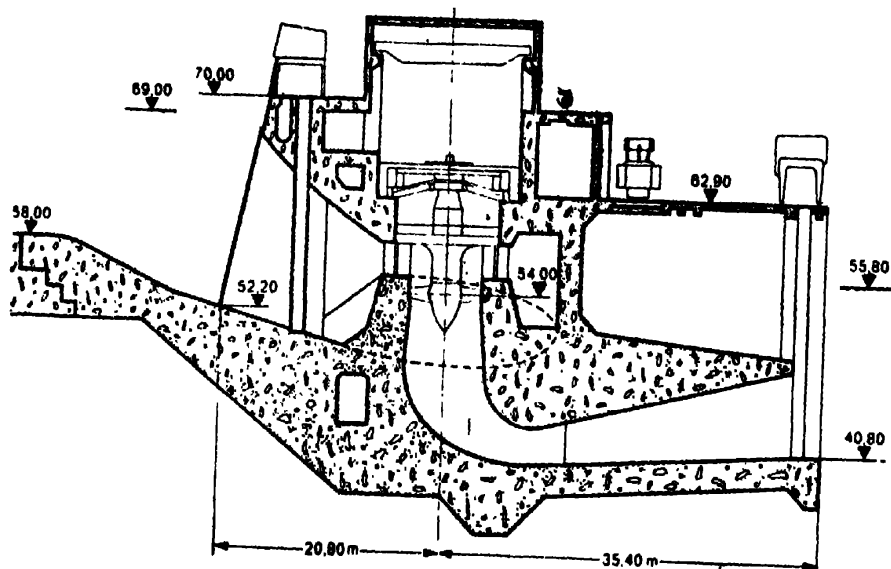


Fig 7.17 87,300 KW Power station under a head of 10 m, provided with Three 29,100 KW conventional Kaplan sets runner at 75 rpm

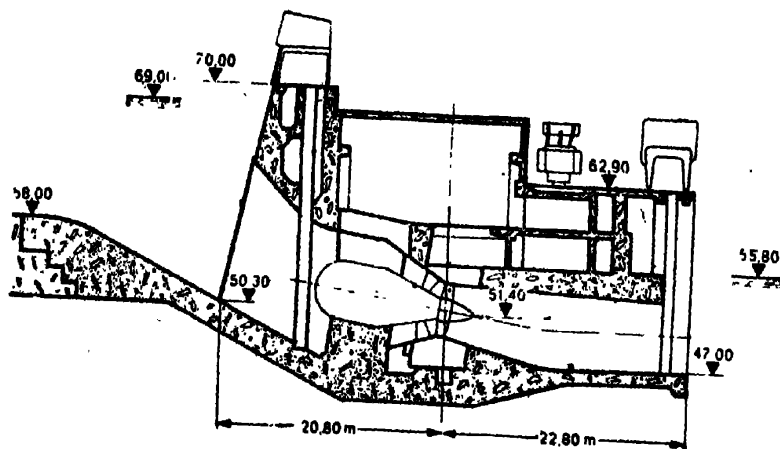


Fig 7.18 87,300 KW Power Station under a head of 10 m, provided with six 14,550 KW Bulb sets running at 125 rpm

recent study illustrates the progress which has been made by using bulb sets in comparison to the conventional Kaplan turbines. Fig 7.17 and 7.18 are drawn to the same scale to give an idea of the difference in the overall dimensions of the plants proposed the saving offered by the bulb sets in the infra structure of the station. In this particular case the total width of the station was the *same* for both solutions.

The reduction in the cost of a bulb type station in comparison with a conventional Kaplan turbine plant each developing the same power, varies according to the site conditions and may be as much as 30%. In this case the overall saving was 15%. Moreover the straight line flow appreciably increases hydraulic efficiency. On intake side the spiral casing is replaced by straight convergent intake duct. This is favourable for the runner operation. On down-stream side the straight line draft tube replaces the elbow type draft tube and with this arrangement it is possible to recover much more of the kinetic energy of the water at the discharge side of the turbine runner than it can be done by conventional method, thus improving the turbine efficiency.

### **7.12 Advantages and Disadvantages of Bulb Sets—**

#### **(a) Advantages of Bulb sets over the use of Kaplan Turbines—**

- (1) The bulb sets can be employed at very low head sites.
- (2) The width of plant is less because of the absence of spiral casing.
- (3) Saving in excavation and civil engineering works owing to the relatively smaller dimensions of the power house; reduction of cost upto 30% as compared to conventional Kaplan installation is possible.
- (4) A higher speed of rotation can be secured through suitable gearing and thus the size of the generator unit and consequently that of the bulb can be reduced.
- (5) Maximum turbine efficiency is increased by about 3% due to almost straight flow and straight draft tube.
- (6) Under equivalent conditions of head, runner diameter and efficiency the bulb units are capable of passing higher discharge than the conventional Kaplan turbine.
- (7) In view of (4), (5) and (6) stated above, the specific speed of the bulb turbine is higher than the Kaplan turbine.
- (8) There is reduced loss of efficiency at part loads.
- (9) The unit is better suited for operation on widely varying heads.
- (10) Dewatering of draft tube is easy. This facilitates the maintenance and the most important is that bulb turbines have been found relatively insensitive from the effect of cavitation factor  $\sigma$  (refer Eqn 6.9) when working under higher heads.

#### **(b) Disadvantages—**

- (1) Since the bulb set is completely submerged under pressure, water leakage into the generator chamber and condensation are source

of trouble. Although improvements in water-proofing and sealing techniques have reduced this problem considerably, this can still become maintenance problem due to the high humidity inside the chamber resulting in gradual deterioration of electrical insulation.

- (2) The erection techniques also involve considerable amount of double handling of equipment and it is doubtful whether any saving in the erection time can be effected as compared to the conventional Kaplan type.

**7.13 Field of Application of Bulb Sets**—Fig 7.19 shows the field of application of bulb sets. On the high head side it is limited to about 15 m. Beyond this head, the bulb turbines have, little advantages over Kaplan type.

On the low head side, where the bulb sets can offer the greatest benefits, the limit is economic in character. The large unit discharges at low head resulting in very low rotational speed and excessive plant cost. The higher rotational speeds can be obtained by sub-dividing the discharge in number of bulb sets. Fig 7.19 shows the desirability of bulb sets in such cases.

The profitability limit for bulb sets is in the region of few hundred *HP* for 2 m head and 10,000 to 30,000 *HP* for 5 to 7 m head.

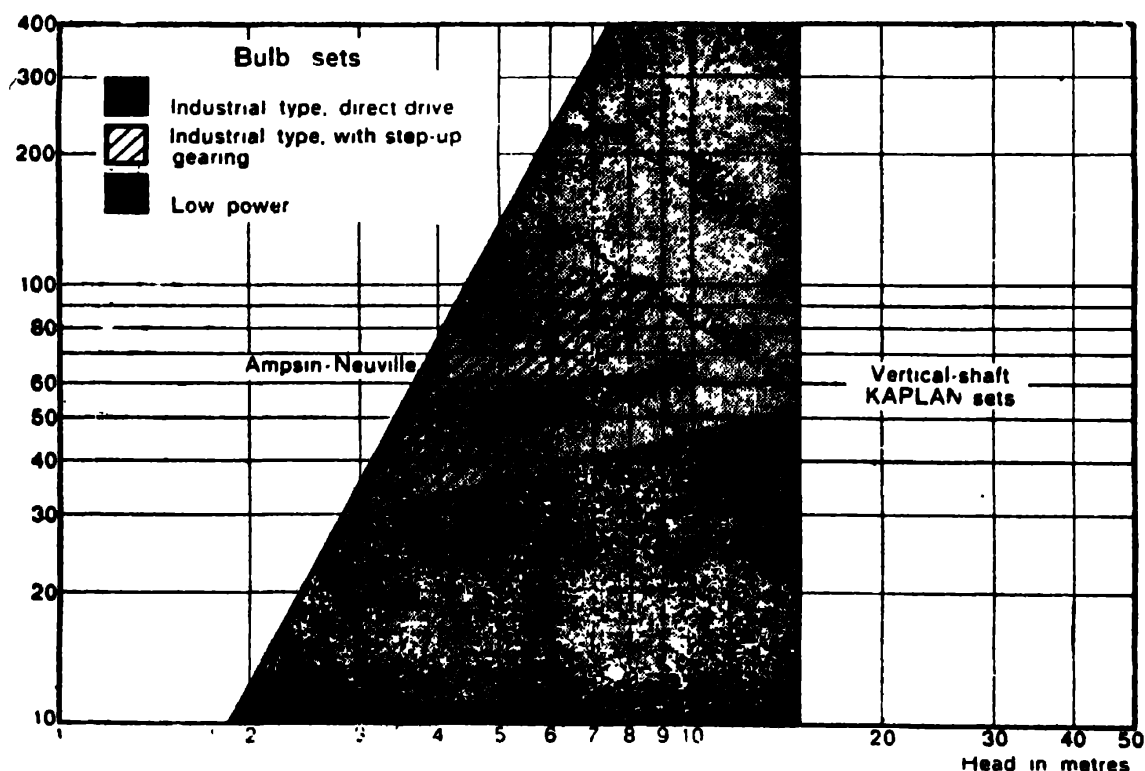


Fig 7.19 Field of Application of Bulb Sets

**7.14 Constructional Details of Bulb Sets**—The bulb sets are axial flow units i.e., the type in which the flow in the turbine is parallel to the centreline of the machine, this centreline being horizontal or sloping slightly down-stream. The water thus passes through the set in a straight

line from the intake of the water chamber to the draft tube outlet (refer Fig 7.20 and Plate 25).

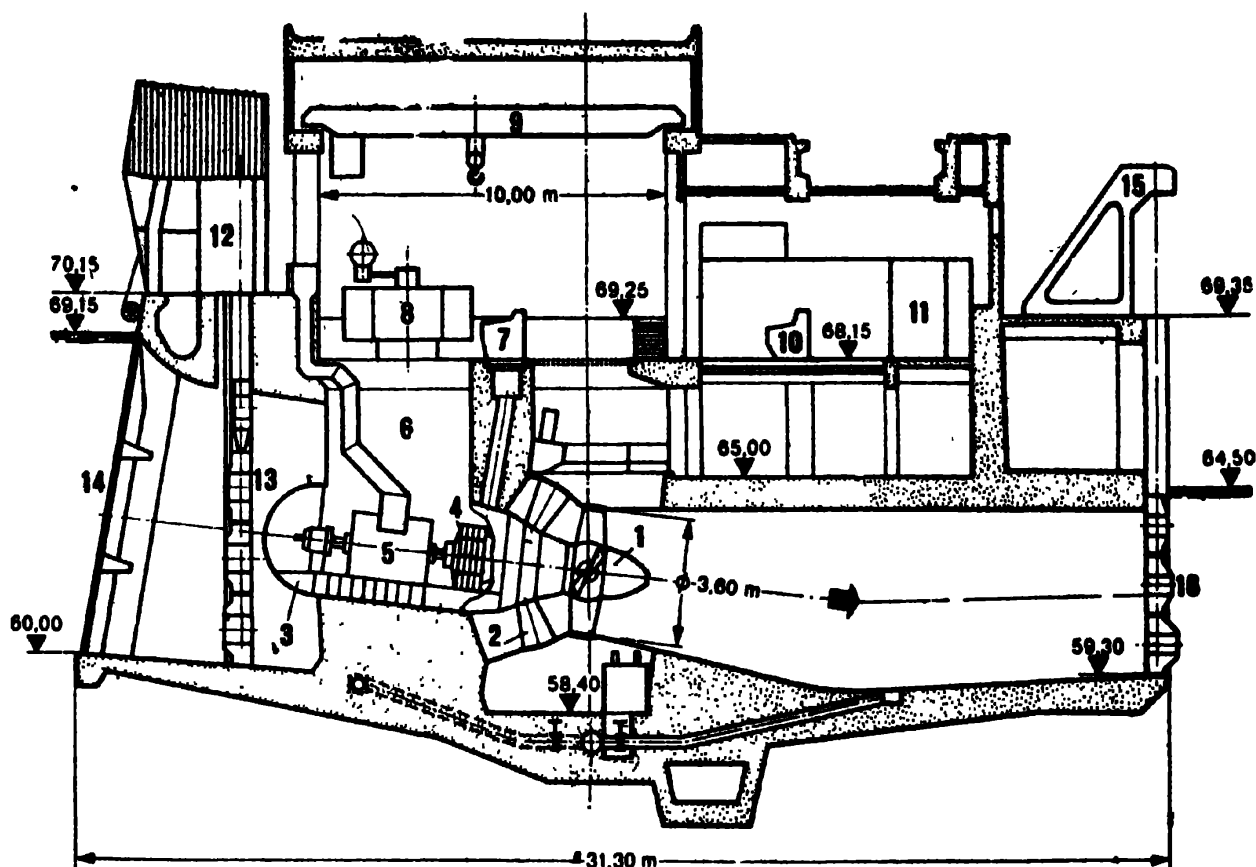


Fig 7.20 Cross-section of Bulb Sets Power Station at Ampsin Neuville (Belgium)

1. Turbine Runner 2. Adjustable Distributor 3. Bulb 4. Step-up Gears 5. Alternator 6. Access Pit 7. Turbine Regulator 8. Transformer 9. Gantry Crane 10. Control Desk 11. Switchboards 12. Rack Cleaning Machine 13. Upstream Stop Logs 14. Downstream Stop Log Operating Gantry Crane 15. Downstream Stop Logs.

A bulb set consists essentially of a nearly horizontal shaft Kaplan turbine, with fixed or movable distributor (guide wheel), driving an alternator mounted in a water-tight bulb. This bulb is situated within the stream of water and the guide vanes of the distributor serve as a support for the bulb assembly.

Two distinct constructional methods are used, each of which has its own proper field. These are :

(1) The bulb set in which the alternator is driven through gears increases speed because turbine speed is less as it is working under low head.

(2) The bulb set in which the alternator is directly coupled to the turbine.

Direct coupling is not possible for speeds below 100 rpm since the alternator size i.e., the bulb size becomes large for lower speeds and with such large bulbs the efficiency of bulb sets is much reduced. It is possible that in the near future same efficiency would be maintained even with large bulb size, then the direct coupling might be applicable at low speeds also.

The gears used to increase the speeds are generally of epicyclic type. With the generator (alternator) running at high speed its size is reduced and thus the cost is also reduced. For large units, direct drive is the only



possibility, since the dimensions of single stage gear system are limited by the size of existing gear cutting machines. The maximum power which can be handled by a single stage multiplying gear system is at present above 10,000 *HP* at 100 rpm which corresponds to bulb set taking a discharge of 130 m<sup>3</sup> and 7 m head. A two-stage gear train would transfer much larger powers but decrease in efficiency is a hinderance in its use.

When turbine runner diameter is small and as a result the bulb is also small with respect to the power developed, which is the case with bulb turbines operating on high heads, the compressed air has to be employed for cooling the alternator. Monobloc bulb sets are explained in Art 7.17.

**7.15 Types of Bulb Sets**—There are generally two types of bulb sets—

- (1) Large bulb sets and (2) small bulb sets.

Large bulb sets are mainly used for developing river power and tidal power whereas the small bulb sets are employed for small rivers and irrigation channels with outputs ranging from 100 to 1,000 *KW*.

On account of the great number of *possible* installations and the relatively restricted demand for small bulb sets the manufacturers had to think to standardise such units for the following two reasons :

- (a) to reduce the cost of design and tooling for each new type ordered,
- (b) to reduce the manufacturing costs.

**7.16 Microstations**—In order to harness numerous small water falls, the standardised bulb sets are used to meet the precise requirements of each place in utilization, construction and operation in order to be economically justified. Such power stations are known as *microstations* using sets with unit output ranging from 100 to 1,000 *KW* working under heads between 1.5 to 10 metres.

To reduce the cost of microstation than that of the cost of conventional installation the following considerations are kept in view—

- (1) The civil engineering work needs to be kept to a minimum and designed to fit in with already existing structures *e.g.*, irrigation channels, locks, small dams etc.
- (2) The machines need to be manufactured in a small range of sizes of simplified design, allowing the use of unified tools and aimed at reducing the cost of manufacture.
- (3) These installations must be automatically controlled to reduce attending personnel.
- (4) The equipment must be simple and robust, with easy accessibility to essential parts for maintenance.
- (5) The units must be light and adequately sub assembled for ease of handling and transport and to keep down erection and dismantling costs.

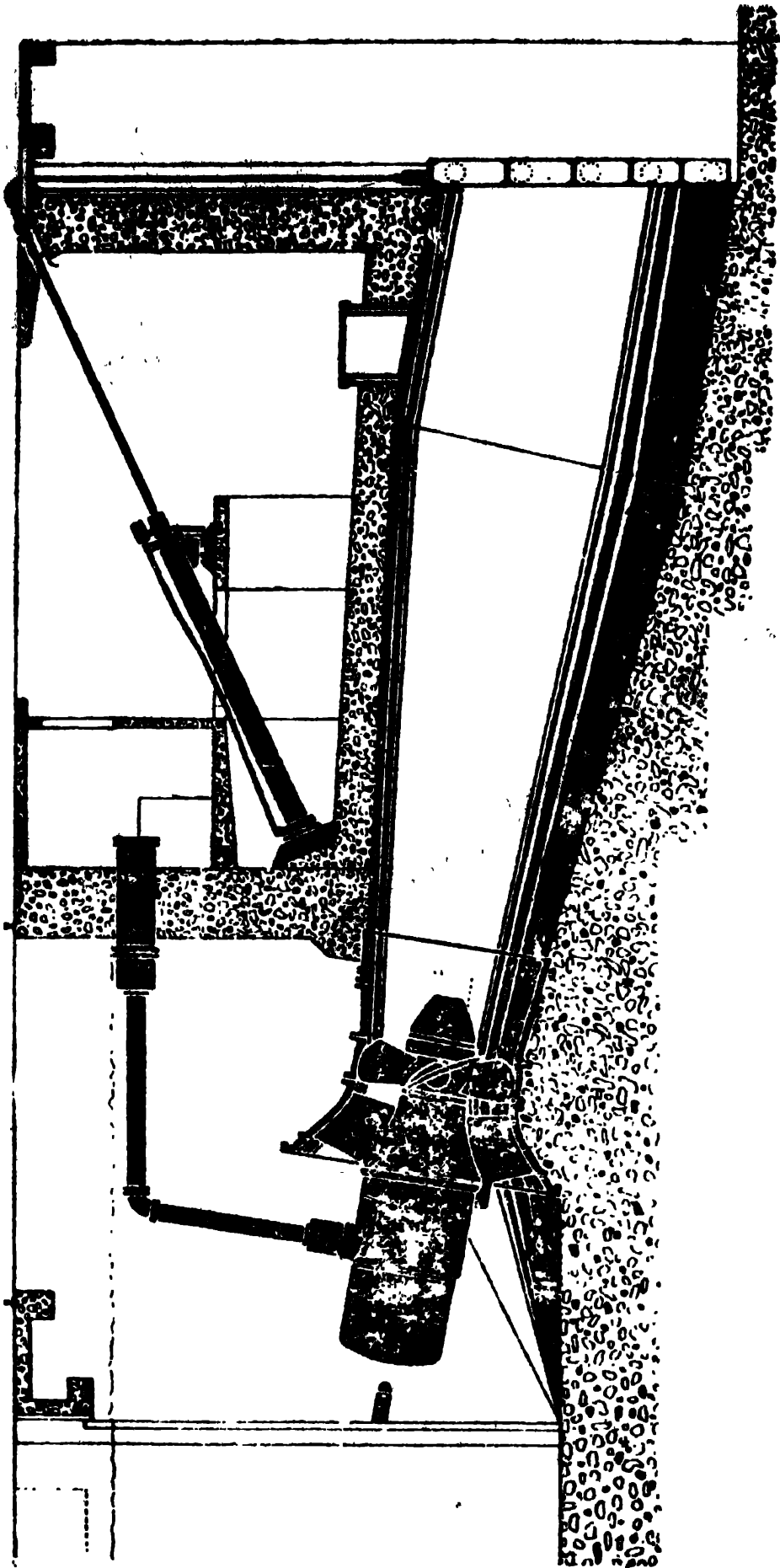


Fig 7.2. Mini-station U<sub>s</sub>      b Set-I      Type

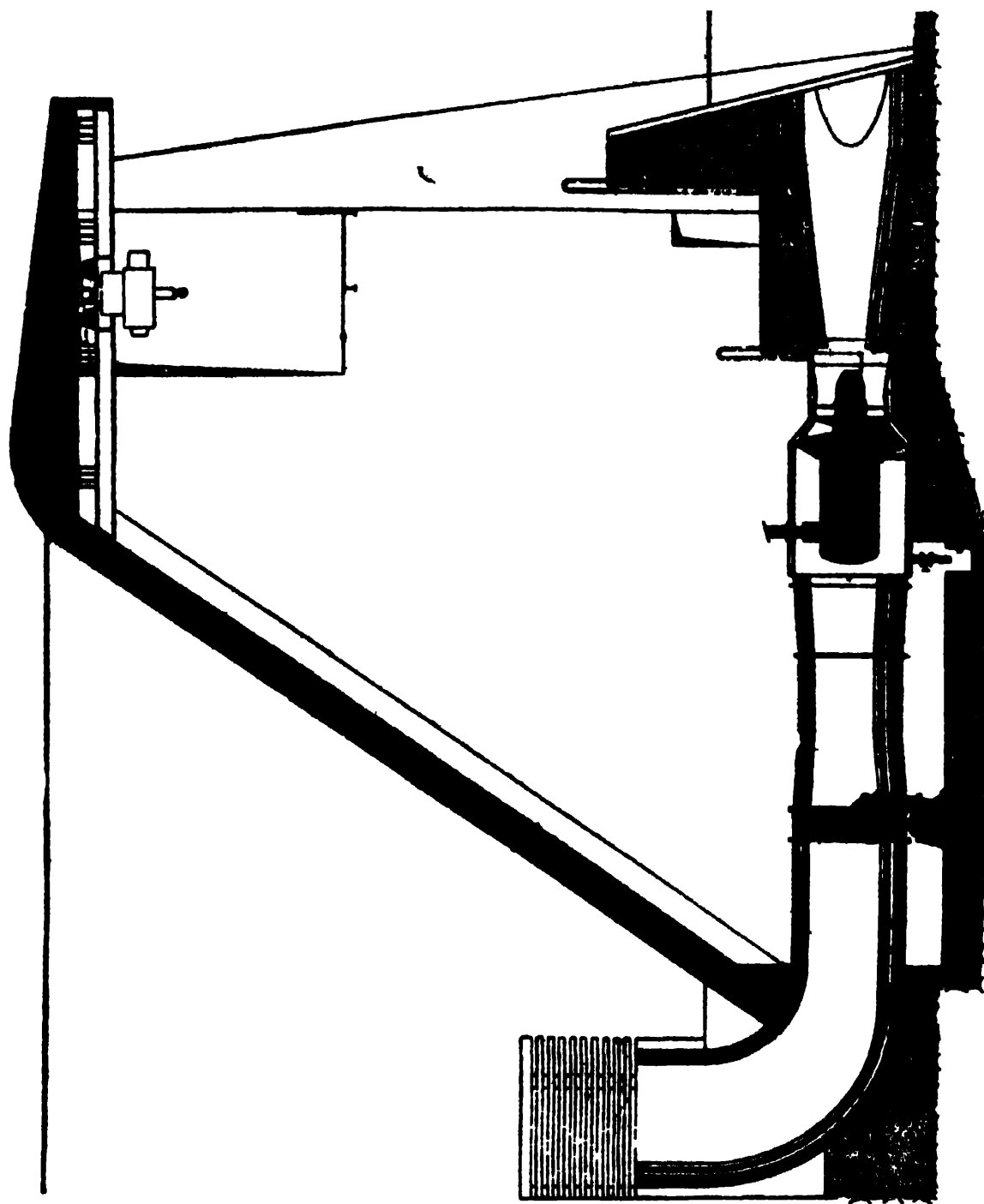


**Fig 7.22 Layout of Mini-station Using Bulb set—Duct Type**

**7.17 Monobloc Bulb Sets**—Research to meet all the five requirements mentioned in the last article, led to an integral assembly known as *monobloc bulb set*. It consists of—

(a) A propeller turbine with either fixed or variable pitch blades. In the latter case the blades are either adjustable when at rest, or else oil operated when frequent changes in flow have to be catered for.

(b) A fixed distributor (guide wheel), the guide vanes of which serve as a support for the bulb assembly.

(c) A generator housed inside a water-tight stream-lined bulb-shaped shell located upstream of the turbine. The bulb diameter is a little smaller than that of the turbine runner, to allow satisfactory axial flow of water round the generator.

For other constructional details, reference may be made to Art 7.14.

**7.18 Medium Head Mini-stations**—There are two types of installations—

(a) Forebay type, and (b) Duct type.

In the 'Forebay' arrangement the unit is totally submerged in the inlet chamber and the water passes directly through the distributor (guide wheel) to the runner (refer Fig 7.21). In 'Duct' arrangement, the unit is again totally submerged, but inside a duct from the upstream reservoir (refer Fig 7.22). In both these arrangements, a closing device upstream and down-stream is required to isolate the unit. The cut off gate may be at upstream or downstream, whilst the other closure device needs only stop logs set in place when at rest.

For the smaller waterfalls, (refer Fig 7.23) a very simple installation is the siphon arrangement, in which the monobloc unit is setup near the elbow of the siphon and is above the water level when shutdown, civil engineering work is reduced to a minimum, a simple reinforced concrete

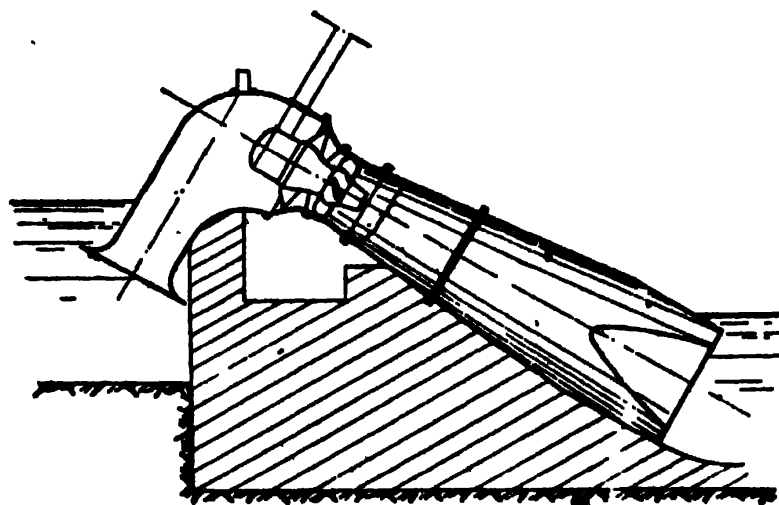


**Fig 7.23** Layout of Mini-Station Using Bulb set-Siphon Arrangement—on Mayenne River (France)  $P_t=89$  HP ;  $H=2.25$ m ;  $Q=4,500$  lit/sec ;  $N=224$  rpm

wall can form the dam and support the unit. If necessary, some additional excavation allows embedding the draft tube.

**Automatic operation of plants**—These sets are automatically operated by an “on-off” device which consists of two electrodes arranged upstream, one above the other a few centimetres apart, at water level that can be adjusted. When the water fall reaches the upper electrode, a relay closes the solenoid valve located at the top of the siphon and starts the priming fan motor. Under the effect of the pressure fall, water rises in the siphon, then begins to flow down-stream, thus starting the turbine. The speed of the unit increases with the water flow, and when it is near the synchronism a special device operates to connect on to the network. The priming fan is stopped and the siphon continues on its own. As long as the water-flow of the unit is less than the river flow, the unit works and feeds the network. If the water-flow of the river is smaller than that of the unit the upstream level will drop and, when the lower electrode is out of the water, a relay opens the solenoid valve and main circuit breaker. The unit is disconnected and at the same time the siphon being no more primed, the unit has to stop. The cycle starts again as soon as the upstream level again reaches the upper electrode.

For heads higher than 5 m the siphon arrangement becomes less suited. Because of their ‘on-off’ operation, such mini-stations can only supply the network with “make-up” power. The mini-stations are connected to the general network and their power is very low as compared to that of the network; the speed of the turbine is then fixed by the network frequency.

**7.19 Tidal Power Turbine**—Another field opened up by bulb turbine is that of tidal power. To be efficient, tidal power requires units which can be operated in either direction as turbine and also in either direction as pumps. The bulb unit lends itself very well to such a difficult type of operation and taking advantage of the new possibilities of these machines, the tidal power plants are being contemplated.

The use of bulb turbines offers the saving in the equipment of low head developments and great flexibility of operation and hence are highly suitable for tidal power station, whether of the single basin, linked multiple-basin or combined multiple-basin types (refer Chapter 10).

Fig 7.24 shows the tidal power turbine of Bulb type designed for the Rance estuary project, having a runner diameter of 5.35 m and producing 10,000 KW at 5.75 m head.

**7.20 Some Bulb Turbines Sets Installations**—Table 7.7 gives some bulb turbine installations throughout the world. Nos. 5 and 6 installations, located in India, belong to Bihar State Electricity Board,

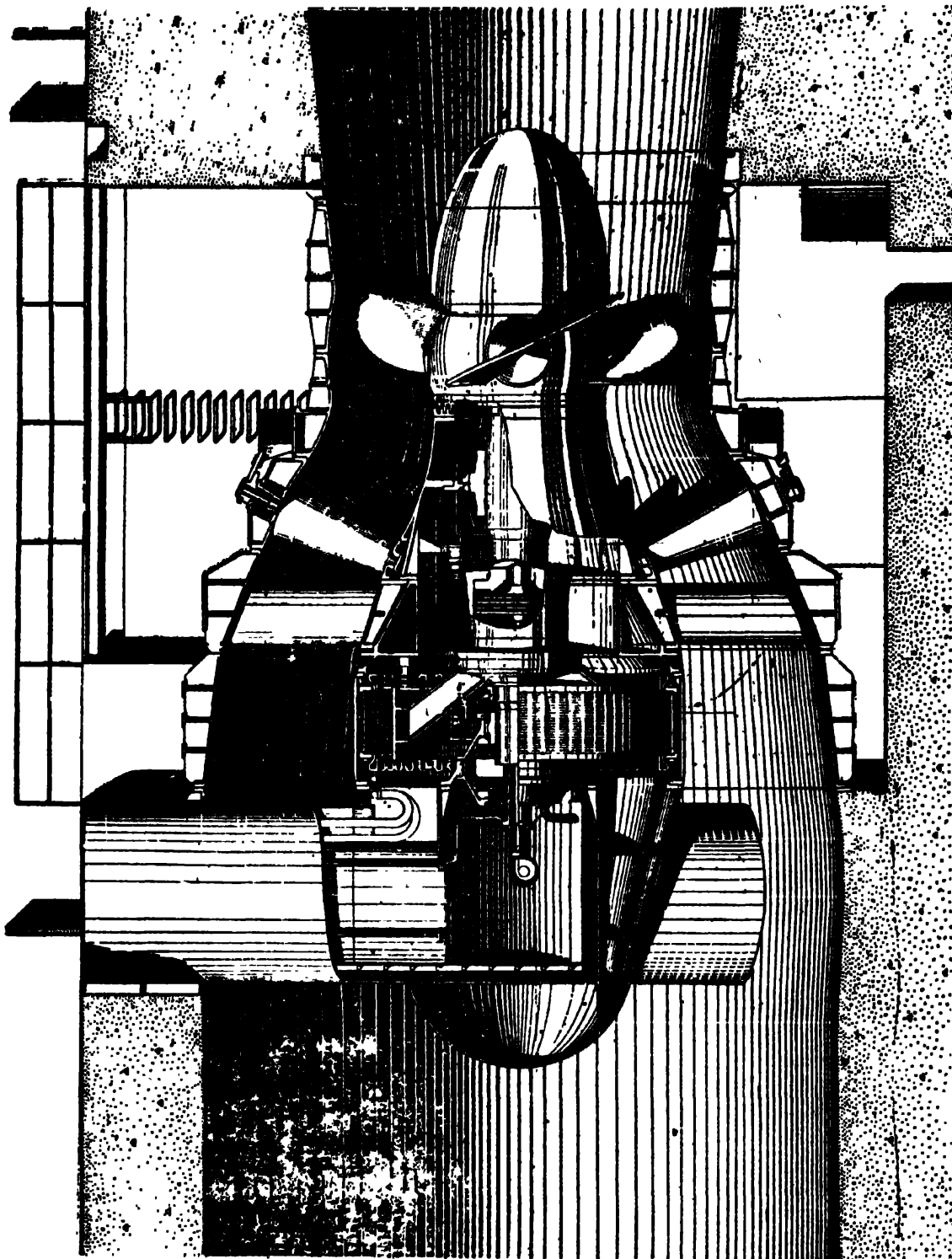


Fig 7.24 Tidal Power Turbine of Bulb Type at La Rance (France),  $P_1 = 13,600$  HP (10,000 KW),  $H = 5.75$  m,  $N = 93.75$  rpm,  $D_1 = 5.35$  m

TABLE 7.7

| St. No. | Power Station                                            | Type of Plant | No. of units | KW per unit | Head m | Turbine speed rpm | Setup Output gear speed rpm | Other Details                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |
|---------|----------------------------------------------------------|---------------|--------------|-------------|--------|-------------------|-----------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1       | Kislogubskaya USSR                                       | Tidal         | 1            | 400         | 1.28   | 72                | Epicyclic 600               | $Q = 37 \text{ m}^3/\text{sec}$ , $D_1 = 33 \text{ m}$ , Bulb dia 27 m, working as two-way turbine and two-way pump both at head from 1 to 2 m.<br>$D_1 = 1.25 \text{ m}$ , $Q = 7.93$ , $Z_2 = 4$<br>$D_1 = 1.65 \text{ m}$ , Bulb dia 1.1 m<br>$D_1 = 3.1 \text{ m}$ , Bulb dia 3.4 m.<br>$Q = 55 \text{ m}^3/\text{sec}$ work turbine both directions under 3 to 18.75 m, Pump both directions under 1 to 3 m head.<br>$D_1 = 4.1 \text{ m}$ , $Q = 104.5 \text{ m}^3/\text{sec}$ |
| 2       | Awe Barrage Scotland                                     | Tubular       | 2            | 443         | 6.85   | 386               | Directly coupled            | $D_1 = 4.5 \text{ m}$ , Bulb dia = 5.4 m,<br>$Q = 103.84 \text{ m}^3/\text{sec}$                                                                                                                                                                                                                                                                                                                                                                                                     |
| 3       | Castet. French National Railways                         | —do—          | 2            | 810         | 7      | 250               | —do—                        | Pump-Turbine                                                                                                                                                                                                                                                                                                                                                                                                                                                                         |
| 4       | Cambeyrac French Electricity Authority                   | —do—          | 1            | 5,150       | 10.75  | 150               | —do—                        | $D_1 = 3.8 \text{ m}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| 5       | Kosi East Canal; India (Bihar State Electricity Board)   | —do—          | 4            | 5,500       | 6.1    | 93.8              | —do—                        | $D_1 = 5.8 \text{ m}$ , two-way turbine under 1 to 11 m and two-way pump 1 to 6 m head                                                                                                                                                                                                                                                                                                                                                                                               |
| 6       | Gandak West Canal, India (Bihar State Electricity Board) | —do—          | 3            | 5,500       | 6.1    | 107.14            | —do—                        | $D_1 = 5.35 \text{ m}$ , $Q = 283 \text{ m}^3/\text{sec}$ , two-way turbine 1 to 11 m, two-way pump 1 to 6 m                                                                                                                                                                                                                                                                                                                                                                         |
| 7       | Detzern, Mosel River. Germany                            | —do—          | 4            | 5,800       | 6.83   | 92.5              | Epicyclic 750               | $D_1 = 3.8 \text{ m}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| 8       | Beaumont-Monteux, Isere River, France                    | —do—          | 1            | 8,800       | 12.5   | 150               | Directly coupled            | $D_1 = 4.6 \text{ m}$ , $Q = 8.5 \text{ m}^3/\text{sec}$                                                                                                                                                                                                                                                                                                                                                                                                                             |
| 9       | St. Malo, French Electricity Authority                   | Tidal         | 1            | 9,000       | 4.8    | 88.25             | —do—                        | $D_1 = 6.1 \text{ m}$ , $Q = 230 \text{ m}^3/\text{sec}$                                                                                                                                                                                                                                                                                                                                                                                                                             |
| 10      | La Rance, French Electricity Authority                   | —do—          | 9            | 10,000      | 5.75   | 93.75             | —do—                        | $D_1 = 5.6 \text{ m}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                |
| 11      | Chandiere, Canada                                        | Tubular       | 1            | 10,300      | 11.6   | 100               | —do—                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 12      | Argentat, Dordogne River, France                         | —do—          | 1            | 14,400      | 16.5   | 150               | —do—                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 13      | Ampsin-Neuville, Belgium                                 | —do—          | 6            | 14,550      | 10     | 125               | —do—                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 14      | Pierre Benite, Rhone River, France                       | —do—          | 4            | 20,000      | 7.95   | 83.23             | —do—                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 15      | Gertstein, Rhine River, France                           | —do—          | 6            | 23,000      | 9.8    | 107               | Directly coupled            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 16      | Ozark Arkansas, USA                                      | —do—          | 5            | 25,000      | 10.67  | 60                | Epicyclic 514               |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 17      | Vallabreques Rhine River, France                         | —do—          | 6            | 34,000      | 14.3   | 93.75             | —do—                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
| 18      | Saratov Hydro-electric Plant                             | —do—          | 6            | 47,300      | 6.3    |                   | —do—                        | $D_1 = 7.5 \text{ m}$                                                                                                                                                                                                                                                                                                                                                                                                                                                                |

Nos. 1, 2, 3, 4, 8, 9, 10, 12, 14, 16—Manufacturers : Neyrpic, Grenoble, France. Nos. 5, 6 Hitachi, Japan, No. 13 Escher Wyss.

**PROBLEMS****(A) Propeller and Kaplan Turbines**

- 7.1 Describe with the help of a sketch the special features of (a) Propeller turbine, (b) Kaplan turbine.
- 7.2 What is the difference between a Propeller turbine and Kaplan turbine ?
- 7.3 What is the maximum permissible head under which a Propeller or Kaplan turbine works ?
- 7.4 How is the evolution of Deriaz runner based on Kaplan's design ?
- 7.5 Where is Propeller turbine used ?
- 7.6 Where is Kaplan turbine used ?
- 7.7 State the differences in construction of Francis and Kaplan turbines.
- 7.8 What is the biggest diameter and the highest head for which a Kaplan turbine is manufactured ?
- 7.9 Describe the most powerful Kaplan turbine of the world so far.
- 7.10 How are the runner blades of a Kaplan turbine rotated ?
- 7.11 Describe the latest development for the rotation of Kaplan runner blades.
- 7.12 How much should be the runaway speed of Kaplan turbine compared with its normal working speed ?
- 7.13 How are guide vanes of Kaplan turbine calculated ?
- 7.14 Draw the outlines of Kaplan runner giving main dimensions.
- 7.15 Describe the places of Propeller or Kaplan runner where the cavitation is likely to take place.
- 7.16 What is meant by the following types of cavitation : (a) cleavage cavitation, (b) throat cavitation, (c) surface cavitation (d) inlet cavitation.
- 7.17 Suggest the empirical relation to find the value of sigma, the cavitation factor.
- 7.18 What is the ratio of hub diameter to the runner external diameter ? On what factors does it depend ?
- 7.19 How are hydraulic losses, volumetric and head losses, determined ?
- 7.20 What is aerofoil theory ? How is the lift of an aeroplane induced ?
- 7.21 What is an aerofoil section and where is it used ?
- 7.22 How is the aerofoil theory applied for the design of axial flow turbomachinery.

**(B) Tubular or Bulb Turbines**

- 7.23 What are tubular turbines ? How have these been developed ? Why are these also known as bulb turbines ?
- 7.24 What are bulb sets ? Why could not these be developed so far ?
- 7.25 Make a comparison of a power station using bulb turbines and the one employing Kaplan turbines with the help of a diagram. What is the overall saving in cost by using bulb sets ?



- 7.26 What are the advantages of bulb sets over Kaplan sets ? Is there any disadvantage also ?
- 7.27 Describe the field of application of bulb sets.
- 7.28 Describe constructional details of bulb sets. Under what circumstances are the epicyclic gears used for such sets ?
- 7.29 What is a monobloc bulb set ? Describe its main constituents ?
- 7.30 Describe main types of bulb sets. Which of them can be standardised ?
- 7.31 What are microstations ? Where and why are they used ?
- 7.32 Describe medium head ministations. Give their types.
- 7.33 How is the automatic operation of ministations carried out ?
- 7.34 How and why has it been possible to use bulb sets for tidal power stations ?
- 7.35 Give some hydro-plants where bulb sets have been used ? Is there any power plant in India employing bulb sets ?
- 7.36 Describe the largest bulb set so far made.

### NUMERICALS

- 7.37 Pathri Hydro Station on the Ganga Canal in U.P. is equipped with three VOITH water turbines, each having the following specifications.

Net working head = 9.76 m ;  
 Discharge = 83.50 m<sup>3</sup>/sec  
 Output = 9,640 BHP  
 Speed = 125 rpm

Determine :

- (a) Overall efficiency of each turbine ;  
 (b) Specific speed and type of turbine ;  
 (c) Speed ratio, if the diameter of the runner is 3.88 m.

Draw the outlines of the above turbine, showing the type of draft tube you would employ.

(88.7% ; 712 ; Kaplan ; 1.84 Elbow) (Roorkee University)

- 7.38 A reaction turbine is to be selected from the following conditions—

Maximum net head = 13 m ;  
 Average net head = 11 m ;  
 Minimum net head = 9 m.

Full output at average head which is also design head is 18,300 BHP. What type of turbine would you suggest in this case ?

The full load total efficiency at average head may be taken as 89% and at maximum and minimum heads as 87% and 88% respectively. The speed ratio is 1.8 at average head and generator speed is 250 rpm.

Determine :

- (a) the diameter of runner ;  
 (b) the designed specific speed ;  
 (c) the full load outputs at minimum and maximum heads.

[Kaplan (a) 2.02 m (b) 1,690 units (c) 22,900 HP, 13,590 HP]

(Panjab University)

- 7.39 A Kaplan turbine has a vertical conical draft tube. The diameter of the tube on the upper side connected to the outlet of the runner is 60 cm and that of the tube outlet is 90 cm. The tube is running full with water flowing downwards, and it is 6 m long with 1.5 m of its bottom length in tail water. The frictional loss between the top and the bottom points is 0.15 times the velocity head at the top point where the water has a velocity of 6 m/sec. Find the water pressure at the top point of the draft tube in kg/cm<sup>2</sup> and metres of water.

(−0.57 kg/cm<sup>2</sup> ; −5.7 m) (*Jadavpur University*)

- 7.40 A Kaplan turbine develops 2,000 HP under a head of 6 m. The turbine is set 2.5 m above tail race level. A vacuum gauge inserted at the turbine outlet records a suction head of 3.1 m. If the turbine efficiency is 85% what will be the efficiency of the draft tube having inlet diameter of 3 m. (68.33%) (*AMIE—Mech*)

- 7.41 The Thoma-factor ( $\sigma$ ) of cavitation can approximately be written as—

$$\sigma = \eta_s K_m^2 + \lambda K_u^2$$

where  $K_m = \frac{v_m}{\sqrt{2gH}}$  and  $K_u = \frac{u}{\sqrt{2gH}}$

$v_m$  and  $u$  being the axial and tangential velocities,  $\eta_s$  is the draft tube efficiency and

$$\lambda = \frac{\frac{p}{\gamma} - \frac{p}{\gamma}}{\frac{2g}{\gamma}}$$

is the dimensionless parameter describing the pressure difference between a point on the runner vane and at the exit.  $\frac{p}{\gamma}$  is the absolute pressure at a point on the vane and  $\frac{p}{\gamma}$  at the exit.

A 250 rpm Kaplan turbine using 25 m<sup>3</sup>/sec at full load under a net head of 20 m has a runner diameter of 2.5 m and hub diameter of 1.25. The difference  $\frac{p}{\gamma} - \frac{p}{\gamma}$  as indicated by the model test can be taken to be equal to 1.825 m of water. Neglect the vapour pressure and determine the maximum permissible suction head for the above machine taking barometric pressure as 8.5 m of water and the draft tube efficiency of 85%. (4.58 m) (*Panjab University*)

- 7.42 The following data were obtained from an efficiency test of a Kaplan turbine—

Diameter of boss of runner = 0.35 times the external diameter.

Speed ratio = 2, Flow ratio = 0.6,

Speed = 75 rpm, Head = 8 m,

Brake horsepower = 18,000.

Find the efficiency of the turbine.

(81%) (*AMIE*)

- 7.43 Kaplan turbine installed at Radhanagri (Gujarat State) Power House develops 1,750 HP at an average head of 29 m. The speed and flow ratios are 2.1 and 0.62 respectively. The diameter of the boss = 0.34 times the external diameter of runner. Overall efficiency is 0.89. Calculate the diameter and speed of runner.

(0.705 m ; 1,335 rpm).

- 7.44 Each Kaplan turbine installed at Vargon (Sweden) is rated at 15,400 HP when working under an average head of 4.3 m. The diameter of the hub is 0.3 times the external diameter of runner. Overall efficiency of turbine is 0.91. Find the speed and diameter of turbine runner. Take the values of speed ratio and flow ratio as 2 and 0.65 respectively.

(42.2 rpm ; 8.38 m) (BHU)

- 7.45 Show that the horsepower required to maintain uniform motion of an aerofoil in a horizontal direction depends upon the value of  $K_D (K_L)^{-1.5}$ .

An aerofoil having an area of 10 m<sup>2</sup> carries a vertical load 1,000 kg. Values of  $K_L$  and  $K_D$  are as follows—

| Angle of incidence | 0°     | 2°     | 4°     | 6°     | 8°    | 10°   |
|--------------------|--------|--------|--------|--------|-------|-------|
| $K_L$              | 0.17   | 0.24   | 0.31   | 0.38   | 0.445 | 0.50  |
| $K_D$              | 0.0080 | 0.0112 | 0.0155 | 0.0212 | 0.028 | 0.035 |

Find the minimum horsepower required to maintain horizontal motion through air having density 1.2 kg/m<sup>3</sup>, together with the corresponding velocity and angle of incidence.

[AMJ Mech E (Lond)]

- 7.46 Sketch a graph showing how the lift and drag coefficients of typical aerofoil section vary with the angle of incidence.

Over the incidence range  $-4^\circ < \alpha < +4^\circ$  the lift and drag coefficients of an aerofoil are given by

$$C_L = 0.1 \alpha + 0.45$$

$$C_D = 0.01 \alpha + 0.1.$$

If the aerofoil has to lift 25 kg/m<sup>2</sup> of surface at an incidence of  $-1^\circ$ , determine the minimum velocity of flight and the horsepower required for a total lift of 500 kg in air with a specific weight of 1.12 kg/m<sup>3</sup>.

[AMI Mech E (Lond)]

- 7.47 A water turbine of Kaplan type is supplied with water under a head of 5 m. The runner of the turbine has four blades of aerofoil section, and moves at 75 rpm. The blade length in radial direction is 60 cm, and the mean radius of blade circle is 1.6 m. The chord of the blade is inclined at  $30^\circ$  to the direction of motion, and the chord length is 2.6 m. The velocity of flow is 5 m/sec. Calculate the horsepower developed by the turbine, and the theoretical efficiency of the turbine. Neglect the area occupied by the blade thickness.  $C_L$  and  $C_D$  for the angle of incidence used are 0.75 and 0.05 respectively.

(1,430 HP : 79.3%)

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## *Governing of Water Turbines*

**8.1 Introduction**—Governing means regulation of speed. The main function of a governor is to maintain a constant speed when load on the turbine fluctuates. The governor is, therefore, the point of co-ordination between the turbine and external control. This is accomplished by controlling the rate of flow of liquid leading to the runner, in proportion to the load. In case of Pelton turbines the governor closes or opens the needle valve, in case of Francis turbines, it closes or opens the wicket gates and in cases of Kaplan turbines it moves the runner blades in addition to closing and opening of wicket gates.

Almost all modern hydraulic turbines are directly coupled to alternators which must run constantly at the designed speeds. The speed of an alternator is fixed by the number of pair of poles and the required frequency (cf Art 5.9). For various reasons the frequency cannot be allowed to vary. The speed must, therefore, remain constant when the load changes, hence the necessity of speed control. Governors are also used for other prime movers such as diesel engines, steam turbines and steam engines. In the former it operates by regulating the fuel supply and in the latter by controlling the steam pressure. The water turbine governor controls the rate of flow of water.

The governor of a diesel engine has to perform much less work than that of a hydraulic turbine. While a small centrifugal governor will suffice to control the supply of a fuel, like oil it needs a very strong governing mechanism to exert forces necessary to divert or to stop the flow of water into a turbine runner.

The turbine governor is called upon to satisfy conflicting conditions. It should be quick acting i.e., it should restore the speed to normal as soon as possible after it has been disturbed due to a variation of load. On the other hand it should not close the pipeline so quickly as to cause a serious hammer blow. Obviously, the governor should be so designed as to keep both speed rise of runner and pressure rise in pipe within limits. In order to save the pipeline from water hammer effects, the turbines are equipped with a safety device operated by the governor. A deflector or diffuser in Pelton turbines and relief valve (i.e., pressure regulator) in reaction turbines will serve this purpose.

### 8.2 Functions of a Water Turbine Governor—

- (a) It controls the speed of turbine set and match it to the hydro-electric system for synchronising.
- (b) It controls the speed and frequency of turbine unit or units to maintain a desired frequency and voltage.
- (c) It sets the amount of load a turbine unit has to carry.
- (d) It helps in starting and shutting down the turbo-unit by opening and closing the nozzle of Pelton wheel and wicket gates of reaction turbine.

**8.3 Types of Water Turbine Governors—**The governing mechanism may operate mechanically, electrically or by fluid pressure. Mechanical governors such as ball and spring governors as well as electric brake governors are used for small turbines. Speed control in large water turbines requires a mechanism capable of relatively heavy duty and therefore, oil pressure governor is the only suitable type.

**8.4 Qualities of a Governor—**The following are the fundamental qualities or performance characteristics of a governor if it is to regulate satisfactorily:

- |                        |                                         |
|------------------------|-----------------------------------------|
| (a) Sensitiveness      | (e) Hunting or Racing                   |
| (b) Rapidity of Action | (f) Capacity                            |
| (c) Stability          | (g) Speed Regulation                    |
| (d) Isochronism        | (h) Regulating Time or Governor's Time. |

(a) **Sensitiveness**—The ratio of the mean speed of action to the difference between the maximum and minimum speeds is a measure of sensitiveness of a governor.

$$\text{i.e., Sensitiveness} = \frac{\omega_2 + \omega_1}{2} \cdot \frac{1}{\omega_2 - \omega_1} \quad \dots(8.1)$$

or in other words sensitiveness of governor means the alteration of position of governor which is caused by the smallest increase or decrease in speed at any turbine gate opening

(b) **Rapidity of Action**—The action of governor must be rapid. If the load increases, the governor's action should be quick to open the gates. However, if the load falls, the quick closing of gates checks the flow of water in the pipeline and may cause water hammer. If the pipeline is long relative to head (i.e., pipe length is more than 5 to 6 times the static head), the problem of pressure rise becomes serious. In order to avoid such a pressure rise, the closing time of governor should be increased, but the delay in shutting of the water allows a greater time for the turbine to speed up and hence the speed rise becomes considerable unless a very heavy flywheel is fitted.

(c) **Stability**—If a governor is stable and its balls are slightly displaced from their equilibrium position, the speed remaining constant, they will have a tendency to return to the normal configuration corresponding to speed. Thus a stable governor helps the gates of water turbine to come to their proper position with the least possible oscillations.

(d) **Isochronism**—A governor is said to be isochronous if, neglecting friction, the equilibrium speed is same for all radii of balls. This is possible if the governor is *infinitely* sensitive and the governor always flies

to one or the other extreme positions. The condition is represented by  $\omega_1 = \omega_2$  in Eqn 8.1.

(e) **Hunting or Racing**—A governor is said to hunt or race if there is periodic speed fluctuations with steady load. This is caused if the governor is too sensitive.

(f) **Capacity**—The capacity of governor which controls the flow of oil under pressure to the turbine servomotor, is determined from the force required to move the turbine wicket gates. The capacity of governor is the sum of the capacities of gate servomotor and blade servomotor (in case of Kaplan turbine only).

The capacity of a servomotor in m-kg is the total force in kg exerted by oil under pressure on the piston or pistons multiplied by the stroke in metres.

$$\text{Capacity of gate servomotor } G_g = G_1 D_2^3 H_{max} \text{ m-kg} \quad \dots(8.2)$$

$$\text{where } G_1 = \text{constant} = 5 \left( \frac{N_s + 45}{45} \right)$$

$D_2$  = runner exit diameter ;

$H_{max}$  = maximum head including water-hammer ;

$N_s$  = specific speed.

$$\text{Blade servomotor capacity, } G_b = \frac{P_t \cdot N_s^{\frac{1}{2}}}{H^{\frac{1}{2}}} \text{ m-kg} \quad \dots\dots(8.3)$$

where  $P_t$  = the rated horsepower ;

$H$  = the rated head.

(g) **Speed Regulation**—The speed variations which are caused by sudden fall or increase of full or partial load, are to be regulated by the governor in order that the normal speed is returned immediately.

(h) **Regulating Time or Governor's Time**—This is the time in seconds required by the governor to move the turbine gates from the closed to the open position or *vice versa*. The governor system is designed to close the wicket gates in 3 seconds for a short penstock and upto 5 seconds for a long penstock.

**8.5 Principal Elements of Oil Pressure Governor**—The principal elements of a water turbine governor which is generally of oil pressure type, are as follows—

(a) Speed-responsive element,

(b) Power element,

(c) Stabilising or compensating element.

(a) **Speed-responsive Element** which starts its action with the change of speed, consists of—

(i) *Pendulum or governor's head* which functions by centrifugal action.

(ii) *Drive of Pendulum or of Governor's head* from the turbine main shaft.

(b) **Power Element** which supplies power to open or to shut the turbine gates according to load, consists of—

(i) *Servomotor*—A piston operating inside a cylinder by oil pressure is made to transmit power to turbine gates through suitable mechanism.

(ii) *Distributor or Relay Valve* supplies oil to either side of the piston of servomotor for its (piston's) to and fro motion.

(iii) *Pressure Oil Supply Unit* supplies oil to distributor or relay valve. It has a pumping unit with an air vessel and a sump tank.

(c) **Stabilising or Compensating Element** which prevents hunting or racing of governor, consists of—

(i) *Dash Pot*—The action of the pendulum is transmitted through a system of floating levers to the dash pot. The action of dash pot is *asymptotic*.

(ii) *Pilot Valve*—Dash pot transmits the action of pendulum to the pilot valve which causes the oil pressure to be transmitted to the distributor or relay valve [(ii) of *b* above].

In addition to above, the stabilising or compensation element consists of the following—

(iii) *Starting and Stopping Gears*—Starting of turbine is accomplished by a lever, operated by hand, which brings the distributing valve gradually into opening position. As soon as the turbine reaches normal speed, the oil pressure is admitted to the emergency cylinder by turning the bypass cock into “to run” position. To shut down, the bypass cock is turned into “to close” position whereby the emergency or safety oil pressure cylinder is exhausted and the turbine shuts.

(iv) *Safety devices* against oil pressure and drive failures.

(v) *Load Limiting Device* with which the normal load can be limited before and/or during operation.

(vi) *Speed Changer or Speed Setting* can be done by hand or by an electromotor.

(vii) *Hydraulic Return Motion Gear* is fitted over the dash pot, which helps in transmitting the motion in one direction only and thus no *lost motion* can develop with such a gear.

**8.6 Outlines of Oil Pressure Governor**—It consists of the following main components (refer Fig 8.1) :

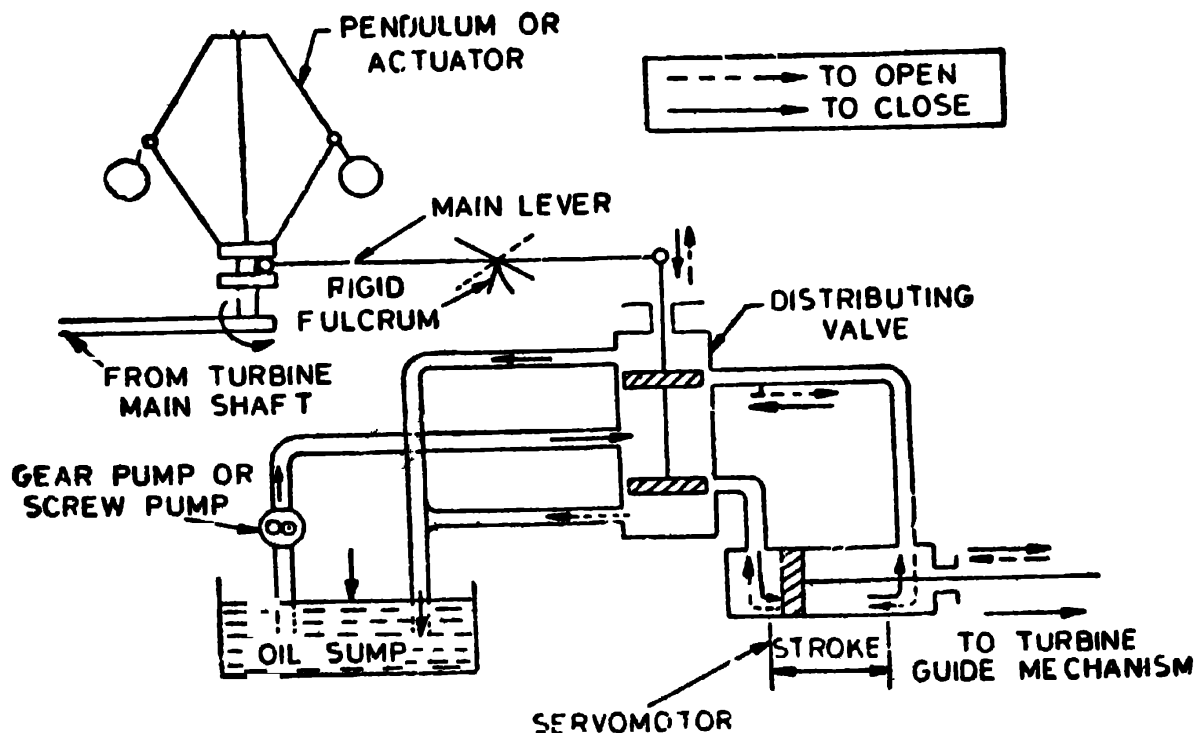


**Fig 8.1 Oil Pressure Governor with Fixed Fulcrum in Neutral Position—Simple Line Diagram**

profile of the cam 10 is such that each position of the deflector corresponds to a given spear opening. When extra load is put on turbine, the deflector 1 is raised and simultaneously the nozzle 2 is opened. When the load is thrown off, the deflector diverts the jet from the runner in less than a second.

**8.11 Governing of Reaction Turbines**—The guide blades of a reaction turbine (refer Fig 8.6) are pivoted and connected by levers and links to the regulating ring. To the regulating ring are attached two long regulating rods connected to a regulating lever. The regulating lever is keyed to a regulating shaft which is turned by the servomotor piston of oil pressure governor.

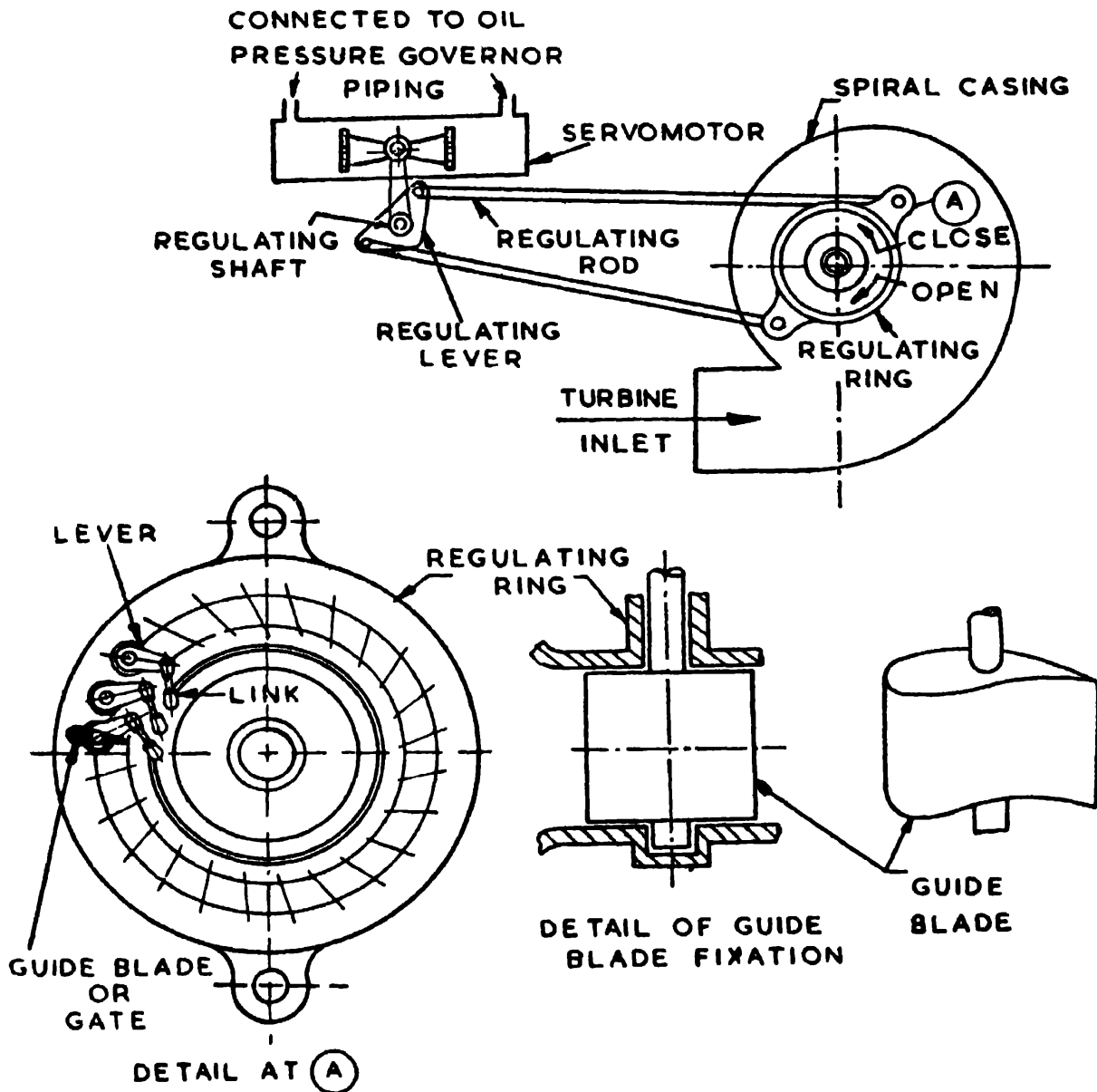


Fig 8.6 Governing of Reaction Turbines—Simple Line Diagram

The penstock, feeding the turbine inlet, has a relief valve better known as '*Pressure Regulator*'. When the guide vanes have to be suddenly closed, the relief valve opens and diverts the water direct to the tail race. Its function is therefore, similar to that of the deflector in Pelton turbines. Thus the double regulation, which is the simultaneous operation, of two elements is accomplished by moving the guide vanes and relief valve in Francis turbines, by the governor.



**8.12 Relief Valve or Pressure Regulator**—It is used as a part of the double regulating gear for speed control in Francis turbines. The object of the pressure regulator is to bypass a portion of turbine discharge when load suddenly drops and in consequence the admission of water to the turbine is suddenly reduced in such a manner that no great increase of pressure can take place in the pipeline and that the waste of water is a minimum.

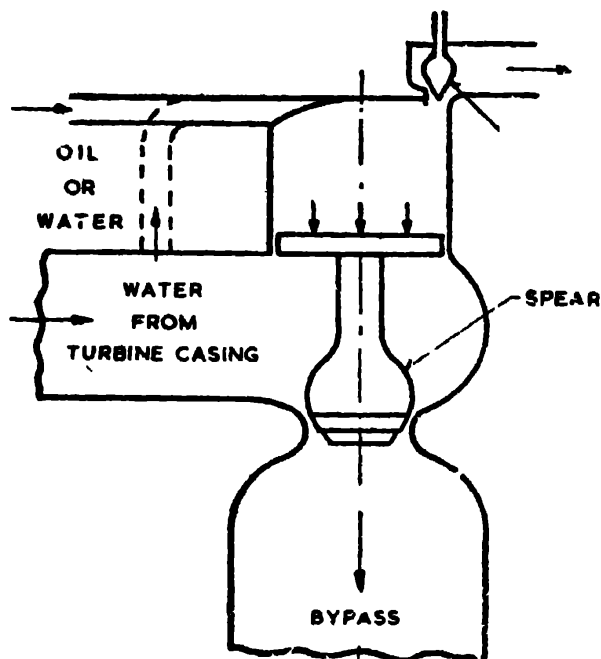


Fig 8.7 Relief Valve—Simple Line Diagram

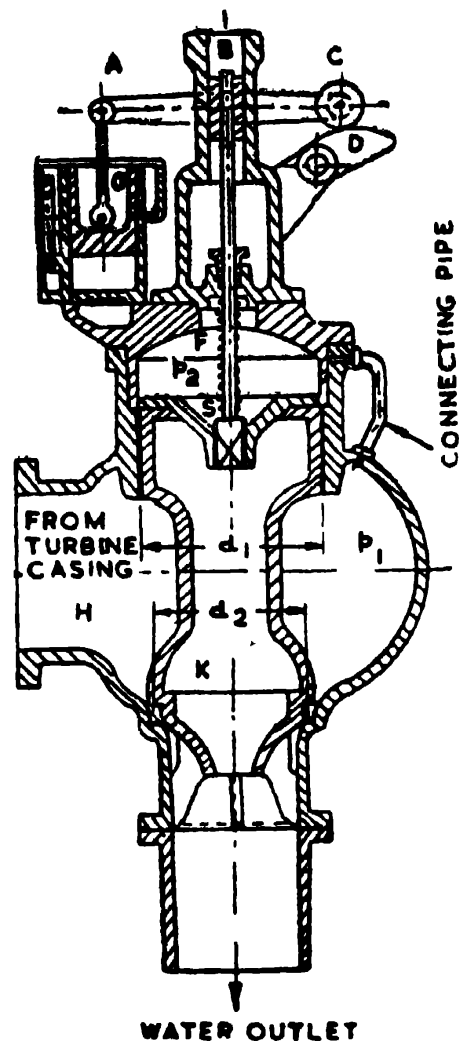


Fig 8.8 Relief Valve or Pressure Regulator (Theoder Bell and Co. Ltd. Kriens, Lucerne, Switzerland) *H*-Casing *K*, Piston Valve, *S*-Regulating Valve *O*-Dashpot, *D*-Pawl, *ABC*-Lever, *F*-Spring

Fig 8.7 shows a simple line diagram of relief valve. Having the same principle as that of a Pelton nozzle, a relief valve consists of a spear which opens the by-pass by a servomotor piston, by oil or water pressure. The release of pressure on the top of piston, by opening the pilot valve, causes the spear to move downwards thus closing the by-pass.

Fig 8.8 shows a pressure regulator manufactured by Theoder Bell and Co Ltd Kriens (Switzerland). The essential parts are the casing *H*, the piston valve *K*, the regulating valve *S*, the dash pot *O*, the pawl *D* and the lever *ABC*.

When the load on the turbine suddenly drops, the automatic governor immediately closes the turbine gates. The pawl *D* quickly raises the

- (ii) Deflector regulation,  
and (iii) Combined spear and deflector control.

The spear and deflector in all cases are operated by the servomotor mechanism.

(i) *Spear Regulation*—To and fro movement of the spear inside the nozzle alters the cross-sectional area of stream, thus, making it possible to regulate the rate of flow according to the load. Spear regulation is satisfactory when a relatively large penstock feeds a small turbine and the fluctuation of load is small. With the sudden fall in load, the turbine nozzle has to be closed suddenly which may create water hammer in the penstock.

(ii) *Deflector Regulation*—The deflector is generally a plate connected to the oil pressure governor by means of levers. When it is required to deflect the jet, the plate can be brought in between the nozzle and buckets, thereby diverting the water away from the runner and directing into the tail race.

Deflector control is adopted when supply of water is constant but the load fluctuates. The spear position can be adjusted by hand. As the nozzle has always a constant opening, it involves considerable wastage of water and can be used only when supply of water is abundant.

(iii) *Combined Spear Deflector Regulation*—As both of the above methods have some disadvantages, the modern turbines are provided with double regulation which is the combined spear and deflector control. Double regulation means regulation of speed and pressure. The speed is regulated by spear and the pressure is regulated by deflector arrangement.

The jet deflector controlled directly by governor, deflects jet from runner within a very short period so that no further energy is imparted to the latter. The deflector engages until the spear has been adjusted to a new position of equilibrium. The closing of the spear can thus be retarded to avoid undue pressure rise, whilst only the short time which the deflector requires to act need be considered in determining speed rise and flywheel momentum. The flywheel momentum should however, be sufficient to deal with a sudden increase in load.

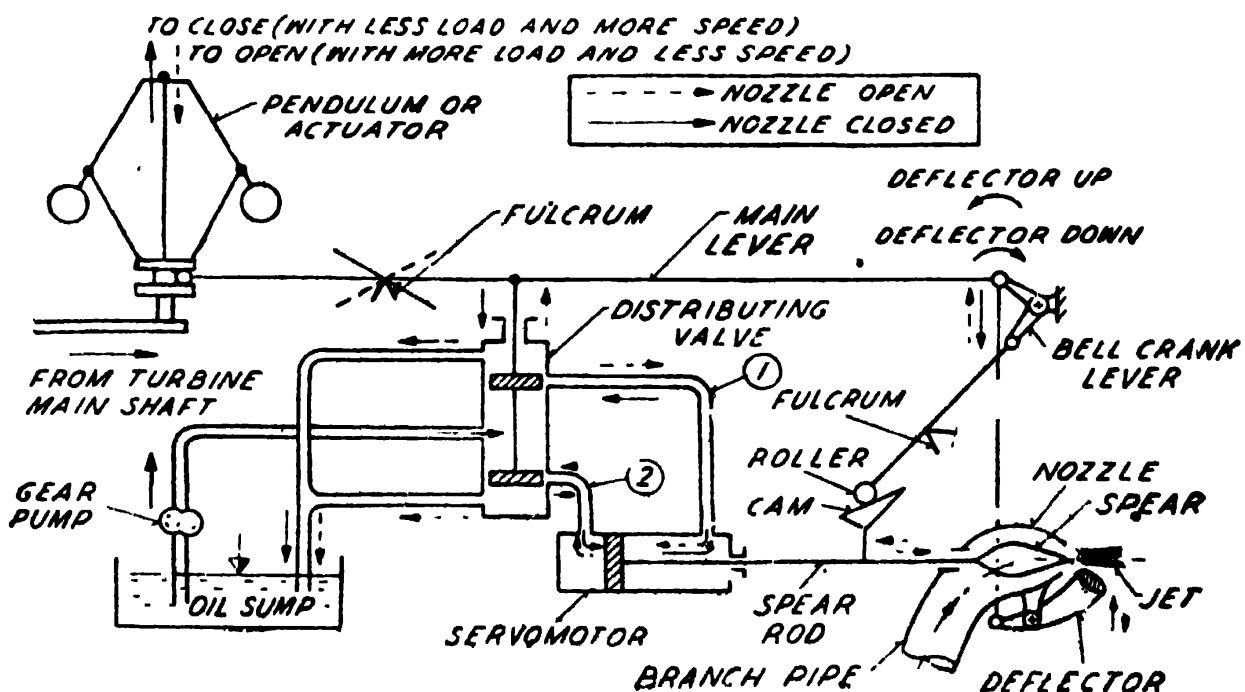


Fig 8.4 Governing Impulse Turbines by Double Regulation System in Neutral Position—Simple Line Diagram

The turbines manufactured by English Electric Co. (U.K.) are equipped with diffuser (Seever patent) instead of deflector (refer Fig 5.2).

**Working**—(refer Fig 8.4)—As the load on the turbine is decreased, the pendulum balls occupy the top position, thus moving the bell-crank lever towards bottom with which the deflector is brought in front of the nozzle, diverting whole or part of the water jet direct to the tail race.

With movement of bell crank lever, the roller on the cam is also raised. The distributing valve also functions simultaneously thus moving the servomotor piston rod towards right hand side (as explained in Art 8.7). This reduces the nozzle outlet by the movement of spear which occupies the position so that the constant turbine speed is maintained. With the movement of spear rod the cam also moves towards right, with which the roller slips and finally with the movement of bell-crank lever the deflector comes to its original position. The whole of the above operation takes place in about 50 to 80 seconds.

Fig 8.5 shows the action of Escher Wyss governor to accomplish double regulation. The governor moves the deflector 1 whilst the spear 2 is controlled by the regulating rods of the deflector. The governor is subject to a closing force exerted by spring 3, for the purpose of safety, so that in the event of any failure in the supply of oil under pressure the deflector 1 is still capable of deflecting the jet away from the runner wheel. The Governor servomotor 4 is controlled by the centrifugal pendulum 5 (refer Fig 8.3). The spear servomotor 6 also

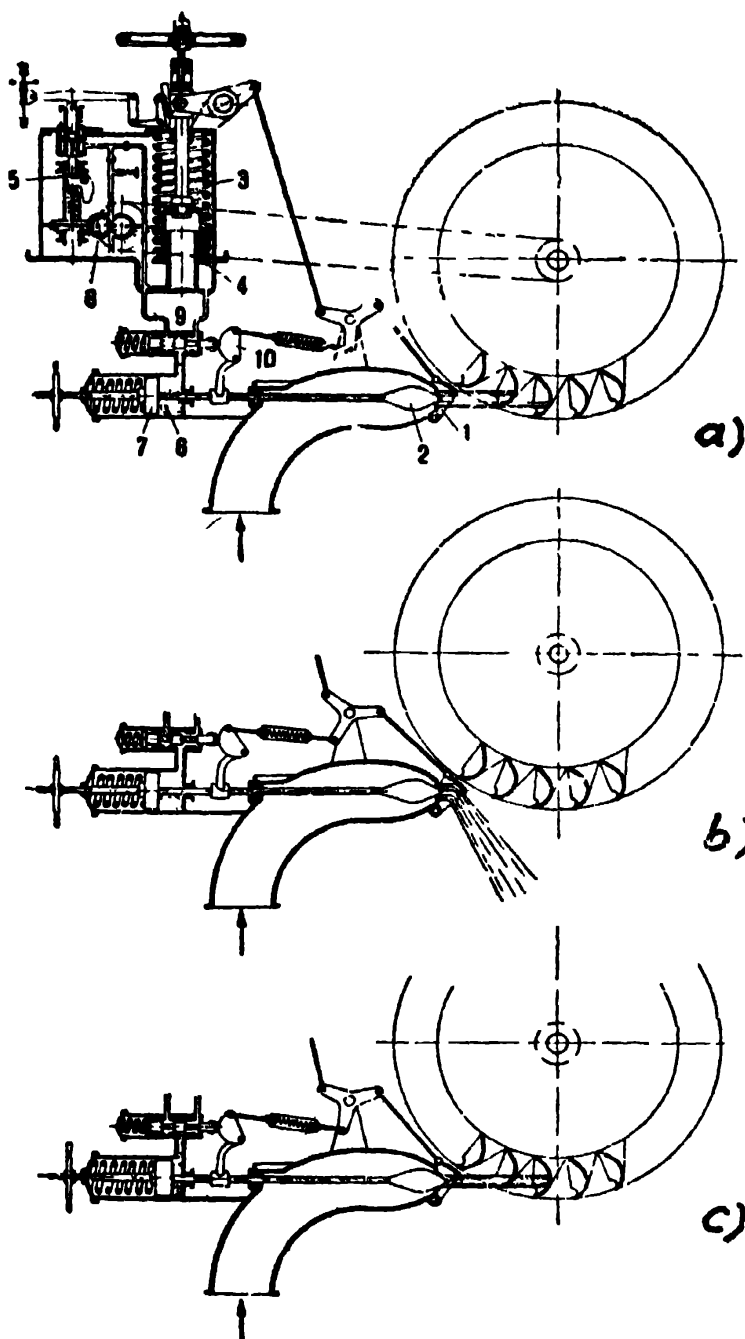


Fig 8.5 Double Regulation of a Pelton Turbine  
(Escher Wyss and Co. Ltd., Zurich)

1. Deflector 2. Spear 3. Spring 4. Servomotor Governor 5. Centrifugal Pendulum 6. Spear Servomotor 7. Piston 8. Gear Pump 9. Control Valve 10. Cam.

has its safety closing spring, whilst for the opening movement oil under pressure from pump 8 is passed through valve 9 in the piston 7. The



**88 Modern Types of Oil Pressure Governors**—The oil pressure governor may be of two types—

(a) **Gate Shaft Type**—(refer Figs 8.2 and 8.3)—In this type all the components of the governor *viz.*, pendulum, dash pot, pilot valve, relay valve, servomotor and oil pressure supply are enclosed in a casing and form a single integrated unit. This is used for smaller units and the governor's capacity is upto about 7,000 m-kG.

(b) **Actuator Type**—In this type the servomotor or servomotors being of large size are separated from the other components. In some cases the governor unit is divided in three parts namely oil pressure unit, servomotor unit and actuator and they are normally located at different

places in the power station. The governor's capacity in this case is not limited. Plate 27 shows Cabinet Actuator Governing Equipment. Localization of all controls and indicators on the front greatly facilitates operation.

**8.9 Different Types of Actuators**—The type of governor's actuator depends upon the principle of pendulum operating mechanism. The different types are as follows:

- (a) Speed-responsive actuator,
- (b) Speed-cum acceleration responsive actuator,
- (c) Electro-hydraulic actuator.

(a) **Speed Responsive Actuator**—(refer Fig 8.2)—Mostly the governors are equipped with speed responsive element i.e., flyballs. The dash-pot acts as stabilizing device.

(b) **Speed Cum Acceleration Responsive Actuator**—The pendulum in this case is sensitive to both speed and acceleration. The sensitivity of the governor is said to increase at the *accelerometer*, connected to the actuator, causes the governor to start its corrective operation immediately the load varies i.e., before the flyballs have responded to a speed change. Some manufacturers like Charmilles and Vavey (Switzerland), and Riva (Italy) are using this type of actuator.

(c) **Electro Hydraulic or Electronic Actuator** (refer Plate 28 and-29)—In the electro-hydraulic governor developed by ASEA (Sweden) the pendulum has been replaced by an electric circuit which is sensitive to frequency. The electrical part is built by ASEA while the electromechanical part, the actuator cabinet, is at present built by **NOHAB, KMW** and Finshytan, all of Sweden.

Plate 29 shows the modern compact actuator cabinet manufactured by ASEA, having the electric circuit and replacing the pendulum. It is located in direct connection to the main distributing valve. It functions only as an electro-hydraulic amplifier. Plate 28 shows the regulator cubicles which contain most of the electrical parts of the governor. The equipment has been separated into plug in unit each performing a special duty e.g., frequency metering unit, feedback units etc.

**Principle of Operation**—The electric regulation is characterised by the fact that all the factors influencing the regulation are transformed into electric impulses, mainly alternating voltages, which are analysed and mixed in an electron tube governor and so combined that its outgoing impulses give positive indications on how the gate mechanism shall be altered with regard to direction and speed, so that the desired regulation will be ideal. In the so-called electro hydraulic operating device the impulses of the electron tube governor are transmitted by an electro-magnetic system to the hydraulic part of the turbine governor.

The voltage, which is fed into the electron tube governor is taken from a source, which has a frequency proportional to the speed of the turbine, for instance, a pendulum generator.

**8.10 Governing of Impulse Turbine**—The quantity of water rejected from the turbine nozzle and from striking the buckets may be regulated in one of the following ways:

- (i) Spear regulation,

(i) **Servomotor or relay cylinder**—A piston operating inside a cylinder is made to transmit power to the turbine gates through suitable mechanisms.

(ii) **Relay valve or distributing valve**—It controls the flow of oil under pressure to either direction of the servomotor cylinder.

(iii) **Actuator or pendulum**—It functions by centrifugal action and it is driven from the turbine main shaft.

(iv) **Dash Pot and Pilot Valve** which applies the motion of pendulum in stabilised form, to the distributing valve.

(v) **Pressure oil supply** including pumping units, supply oil to distributing valve

(vi) **Casing**—It encloses all the above components and also serves as an oil pressure tank.

(vii) **Governor driving mechanism**—The function of each part is explained below—

(i) **Servomotor**—It consists of a cylinder inside which a piston moves under the action of oil pressure. For large units, two cylinders may be employed.

The motion of the piston is transmitted to the spear rod in Pelton turbines and the guide vanes in reaction turbines. An auxiliary governing device is the hand wheel driving the piston. This is used either when starting the unit or when automatic governing is not required.

(ii) **Relay Valve**—This is generally a slide valve of piston type. Distribution of oil supply to the chambers of the servomotor cylinder is regulated by the relay valve operated by a pilot valve which itself is controlled by some speed responsive elements such as flyballs. The relay valve allows the oil under pressure to flow to either side of the servomotor piston to push it for moving the guide vanes to opening or closing positions.

(iii) **Actuator or Pendulum**—It is usually the flyballs mechanism which responds quickly to speed variation. The centrifugal governor and the pumps are driven by tapping power from the turbine mainshaft through belts, gears or an electric motor. The centrifugal governor also operates the runner blade adjusting mechanism in Kaplan turbines.

(iv) **Dash Pot and Pilot Valve** are the stabilising and the compensatory elements. The motion of pendulum's flyballs is transmitted to the relay valve through dash pot and pilot valve. The dash pot presents sudden or too large opening out or closing in of flyballs thus acting as a damping device. The pilot valve receives the damped motion from the pendulum and transmits it to the relay valve.

(v) **Pressure Oil Supply**—Oil under pressure, ranging from 10 to 20 kg/cm<sup>2</sup> is supplied to the distributing valve by a pumping unit. It consists of a rotary (gear or screw type) plunger pump, an oil sump tank, and a pressure tank with suitable valves and pipings. In some cases there are two gear pumps of different sizes each running continuously and connected to a common shaft. The smaller unit supplies oil to counteract gradual load fluctuations while the larger operates in case of sudden load drops or increases. No pressure tank is then required,

(vi) **Casing**—The mechanisms enumerated above are enclosed in a casing which protects them against dust and damage. It also serves as an oil sump.

(vii) **Governor Driving Mechanism**—The centrifugal governor and the rotary gear pump are driven by the turbine main shaft through belts or gears. In modern units the governor incorporates an electric motor which is energised by a small alternator coupled to the main shaft. The motor is designed to respond to any fluctuation of frequency caused by a variation of speed of the main unit.

In very modern installations the alternator and motor have been replaced by electrical devices responsive to frequency variation. Precision control can be effected by employing electric apparatus extremely sensitive to variation of frequency.

**8.7 Working of Oil Pressure Governor** (refer Fig 8.1)—When the load on the generator increases, the speed of turbine which is directly coupled to generator falls. Now the pendulum or the actuator is driven by the main shaft of the turbine. Therefore on reduction of speed of the turbine, the pendulum balls occupy the bottom position with which the lever at right hand end, is raised. This raises the pistons of the distributing valve with which the passage for pipeline opens for the high pressure oil from the gear pump to pass and to strike to the right hand side of the servomotor piston, allowing the servomotor rod to move towards left. This movement can be transmitted to open the nozzle of an impulse turbine or guide vanes of reaction turbine. If the load on the turbine is decreased, the operation explained above will be reversed.

A simplified operating diagram of Turbine Speed Governor manufactured by J.M. Voith, Heidenheim Germany is shown in Fig 8.2. Its pictorial view is given in Plate 26(a) and sectional drawing in Plate 26(b). Fig 8.3 shows turbine speed governor manufactured by Escher Wyss and Co., Ltd Zurich, (Switzerland).

- 1 Plate-spring Flyballs.
- 2 Flyball Drive,
- 3 Flyball Collar with P.I. : Ste
- 4 Pilot Oil Circuit.
- 5 Relais Valve
- 6 Servomotor.
- 7 Pilot Valve Retardation,
- 8 Pilot Valve,
- 9 Retardation Cam,
- 10 Retardation Gear,
- 11 Dashpot Cylinder,
- 12, 13 Coil Springs,
- 14 Dashpot Piston.
- 15 Connecting Rod,
- 16 Fulcrum of Lever 7

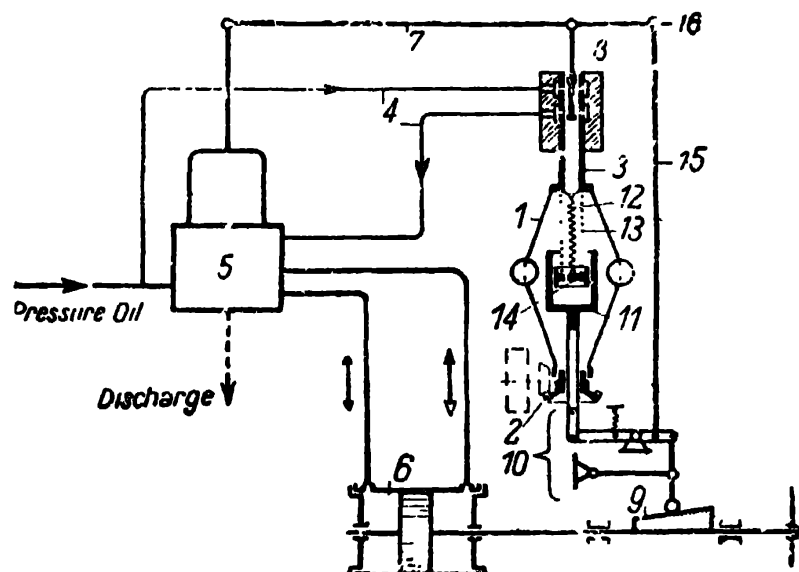


Fig 8.2 Water Turbine Speed Governor and its Operating Diagram (Type D. Manufactured by J. M. Voith, Heidenheim, Germany)

When there is a sudden demand of water by the turbine, the water level in the riser falls rapidly, thus developing an immediate considerable accelerating head on the pipe line. The additional water required is supplied by the tank slowly through the ports at the base of the riser.

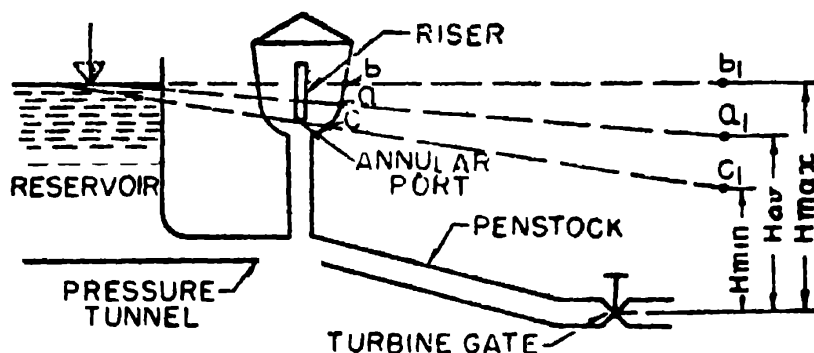


Fig 8.15 Differential Surge Tank

Referring to Fig 8.15—

For *steady* turbine load, the velocity variations in the penstock do not exist, then the pressure gradient is normal i.e.  $Oaa_1$ . For *increasing* turbine load, the water is being drawn and therefore the gradient takes the shape of  $Occ_1$ .

For *falling* turbine load, extra water is pushed to the tank and then the gradient is shown by  $Obb_1$ .

**Largest Differential Tank of the World** till today is at Wallenpanpack (Pennsylvania, USA), made of steel, 16.75 m diameter, 41.25 m high, having a storage capacity of nine million litres.

#### Differences between Simple and Differential Surge Tanks—

(1) The capacity of the differential surge tank is less than the simple surge tank for the same stabilising effect, because a considerable retarding head is available in the former, while in case of latter the head builds up gradually as the tank fills.

(2) In the case of simple surge tanks the head which builds up is the accelerating head, but in case of a differential tank the head in the surrounding tank is independent of the accelerating head as well as of the head acting on the turbines. This is due to the restricted flow of water through the riser.

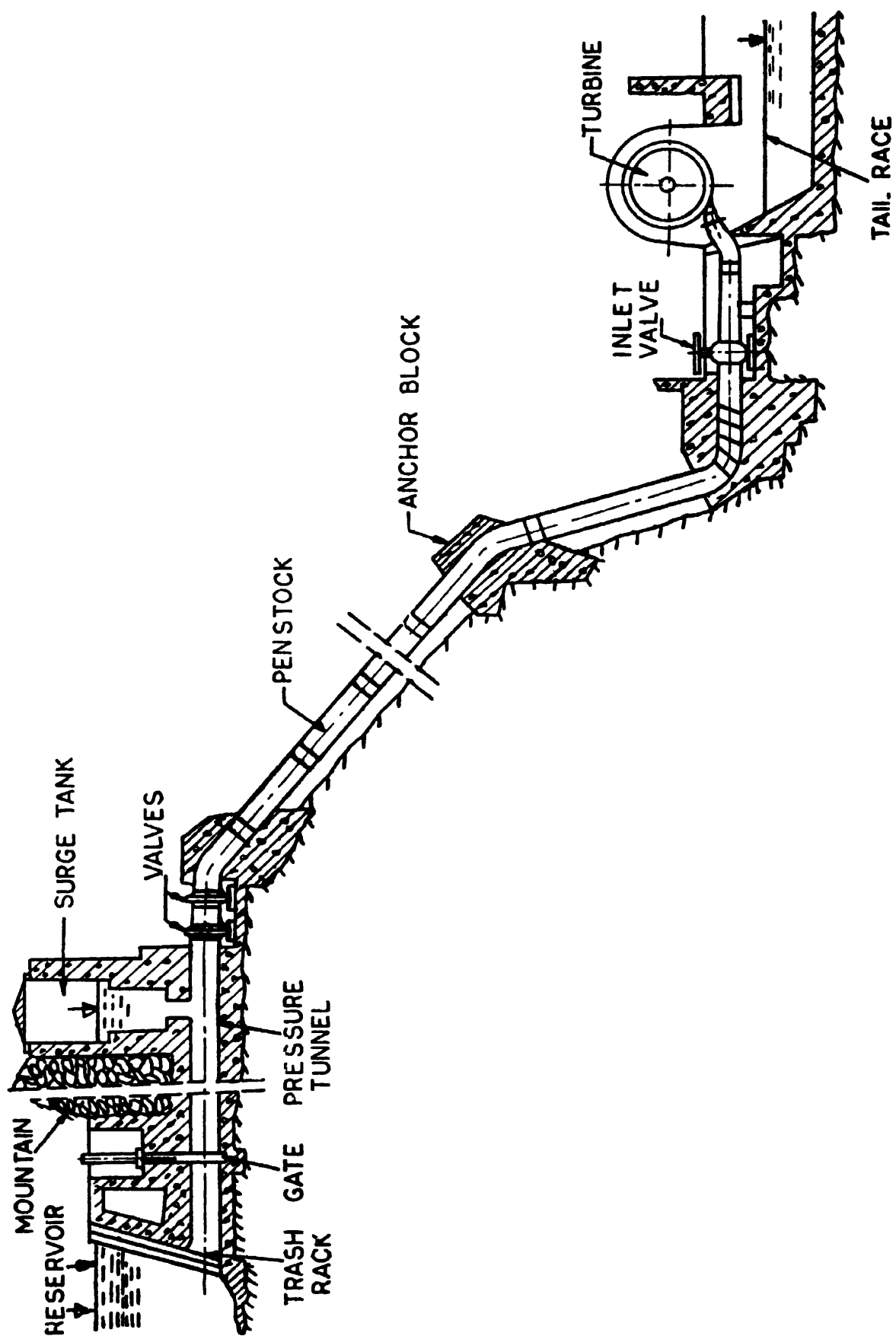
**8.16 Forebay**—Like a surge tank, a forebay is a storage reservoir at the beginning of a penstock. When a hydro-power plant is located at the end of a canal (refer Fig 4.6), a forebay is made by enlarging the canal at the intake to the penstocks. The functions of the forebay will thus be to store the rejected water when the load on the turbine is reduced and to supply the initial water for an increasing load, while the water in the penstock is being accelerated.

### UNSOLVED PROBLEMS

- 8.1 What is governing and how is it accomplished for different types of water turbines?
- 8.2 State the difference between the governing of different prime movers.
- 8.3 State the different types of water turbine governors.



- 8.4 What is the function of a governor in a water power plant ?  
(Jadavpur University)
- 8.5 What should be the qualities of a governor ?
- 8.6 Describe the following qualities of a governor—
- |                         |                        |
|-------------------------|------------------------|
| (a) Sensitiveness,      | (e) Hunting or Racing, |
| (b) Rapidity of Action, | (f) Capacity,          |
| (c) Stability,          | (g) Speed Regulation,  |
| (d) Isochronism,        | (h) Governor's Time.   |
- 8.7 State the principal elements of a water turbine governor. State each element together with its components.
- 8.8 What is hydraulic return motion gear in a water turbine governor ?
- 8.9 What is the function of the following in water turbine governor—  
Relay valve, Dashpot, and Pilot valve ?
- 8.10 What is an oil pressure governor ? Draw a line sketch and describe its principal components. (AMIE)
- 8.11 Why is the oil pressure governor employed to control the speed of a water turbine ? Show by the help of a line sketch the different parts of the oil pressure governor and describe them briefly.
- 8.12 Sketch and describe a modern method of regulation to maintain a constant speed for either  
(a) a Pelton wheel or (b) a Francis turbine. (AMI Mech E—Lond)
- 8.13 Show with the help of a line sketch as to how the speed of a reaction water turbine is governed by servomotor. (AMIE)
- 8.14 What are the different types of oil pressure governors ?
- 8.15 What are the different types of actuators ? Describe them briefly.
- 8.16 Describe how the electro-hydraulic or electronic actuator works.
- 8.17 What is the necessity of double control regulation and how is it effected in different types of water turbines ?
- 8.18 What is the difference between a deflector and a pressure regulator in a water turbine ?
- 8.19 What is the function of a pressure regulator ? (AMIE)
- 8.20 Describe briefly how the governing of a Kaplan turbine is carried out.
- 8.21 What are surge tank and forebay and what are their functions ?
- 8.22 Describe a differential type surge tank. How will you differentiate it with that from simple or restricted orifice type ?



**Location of Surge Tank**—Theoretically a surge tank should be located as close to a power or pumping plant as possible. The ideal place in case of power plants is at the turbine inlet, but it is seldom possible in case of medium and high head plants because it will have to be made very high. In order to reduce its height, it is generally located at the junction of pressure tunnel and penstock (refer Fig 8.12) or on the side of the mountain.

**Surge Tank Types**—There are three main types : Simple, Restricted orifice and Differential.

(a) **Simple Surge Tank** is a reservoir directly connected to a pipe line (penstock) as shown in Fig 8.13. In this type of surge tank the accelerating and retarding heads induced by a change of water surface accumulate slowly, therefore its action is sluggish. It is liable to set up considerable oscillations unless it is of such a large size as to render its cost prohibitive. Now-a-days it is seldom used.

(b) **Restricted Orifice Type Surge Tank**—It consists of a restricted orifice (refer Fig 8.14) installed between the pipe line and the simple tank. With the help of orifice the water storage function of surge tank can be separated from accelerating and retarding functions. When the turbine gates are closed suddenly, a retarding head develops by the orifice more quickly than with a simple surge tank. Similarly the orifice will create a rapid accelerating head when additional flow has to be supplied by the surge

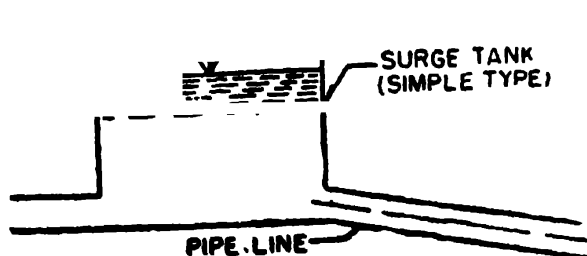


Fig 8.13 Simple Surge Tank

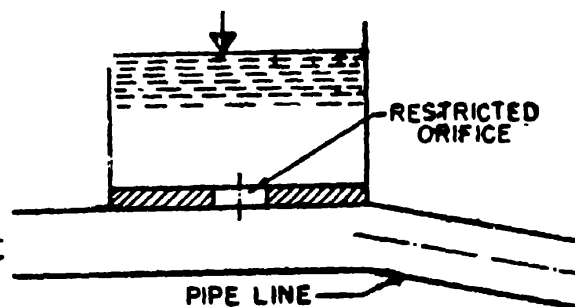


Fig 8.14 Restricted Orifice Type Surge Tank

tank. With the development of sudden retarding or accelerating heads, the head on the turbine is liable to fluctuate thus making the governor mechanism more cumbersome and costly.

(c) **Differential Surge Tank**—It is a combination of simple and restricted orifice type, with an addition of an internal riser (refer Fig 8.15).

The riser is of smaller diameter than the connection to the pipe line. At the base of the riser there is an annular port, communicating with the tank, the area of which is proportional to suit the conditions under which the tank is to operate. The water enters the tank through the annular port, therefore, the function of the tank depends upon the area of this port.

When there is a sudden drop in load on the turbine, the water from the pipe line rises immediately in the riser and spills over from its top into the tank. This establishes an immediate retarding head on the pipe line, as well as a differential head on the port. The differential head is responsible for forcing the water by the turbine, through the port into the tank. Due to restriction of flow through the riser there will be frictional loss which dissipates a part of increased pressure.

upon the load, adjusting the rate of flow of water entering the runner. Whenever the turbine gates are suddenly closed the water flowing in the long penstock which is on its way to the turbine has to be retarded. This leads to a phenomenon known as *water hammer* which causes excessive pressure in the pipe line with which the pipe may burst. It is necessary, therefore, to provide the plant with a safety device. Such a device may be *relief valve* or *pressure regulator* in case of Francis turbines (refer Art 8.11 and 8.12). However to relieve the penstock of its excessive pressure the following devices are also employed—

(a) **Surge tank** in case of high and medium head power plants, where the penstock is very long.

(b) **Forebay** in case of medium and low head power plants, where the length of penstock is short.

Both of them will be described in the following articles.

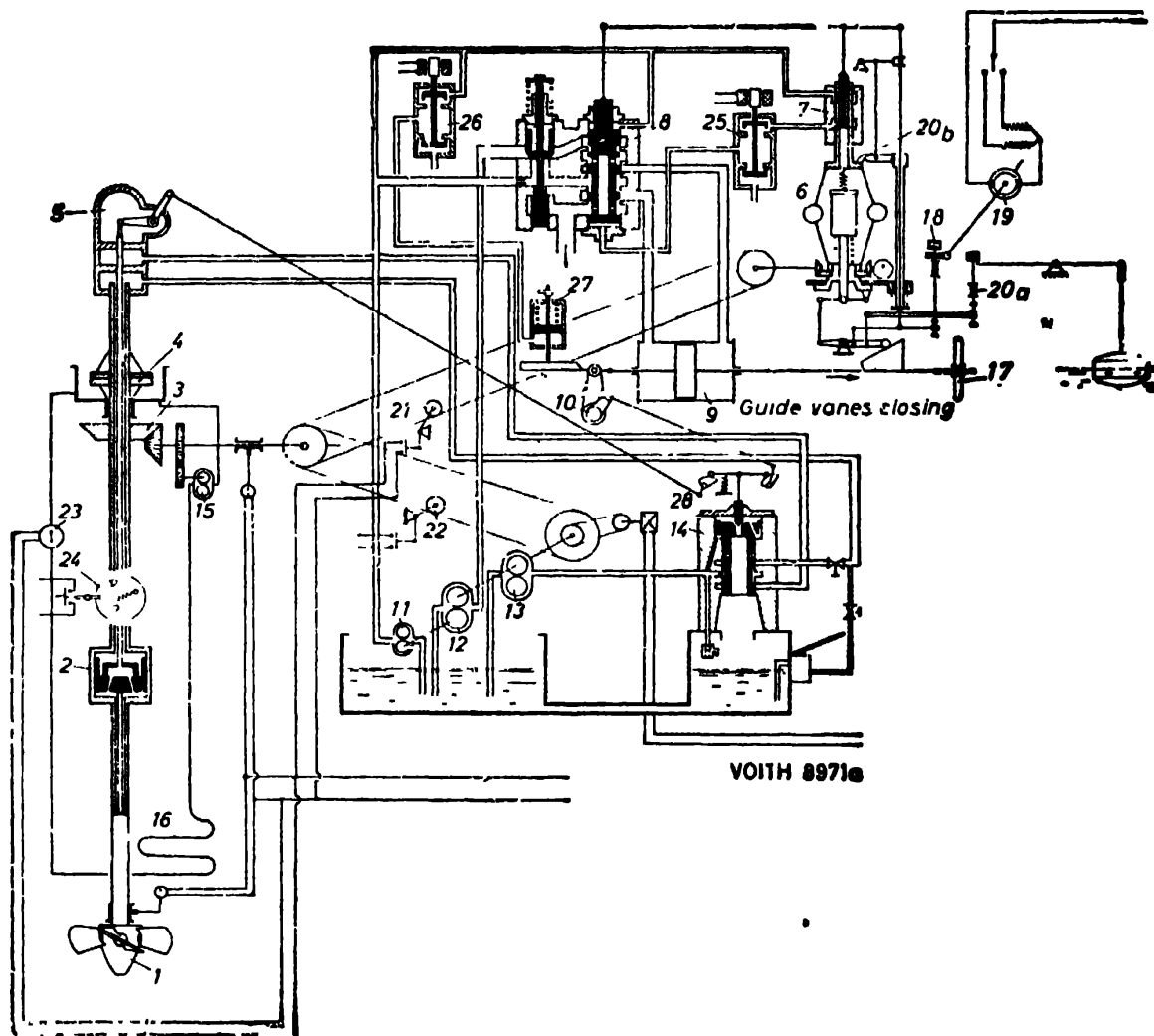


Fig 8.11 Governing Mechanism of Kaplan Turbine by Voith, W. Germany

1. Kaplan Runner 2. Runner Servomotor 3. Bevel Gears 4. Thrust Bearing 5. Relay Valve for Runner 6. Pendulum 7. Pilot Valve 8. Relay Valve 9. Relay Valve for Guide Wheel 10. Guide Wheel Servomotor 11. Connection Between Guide Wheel and Runner Operations 12. Small Oil Pump for Guide Wheel Governor 13. Big Oil Pump for Guide Wheel Governor 14. Oil Pump for Runner Regulator 15. Distributing Valve for Relative Positions of Guide Vanes and Runner Blades 16. Hand Pump for Starting 17. Runner Regulator 18. Guide Vanes Connections 19. Opening Reducer 20. Electrical Speed Regulator 21. Hand Wheel 22. Safety Device for Pendulum Belt 23. Safety Device for Pump Driving Belts 24. Connections of Safety Governor 25. Safety Governor.

**8.15 Surge Tank** (refer Fig 8.12)—A surge tank is a storage reservoir fitted at some opening made on a long penstock to receive the rejected flow when the penstock is suddenly closed by a valve fitted at its steep end. Surge tank, therefore, relieves the pipe line of excessive pressure produced due to closing of penstock, thus eliminating positive water hammer effect by admitting in it a large mass of water which otherwise would have flown out of the pipe line. It is necessary for medium and high head water power plants, especially when the water has to travel a long way from the intake to the power house. It is also used in a large pumping plant to control the pressure variations resulting from rapid changes in the flow.

Surge tank also serves the purpose of supplying initial water for an increasing load on the turbines while the water in the pipe line is being accelerated.

In case of a water power plant, when there is sudden reduction of load on the turbine, it becomes necessary for the governor to close the turbine gates for adjusting the flow of water in order to keep the speed of the turbine constant. However, the water is already on its way to the turbine. When the turbine gates are closed, the moving water has to go back. A surge tank would then act as a receptacle to store the rejected water and thus avoid water hammer. In other words as the demand of water by the turbine is less, the pipe line velocity must be decreased according to the requirement. A surge tank would then build up a retarding head which, in turn, reduces the pipe line velocity until the flow therein no longer exceeds than that required by the turbine. On the other hand when there is an immediate demand on the turbine for more power, the governor re-opens the gates in proportion to increased load, thus making it necessary to supply more water. For a long pipe it takes a considerable time before the entire mass of water can be accelerated. Surge tank which should be located near the turbine will meet the sudden increased demand for water till such time the velocity in the upper portion of the line assumes a new value.

For a large pumping plant with a long delivery pipe a surge tank can also be employed to control the pressure variations on the delivery side, which are produced due to sudden shut down or starting up of pump. When the pump is started, most of the initial flow from the pump enters the surge tank thus reducing the water hammer effects in the delivery pipe. On the other hand, when the pump is shut down suddenly, the surge tank provides extra space to accommodate water which would come back, thus relieving water hammer pressure.

**Functions of Surge Tank** can be summed up as follows—

(a) Control of pressure variations resulting from rapid changes of flow in penstock, relieving the line of excessive pressure, thus eliminating water hammer effects.

(b) Regulation of flow in power and pumping plants by providing necessary accelerating or retarding head. The more effectively the accelerating or retarding head is applied the shorter will be the duration of surge, less amount of water will then have to be stored or given up by the tank, thereby smaller will be the size of the tank required.

(c) Regulation of turbine speed with the help of (b)

end  $C$  of the lever. The other end  $A$  cannot follow quickly because of the brake action of the dash-pot ; and it may be assumed that during this period the lever turns around  $A$  lifting the valve  $S$  with which it is connected at  $B$ . Some water escapes through valves  $S$  and  $K$ , the pressure  $p_2$  falls below  $p_1$  because of a filter in the small connecting pipe. This difference of pressure raises the piston valve  $K$  and water is discharged through the pressure regulator, without passing through the runner.

Force exerted by the spring  $F$  on valve  $S$  pulls the lever down at  $B$ , and while the end  $C$  of the lever rests on the pawl, piston of the dash-pot slowly descends. Valves  $S$  and  $K$  slowly follow this motion and pressure regulator closes gradually.

If only a small part of the load is taken off, the regulating mechanism of turbine moves slowly, the piston of the dash-pot can follow this motion and as the lever turns about  $B$ , the pressure regulator does not act.

If the load rises suddenly the dash pot exerts no brake action because a non-return valve permits the piston to move freely in one direction. Again  $B$  is at rest and pressure regulator does not operate.

Plate 30 shows a Francis turbine equipped with a relief valve, manufactured by Escher Wyss and Co. Ltd., Zurich.

Fig 8.9 shows pressure regulator manufactured by Allis Chalmers Mfg. Co., USA for Pelton turbines of Caribou plant. It acts as a relief valve opening quickly to discharge water from the penstock to safeguard against rise of pressure.

**8.13 Governing of Kaplan Turbines by Double Regulation—**Double regulation system for Kaplan turbines comprises the movement of guide vanes as well as runner vanes. It is employed to ensure the most satisfactory relationship between the relative positions of guide wheel and runner blades. The single weight pendulum 1 (refer Fig 8.10) operates through valve 2, the guide wheel servomotor 3 and from the regulating rods of the guide mechanism, the runner blades 6 are controlled through valve 4 and servomotor 5. Each position of the runner blades is brought into an exact relationship with the guide wheel by means of cam 7, arranged before the runner distributing valve 4. When the load increases the guide wheel and the runner come into play simultaneously whereas when the load being thrown off, the guide wheel goes ahead and is slowly followed by the runner. The larger sized turbines are equipped with emergency pressure oil pump 8. As long as the service oil pressure supply 9 is in order, the emergency pump 8 supplies oil without pressure, however, if any breakdown occurs, the valve 10 is automatically switched off and the emergency pump 8 delivers oil under high pressure immediately to the closing side of runner servomotor. The servomotor piston 11 is displaced upwards and turns the runner blades 6 into the closing position quite independently from the prevailing position either of the guide wheel or of runner control gear. This valve 10 is, however, not only controlled by the pressure oil supply, but also by the speed of the turbine. If this speed exceeds a given limit, the safety governor 12 closes a circuit and with the aid of magnet 13 switches the change-over valve 10 to its closing position.

Fig 8.11 shows the arrangement of double regulation of Kaplan turbine by Voith, West Germany.

**8.14 Safety Devices for Penstocks—**As the load on the generating units varies, openings of the turbine gates are to be changed, depending

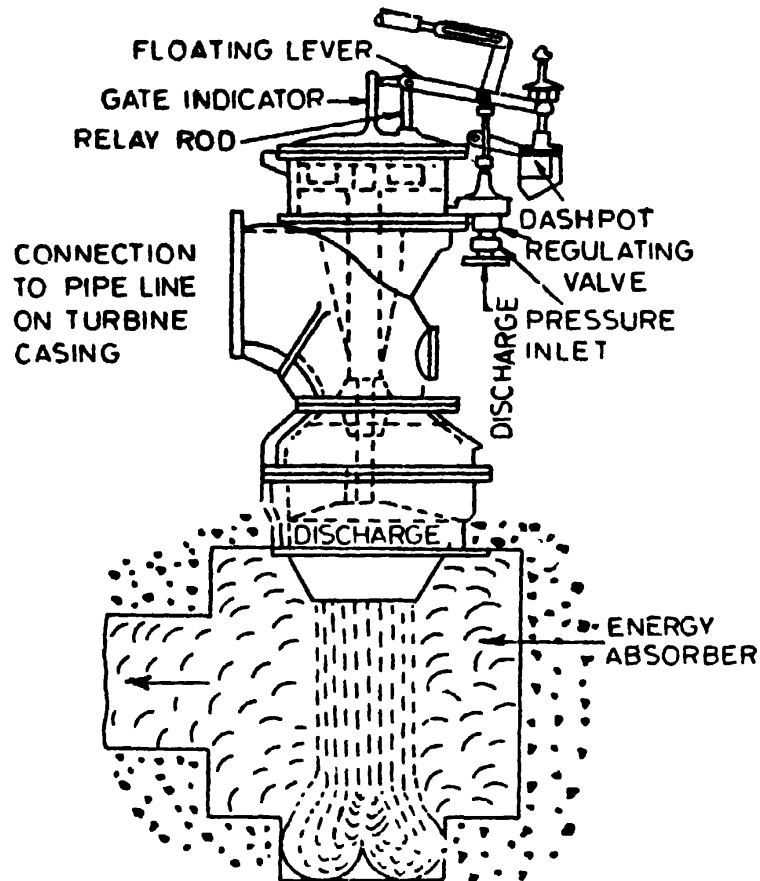


**Fig 8.9** Pressure Regulator for Pelton Turbines at Caribou Plant.  
Manufacturers : Allis Chalmers Mfg Co., USA

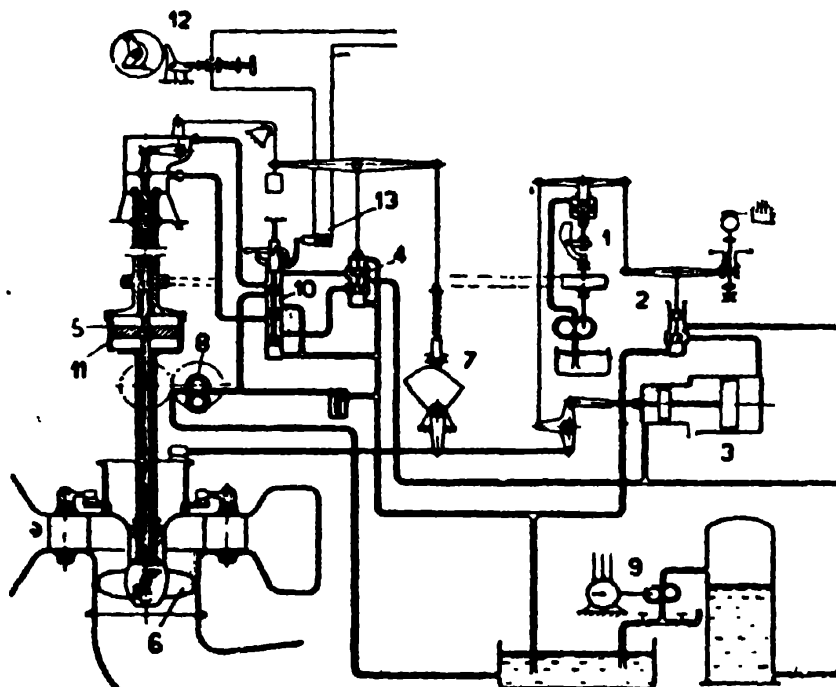


**Fig 8.10** Double Regulation of a Kaplan Turbine  
(Escher Wyss and Co. Ltd., Zurich)  
1. Single Weight Pendulum, 2. Valve, 3. Guide Wheel Servomotor, 4. Valve, 5. Runner Blades Servomotor, 6. Runner Blades, 7. Cam, 8. Emergency Pressure Oil Pump, 9. Service Pressure Oil Supply, 10. Valve, 11. Servomotor Piston, 12. Safety Governor, 13. Magnet.

# *Models and Selection of Turbines*

## **A. Turbine Models**

**9.1 Turbine Models and Their Testing**—Water turbines are usually large-sized units. They are manufactured for specific conditions, therefore, it is not possible to produce them on mass scale. If the turbines are tested after the manufacture is complete, then ;

- (a) the extensive tests on prototype machines are very costly and take a long time to conduct them :
- (b) the test results on prototype machines may not tally with the design data, supplied to the manufacturers. To change the dimensions and size of the turbine at this stage will be very costly.
- (c) There are hydraulic limitations to the tests performed on the prototype—
  - i.e., (i) it is not possible to vary the head,
  - (ii) the speed of the unit is constant,
  - (iii) the load available may not be as steady as is desired for accurate observations.

Hence due to these reasons it is essential to make a turbine model, which is geometrically similar for carrying out the tests in order to predict the behaviour and working conditions of the full sized turbine. The actual turbine and the model are identical in shape and all parts viz., spiral casting, guide mechanism, runner and draft tube of the one are geometrically similar to the corresponding parts of the other.

Model should be of a suitable size so that it can be conveniently tested in the laboratory. Too small a model may be difficult to manufacture accurately and also it may not give accurate results. It is therefore desirable to make a turbine model having an output of not less than 5 BHP and not more than 50 BHP. Diameters of runners of model turbines vary from 250 to 600 mm and the test head varies from 4 m to 100 m.

Hence model tests as a substitute for tests in the field are justified, on the ground of accuracy, comprehensiveness and convenience.



**9.2 Similarity Considerations of Model and Prototype Turbines**—To ensure similarity in the model and prototype flows in hydraulic turbines, and kinetic energy exchange it is necessary to perform tests based on the similarity laws *viz*, geometric kinematic and dynamic given below which are based on discussions of Chapter 3.

(a) **Geometric Similarity** implies that the model must be similar to the prototype turbine *i.e.*, all linear dimensions of the two machines compared are proportional and the *angles* between the corresponding elements are *equal*.

Taking runner exit and entrance diameters to be proportional—

$$\left( \frac{D_2}{D_1} \right)_m = \left( \frac{D_2}{D_1} \right)_p \quad \dots (9.1)$$

(b) **Kinematic Similarity** requires the similitudes of flow fields in the passages of model and prototype turbines. It means that velocities at the corresponding points in flows compared have to be proportionate and have the *same directions*. In hydraulic turbines kinematic similitude expresses the similarity of velocity triangles at corresponding points of flow. If the velocity triangles are drawn by using non-dimensional velocity coefficients then these are congruent. Thus for inlet velocity triangles—

$$(K_{u_1})_m = (K_{u_1})_p \quad \dots (9.2a)$$

$$(K_{v_1})_m = (K_{v_1})_p \quad \dots (9.2b)$$

$$(K_{v_{u_1}})_m = (K_{v_{u_1}})_p \quad \dots (9.2c)$$

$$(K_{v_{m_1}})_m = (K_{v_{m_1}})_p \quad \text{etc.,} \quad \dots (9.2d)$$

(c) **Dynamic Similarity** demands the proportionality of forces acting on the streamlined elements of turbine passage *e.g.*, proportionality of the forces acting on the blades of the two similar turbines *i.e.*, model and prototype.

$$P_m = P_p \quad \dots (9.3)$$

The forces which act simultaneously are the inertia, gravity, pressure and friction forces in such cases. They include body and surface forces. Since it is not possible to consider all the forces together, the proportionality of those forces which predominate in the flows and influence sufficiently the turbine performance characteristics, are ensured (refer Author's book on Hydraulics and Fluid Mechanics, Chapter on Dimensional Analysis).

For complete dynamic similarity the relative losses in the flows and efficiencies should be same which is difficult to realise in practice.

The conditions required for the fulfilment of the dynamic similarity are that the dimensionless numbers *viz.*, Reynolds number  $R_e$ , Froude number  $F_r$ , Euler number  $E_u$  and Weber number  $W$  (refer Author's book on Hydraulics and Fluid Mechanics, Chapter on Dimensional Analysis) are equal for model and prototype water turbines. In addition to these the Strouhal number (refer Art 9.5a and Eqn 9.15) will also be equal.

**9.3 Incomplete Similarity in Turbine Flows**—In practice it is difficult to provide neither complete geometric nor mechanical similarity. (refer Art 3.12). For example it is practically not possible to keep the strict similarity based on Reynolds' and Froude's numbers simultaneously. If  $R_e$  is same for model and prototype.

$$v_m > v_p \quad \text{as } (D_1)_m < (D_1)_p \quad (D_1)_m < (D_1)_p$$

On the other hand if  $F_r$  is same, than  $v_m < v_p$  as. It is for these reasons that only one of the foregoing criteria for similarity can be satisfied at a time. For example when testing low head Kaplan turbine generally  $F_r$  is required to be equal for its model and prototype whereas in case of Francis turbine it is  $R_e$  which is taken into account.

Further, some criteria of similarity in the flow have to be fulfilled because of their essential influence on efficiency and cavitation performance characteristics. These are discharge  $Q$ , power output  $P_i$ , speed of rotation  $N$ , efficiency  $\eta$ , cavitation parameters etc. The two efficiency and cavitation parameters are determined mainly by the distribution of pressure on the blades. The similarity of pressure distribution in the passage of geometrically similar turbines is possible when Euler number  $E_u$  is constant. Euler number is given by

$$E_u = \frac{\rho \cdot v^2}{p} \approx \frac{\rho \cdot Q^2}{\gamma H \cdot D_1^4} \approx \frac{Q_{11}^2}{g} \quad \dots (9.4)$$

where  $H$  is the head working on the turbine.

$$v = \frac{4Q}{\pi D_1^2} \text{ is the average characteristic velocity of the flow in the turbine runner.}$$

The basic parameters of a turbine  $H$ ,  $Q$  and  $D_1$  are connected to form a new parameter known as *specific flow*  $Q_{11}$  and

$$Q_{11} = \frac{Q}{D_1^2 H^{\frac{1}{2}}} \quad (\text{refer Eqn 3.6 and Art 3.4})$$

which is equal to the discharge through turbine having unit (i.e., one metre) runner diameter and working under unit (i.e., one metre) head.

$$\begin{aligned} \text{The dimensions of specific flow } Q_{11} &= \frac{Q}{D_1^2 H^{\frac{1}{2}}} = \frac{\text{m}^3}{\text{sec. m}^2 \text{m}^{\frac{1}{2}}} \\ &= \text{m}^{\frac{1}{2}} \text{ per sec.} \end{aligned}$$

**9.4 Conditions of Similarity for Turbine Model**—Summing up the various considerations from the previous articles, the following conditions will be required to be satisfied for making models of the turbines.

$$(a) \text{ From Eqn 9.1} \quad \left( \frac{D_2}{D_1} \right)_m = \left( \frac{D_2}{D_1} \right)_p$$

$$\text{Also, since } \frac{D_2}{D_1} = \frac{K_{u_2}}{K_{u_1}} \text{ i.e., speed ratios are proportionate,}$$

$$\left( \frac{K_{u_2}}{K_{u_1}} \right)_m = \left( \frac{K_{u_2}}{K_{u_1}} \right)_p \quad \dots (9.5)$$

(b) From the velocity triangles and Eqn 9.2 (a) the speed ratios for the two similar turbines are constant.

$$(K_{u_1})_m = (K_{u_1})_p \quad \dots (\text{refer 9.2a})$$

(c) Since the Euler number of the two similar turbines is constant, the specific flows (refer Eqn 9.4) or the rates of flow per unit head and per unit inlet runner diameter should be same for the model and its prototype.

$$\text{Thus} \quad (Q_{11})_m = (Q_{11})_p \quad \dots (9.6)$$

$$\text{where} \quad Q_{11} = \frac{Q}{D_1^3 \cdot \sqrt{H}} \quad (\text{refer Eqn 3.6})$$

(d) Theoretical specific speeds of two similar turbines are equal. The specific speed depends upon the brake horsepower (*BHP*) which is obtained by multiplying the available power by the turbine overall efficiency ( $\eta_t$ ). Now,  $\eta_t$  of model and that of prototype are not exactly the same, as proved below. Therefore strictly speaking the specific speed of model and prototype should not be equal. If it is assumed that the overall efficiencies of model and prototype are equal, then it can be said that theoretical specific speeds must be equal.

$$(N_{s_{th}})_m = (N_{s_{th}})_p \quad \dots (9.7)$$

$$\text{or} \quad (N_s)_m = (N_s)_p \left( \frac{\eta_m}{\eta_p} \right)^{\frac{5}{4}} \quad \dots (9.8)$$

If instead of specific speed, the non-dimensional factor  $K_s$  (refer Art 4.19) is used, then

$$(K_s)_m = (K_s)_p \quad \dots (9.9)$$

since  $K_s$  does not contain the efficiency parameter.

(e) The efficiencies  $\eta_H$  and  $\eta_Q$  for the two similar turbines should theoretically be equal. In practice, however, it is not possible to realise, since the frictional factor of the turbine is different from that of its model.

Usually  $f_m > f_p$  also relative roughness  $\delta_m > \delta_p$ , and  $(R_s)_m < (R_s)_p$  because of the limitation of the size of model and the rate of flow.

$$\eta_{H_m} < \eta_{H_p} \quad \dots (9.10)$$

Since the leakage loss in the model is relatively more than in its prototype, then

$$(\eta_Q)_m < (\eta_Q)_p \quad \dots (9.11)$$

Similarly the mechanical losses of the model are relatively more than those of the full size unit,

$$\therefore (\eta_{mech})_m < (\eta_{mech})_p \quad \dots (9.12)$$

$$\text{Hence} \quad (\eta_t)_m < (\eta_t)_p \quad \dots (9.13)$$

**9.5 Conditions for Similarity of Inertia Forces in Unsteady Flows—Strouhal Number**—In the previous articles the similarity conditions have been derived assuming the flow to be steady in which the average magnitudes of velocity, pressure etc., do not depend on time. However, in practice the flow through the turbine is unsteady. In the unsteady absolute flow the resultant acceleration of a particle (or force per unit mass) is a function of coordinates and time. For such cases Strouhal number for the model and prototype is taken as constant. It is denoted by  $Sh$  and where

$$Sh = \left[ \frac{\frac{v^2}{L}}{\frac{v}{T}} \right]_m = \left[ \frac{\frac{v^2}{L}}{\frac{v}{T}} \right]_p$$

$$\left( \frac{vT}{L} \right)_m = \left( \frac{vT}{L} \right)_p = Sh = \text{constant} \quad \dots (9.14)$$

$T$  is the characteristic interval of time for the motion considered and  $D_1$  is the inlet runner diameter which is of linear dimension  $L$ . At a point in the turbine runner flow fixed with respect to the absolute system of coordinates, the flow is periodical with the time interval  $T = \frac{1}{N}$ , where  $N$  is the speed of the runner in rpm.

Since the flow in a turbine runner is essentially unsteady, the Strouhal's number is an important criterion as the similarity of inertia forces is maintained by the equality of Strouhal's number.

$$Sh = \frac{vT}{D_1} = \frac{K_{v_1} \cdot \sqrt{2gH}}{D_1 N}$$

or  $Sh \propto \frac{\sqrt{H}}{D_1 N} = \frac{1}{N_{11}} \quad \dots(9.15)$

where  $N_{11}$  is the speed of turbine runner per unit (or 1 m) head and per unit (or 1 m) inlet diameter.

or  $N_{11} = \frac{ND_1}{\sqrt{H}} \text{ m}^{\frac{1}{2}}/\text{sec} \quad \dots(9.15a)$

Thus  $(N_{11})_m = (N_{11})_p \quad \dots(9.16)$

Therefore the speed of the model may be determined by Eqn 9.16 instead of Eqn 9.2(a). Eqn 9.16 can also be derived from Eqn 9.2(a) as given below :

$$(K_{u_1})_m = (K_{u_1})_p$$

or  $\left( \frac{u_1}{\sqrt{2gH}} \right)_m = \left( \frac{u_1}{\sqrt{2gH}} \right)_p$

or  $\left( \frac{\pi D_1 N}{60 \sqrt{2gH}} \right)_m = \left( \frac{\pi D_1 N}{60 \sqrt{2gH}} \right)_p$

or  $\left( \frac{D_1 N}{\sqrt{H}} \right)_m = \left( \frac{D_1 N}{\sqrt{H}} \right)_p$

or  $(N_{11})_m = (N_{11})_p$

The Strouhal's number is used when there are vibrations due to the unsteady flow, otherwise the velocity ratio (refer Eqn 9.2a) is employed.

**9.6 Correction Factors due to Scale Effect**—As stated above that in practice, it is not possible to maintain the Reynolds' number of the model and its prototype as constant. Perhaps it might be possible to a certain extent if the size of the model is greater in which case it is expensive because of large model diameter and high testing head. But the model investigations are carried out with a smaller size where  $(R_s)_m < (R_s)_p$  in which case the complete geometrical similarity will not be maintained. This is due to the fact that relative roughness in both cases will be different (i.e.,  $\delta_m > \delta_p$ ). Thus friction factor  $f_m > f_p$  with which the relative losses of energy of model are more. Finally the efficiency  $\eta_m < \eta_p$ . In order to avoid or rather decrease the influence of  $R_s$  on efficiency, the model experiments are carried with very high value of  $R_s$  or  $R_s > 10^6$ .

The difference in efficiency of model and prototype is taken into account by making *necessary correction* in the formulae already derived.

As stated above, this is necessary due to the fact that the size of the model is much less. This is known as *scale effect*. Then the above Eqns 9.16 and 9.6 are modified as follows—

$$\frac{(N_{11})_p}{(N_{11})_m} = \sqrt{\frac{\eta_p}{\eta_m}} \quad \dots(9.17)$$

and 
$$\frac{(Q_{11})_p}{(Q_{11})_m} = \sqrt{\frac{\eta_p}{\eta_m}} \quad \dots(9.18)$$

where 
$$N_{11} = \frac{ND_1}{\sqrt{\eta_h \cdot H}} \quad \dots(9.19)$$

and 
$$Q_{11} = \frac{Q}{\sqrt{H}} \quad \dots(9.20)$$

In these equations  $\eta_h$  is the hydraulic efficiency.

The turbine overall efficiency in terms of  $Q_{11}$  and  $N_{11}$  is given in Fig 9.1.

Further 
$$u_1 = K_{u_1} \cdot \sqrt{2gH\eta_h} = \frac{\pi D_1 N}{60}$$

If  $D_1 = 1$  m and  $H = 1$  m,

$$N_{11} = \frac{ND_1}{\sqrt{H}} = 60 K_{u_1} \cdot \sqrt{2g\eta_h} \quad \dots(9.21)$$

The rate of flow through the runner

$$Q = \text{area} \times \text{relative velocity of flow} = \left( K \cdot \frac{\pi}{4} D_1^2 \right) w$$

where  $K$  = percentage of net flow area between the blades.

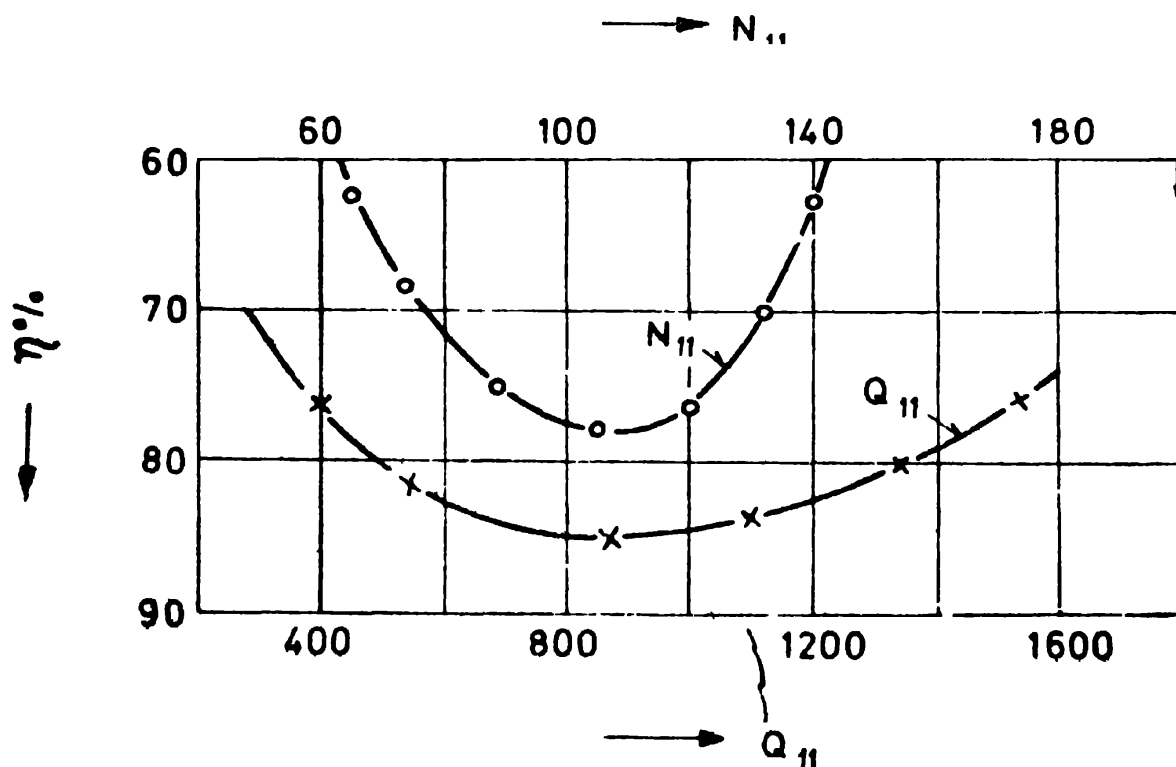


Fig 9.1  $\eta$  in % versus  $Q_{11}$  and  $N_{11}$  both in  $m^{1/2}/sec$

Let  $K' = K \cdot \frac{\pi}{4},$

$K_w =$  relative velocity coefficient,

then  $Q = \frac{K' D_1^3 \cdot K_w \cdot \sqrt{2gH\eta_h}}{\eta_Q}$

where  $\eta_Q$  is the volumetric efficiency

$$\therefore Q_{11} = \frac{Q}{D_1^3 \sqrt{H}} = \frac{K' \cdot K_w \cdot \sqrt{2g\eta_h}}{\eta_Q} \quad \dots(9.22)$$

and turbine specific power

$$\begin{aligned} P_{11} &= \frac{P}{H^{\frac{3}{2}} D_1^3} = \frac{\gamma Q \cdot H \eta_t}{75 H^{\frac{3}{2}} \cdot D_1^3} = \frac{\gamma Q \cdot \eta_t}{75 H^{\frac{1}{2}} D_1^3} \\ &= \frac{\gamma Q_{11} \cdot \eta_t}{75} = \frac{\gamma K' \cdot K_w \sqrt{2g\eta_h}}{75 \eta_Q} \eta_t \quad \dots (9.23) \end{aligned}$$

**9.7 Relation Between the Characteristic Data of a Turbine and that of its Model**—Let the scale of model be  $n$ . Then the full size turbine is  $n$  times bigger than the model.

And  $n = \frac{D_{1p}}{D_{1m}}$ . Also sometimes  $n = \frac{H_p}{H_m}$

The following quantities are measured and calculated from the tests.

$$H_m, Q_m, N_m, P_{a_m}, P_{t_m} \text{ and } \eta_{t_m}$$

From these, the corresponding values for actual turbine can be obtained as hereunder provided the efficiency of model is same as that of prototype. For similar turbines, the speed ratios taken from velocity triangles are equal.

(i) **Speed of Turbine :**

$$\therefore (K_{u1})_p = (K_{u1})_m$$

or  $\frac{\pi \cdot D_{1p} \cdot N_p}{60 \cdot \sqrt{2g} \cdot \sqrt{H_p}} = \frac{\pi \cdot D_{1m} \cdot N_m}{60 \cdot \sqrt{2g} \cdot \sqrt{H_m}}$

or  $N_p = N_m \cdot \frac{D_{1m}}{D_{1p}} \cdot \sqrt{\frac{H_p}{H_m}} \quad \dots(9.24)$

(ii) **Rate of Flow :**

For similar turbines, the specific flows are equal

$$\therefore (Q_{11})_p = (Q_{11})_m$$

or  $\frac{Q_p}{D_{1p}^3 \cdot \sqrt{H_p}} = \frac{Q_m}{D_{1m}^3 \cdot \sqrt{H_m}}$

or  $Q_p = Q_m \cdot \frac{D_{1p}^3}{D_{1m}^3} \cdot \sqrt{\frac{H_p}{H_m}} \quad \dots(9.25)$

(iii) **Available Power :**

and  $(P_a)_p = \gamma_p \cdot Q_p \cdot H_p / 75$   
 $(P_a)_m = \gamma_m \cdot Q_m \cdot H_m / 75$

$$\therefore \frac{(P_a)_p}{(P_a)_m} = \frac{\gamma_p}{\gamma_m} \cdot \frac{Q_p}{Q_m} \cdot \frac{H_p}{H_m}$$

If same fluid is used, i.e.,  $\gamma_a = \gamma_m$

$$\therefore \frac{(P_a)_p}{(P_a)_m} = \frac{Q_p}{Q_m} \cdot \frac{H_p}{H_m} = \frac{D_{1p}^2}{D_{1m}^2} \cdot \left( \frac{H_p}{H_m} \right)^3 \quad \dots(9.26)$$

Thus specific powers are equal.

(iv) **Brake Horse Power :**

In general  $P_t = P_a \cdot \eta_t = P_a \cdot \eta_H \cdot \eta_Q \cdot \eta_{mech}$

$$\begin{aligned} \therefore \frac{(P_t)_p}{(P_t)_m} &= \frac{P_{a_p}}{P_{a_m}} \cdot \left( \frac{\eta_{H_p}}{\eta_{H_m}} \right) \cdot \left( \frac{\eta_{Q_p}}{\eta_{Q_m}} \right) \cdot \left( \frac{\eta_{mech_p}}{\eta_{mech_m}} \right) \\ &= \frac{D_{1p}^3}{D_{1m}^3} \cdot \left( \frac{H_p}{H_m} \right)^{\frac{3}{2}} \cdot \left( \frac{\eta_{H_p}}{\eta_{H_m}} \right) \cdot \left( \frac{\eta_{Q_p}}{\eta_{Q_m}} \right) \cdot \left( \frac{\eta_{mech_p}}{\eta_{mech_m}} \right) \end{aligned} \quad \dots(9.27)$$

(v) **Efficiency :** Efficiency of the turbine is usually calculated from the model efficiency by means of empirical relations. The following relations are in vogue today :

**Head Efficiency :**

(a) Dr. J. Ackeret (Switzerland), 1930, formula—

$$\eta_{H_p} = 1 - (1 - \eta_{H_m}) \left\{ 0.5 - 0.5 \left( \frac{R_{e_m}}{R_{e_p}} \right)^{\frac{1}{5}} \right\} \quad \dots(9.28)$$

(b) Prof. Moody (USA), 1942, modified formula—

$$\eta_{H_p} = 1 - (1 - \eta_{H_m}) \left( \frac{D_{1m}}{D_{1p}} \right)^{\frac{1}{5}} \quad \dots(9.29)$$

(c) Dr. R. Gregorig (Switzerland), 1933, formula—

$$\eta_{H_p} = 1 - (1 - \eta_{H_m}) \left( \frac{R_{e_m}}{R_{e_p}} \right)^{\frac{1}{4}} - K_{v_{m_3}} \left\{ 1 - \left( \frac{R_{e_m}}{R_{e_p}} \right)^{\frac{1}{4}} \right\} \quad \dots(9.30)$$

(d) Dr. S.P. Hutton\* (England), 1954, formula—

$$\eta_{H_p} = 1 - (1 - \eta_{H_m}) \left\{ 0.3 - 0.7 \left( \frac{R_{e_m}}{R_{e_p}} \right)^{\frac{1}{5}} \right\} \quad \dots(9.31)$$

Ackeret and Hutton formulae would be more useful for Kaplan turbines. In the above formulae—

$$R_e = \frac{\sqrt{2gH} \cdot D_1}{\nu} \quad \text{where } \nu = \text{kinematic viscosity,}$$

and  $K_{v_{m_3}} = \frac{v_{m_3}}{\sqrt{2gH}}$  where  $v_{m_3}$  = velocity of flow of water at the outlet of draft tube.

**Leakage Losses†**

$$\eta_Q = 1 - \frac{\Delta Q}{Q}$$

\* I. Mech. E. Proceedings 1954—Vol. 168, No. 28.

† I. Mech. E. Proceedings 1969-70—Vol. 184, Part I, No. 20.

Mosony Formula

$$\Delta Q = \mu \sqrt{2gHD_1} \pi S \quad \dots(\mu = 0.9) \quad \dots(9.32)$$

where  $S$  = clearance width between inner blades and turbine casing.

$$\eta_{Q_p} = 1 - (1 - \eta_{Q_m}) \left( \frac{D_{1_m}}{D_{1_p}} \cdot \sqrt{\frac{H_m}{H_p}} \right)^{0.2} \quad (9.33)$$

For overall efficiency†

Dubs-Moody Formula

$$\eta_{t_p} = \left\{ 1 - (1 - \eta_{t_m})^2 \sqrt{\frac{D_{1_m}}{D_{1_p}}} \right\}^2 \quad \dots(9.34)$$

**Problem 9.1** A turbine model having the following specifications was made to predict the behaviour of the actual turbine :

$$P_t = 15.8 \text{ HP} \quad H = 12 \text{ m} \quad N = 474 \text{ rpm}$$

Model scale  $n = 10$

Determine the speed of the actual turbine runner and its power developed if the overall efficiencies of the turbine and its model are equal. Find the type of turbine of which the model was made.

**Solution**

$$\text{Model scale } n = \frac{D_{1_p}}{D_{1_m}} = 10$$

where 'p' and 'm' denote prototype turbine and model turbine respectively.

Assuming the ratio of heads of prototype turbine and that of model are equal to scale ratio,

$$\text{then, } H_p = 10 \times H_m = 10 \times 12 = 120 \text{ m} \quad \text{Answer}$$

(a) As the overall efficiencies of the turbine and its model are same, their specific powers must be equal.

$$(P_{t_{11}})_p = (P_{t_{11}})_m$$

$$\text{or } \frac{P_{t_p}}{D_{1_p}^3 \cdot H_p^{\frac{3}{2}}} = \frac{P_{t_m}}{D_{1_m}^3 H_m^{\frac{3}{2}}}$$

$$\begin{aligned} \therefore P_{t_p} &= P_{t_m} \left( \frac{D_{1_p}}{D_{1_m}} \right)^3 \left( \frac{H_p}{H_m} \right)^{\frac{3}{2}} \\ &= P_{t_m} \cdot n^3 \cdot n^{\frac{3}{2}} = P_{t_m} \cdot n^{\frac{7}{2}} \\ &= 15.8 \times 10^{\frac{7}{2}} = 15.8 \times 1,000 \times 3.16 \\ &= 50,000 \text{ HP} \quad \text{Answer} \end{aligned}$$

(b) For similar turbines the speed ratios are equal

$$\therefore (K_{u_1})_p = (K_{u_1})_m \quad \dots(\text{refer Eqn 9.2a})$$

$$\text{or } \frac{\pi \cdot D_{1_p} \cdot N_p}{60 \cdot \sqrt{2g} \cdot \sqrt{H_p}} = \frac{\pi \cdot D_{1_m} \cdot N_m}{60 \cdot \sqrt{2g} \cdot \sqrt{H_m}}$$

† I. Mech. E. Proceedings 1969-70—Vol. 184, Part I, No. 20.



or

$$N_p = N_m \times \frac{D_{1m}}{D_{1p}} \times \sqrt{\frac{H_p}{H_m}} = N_m \cdot \frac{\sqrt{n}}{n}$$

...(refer Eqn 9.24)

$$= \frac{N_m}{\sqrt{n}} = \frac{474}{\sqrt{10}} = \frac{474}{3.16} = 150 \text{ rpm Answer}$$

(c) Specific speed of any of the turbines :

$$N_{s,m} = N_{s,p} = \frac{150 \times \sqrt{50,000}}{120^{\frac{3}{4}}} = \frac{150 \times 223.5}{120 \times 3.31} = 84.5$$

A the turbine is of **Kaplan** type Answer

**Problem 9.2** The runner of a Pelton turbine installed at Tata Hydro-Electric Co. Ltd., Khapoli, was built according to the following specifications :

$$P_t = 17,100 \text{ HP}$$

$$H = 515 \text{ m}$$

$$Q = 2,940 \text{ lit/sec}$$

$$N = 300 \text{ rpm}$$

Find the specifications of a one-ninth scale model of this turbine, assuming the efficiencies of the two turbines to be the same.

**Solution**

$$\text{Model scale } n = 9 = \frac{D_{1p}}{D_{1m}} \quad \text{Assume } \frac{H_p}{H_m} = n = 9$$

(a) Head under which the model turbine would work :

$$\frac{H_p}{H_m} = \frac{515}{H_m} = 9$$

$$H_m = \frac{515}{9} = 57.2 \text{ m Answer}$$

**Note :** This head for a model turbine is just an assumption.

This may be different depending upon the site conditions. In that case the speed, discharge and power for the model calculated below will be different.

(b) Speed of the model turbine :

$$(K_{u_1})_p = (K_{u_1})_m \quad \text{...(refer Eqn 9.2a)}$$

$$\therefore N_m = N_p \cdot \frac{D_{1p}}{D_{1m}} \cdot \left( \frac{H_m}{H_p} \right)^{\frac{1}{2}}$$

$$= N_p \cdot \frac{n}{\sqrt{n}} = N_p \cdot \sqrt{n} = 300 \times \sqrt{9}$$

$$= 900 \text{ rpm Answer}$$

(c) Rate of flow for the model turbine :

$$(Q_{11})_p = (Q_{11})_m \quad \text{...(refer Eqn 9.6)}$$

$$\text{or } \frac{Q_p}{d_{1p}^3 \cdot \sqrt{H_p}} = \frac{Q_m}{d_{1m}^3 \cdot \sqrt{H_m}}$$

$$\therefore Q_m = Q_p \cdot \frac{d_{1p}^3}{d_{1m}^3} \cdot \frac{H_m^{\frac{1}{2}}}{H_p^{\frac{1}{2}}} = Q_p \cdot \left( \frac{1}{n} \right)^3 \left( \frac{1}{n} \right)^{\frac{1}{2}}$$

$$= \frac{Q_p}{n^{\frac{1}{2}}} = \frac{2,940}{9^{\frac{1}{2}}} = \frac{2,940}{81 \times 3}$$

$$= 12.1 \text{ lit/sec} \quad \text{Answer}$$

(d) Power developed by the model turbine

$$(P_{t_{11}})_p = (P_{t_{11}})_m \quad \text{or} \quad \frac{P_{t_p}}{d_{1_p}^3 \cdot H_p^{\frac{3}{2}}} = \frac{P_{t_m}}{d_{1_m}^3 \cdot H_m^{\frac{3}{2}}}$$

$$\therefore P_{t_m} = P_{t_p} \left( \frac{d_{1_m}}{d_{1_p}} \right)^3 \left( \frac{H_m}{H_p} \right)^{\frac{3}{2}} = P_{t_p} \cdot \frac{1}{n^{\frac{3}{2}}}$$

$$= \frac{17,000}{9^{\frac{3}{2}}} = \frac{17,100}{81 \times 27} = 7.82 \text{ HP} \quad \text{Answer}$$

$\therefore$  Specifications of the turbine model—

$$\left. \begin{array}{ll} P_t = 7.82 \text{ HP}, & H = 57.2 \text{ m}, \\ Q = 12.1 \text{ lit/sec}, & N = 900 \text{ rpm} \end{array} \right\} \text{Answer}$$

**Problem 9.3** A model turbine is provided with a runner 2 m in diameter. It is found to develop 64 HP under a head of 100 m when running at a speed of 4,000 rpm. Calculate the specific speed and unit speed for this model. It is required to build a similar turbine to develop 200 HP under a head of 120 m. Calculate the required diameter. Prove any formula used.  
(Panjab University)

**Solution**

$$\begin{array}{lll} D_m = 2 \text{ m} & H_m = 100 \text{ m} & H_p = 120 \text{ m} \\ P_{t_m} = 64 \text{ HP} & P_{t_p} = 200 \text{ HP} & N_m = 4,000 \text{ rpm} \end{array}$$

$$\text{Specific speed of model} \quad N_{s_m} = \frac{N_m \cdot \sqrt{P_{t_m}}}{H_m^{\frac{5}{4}}}$$

$$= \frac{4,000 \times \sqrt{64}}{100^{\frac{5}{4}}} = 101.3 \text{ Units} \quad \text{Answer}$$

$$\text{Unit speed of model,} \quad N_{1_m} = \frac{N_m}{\sqrt{H_m}} = \frac{4,000}{\sqrt{100}}$$

$$= 400 \text{ Units} \quad \text{Answer}$$

Now, the specific powers of the similar turbines are equal,

$$\therefore \frac{P_{t_p}}{D_p^3 \cdot H_p^{\frac{3}{2}}} = \frac{P_{t_m}}{D_m^3 \cdot H_m^{\frac{3}{2}}}$$

$$\text{or} \quad D_p = \sqrt[3]{D_m^3 \cdot \left( \frac{H_m}{H_p} \right)^{\frac{3}{2}} \left( \frac{P_{t_p}}{P_{t_m}} \right)}$$

$$= \sqrt[3]{2^3 \times \left( \frac{100}{120} \right)^{\frac{3}{2}} \times \frac{200}{64}}$$

$$= 3.09 \quad \text{Answer}$$

Refer Art 3.3 and 3.4 for the proofs of the formulae used above.

**Problem 9.4** A quarter scale turbine model is tested under a head of 36 m. The full scale turbine is required to work under a head of 100 m

and to run at 428 rpm. At what speed must the model be run and if it develops 135 HP and uses 0.324 m<sup>3</sup>/sec of water at this speed, what power will be admitted from the full scale turbine assuming that its efficiency is 3 per cent better than that of the model?

State the type of runner used in this turbine and explain briefly why the efficiency of the full scale turbine should be better than the efficiency of the model. (London University)

**Solution**

$$\text{Scale ratio } n = 4 = \frac{D_p}{D_m}$$

$$H_m = 36 \text{ m}$$

$$H_p = 100 \text{ m}$$

$$P_{t_m} = 135 \text{ HP}$$

$$N_p = 428 \text{ rpm}$$

$$Q_m = 0.324 \text{ m}^3/\text{sec}$$

$$\eta_a = \eta_m + 3\%$$

(a) Speed ratios of the model and the prototype are equal,

$$\therefore (K_{u_1})_m = (K_{u_1})_p \quad \dots(\text{refer Eqn 9.2a})$$

$$\begin{aligned} \text{or } N_m &= N_p \cdot \frac{D_p}{D_m} \sqrt{\frac{H_m}{H_p}} = 428 \times 4 \sqrt{\frac{36}{100}} \\ &= 1,027 \text{ rpm} \quad \text{Answer} \end{aligned}$$

(b) Efficiency of model and prototype :

$$P_{t_m} = \frac{\gamma \cdot Q_m \cdot H_m}{75} \cdot \eta_{t_m}$$

$$\text{or } \eta_{t_m} = \frac{75 \cdot P_{t_m}}{\gamma \cdot Q_m \cdot H_m} = \frac{75 \times 135}{1,000 \times 0.324 \times 36} = 0.87$$

$$\therefore \eta_{t_p} = 0.87 + 0.03 = 0.9 \text{ or } 90\%$$

Now specific flows are equal,

$$\therefore Q_p = Q_m \cdot \left( \frac{D_{t_p}}{D_{t_m}} \right)^2 \cdot \left( \frac{H_p}{H_m} \right)^{\frac{1}{2}} \quad \dots(\text{refer Eqn 9.25})$$

$$\begin{aligned} \text{or } Q_p &= 0.324 \times 4^2 \times \sqrt{\frac{100}{36}} \\ &= 8.65 \text{ m}^3/\text{sec} \end{aligned}$$

$\therefore$  Required output of full-scale turbine,

$$\begin{aligned} P_{t_p} &= \frac{\gamma Q_p \cdot H_p}{75} \cdot \eta_{t_p} \\ &= \frac{75 \times 8.65 \times 100}{75} \times 0.9 \\ &= 10,400 \text{ HP} \quad \text{Answer} \end{aligned}$$

(c) Specific speed

$$N_s = \frac{N_m \cdot \sqrt{P_{t_m}}}{H_m^{\frac{5}{4}}} = \frac{1,027 \times \sqrt{135}}{36^{\frac{5}{4}}} = 136$$

With  $N_s = 136$ , type of runner used is **Francis** Answer

(d) For explanation to  $\eta_{t_p} > \eta_{t_m}$ , refer to Art 9.13

## B. SELECTION OF HYDRAULIC TURBINES

**9.8 Preliminary Data**—There are three main types of hydraulic turbines viz. Pelton, Francis and Kaplan. Lately Deriaz and Bulb turbines are being used. Propeller turbines are also employed at some places. A hydraulic turbine is always selected and designed to match the specific conditions under which it has to operate in order to attain a high order of efficiency which is expected in present-day installations. The following preliminary data are required while selecting the different types of water power plants, which will ultimately effect the right type of selection of hydraulic turbine.

(a) *Selection of site* depends upon—

- (i) Availability of head and discharge ;
- (ii) Places of power shortage ;
- (iii) Power-deficient periods such as peak loads.

(b) *Field Information* depends upon—

- (i) Estimation of power available from rainfall and stream flow records ;
- (ii) Ultimate development of power—The power plants should not be constructed just to meet the immediate needs, but ultimate development of the place must be kept in mind ;
- (iii) Proportion of available potential that can be economically developed ;
- (iv) Supply of accurate field data.

*Improper and unsuitable selection* means—

- (i) High initial costs ;
- (ii) Low efficiency which will reduce the revenue thus impairing the earning repayment-ability of plant ;
- (iii) Unit may be such as has excessive operating cost and is difficult to control.

**9.9 Selection of Type of Hydraulic Turbine**—The following points are important for the selection of right type of hydraulic turbine which will be discussed separately :

- |                         |                                  |
|-------------------------|----------------------------------|
| (a) Specific Speed      | (f) Cavitation ,                 |
| (b) Rotational Speed    | (g) Disposition of turbine shaft |
| (c) Efficiency          | (h) Number of units              |
| (d) Part Load Operation | (i) Head.                        |
| (e) Overall Cost        |                                  |

The selection of turbine is made from two sets of curves. The first set (refer Fig 9.2) gives the value of specific speed of various types of turbines for a given head and the second set (refer Fig 9.3) shows the maximum efficiency which can be obtained for different values of specific speed.

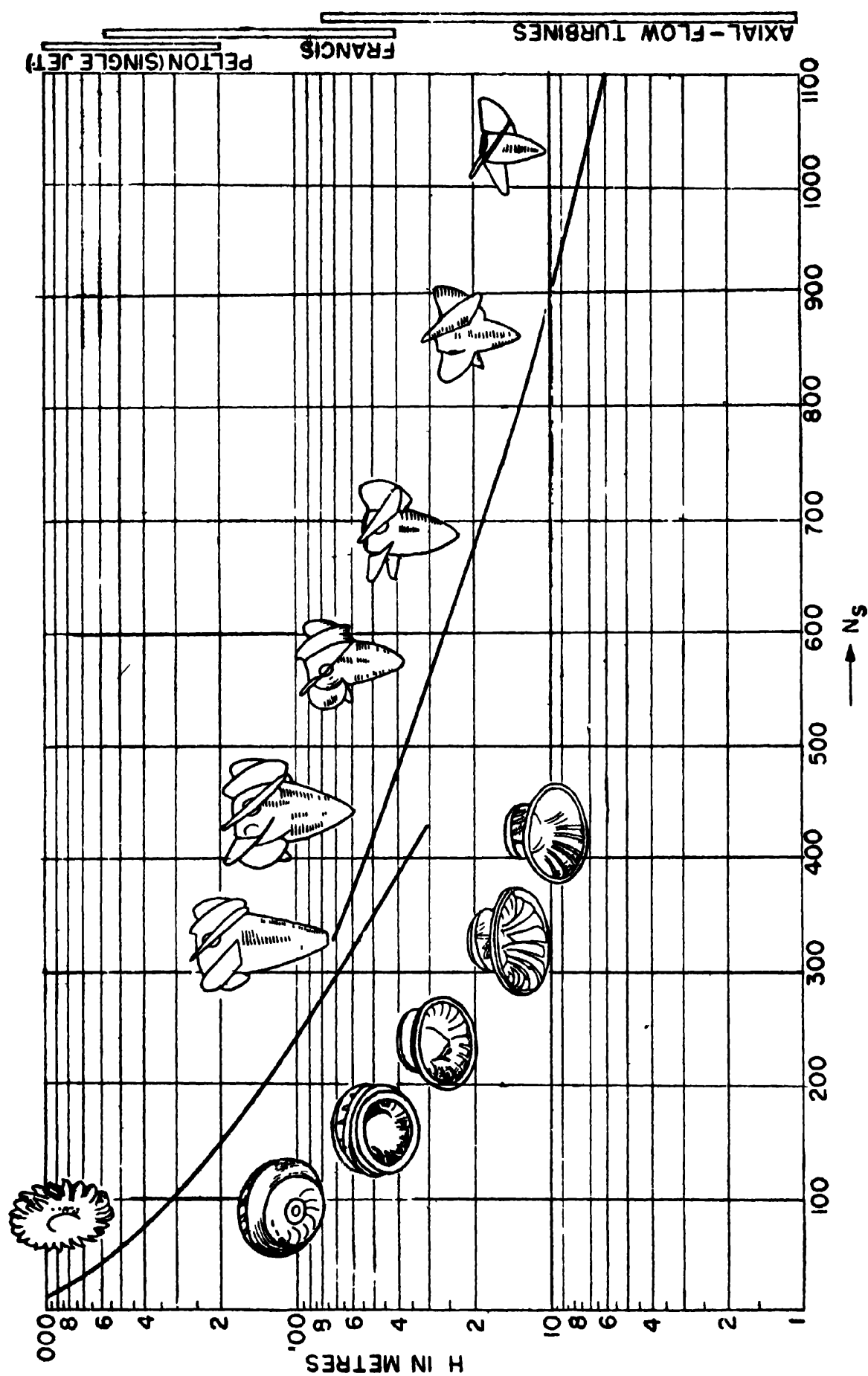


Fig. 9.2 Specific speed versus Head Adopted From CKD Blansko USSR

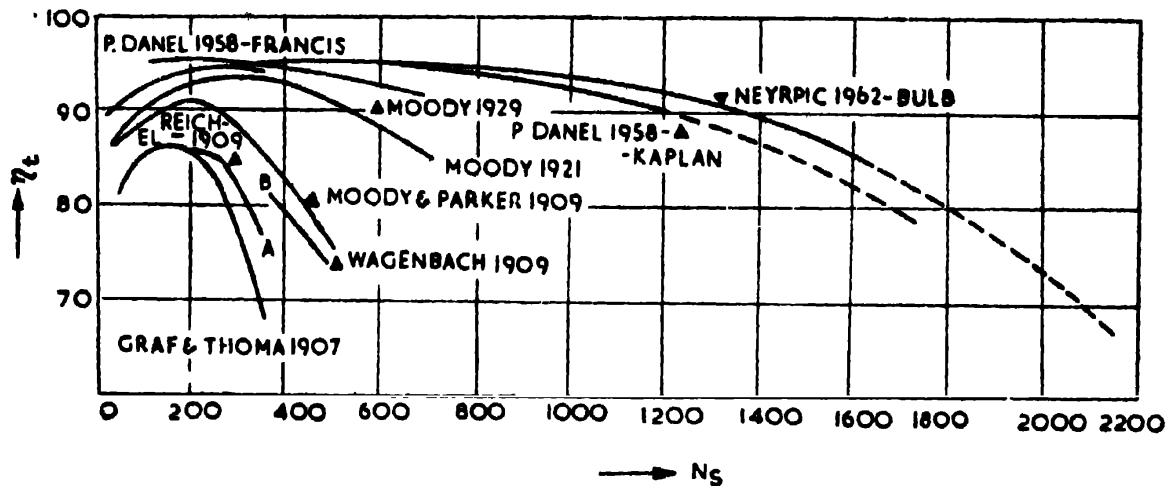


Fig 9.3 Specific-Speed versus Optimum Efficiency for Various Turbines

Further for the selection of number of jets for a Pelton turbine curves given in Fig 9.4 may be used. N.N. Kovalev gave the basis of selection

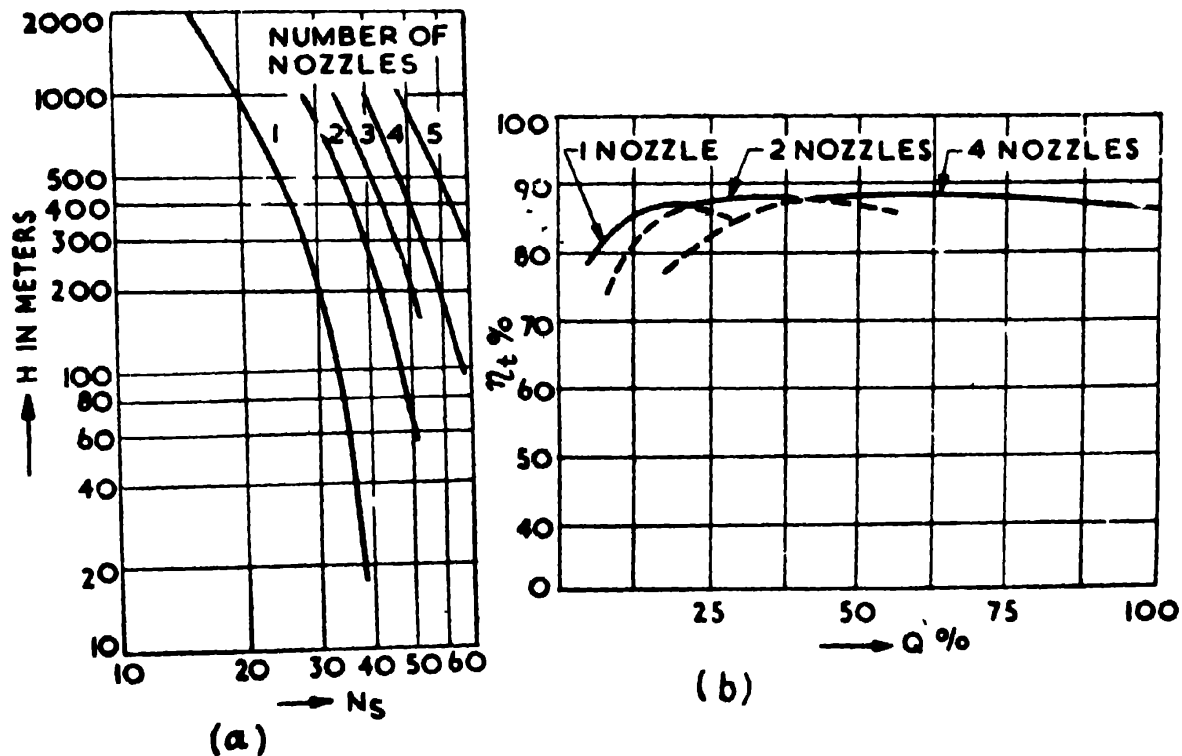


Fig 9.4 Determination of Number of Nozzles for Pelton Turbine

(a)  $N_s$  versus  $H$

(b)  $Q\%$  versus  $\eta_t\%$  for various Nozzles

of reaction turbines in his book on "Hydro Turbines—Design and Construction." Fig 9.5 taken from his book shows the output of Francis and Kaplan turbines for various values of head.

**9.10 Specific Speed**—From the field the main data available are—

(a) Head and (b) Discharge.

It has been found from experience that there is a range of head and specific speed at which each type of turbine is most suitable. This is given

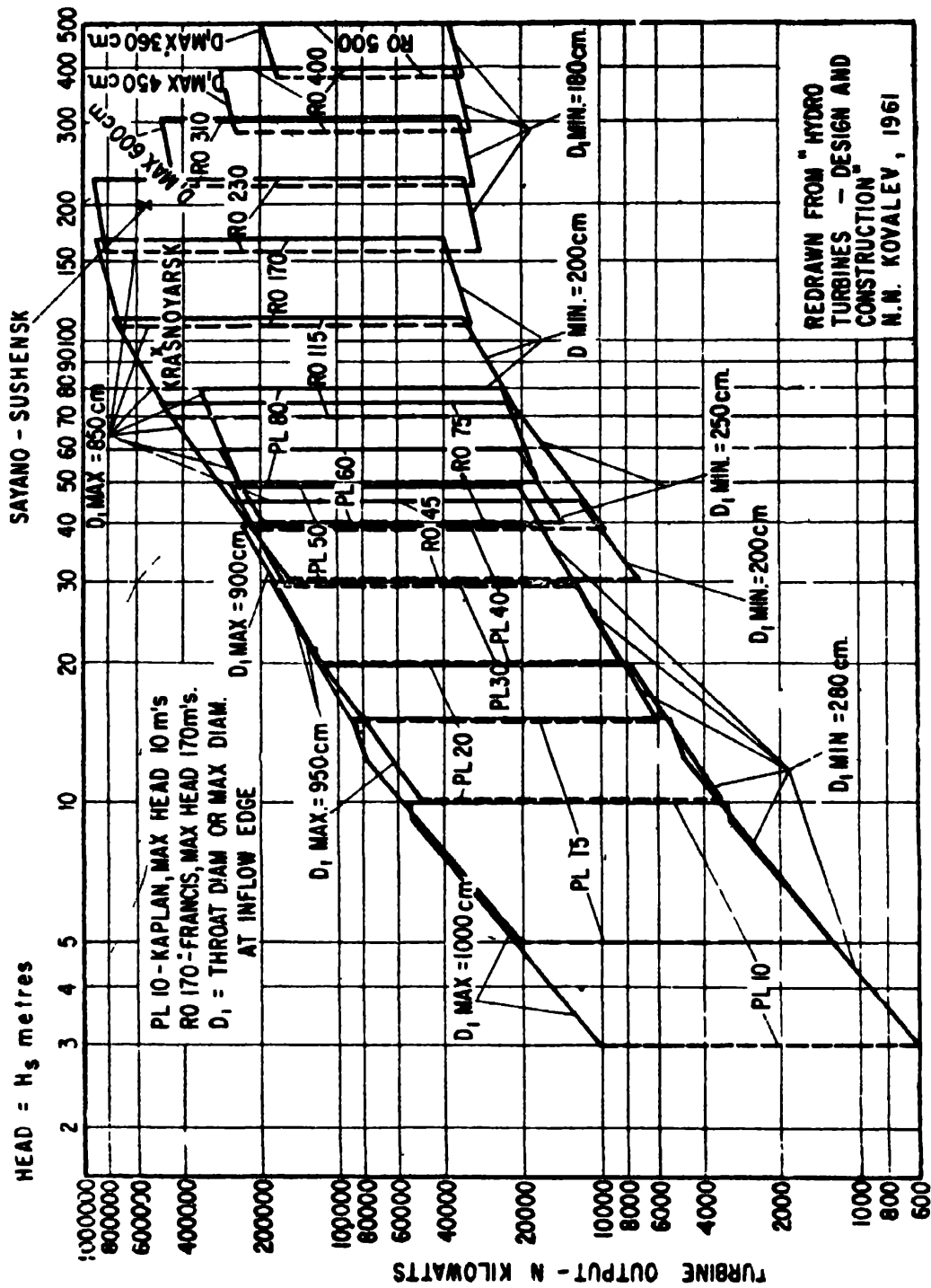


Fig 9.5 New Turbine Classification in USSR by N.N. Kovalev  
Redrawn from his Book on "Hydro Turbines—Design and Construction."

in Fig 9.1, representing  $N_s$  vs  $H$  which is drawn from experimental data for all types of water turbines. M/s Hitachi Ltd, Japan give the following empirical relation to find specific speed if the head is known. This is applicable for *reaction turbines* only.

$$N_s = \frac{13,000}{H+20} + 50 \quad \dots(9.35)$$

where  $H$  = head in metres

and  $N_s$  = specific speed in *m-KW*

i.e., in calculating  $N_s$ , the power to be substituted is in *KW* instead of HP.

From the USA practice the following empirical relation is derived in 1935 for reaction turbines.

$$N_s = \frac{2,080}{\sqrt{H}} \quad \dots (9.36)$$

where  $H$  and  $N_s$  are in MKS units.

The head  $H$  being known from the field data, an appropriate value of  $N_s$  can be determined from the above curves or Eqn 9.35. However, for a particular head, different values of specific speed may be available. The selection of right type of specific speed is made from the curve  $\eta_r$  versus  $N_s$  which indicates the maximum efficiency for various values of specific speed. It is a common practice to select a high specific speed runner which is always economical because the size of the turbo-generator as well as that of the power house will become smaller.

High specific speed is essential where head is low and output (BHP) is large, because otherwise the rotational speed ( $N$ ) will be very low which means cost of turbo-generator and power houses will be high. On the other hand there is practically no need of choosing a high value of specific speed for high head installations, because even with low specific speed, high rotational speed can be attained with medium-capacity plants.

High specific speed means greater value of cavitation factor, explained later.

**9.7 Rotation Speed** depends on specific speed. From the equation of specific speed (refer Eqn 3.10) it is evident that the specific speed is directly proportional to the rotational speed of the turbine. Also the rotational speed of an electrical generator with which the turbine is to be directly coupled, depends on the frequency and number of pair of poles (refer Eqn 5.9). The value of specific speed adopted should be such that it will give the synchronous speed of the generator.

If the rotational speed is high, then for the same power the following considerations are important—

- (a) *Civil Engineering Considerations*—The hydraulic turbine will have a smaller size, which is less costly. Also the size of the generators is small, because the number of pair of poles is less. Smaller units will occupy less space and the installation and excavation costs are less.
- (b) *Hydraulic Engineering Considerations*—Higher specific speed turbine is generally more liable to cavitation, thus involving deeper excavation and consequently foundations.
- (c) *Mechanical Engineering Considerations*—Mechanical design is influenced by centrifugal forces which are set up at higher speed



in the revolving parts, which will have to stand more severe stresses.

- (d) *Metallurgical Considerations*—Suitable material may have to be manufactured by special metallurgical processes.

**9.12 Efficiency**—The turbine selected should be such that it gives the highest overall efficiency for various operating conditions. The turbine is normally designed to give maximum efficiency at the rated head which is estimated as annual mean net head available. The optimum turbine efficiency can be known from Fig 9.4 for a value of specific speed.

**9.13 Part-Load Operation**—The turbine may have to cope with considerable load variations. As the load deviates from normal working load the efficiency would also vary. In general the efficiency at part-loads and over-loads is less than normal. For the sake of economy the turbine should always run with maximum possible efficiency to get more revenue.

With part-load operations, the maintenance cost is also increased.

Deriaz turbine is employed instead of Francis turbine when the turbine has to run at part or overload conditions. Similarly Kaplan turbine will be useful for such purposes in place of propeller turbine for low heads.

**9.14 Overall Cost**—Cost is a major item. Cost may be subdivided broadly into initial cost and running cost. In the estimation of initial cost, both manufacture and installation should be kept in mind. The running cost includes the maintenance and overhead charges, the cost of operation, repair and replacement. The influence of speed, efficiency and part-load operations described above, is a guiding factor in the selection of water turbine.

**9.15 Cavitation**—Cavitation effects the installation of water turbines of reaction type, over the tail race level. The critical value of cavitation factor must be obtained to see that the turbine works in safe zone. Such a value of cavitation factor also effects the design of turbine, especially of Kaplan, propeller and bulb types.

High specific speed increases the value of cavitation factor. Therefore the specific speed and the suction head must have proper relation to the net working head, if satisfactory performance and life of vital parts are to be assured. The cavitation factor determines whether the turbine is to be placed above or below tail race water level. It is always advisable to avoid the undue deep excavation for the power house foundations, so that the cost may not be very high. If the turbine is installed below tail race level, the water has to be pumped out every time the inspection or repair is made. However for Kaplan or propeller turbine, in order to bring the working within safe zone, it becomes essential to install the runner below tail race level. It has been explained above that high specific speed is necessary to reduce the size of turbine, generator and power house, but on the other hand such a turbine has to be installed below tail race level. The experience has shown that the economy secured by increase in specific speed is so high especially in case of low head plants, where the size of machinery is relatively large, that it becomes necessary and advantageous to adopt high specific speed turbines and to sacrifice the easy access to runner.

**9.16 Disposition of Turbine Shaft**—A vertical shaft turbine will require deeper foundations and a high building. On the other hand horizontal shaft turbine will need greater floor area. Experience has shown

that the vertical shaft arrangement is better for large-sized reaction turbines, therefore it is almost universally adopted. In case of large size impulse turbines, horizontal-shaft arrangement is mostly employed.

In order to avoid deeper foundations and Z type flow occurring in Kaplan turbine, the application of bulb turbine is gaining ground for low head installations. The shaft of bulb unit is slightly inclined.

**9.17 Number of Units**—The following points must be borne in mind to determine whether or not a multi-unit plant is required—

(a) **Cost**—If the units are large, the cost per kilowatt for hydroelectric power of a given capacity does not vary much with the number of units. Generally speaking, the cost of power house and its foundations increases with the number of machines installed, but not in direct proportion. When more than one machine is installed they should be of the same capacity so that their parallel running is more stable and also some economy may be achieved by standardization of spare parts.

The running costs will be higher for a multi-plant.

(b) **Load Supply**—Due to the variation of load, the turbine may be required to operate at low efficiency. Multi-unit plants can efficiently meet large variations of load by varying the number of units in service.

The multi-unit plant is useful for peak load conditions.

For a plant belonging to inter-connected system the multi-unit plant is more economical.

**9.18 Head**—(a) **Very High Head (above 350 m and above)** : For heads greater than 350 m, Pelton turbine is generally employed and there is practically no choice except in very special cases. Whether a single jet or multijet Pelton turbine is employed can be seen from the curve (refer Fig 9.4)

(b) **High Head (150 m to 350 m)** : In this range either Pelton or Francis turbine may be employed. Fig 9.2 shows approximate values of specific speed ( $N_s$ ) against head ( $H$ ). The curve has been drawn for existing installations. For higher specific speeds Francis turbine is more compact and economical than the Pelton turbine which for the same working conditions would have to be much bigger and rather cumbersome.

For heads ranging from 250 to 350 m, discharge velocity of water is rather high and to restrict the loss of head, necessary draft tube installations become complicated. Pelton turbine is therefore preferable, as it is also stronger mechanically. Further, the efficiency of Pelton turbine is less sensitive than that of Francis towards variation of load and head. Fig 9.6 illustrates the variation of turbine efficiency with load. Whenever the operating conditions vary over a wide range, Pelton turbine is preferable.

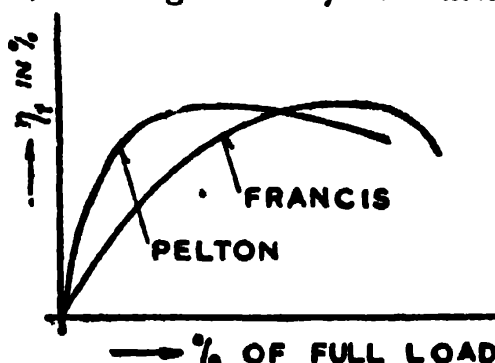


Fig 9.6 Comparison of Performance of Pelton and Francis Turbines

(c) **Medium Heads (60 m to 150 m)** : In this range a Francis turbine is usually employed. Whether a high or low specific speed unit would be used depends on the selection of speed which has been previously discussed.

(d) **Low Heads (under 60 m)** : Both Francis and Kaplan turbines may be used for heads from 30 to 60 m. The latter is more expensive but yields a higher efficiency at part-loads and overloads. It is therefore preferable for variable loads (refer Fig 9.7).

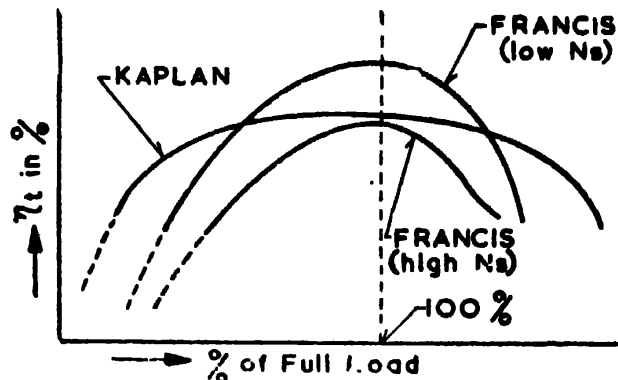


Fig 9.7 Comparison of Performance of Francis and Kaplan Turbines

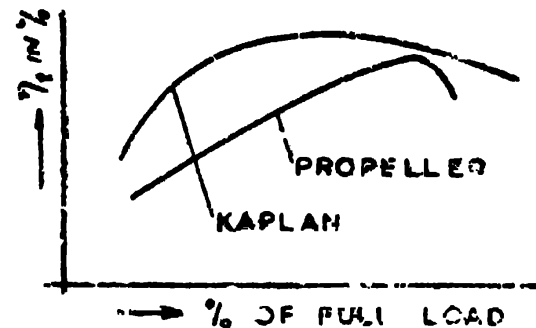


Fig 9.8 Comparison of Performance of Kaplan and Propeller Turbines

Kaplan turbine is generally employed for heads under 30 m. In this range Francis turbine would be an exception. Propeller turbines are, however, commonly used for heads up to 15 m. They are adopted only when there is practically no load variation (refer Fig 9.8). Propeller turbines are definitely cheaper than Kaplan turbines because the former dispenses with the runner blade adjusting mechanism.

(e) **Very Low Heads**—Bulb turbines are now employed for very low heads ranging from 1 m to 15 m. Though Kaplan turbines can also be employed for heads from 2 m to 15 m, but they are not economical.

**Problem 9.5** The quantity of water available for hydroelectric station is  $260 \text{ m}^3/\text{sec}$  under a head  $1.7 \text{ m}$ . Assuming the speed of the turbines to be  $50 \text{ rpm}$  and their efficiency  $82\%$ , determine the least number of turbines of Kaplan type, all of the same size, that will be needed if turbine specific speed does not exceed  $800$ . What would be the output of each turbine?

(Panjab University)

#### Solution

$$Q_{tot} = 260 \text{ m}^3/\text{sec}$$

$$N = 50 \text{ rpm}$$

$$N_s = 800$$

$$H = 1.7 \text{ m}$$

$$\eta_t = 82\%$$

$$\text{Specific speed } N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$\text{or } P_t = \left( \frac{N_s \cdot H^{\frac{5}{4}}}{N} \right)^2 = \left( \frac{800 \times 1.7^{\frac{5}{4}}}{50} \right)^2$$

$$= 965 \text{ HP per turbine Answer}$$

$$\text{Turbine output } P_t = \frac{\gamma \cdot Q \cdot H}{75} \cdot \eta_t \text{ BHP}$$

$$\text{or } Q = \frac{75 \cdot P_t}{\gamma \cdot H \cdot \eta_t} = \frac{75 \times 965}{1,000 \times 1.7 \times 0.82} = 51.5 \text{ m}^3/\text{sec}$$

$$Q_{tot} = 260 \text{ m}^3/\text{sec}$$

$$\therefore \text{ Number of Kaplan turbines required} = \frac{260}{51.5} = 5.05 \text{ or } 5$$

Answer

**Problem 9.6** A power plant is to be built on a river having rate of flow of  $67 \text{ m}^3/\text{sec}$  and utilising  $12.8 \text{ m}$  of head. The speed fixed by the electrical company is  $166.67 \text{ rpm}$ . The efficiency of the turbine is  $90\%$ . It is further seen that the specific speed of  $342 \text{ rpm}$  would be best for the hydro-plant. Selection of water turbine is to be made from the following data meant for the required specific speed and calculated for the unit head.

$$N_s = 342$$

$$H = 1 \text{ m}$$

| Dia of runner in cm | 137.5 | 145  | 152.5 | 160  | 167.5 |
|---------------------|-------|------|-------|------|-------|
| HP                  | 47.5  | 53   | 58.5  | 64.5 | 71    |
| rpm                 | 49.5  | 47.0 | 44.6  | 42.5 | 40.6  |

Find out—

- (a) the type of the turbine runner ;
- (b) the number of turbines required ;
- and (c) the diameter of the runner selected.

**Solution**

$$Q = 67 \text{ m}^3/\text{sec}$$

$$H = 12.8 \text{ m}$$

$$N = 166.67 \text{ rpm} \quad N_s = 342 \quad \eta_t = 0.9$$

$$\text{Now specific speed } N_s = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$\text{or} \quad 342 = \frac{166.67 \times \sqrt{P_t}}{12.8^{\frac{5}{4}}}$$

$$\text{or} \quad P_t = 2,500 \text{ HP per turbine}$$

$$\text{Unit power} \quad P_t = \frac{P_t}{H^{\frac{3}{2}}} = \frac{2,500}{12.8^{\frac{3}{2}}} = 54.4 \text{ HP. m}^{\frac{3}{2}}$$

$$\text{Unit speed} \quad N_1 = \frac{N}{H^{\frac{1}{2}}} = \frac{166.67}{12.8^{\frac{1}{2}}} = 46.5 \text{ rpm. m}^{-\frac{1}{2}}$$

(a) Type of runner is Kaplan, because specific speed is 342 **Answer**

$$(b) \text{ Turbine output } P_t = \frac{\gamma \cdot Q \cdot H}{75} \cdot \eta_t$$

$$\text{or} \quad Q = \frac{75 P_t}{\gamma \cdot H \cdot \eta_t} = \frac{75 \times 2,500}{1,000 \times 12.8 \times 0.9} = 16.3 \text{ m}^3/\text{sec}$$

$$\therefore \text{ Number of units} = \frac{67}{16.3} = 4.1 \text{ or } 4 \text{ units} \quad \text{Answer}$$

(c) For a unit power of  $54.4 \text{ HP}$ , select the diameter of runner as  $152.5 \text{ cm}$  from the above table. If an intermediate runner is available, then calculate the diameter by interpolation.

Let  $x$  be the required diameter of runner in cm.

$$\text{Then } \frac{47.0 - 44.6}{46.5 - 44.6} = \frac{152.5 - 145}{152.5 - x}$$

$$\text{or } \frac{2.4}{1.9} = \frac{7.5}{152.5 - x}$$

$$\text{or } 152.5 - x = 5.95$$

$$\therefore x = 152.50 - 5.95 = 146.55 \text{ cm } \text{Answer.}$$

### UNSOLVED PROBLEMS

#### (A) Turbine Models

- 9.1 Why is the turbine model manufactured ?
- 9.2 What are the conditions of making the turbine models ?
- 9.3 Show that the speed of a turbine model is  $\sqrt{n}$  times the speed of its prototype. The prototype turbine is  $n$  times bigger than that of its model.
- 9.4 Show which one is correct and give reasons in favour of your answer. "The efficiency of a turbine model is less, equal or more than its proto type."

#### Numericals

- 9.5 A model of a water turbine develops 25 HP when working under a head of 5 m and running at 480 rpm.  
Determine the HP of the actual turbine if the working head is 40 m and model scale  $\frac{1}{10}$ th actual turbine. What is the rpm of the actual turbine ? Assume the efficiency of the model as well as that of actual turbine to be the same.  
(56,500 HP ; 136 rpm) (*Jadavpur University*)
- 9.6 A model Francis turbine made to  $\frac{1}{8}$ th scale, when tested gave 4.1 BHP at a speed of 360 rpm under a head of 1.8 m. Calculate the equivalent speed and power of full size turbine when working under a head of 6 m. Find also the ratio of quantities of water flowing through the turbine per second.  
(131.8 rpm ; 623 BHP ; 45.6) (*Madras University*)
- 9.7 A model having a scale ratio of  $\frac{1}{10}$  is constructed to determine the best design for a Kaplan turbine to develop 10,000 BHP under a net head of 12 m when running at 100 rpm. If the head available at laboratory is 8 m and the model efficiency is 80% find—  
(a) the correct running speed of the model ;  
(b) the flow required in laboratory in  $\text{m}^3/\text{sec}$  ;  
(c) the BHP of the model ;  
(d) the specific speed in each case.  
Prove any formula, you use.  
(816 rpm ; 0.58  $\text{m}^3/\text{sec}$  ; 54.4 BHP ; 448) (*Delhi University*)
- 9.8 Laboratory tests are to be carried out on a model to determine the best design speed for a Francis turbine to develop 40,000 HP under a net head of 240 m when running at 500 rpm. If the available head at the laboratory was 30 m and discharge 150 lit/sec, assuming an overall efficiency of 88%, find—

- (a) suitable scale ratio for the model ;
- (b) running speed of the model ;
- (c) the HP of the model.

Prove any formula, you use.

(5·87 or 6 : 1,040 rpm ; 52·8 HP) (*AMI Mech E—London*)

- 9.9 A model test of 40 cm water turbine runner was carried out under a head of 7·5 m and gave the following best results :

Discharge = 0·5 m<sup>3</sup>/sec ;

Speed = 400 rpm ;

Output = 40 HP.

- (a) Find the specific speed, unit speed, unit power, unit discharge and efficiency of the runner ;

- (b) State the type of runner ;

- (c) If a 100 cm runner of the same design is used under a head of 50 m, determine the speed, discharge and horsepower for the same.

[(a) 204 ; 14·65 rpm ; 1·95 HP ; 0·0183 m<sup>3</sup>/sec ; 80%

(b) Fast Francis ; (c) 412 rpm ; 8·05 m<sup>3</sup>/sec ; 4,320 HP]

(*Jadavpur University*)

### (B) Selection of Turbines

- 9.10 What kind of preliminary data is required for the selection of water turbines ?

- 9.11 On what factors the selection of site for hydro-power plant depends ?

- 9.12 What kind of field information is needed to make the right type of selection of a hydro-power plant ?

- 9.13 What difficulties will arise, if the selection of turbine is unsuitable ?

- 9.14 On what factors the selection of different types of water turbines depend ?

- 9.15 How do the rotational speed and cavitation effect the selection of a water turbine ?

- 9.16 What type of turbine will you select for head of (i) 200 m (ii) 60 m (iii) 20 m (iv) 5 m ?

(*BITS—Pilani*)

- 9.17 Discuss the considerations which govern the type and size of turbines to be installed in a hydro-electric power station. What considerations are favourable for the use of

- (a) Propeller turbines, and

- (b) Other types of reaction turbines ?

(*Pajab University*)

- 9.18 Sketch a typical layout of a low head hydro-electric power station fitted with Kaplan turbines, i.e., having variable pitch propeller blades. Compare this type of turbine with a fixed blade propeller turbine and Francis turbine for this application.

(*AMI Mech E—London*)

### Numericals

- 9.19 It is required to develop 1,00,000 BHP by a number of water turbines. Each of the turbine runs at 166·7 rpm when working under a head of 30 m. Find the number of turbines to be employed if the specific speed of each turbine is 335.

(3) (*Roorkee University*)

- 9.20 The quantity of water available for a hydro-electric station is  $260 \text{ m}^3/\text{sec}$  under a head of  $1.7 \text{ m}$ . Assuming the speed of the turbines to be  $50 \text{ rpm}$  and their efficiency  $82.5\%$ , determine the number of turbines required. Assume a specific speed of  $890$ .

(*Madras University*)

- 9.21 A hydro-electric station is installed to utilise  $30 \text{ m}^3/\text{sec}$  of water of a river at a head of  $7.5 \text{ m}$ . If  $85\%$  turbine efficiency can be counted on,

- (i) is it feasible to develop this power by two turbines with rpm not less than  $50$ , the turbine having specific speed not greater than  $540 \text{ rpm}$ ?
- (ii) what type of runner will be used?
- (iii) what is the diameter of runner if the speed ratio is  $0.85$ ?

(*Panjab University*)

[(i)  $N = 156.5 \text{ rpm}$ ,  $\therefore$  yes (ii) Kaplan (iii)  $1.25 \text{ m}$ ]

- 9.22 In a water power site the available discharge is  $340 \text{ m}^3/\text{sec}$  under a net head of  $30 \text{ m}$ . Assuming a turbine efficiency of  $88\%$  and rotational speed of  $166.7 \text{ rpm}$ , determine the least number of machines, all of the same size, that may be installed if the selection rests with—

- (a) Francis turbine with  $N_s$  not greater than  $270$ ;
- (b) Kaplan turbine with  $N_s$  not greater than  $800$ .

What will be the output of each unit? Which of the two installations will be more economical?

( $10$  ;  $2$  ;  $12,000 \text{ HP}$ ,  $60,000 \text{ HP}$ )

(*Panjab University*)

- 9.23 Following is the data for a hydro-electric development :

Net Head =  $300 \text{ m}$

Turbine Output =  $10,000 \text{ HP}$

- (i) What type of turbine would you suggest?
- (ii) Assuming the following specific speeds as the suitable ones, what would be the nearest synchronous speeds?  
 $N_t = 24$  per jet for a Pelton wheel with two nozzles,  
 $N_s = 80$  for a Francis type runner.
- (b) Which of the two possibilities viz ; a twin jet Pelton or a single Francis type turbine as described above under (ii) would you favour? Support your arguments in the light of the relative advantages and disadvantages.

(*Panjab University*)

( $428.58 \text{ rpm}$  ;  $1,000 \text{ rpm}$  ; Twin-jet Pelton better)

## ***Recent Trends in the Development of Water Turbines and Modern Hydro Power Plants***

**10.1 Introduction**—In the foregoing chapters, fundamentals of water turbines were dealt with. In this chapter a review of recent developments touching upon the design and performance of hydraulic turbines will be made. It is also concerned with different kinds of modern hydro-electric schemes which are being carried out in the field. The articles on recent developments of water turbines are based mainly on the following :

(a) A Research Committee was appointed by the Institution of Civil Engineers, London, to prepare a review outlining the recent developments in the sphere of Hydraulics. The developments in different types of hydraulic turbines, have been prepared by the above Committee and published in the Proceedings of the Institution of Civil Engineers, Part III, Vol. 4, December 1955, Number 3.

(b) The paper presented at Joint *ASME—EIC* conference Denver, Colorado, USA in 1966 on “Large Hydraulic Prime Movers : Recent Developments and Current Trends” by J.L. Haydock, D.M. Coulson and R.I. Gilchrist.

(c) Notes on Hydraulic Turbines in two volumes by Dr. V.V. Barlit, Unesco Expert at Maulana Azad College of Technology, Bhopal delivered in 1969.

### **(A) RECENT TRENDS IN THE DEVELOPMENT OF WATER TURBINES**

**10.2 Recent Developments in Water Turbines in General**—Three types of hydraulic prime movers are essentially the post World War (II) developments : (1) Francis type pump turbines, (2) Deriaz turbines and pump-turbines and (3) Bulb and tubular turbines and pump turbines.

Considering the conventional turbines *viz.*, Pelton, Francis and Kaplan turbines, the current trends have been to use large hydraulic prime movers.

Taking Francis turbines, in Siberia, USSR, ten turbines each rated at 685,000 HP under a head of 95 m have been installed. Further 740,000 HP turbines for Syano-Sushensk are being worked out. Fig 9.5 gives the new Soviet Union classification of turbines covering outputs upto 1,000,000 KW and heads upto 500 metres. In USA, Grand Coulee



will have additional Francis turbines, each rated at 685,000 HP at 100.6 m head. In Canada ten Francis turbines, each rated at 610,000 HP under 317 m head are under manufacture. Fig 10.1 shows the increase in the

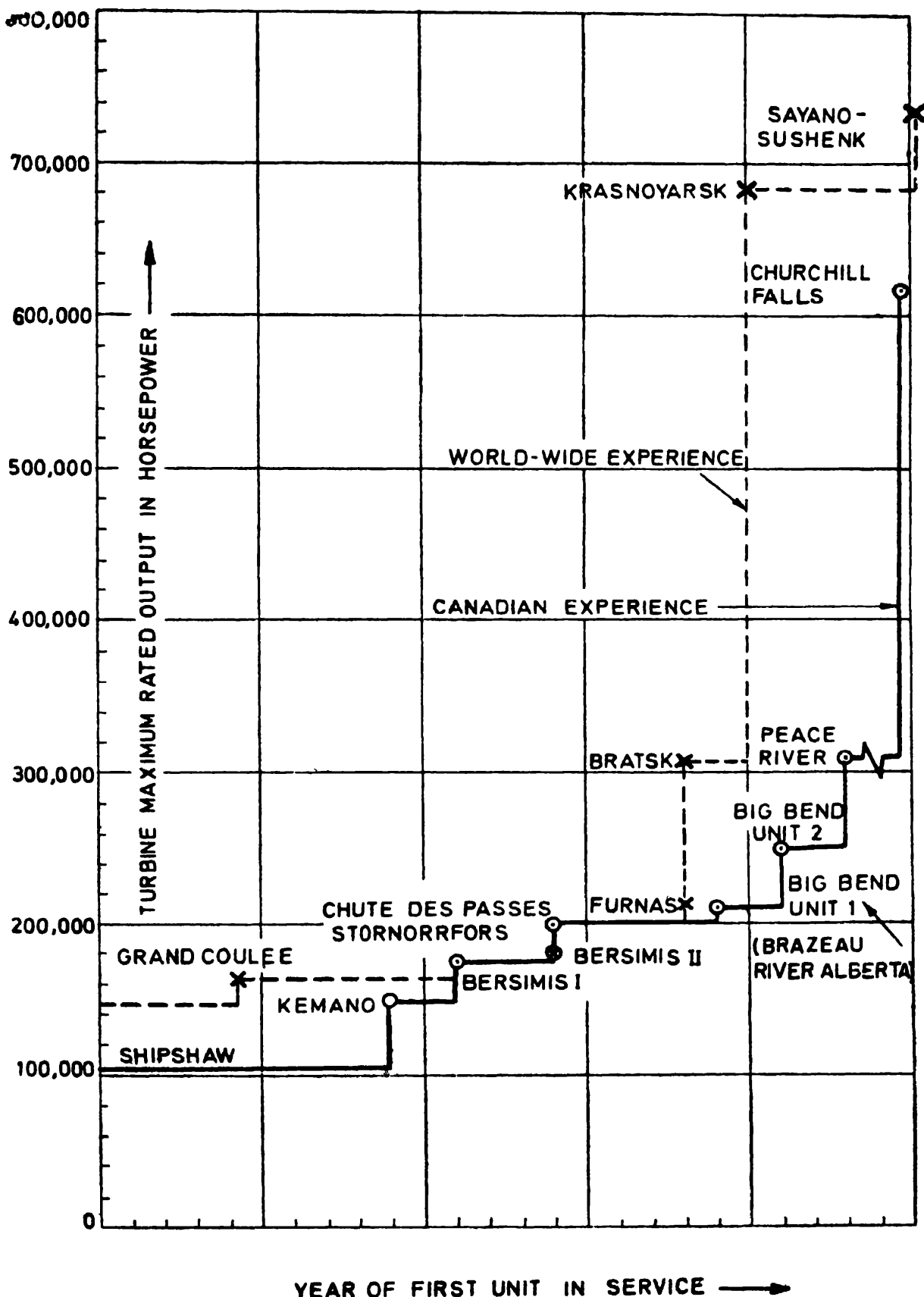


Fig 10.1 Turbine Maximum Ratings versus Year of First Unit in Operation

turbine maximum rating during the post World War II period. Taking Krasnoyarsk units the increase is 600% over Hoover turbines installed before the War.

The details of other notable turbines have already been given in the relevant chapters.

The recent developments of water turbines in general may be explained under different headings as given below :

(a) *Weight and Cost of Turbine*—The total weight of a turbine tends to vary as  $D^{2.3}$  to  $D^{2.7}$  where  $D$  is the outside diameter of turbine runner. The lower figure apply to relatively small machines in which the power 2.7 appears to represent, at present, a limiting figure for large machines in which further increase in size involve additional flange joints and additional weight resulting from the desire to achieve greater rigidity. These figures do not make any allowance for such factors as the use of special materials, such as quenched and tempered steels for spiral casings. This would reduce the weight considerably and could, in fact, have a bearing on feasibility, but since such materials invariably cost more per kg than conventional materials the effect on cost variation is normally much smaller.

The cost per kg of fabrication and castings also decreases as the unit weight of each component goes up. In case of castings, the price is made up of pattern, moulding and melt costs.

Considering the above, the monetary cost of turbines per HP decreases as the unit gets larger. This statement ceases to be true only when the unit size reaches a point where additional requirements for additional rigidity outweigh the considered effects.

(b) *Mechanical Design and Manufacturing*—The mechanical design of a turbine is always influenced by its size, type, and availability of the various physical facilities required to manufacture the components. The manufacturing facilities available today are impressive. Foundries can cast runners in mild carbon steel weighing upto 150 tonnes with diameters over 7 m. Chromium steel runners can be cast upto 6 m diameters with finished weights upto 140 tonnes. Forge shops can provide one-piece forgings upto approximately 100 tonnes with maximum flange diameters upto 2.75 m.

In machine shop, the largest vertical boring mill in Canada can handle components upto 17 m in diameter. For turning of main shafts the biggest horizontal lathe in USA can turn 2.4 m diameter by 15 m long job.

(c) *Shafts*—Welded tubular shafts upto about 2 metres in diameter, with cast or separately forged flanges, are now in the service in the Soviet Union. The successful development of electro-slag welding has permitted the use of shafts larger than those it is possible to forge with present facilities. It is seen that fabricated shafts are competitive in cost with solid forged shafts even in moderate size ranges.

(d) *Governor Oil-Systems*—During the past two decades, the operating oil pressures for governor system have increased from 15 to 40 kg/cm<sup>2</sup> and in isolated instances 70 kg/cm<sup>2</sup>. High oil pressures will permit the use of smaller servomotors, requiring less space and thus advantageous in many modern installations.

(e) *Hydraulic Design*—The hydraulic turbine designer improves existing designs to achieve higher speeds, unit powers, efficiency etc. and also develops basically new designs. By using higher specific speeds, it is possible to obtain higher rpm's and smaller, more economic turbines and generators with the same output. Conversely greater output can be obtained from a given unit size at the same rpm.

(f) *Shipping and Handling*—It is difficult and expensive to produce split turbine runners, therefore it will be better to make full use of railway and roadway facilities for shipping. Some manufacturers provide their own railway wagons to carry the runners. Kransnoyarsk (USSR) turbine runner weighs 280 tonnes and is 8.24 m in diameter and just under 4.27 m high. The runner is split into four pieces. The Churchill Falls (Canada) turbine runner is a single piece casting weighing 117 tonnes with 5.6 m diameter.

(g) *Choice of Optimum Point*—The choice of the position of best efficiency point relative to the normal operating point, hinges on a number of factors, including :

1. Shape of load-efficiency curve ;
2. Machine cost variations with physical size ;
3. Civil cost variations with physical size ;
4. Incremental value of energy ;
5. Spare, reserve, or overload capacity required beyond the normal operating range of loads ;
6. Unit load duration weighing factors.

From an economic study taking into account these variables, it is possible to arrive at an optimum point or, at least, a relatively narrow band of load within which the point of best efficiency should lie.

(h) *Efficiencies*—Peak efficiencies of modern Francis turbine in the medium head range are quite commonly 94 to 95.5% and such gains in maximum efficiency as have been made in recent years are the result of research, model testing and refined design and manufacturing practices. Clearly, when total losses not exceeding 5 or 6% are common, the law of diminishing returns applies rather strictly to efforts to improve still further, although possibilities still remain and are being exploited.

**10.3 Recent Developments in Pelton Turbines**—The present trend of research on Pelton turbines is towards the improvement of efficiency by studying the bucket surface resistance, friction and windage losses, water rebound in turbine casting and residual energy in the water leaving the wheel. Attempts have been made to recover the unused head at the runner exit, by hydro pneumatization, which have not proved successful. The needle and nozzle profiles forming the jet have resulted from experimental work based on minimum surface resistance and the long stroke required for precision discharge adjustment and governing where balancing of needle forces has become important. Short jet lengths have been suggested to avoid energy loss by jet spreading and disintegration.

During recent years the cavitation at the Pelton buckets has been eliminated by having more efficient bucket tips and edges. Higher specific speeds per nozzle for a given head has been attained due to the improved hydraulic design.

Since the Pelton turbine runs faster, the runner is subjected to a greater number of impacts during a given time. The limiting factor is metal fatigue. As the permissible number of impacts is also inversely proportional to the stress, it is possible to design the bucket with high limiting impact number. The current practice is 60 impacts per second for 6 nozzles and with 600 rpm. Therefore the possibility of increasing the number of impacts leads to higher runner speeds and more nozzles. Finally, with this, considerable saving in the weight and cost of hydro-units is achieved.

The water containing large quantities of sand damages needle and nozzle and reduces the turbine efficiency. Now stainless steel design permit rapid replacement. Welding and annealing allow worn bucket surfaces to be restored to the normal profile. Pressed steel bowls with forged-steel splitters and lugs welded on the bucket, are a new departure on a bucket construction.

**10.4 Recent Developments in Francis Turbines**—The recent trend had been to produce very large Francis turbines as already explained in Art 10.2. Francis turbines are being used for high heads, upto 300 m, which were formerly confined to Pelton turbines. High rotational speed has reduced the turbine overall dimensions as well as that of the power house. Research on cavitation and availability of material to withstand the stresses has led to increase the turbine rotational speeds.

At 40% to 80% of gate opening of Francis turbine, violent fluctuations in the power output occur. This has attributed to the existence of a rotating vortex core in the draft tube. The research being carried out now<sup>1</sup> shows that the speed of rotation of this vortex core varies from one-third to one-seventh of the turbine rotational speed. Flow surveys down-streams of the runner in the draft tube are envisaged.

In order to obtain high efficiency, research has been directed towards the smooth welded construction for spiral casing, stay rings, guide vanes, runner and other components.

Modern measuring instrument techniques have been developed to check vibrations causing cracked blades and reduced outputs. Such equipments include surface recessed pressure cells, strain and acceleration gauges, strain amplifiers and oscillographs.

**10.5 Recent Developments in Kaplan Turbines**—Kaplan turbines have taken the place of Francis turbines for certain medium head installations. Kaplan turbines with sloping guide vanes to reduce the cover dimensions have been used recently.

Research is being carried out on draft tubes to recover the residual velocity head at the runner exit. The revolving parts of the turbo-set are designed to withstand runaway speed of 2.5 to 3.5 times the working speeds. Emergency oil pressures systems and runner braking vanes have been provided to prevent the maximum runaway speed being attained, particularly where intake gates are omitted.

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<sup>1</sup> Research carried out at Indian Institute of Science, Bangalore under the guidance of Dr. Rama Prasad.

Research has been carried out to find the flow phenomena in the space between guide vanes and the runner by the aid of conformal transformation. Such investigations have aided the development of efficient cavitation-free high head Kaplan turbine blading for a wide range of specific tip speeds.

**10.6 Recent Research on Cavitation**—The cavitation problems due to the presence of solid bodies entering the water, are being studied. One modern hypothesis envisages "Cavitation nuclei" consisting of minute dust-sheltering bubbles in crevices of metallic surfaces.

(a) **Cavitation resistive Materials**—The following special cavitation damage study devices are used to investigate cavitation resistance of different materials on special specimens and to choose the most suitable steel :

(i) Magneto-strictive device ; (ii) Rotating disc device ; (iii) Venturi device.

As a result of the above study the following types of steels are being used now :

(1) Low-carbon steels with welding or coating by special materials on the places attacked by cavitation.

(2) Carbon manganese steels with improved mechanical properties.

(3) Stainless steel—12% to 13% Cr and 1% Ni.

(4) Aluminium brass alloys.

At the power stations the welding of turbine at places damaged by cavitation is done by special techniques with electrodes of the following composition—

| C%   | Cr% | Ni% | MO% |
|------|-----|-----|-----|
| 0.05 | 20  | 10  | 3.5 |
| 0.05 | 18  | 8   | 3   |
| 0.05 | 17  | 7   | 3   |

In order to have coating of stainless steel, special preheating of the parts is required to avoid the formation of cracks and inner tensions.

The research for the development of new steels, alloys, special coatings to resist cavitation is still carried out. Pressed plates appear to behave better than castings. For relatively small thin bladed Kaplan turbines, welding distortion can be avoided with a thin cast stainless steel sheath over a carbon-steel case which requires only light grinding and polishing.

(b) **Measures Adopted During Operation for Cavitation Reduction**—

(i) *Air injection* into flow behind the runner at the places where vortex cores are formed. It is done through hollow shafts and at high pressures.

(ii) *Cathodic Protection* of runner blades by electrical current of certain voltage to evolve hydrogen from the passing water to give cushioning effect of hydrogen at the boundaries of blades.

(iii) *Prevention of separation by suction of air from draft tube*—When the value of diverging angle of the draft tube is made more than  $4^\circ$  to

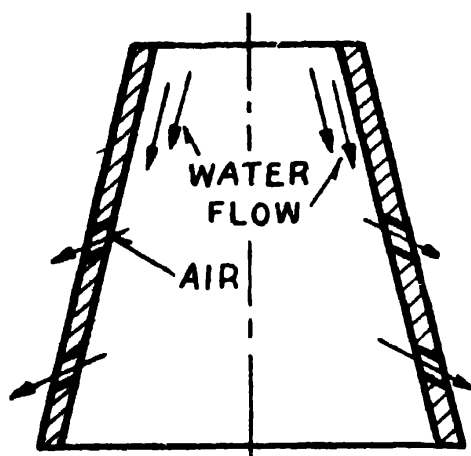


Fig 10.2 Suction of Air from Draft Tube

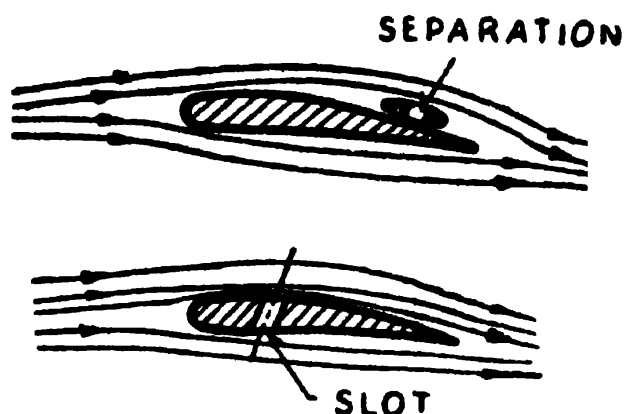


Fig 10.3 Provision of Slot in an Aerofoil Section

reduce its length, the air is sucked from the inside of draft tube as shown in Fig 10.2 so that the water may not detach itself from the walls and create separation. The efficiency of draft tube was raised from 50 to 81% by this method in certain cases.

(iv) *Slot provision*—If a slot is provided (refer Fig 10.3) in the aerofoil section it, causes suction by which separation can be controlled.

(c) **Methods for cavitation study**—In addition to the use of stroboscope and high speed photography as visual methods, the following new methods are employed—

(i) *Chemical method*—The chemical resistance of flow is a function of cavitation intensity. The chemical resistance will increase with the saturation of flow made by bubbles and vapour pocket at the places of cavitation. This is done by installing two pairs of electrodes, one behind the runner, where cavitation is expected and the other in the non-cavitating flow. The inception of cavitation can be detected easily.

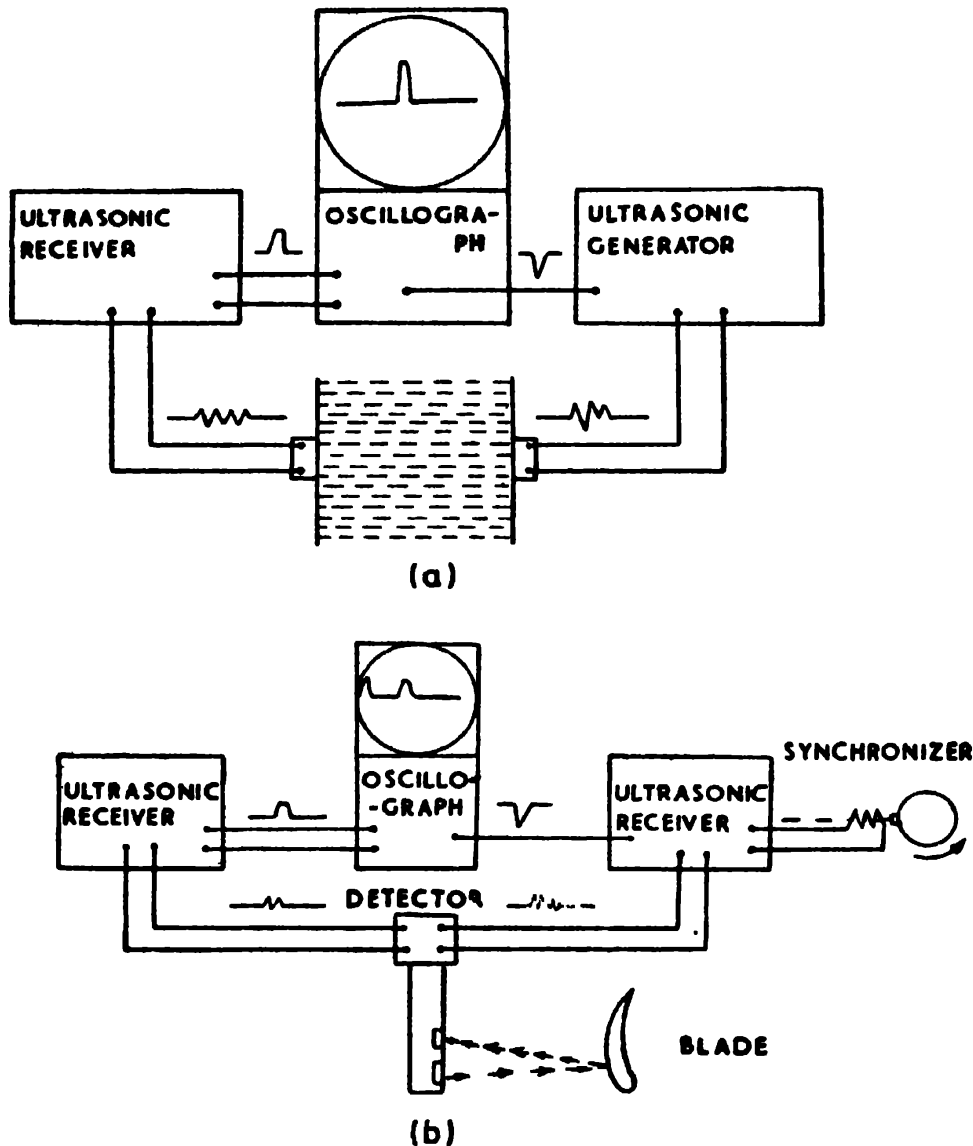
(ii) *Ultrasonic method*—If ultrasonic rays are passed through the flow where presence of bubbles are expected, the value of their energy at the receiver can be used as a measure of cavitation. Ultrasonic generator (refer Fig 10.4a and b) produces impulses which are sent into the flow by special ultrasonic radiator. These impulses are converted into electrical impulses by barium titanate plate, which are supplied to oscillograph. The energy difference of the supplied and received rays gives the measure of cavitation.

*Ultrasonic analyser* investigates the whole flow after the prototype turbine runner by penetration method and detects the cavitation zones at the rotating runner by detection method.

(iii) *Acoustic method* generates the ultrasonic oscillation in the cavitating flow during collapse of bubbles. This method has been used successfully at the hydro-power plant.

**10.7 Recent Developments on Models Tests**—There exists a persistent trend to develop new types of hydraulic turbines with larger values of specific speed, better efficiency, cavitation, pulsation and other characteristics. This is materialised through reliable model investigations

for which the demands for a good test stand and the measuring instruments are increasing. The results of model tests investigations are employed :



(a) Penetration Method

(b) Detection Method

Fig 10.4 Ultrasonic Method of Cavitation Investigation

(a) for the prediction of efficiency, cavitation, force and pulsation characteristics of prototype turbine and their selection ;

(b) for the stress analysis calculations of the turbine and generator coupled to it ;

(c) for the improvement of the applied hydrodynamic methods of calculations ;

(d) for checking and improving, if possible, the existing efficiency formula.

The test laboratories of various manufacturing firms have been improved. The modern hydroturbine test-stands are classified as under :

(i) Rig with open circuit ;

(ii) Universal test-stand with closed circuit which is used for complex investigations and is preferred now a days ;

- (iii) Cavitation test rig ;
- (iv) Turbine test stand with air or freon as working medium.

Now there exist two trends in model experiments. One trend is to test the models with largest possible diameter (6 to 8 cm) but at the moderate testing heads. In other trend, it is preferred to investigate the models at prototype heads and with moderate size model having a runner diameter of 25 to 50 cm.

About 15% of the cost of total equipment is generally spent for model investigation.

The most modern test bed till today is in USSR at TGZ (Charkov) where all kinds of model investigations can be made by simulating field conditions.

In pumping installations, model studies have been very useful for the development of intake, as it is necessary to prevent vortex formation in the sump because head falls seriously. It has been seen that as little as 2% of air will reduce the head obtainable by 50% and 10% of air will de-prime the pump. Model tests are also very useful for centrifugal and propeller pumps of large size which are not manufactured on mass production.

## (B) MODERN HYDRO POWER PLANTS

**10.8 Development of Water Power Resources**—Integration, comprehensive planning and development of power resources, and the size of the annual increment in demand for electric power and energy in industrially developed or rapidly developing countries, renders it not only possible but economically necessary to plan in terms of the development of water power resources and/or construction of pumped-storage facilities in large blocks, the whole output of such blocks being absorbed by the system as soon as it becomes available. Increasingly, rapid, full development of a site or river system is required. At the same time, individual systems grow and are interconnected to form large power pools, the systems become able to tolerate longer and larger units.

Under these circumstances it becomes practical and economically viable to consider the installation of very large units since, by so doing, capital costs of a project can usually be substantially reduced.

As mentioned in Art 10.2, three essentially post war developments in the hydraulic prime mover field have very quickly move into a position of some prominence, namely, the reversible Francis pump turbine, the Deriaz turbine/pump-turbine and the bulb turbine/pump-turbine.

With the growth of power systems there has been a spectacular increase in the need for peak load capability. In some systems the ratio of power demands at 'peak' period and 'off peak' periods is also becoming greater. The use of reversible pump-turbines is one of the means by which this growing demand for economical peaking power is being met.

The evolution of the bulb turbine/pump-turbine constitutes a logical step in the search for greater economy in the development of low-head river and tidal power projects.

**10.9. Pumped Storage Plants\***—The load on a power station fluctuates within large limits during the twenty-four hours of the day as



well as during the twelve months of the year. To supply peak loads the installed capacity of the station needs to be high, most of which would remain idle during off-peak hours. It means that machines which work only during peak hours, do not bring good revenue. It is necessary, therefore, to devise some way to achieve the economical loading of the plant by levelling-up the load curve. The following methods may be employed—

(a) *Commercial Method*—By selling electric current at a higher rate during peak hours than during off-peak hours. In Zurich (Switzerland) the current rate from 6.30 PM to 9.30 PM is more than during the remaining 24 hours of the day. With this system most of the people refrain themselves from using extra electric energy during these three peak hours.

(b) *Technical Method*—The following two means may be used :

(i) By *installing special peak load power plant* in which the fixed charges are low. Such a station may be either thermal or hydraulic. In thermal station a relatively long period is needed before they can take up load because both the boiler and the turbine must be made ready for operation as soon as considerable increase in load has to be expected. This fact makes the efficiency of the current production very poor. Therefore, hydraulic power station is preferred for the purpose.

(ii) By *storing energy* produced during off-peak hours. Such a system is known as **Pumped Storage Plant**. A pumped storage hydro-electric plant pumps water at off-peak hours of electric demand by means of surplus power (which is not being supplied) into a high level, natural or artificial reservoir, in order to utilise the stored energy at periods when it is most needed. Hence pumped storage does not supply the electric energy to consumers, but it merely transforms the surplus electrical current, which is not utilised during off peak periods (night during 24 hours and winter during the year) into hydraulic energy, stored to be used during peak period. Thus pumped storage plants act like large energy flywheels.

**10.10 Purpose of Pumped Storage Plants**—The following purposes may be served by the pumped storage plants—

(a) Combined with steam power stations, they represent the best compensation for it, in that they reduce the load fluctuations to within the narrow limits necessary for their economy and act as economic peak load stations throughout the year.

(b) Combined with hydro electric installations in addition to daily peak load, it can also cater to seasonal variations in water. The firm power of the installation, which is the power available at all times, is increased as during low water periods, water can be drawn from storage. Increase in firm power increase the earning capacity of the installation.

(c) In some cases storage plants may not have current generating unit. It may consist of pump and motor and no turbine. The utility of such a plant is that the pump increases the head in the feeder reservoir of a separate hydro-electric plant while the motor acts as phase advancer, improving the power factor in electrical supply net work.

**10.11 Classifications of Pumped Storage Plants**—The following are the ways by which the pumped storage plants may be classified—

(a) *Daily, weekly or seasonal storage plants.*

(b) *High, medium or low head plants.* Low head plants are not common.

(c) *According to the type of turbine used in the plant.* Francis turbine is very common because of its high efficiency and high speed. Pelton and Kaplan turbines have also been used.

(d) *Pure or mixed storage plants*—If the total quantity of water passing through the turbine is equal to the total quantity of pumped water and both the hydraulic machines working under the same gross head, the plant is called a pure storage plant. On the other hand, if the water passing through the turbine is more than the pumped water and/or where the turbine head is more than pump gross head, the plant is known as mixed storage plant.

(e) *Horizontal or vertical storage plants*, according to the disposition of shafts. Generally the horizontal plants (refer Fig 10.5) are preferred over the vertical due to good visibility, favourable conditions of erections and dismantling for repairs and check-up.

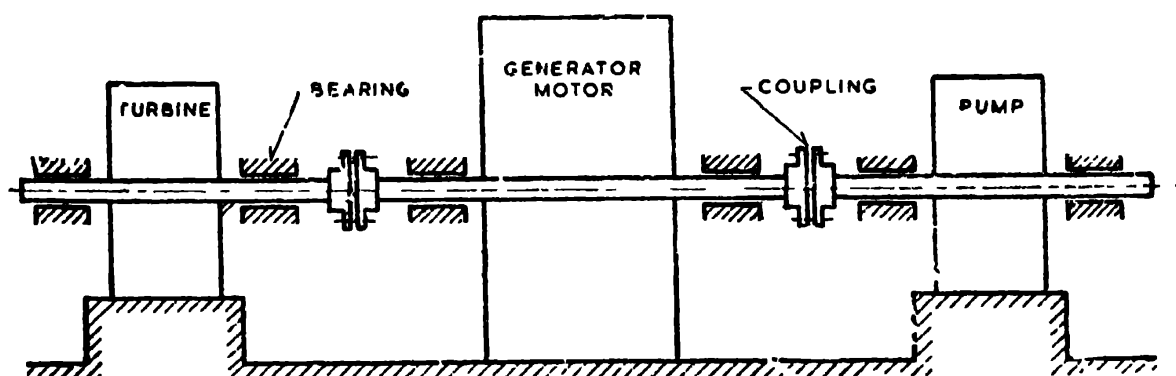


Fig 10.5 Horizontal Arrangement of Pumped Storage Plant

**10.12 Starting of Pumped Storage Plants**—The layout of a pumped storage plant is shown in Fig 10.5. The set consists of a turbine, generator/motor and a pump. There are several methods employed for starting the plant. The most common method is given below :

The pump is primed by means of an ejector, and started by means of the turbine, the generator being disconnected from the system and pump delivery valve being closed. When the synchronous speed is reached the generator which acts as motor now, is closed to the system, the turbine inlet valve shuts off and pump delivery valve is gradually opened. Turbine shaft with runner will be revolving idle while pumping takes place.

In some cases there is a small Pelton turbine coupled with the same shaft to which the main turbine, generator/motor and pump are connected. The Pelton turbine rotates the shaft in the opposite direction. As soon as the generating is to be shut off, the Pelton turbine nozzle is opened which first stops the main turbine and then by revolving the shaft in opposite direction, brings the pump and motor to a synchronous speed. The motor is then connected to the system and the Pelton turbine inlet is closed. In high head installations, the Pelton turbine brake nozzle serves the above purpose. The research\* conducted by the author on this subjected showed

\*"Characteristics of Free Jet Water Turbine in Working and Brake Regions (in German), by Dr. Jagdish Lal, Published by Springer Verlag, Vienna, 1952.

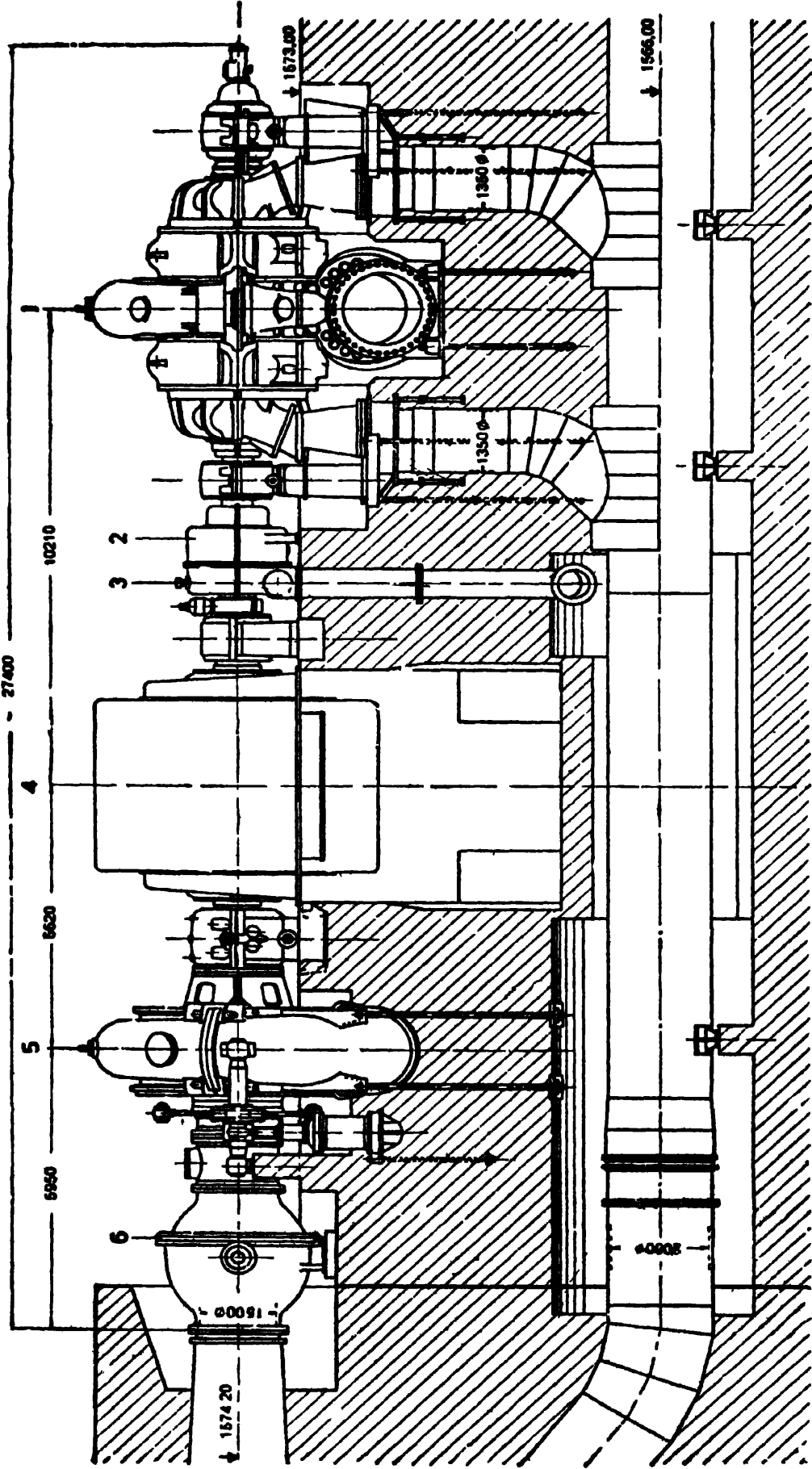


Fig 10.6 Limberg (Austria) Pumped Storage Plant (Escher Wyss)

|                     |                    |                    |
|---------------------|--------------------|--------------------|
| 1. Storage Pump     | 3. Braking Turbine | 5. Francis Turbine |
| 2. Toothed Coupling | 4. Motor-Generator | 6. Rotary Valve    |

that the process of "Generating to Pumping" can be carried out within 60 to 80 seconds.

**10.13 Notable Installations of Pumped Storage Plants**—One of the biggest installation of pumped storage plant is at Limberg Power Station (Austria), which is equipped with the brake turbine of Pelton type. This plant has pumps, toothed coupling braking turbine, motor-generator, Francis turbine and rotary valve all six connected, horizontally with a total length of 27,400 mm as shown in Fig 10.6. The hydraulic machines were manufactured by Escher Wyss & Co Ltd. The pump rotor, comprising of two single impellers with double admission weighs 41 tonnes. The length of the shaft is 9 m with a maximum diameter of 820 mm. The input of the pump is 83,000 HP. The main turbine is of Francis type developing 79,000 HP under 364 m head at 500 rpm, using  $18.75 \text{ m}^3/\text{sec}$ .

Fig 10.7 shows layout of machine house of a pumped storage plant of Villa Gargnano (Italy). The station consists of two sets manufactured by Escher Wyss, Zurich each consisting of a Francis turbine, a motor generator and a two stage storage pump. The suction pipes of both the turbine and the pump are connected to the Lake of Garda through a tunnel 328 m long. The pump has delivery head of 417 to 375 m, discharge 14 to  $16.24 \text{ m}^3/\text{sec}$ , input 85,400 to 91,000 HP, impellers diameter 2 m each.

Plate 31 and 32 show Storage Pump manufactured by Escher Wyss for Luenersee (Austria) single flow, 5 stage having maximum input of 57,600 HP; delivery head of 895 to 1,005 m; discharge  $Q=4.11$  to  $3.49 \text{ m}^3/\text{sec}$ ; speed=750 rpm with overall height of complete set of 28 m.

A Voith Storage Pump for the largest pumped Storage Scheme of the World (Vianden, Luxemburg) has max absorbed HP=103,000 and max delivery head=300 m. The Francis turbine installed has max output of 150,000 HP at 300 m head.

Table 10.1 shows particulars of some important pumped storage plants operating at present.

**10.14 Reversible Turbine Pump**—In a pumped storage plant, there is only one electrical machine which acts as a generator as well as a motor. However, the plant, as described in the above articles, consists of two hydraulic machines *viz.*, a turbine and a pump. If these two machines are combined in one pump/turbine unit, it will not only save the cost of one full machine, but also eliminate elaborate hydraulic connections and couplings etc. Such a machine is possible because Francis turbine is just a reverse of a centrifugal pump. It functions in one direction as a motor driven pump and in the reverse direction as a turbine-driven alternator. According to model tests, the efficiencies of pumping and generating have exceeded 85%. As the specific speed of a pump is greater than that of geometrically similar turbine two different rotational speeds are necessary for the two machines if the best efficiency is to be obtained in each direction of rotation. The same speed is possible only with some sacrifice of the efficiency.

Comparing the Francis turbine and the pump for the same head and horsepower, the turbine runner size is larger than the pump impeller and the turbine casing is smaller than the pump casing. The resulting

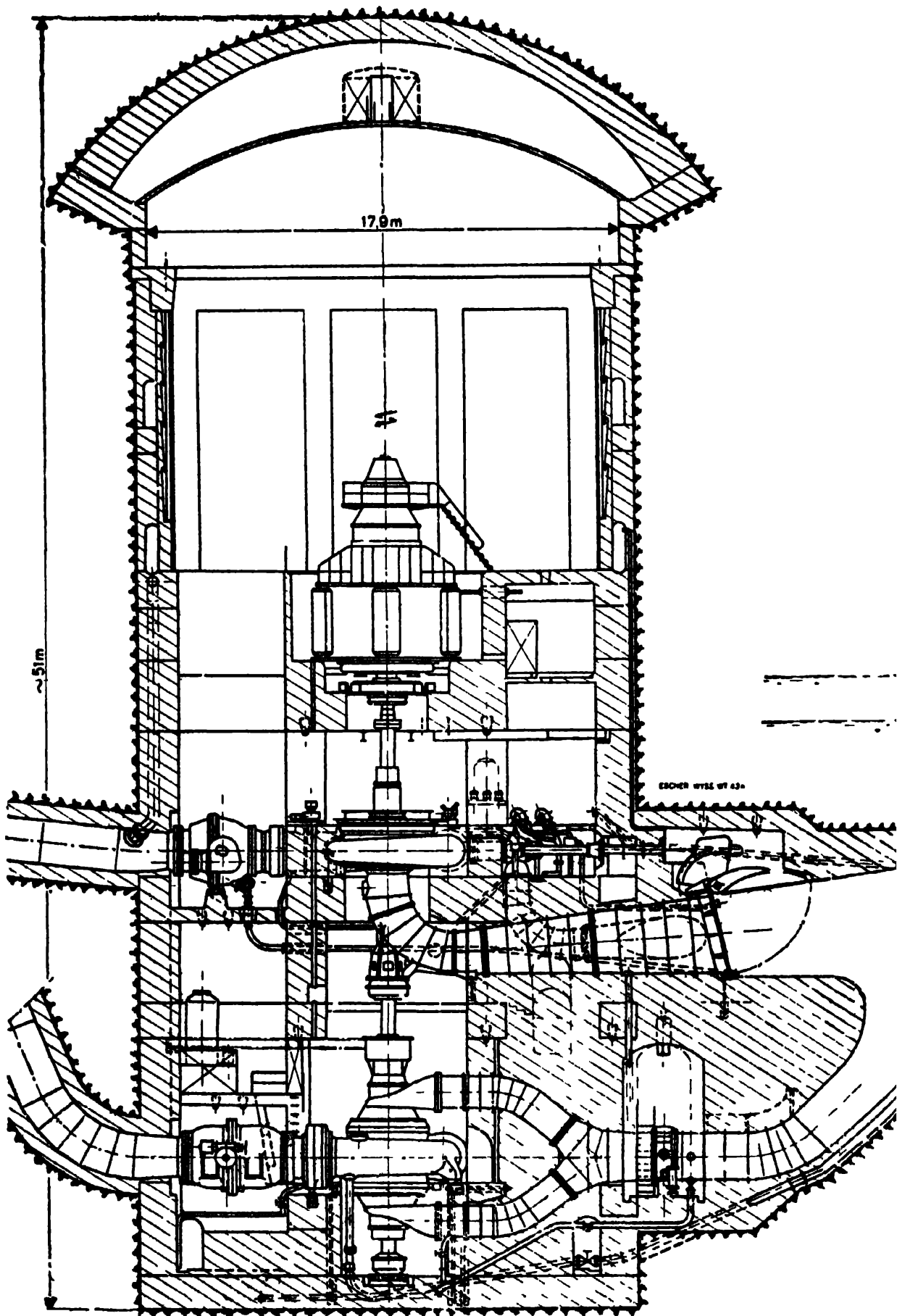


**Fig 10.7**  
**Section Through Machine Hall of Villa Gargnano (Italy) Storage Station with**  
**Overall Height of 51 m. Manufacturers : Escher Wyss, Zurich**

TABLE 10.1  
Some More Pumped Storage Plants  
(Arranged according to Pump Delivery Head)

| Sl. No. | Pumped Storage Station | Country     | TURBINE         |                 |                 | STORAGE PUMP |                    |              | Ratio turbine HP pump HP | Type of unit      | Type of Coupling        |
|---------|------------------------|-------------|-----------------|-----------------|-----------------|--------------|--------------------|--------------|--------------------------|-------------------|-------------------------|
|         |                        |             | No. of turbines | Type of turbine | Turbine HP each | No. of Pumps | No. of Pump stages | Pump HP each | Max. Delivery Head in m  | Pump speed in rpm |                         |
| 1       | Riseck                 | Austria     | —               | —               | —               | 3            | —                  | 7,550        | 1,070                    | —                 | —                       |
| 2       | Oberems                | Switzerland | 2               | —               | 5,300           | 1            | —                  | 7,200        | 1,000                    | —                 | —                       |
| 3       | Luensersee             | Austria     | 6               | —               | 48,500          | 6            | —                  | 57,600       | 895                      | —                 | Vertical                |
| 4       | Tremorgio              | Switzerland | 1               | Pelton          | 15,000          | 2            | 9                  | 6,400        | 900                      | 1,000             | Horizontal              |
| 5       | Belleville             | France      | 2               | Pelton          | 4,500           | 2            | 6                  | 3,800        | 525                      | 1,000             | Horizontal              |
| 6       | Etzel                  | Switzerland | 6               | Pelton          | 20,000          | 5            | 2                  | 21,500       | 490                      | 500               | Vertical                |
| 7       | Pfestiniog             | UK          | 4               | —               | 100,000         | 4            | —                  | 83,000       | 325                      | —                 | —                       |
| 8       | Vian den               | Luxemburg   | 5               | Francis         | —               | 5            | —                  | 92,800       | 268                      | —                 | Horizontal              |
| 9       | Providenza             | Italy       | 3               | —               | 67,000          | 2            | —                  | 61,600       | 240                      | —                 | Horizontal              |
| 10      | Sallre                 | Sweden      | 1               | Francis         | 10,000          | 1            | 2                  | 8,500        | 225                      | 600               | Vertical                |
| 11      | Hausern                | Germany     | 4               | Francis         | 47,000          | 4            | 2                  | 28,000       | 220                      | 333               | Vertical                |
| 12      | Our                    | Luxemburg   | 4               | —               | 107,090         | 4            | —                  | 53,750       | 128                      | —                 | —                       |
| 13      | Flatiron               | USA         | 1               | Francis         | 10,720          | 1            | —                  | 11,793       | 91.4                     | 300               | Reversible turbine/pump |
| 14      | Sron Mor (Glenshire)   | UK          | 1               | —               | 5,000           | 1            | —                  | 7,000        | 48.7                     | —                 | —                       |
| 15      | Niagara                | Canada      | 6               | —               | 46,300          | 6            | —                  | 46,000       | 27.4                     | —                 | —                       |

overall dimensions of reversible turbine/pump are taken as greater of the two. For pump/turbine the impeller is submerged to maintain proper flow while pumping. If the centre line is set above the tail race, priming is necessary, therefore impeller-runner is submerged and depth of submergence depends upon the critical value of cavitation factor.

### 10.15 Reversible Turbine-Pump Installations—

(a) *Flatiron Station (USA)*—The first large-capacity reversible turbine-pump was installed in USA at the Flatiron Power Station of Colorado Big Thomson Project, about 80 km north of Denver. In addition to one reversible turbine-pump there are two high-head conventional Francis turbines. The water discharged from these turbines is used for irrigation all the year round. To meet the seasonal irrigation requirements the discharged water is stored by the reversible turbine/pump in Caster Lake about 1.5 km south of the plant.

The Flatiron station is a peak load plant. It is designed for two speed operations and generator/motor of special design, to realise high efficiency. The specifications of the unit are given below :

| Machine | Rotation          | N<br>rpm | H<br>m                 | Q<br>m <sup>3</sup> /sec | P<br>HP | $\eta$<br>% |
|---------|-------------------|----------|------------------------|--------------------------|---------|-------------|
| Pump    | Counter clockwise | 300      | 73.3<br>(51.8 to 91.5) | 11.17                    | 12,000  | 91          |
| Turbine | Clockwise         | 257      | 76.3<br>(88.5 to 92.7) | 122                      | 10,900  | 88          |

Runner impeller diameter = 2.57 m with blades. The other Francis turbine pump installations are given in Table 10.2.

### 10.16 Use of Deriaz Runner in Pumped Storage Plants—

1. Deriaz runner can be used as one machine turbine-pump (refer Art 10.14) without lowering the efficiency of either turbine or pump.
2. It can be employed for peak load conditions where its efficiency as part load even at 30% load (refer Fig 6.25) is nearly the maximum.
3. Deriaz runner turbine can be started at a very short notice. This feature forms the basis of storage schemes. The advantage of short-notice availability can be extended to an instantaneous availability of keeping the water turbine running at part load in readiness for a sudden increase in demand. This requires good part-load efficiency which Deriaz-runner has achieved.
4. For operation as a turbine, the machine rotates in the opposite direction to that for operation as a pump. The change to pump operation necessitates starting in reverse from rest. In view of the very large size of the motor-generator, synchronous machines as a rule have a relatively low torque near synchronism, resulting in difficulty in running up to speed because of the heavy torque required by the fixed-blade pump when submerged even with shut gate. This difficulty

TABLE 10.2  
Francis Type Pump-Turbines

| Sl. No. | Plant       | Location           | No. of Units | Unit Capacity HP | Head m | Speed rpm |
|---------|-------------|--------------------|--------------|------------------|--------|-----------|
| 1       | Cornwall    | New York, USA      | 6            | 350,000          | 320    | 257       |
| 2       | Taum Sank   | Missouri, USA      | 2            | 243,000          | 227    | 200       |
| 3       | Cabin Creek | Colorado, USA      | 2            | 226,000          | 353    | 360       |
| 4       | Yards Creek | New Jersey, USA    | 3            | 152,000          | 200    | —         |
| 5       | Cruachan    | Loch Awe, Scotland | 4            | 139,000          | 350    | 500*      |
| 6       | Roenkhausen | Germany            | —            | 101,000          | 263    | 600*      |
| 7       | Hiwassee    | N. Carolina, USA   | 1            | 81,000           | 58     | 500       |
| 8       | San Luis    | California, USA    | 8            | 71,000           | —      | 105.9     |
| 9       | Providenza  | Italy              | 1            | 71,000           | 259    | 120,150** |
| 10      | Hatanagi    | Japan              | 1            | 70,500           | 102    | 375       |
| 11      | Chubu       | Japan              | 1            | 70,500           | 102    | 200       |
| 12      | Robier      | Switzerland        | —            | 53,800           | 390    | 200       |

\*Two units by Boving of 500 rpm and two English Electric-Sulzer of 600 rpm

\*\*Two speeds for wide head range.



can be overcome by pneumatisation, which consists in using compressed air to lower the water level in the pump suction pipe so as to permit starting the pump with the runner clear of the water. With the variable-Darrieux runner this complication disappears because the torque required to drive the turbine runner with the blades in the shut position is a very small fraction of the normal load torque. Provision for compressed air, with its bulky tanks and compressors, becomes then unnecessary, and starting can be accomplished in shorter time.

**10.17 Economical Aspects of Pumped Storage Plants**—More energy is required to pump the water up to the lake. Also some energy is lost when hydraulic energy is converted into electrical energy. The overall efficiency of dual conversion of pumped storage energy is about 67% excluding transmission losses, which means the load can be supplied at about two-thirds the efficiency of direct generation. Now one-third energy losses must be compensated to make the plant economical. This can be done by charging more for the on-peak energy than for the off-peak energy as explained in Art 10.9, or otherwise the sum of the generating costs, interest on capital and sinking fund, cost of power used for pumping, must be less than the cost of producing peak load.

**10.18 Underground Power Stations**—Underground hydro-electric power plants have been extensively used in Europe. More interest has been developed in this direction due to the following advantages\*—

- (a) They are safe against bomb attacks, rock and earth slips, and snow avalanches.
- (b) Air-conditioning to maintain comfortable temperature all the year round is possible.
- (c) They allow greater operating heads.
- (d) Since excavation is in the hard rock, draft tubes, penstocks and surge tank may be cut in it, thus effecting a considerable savings in the cost of power houses and the fabrication and erection of accessories,
- (e) They do not disfigure the scenic beauty of the site.

Some of the important underground power stations are—

(a) *Kemano (Canada)*—This power station has a record size underground plant, constructed for the Aluminium Co. of Canada Ltd, located in Central British Columbia. The pressure tunnel and the penstock system about 17.7 km long, laid underground throughout, conducts water from Nechako River to Kemano power house which discharges to the Pacific Ocean. The underground power station chamber is 300 m long, 24.85 m wide and 42.4 m high from the bottom of turbine pits to the crown of roof arch. This station is equipped with 16 sets of Pelton turbines each developing 142,200 BHP (refer also Art 5.7).

(b) *Innert Kirchen and Handeck II (Switzerland)*—These two stations belong to the system of four power houses of Oberhasli Power Station Co, Innert Kirchen. Fig 10.8 shows the layout of Innert Kirchen Underground Power House, built in 1940-42, together with Handeck I which is not underground. The pressure tunnel has a length of 2 km and diameter of 2.4 to 2.6 metres. It is reinforced along its whole length with

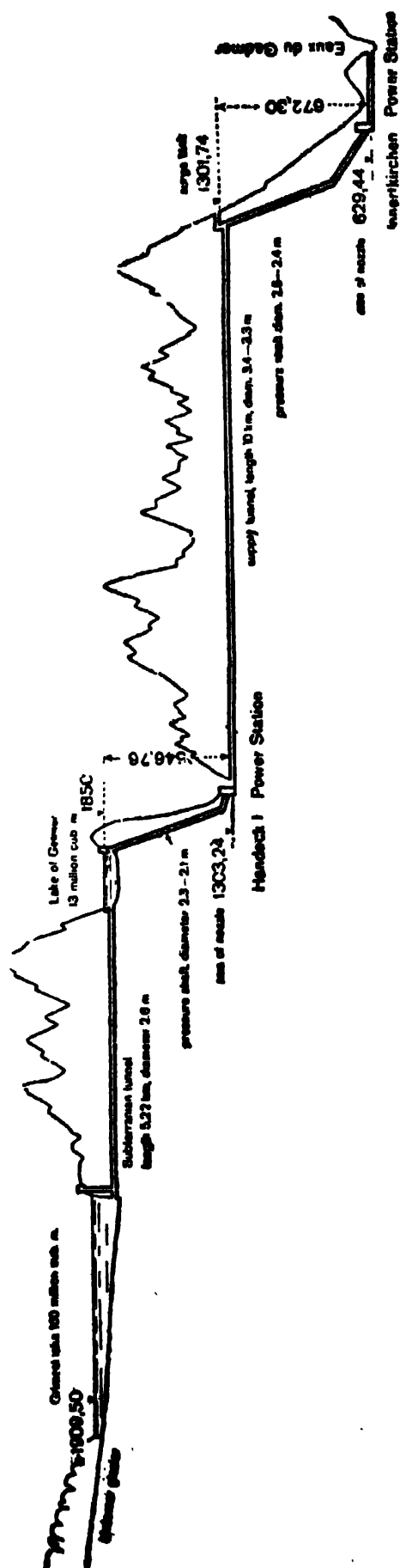


Fig 10.8 Layout of Innert-Kirchen (Switzerland) Underground Power Station together with Handeck I Power House

steel plates. The power house is equipped with five Pelton Turbines supplied by Escher Wyss, each developing 65,000 HP under a gross head of 644 metres and running at 428.6 rpm.

The Handeck II underground power station, built in 1947-1950 has four vertical Pelton turbines, each developing an average output of 40,000 HP under a net head of 450 m and at 300 rpm. This turbine works under a net head of 460 m, having two jets (also seen in the casing), deflectors, one piece cast steel runner with overall diameter of 3.56 m and weighing 16 tonne.

**10.19 Underwater Power Stations**—The underwater power stations are those which are constructed below the upper level of water. These are low head plants. Such an installation consists of weir across the river to be harnessed and housing the power station within the weir. The head race is connected to the tail race by a straight passage in which the tubular

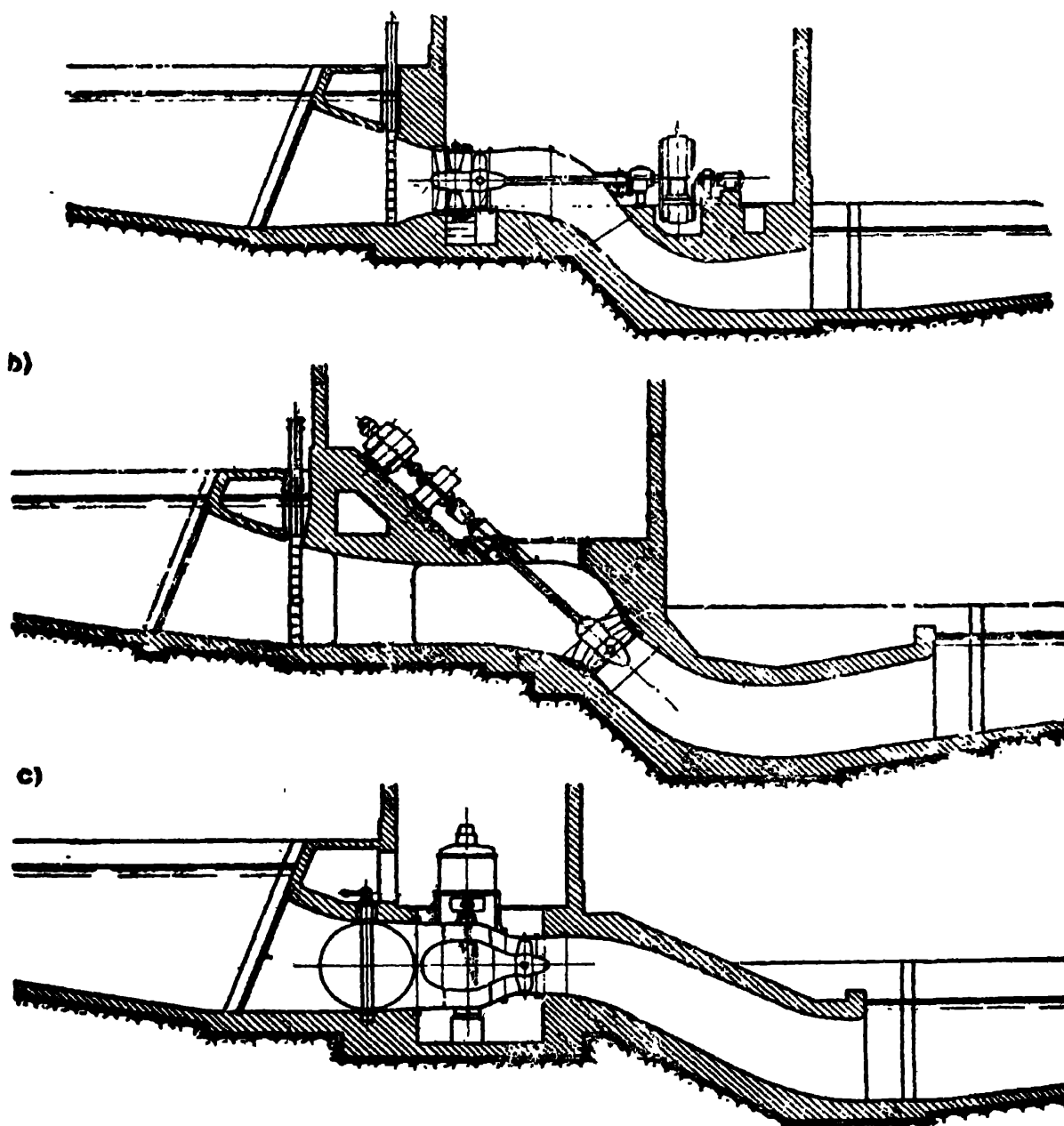


Fig 10.9 Different Layout of Tubular Turbine Power Station (a) Horizontal (b) Inclined (c) Horizontal Turbine and Vertical Generator

or bulb turbine operates. The turbine is connected to the generator by a horizontal or inclined shaft, depending on convenience. Fig 10.9(a) shows a turbine coupled to a generator by a horizontal shaft. The water flows axially without having deflection losses between inlet and outlet. The dismantling of turbine can be carried out by opening the coupling, without touching the generator. Fig 10.9(b) shows the coupling of the turbine and generator by an inclined shaft. In Fig 10.9(c) horizontal turbine drives a vertical generator through bevel gears. In such a case the generator being vertical, the dimensions of the power house building will be less, however, the gearing is to be properly sealed as shown in the figure.

**10.20 Tidal Power**—As explained in Art 4.11 tidal power is not classified as hydro-power. For the development of power two parameters are required which are discharge and head of water and these are made available due to the tide which further is due to the rotation of earth.

(a) **Tides**—Tides flood vast areas and raise the sea level along the seashore by amounts varying from a few centimetres to several metres. The highest tide anywhere on the earth is 19.6 m and is observed in Bay of Fundy (New Brunswick, Canada). The highest tide at the points of the globe are—

Britain—Seven Estuary 16.3 m,      France—Granville 14.7 m,  
 Argentina—Fuerto Gallegos 13.3 m, Australia—Fitzroy Estuary 10.0m,  
 India—Hooghly River 9.3 m and Bombay 9.0 m.

These tidal currents are used by ships to enter the river during flood. They are also used by inhabitants of seashore who go down the river with the ebb and return with the flood. The tidal current penetrates very far in some rivers e.g.,

120 km along Northern Dvina ; 250 km along Anadyr ; 500 km along Khatanga ; 700 km along Lawrance River ; 1,400 km along Amazon River.

Today every navigator has tide tables for determination of tidal height at any point and at any time of the day or night. The height prediction is accurate to one tenth of a metre and time prediction is 10 to 15 minutes.

(b) **Tidal Energy**—A tremendous amount of energy is stored in the earth in this process. The forces exerted are such as to slow down the earth rotation.

One billion KW of tidal power is dissipated by friction and eddies alone. For comparison, the economically exploitable power potential of all rivers in the world is only slightly more than this. 350 billions kwhr/year of tidal power is estimated in the world.

(c) **Tidal Theory**—The tide advances for 6 hours 12 minutes on to the land and then retreats during the next 6 hours 12 minutes. Tidal phenomenon is based on *Newton's static theory*. According to this theory the tide generating force has its origin in the interaction between sun, earth and moon. The lunar tide-generating force at any point on the globe is

equal to the difference between the force of attraction (pull) of the moon and the centrifugal force caused by the rotation of earth-moon system. If the earth were covered by a continuous layer of water, this body of water would acquire an ellipsoidal shape, its largest axis always pointing towards moon. While the ellipsoid together with moon makes one revolution about the earth, the earth itself revolves 29.53 times about its own axis, therefore due to this rotation each point on the earth surface continuously alters its distance.

**(d) Historical Development**—Man has been attempting for a time to produce power from the tides. Primitive tidal mills were installed on the shores of Gaul and Britain during the 11th century. In Britain such a tide mill, mentioned for the first time in 1170 in the records of the Parish of Woodbridge is still in operation in Deben Estuary. After changing four times, a new wooden water wheel of 6 metres diameter with 6 rpm was installed in 1932 using water of basin of 30,000 m<sup>2</sup> area. The enormous wooden wheels of tidal water lifting machine which supplied London with water in the past century remains as a living reproach to man who has harnessed the energy of atom but has not conquered the tides. In 1918 Marynard in France published a list of 218 patents on the utilization of tidal power. In addition several designs of tidal power plants were worked out and approved shortly before World War II. The French Parliament approved the building and equipment of a pilot plant at L' Aber Wrack in 1928. In USA, the building of a large tidal power plant at Quoddy was started in 1935. Work on both projects, however, was discontinued soon afterwards. In USSR a project for the building of the Kislogubskaya tidal plant was worked out in 1940. It was found that power of this plant would be twice as expensive as power supplied by a river power plant. Power of French plant came out to be 10% more, that of Severn plant UK tidal project 20% more and of Quoddy USA 100% more than the thermal power. Technically speaking a turbine operating under the head formed by the flood and ebb will have its direction of rotation changing periodically. The tidal head is low and therefore requires a large discharge which raises the cost. Further the tidal range is not constant, the plant will therefore operate under a variable head and have a variable output. The period of tide is 24 hours 50 minutes, the lunar day is 50 minutes longer than the solar day.

With the evolution of bulb turbine during the past few years the engineering thought is turning again in spite of all difficulties, to harness the tidal power.

**(e) Proper Utilization of Tide**—It has been proved by experiments in France that tidal energy can be increased by artificial arrangements *e.g.*,

(1) If the gates connecting the tidal basin with the sea are suddenly opened at high water, the tidal wave rapidly begins to raise the basin level. After reflection the filling wave returns to the dam, its crest being above the initial sea level. If the gates are quickly shut at exactly this instant, the plant can operate during the ebb under the additional head created by reflection of this flood wave.

(2) Pumping into the basin from the sea at high water and from the basin into the sea at low water is also another method of proper utilization of tidal power.

**10.21 Tidal Power Plants (TPP)**—The turbine used for TPP must operate under heads of different sign, alternatively from the sea and

from the basin, and that they are used for pumping and for idle discharge at level equalization. Cavitation limits provide high rotational speeds e.g., 1600 rpm under a head of 1.5 m. It is advantageous to use super high speed turbines of an entirely new design with a small number of narrow blades derived from wind turbine or screw turbine.

(1) Conventional Kaplan turbines are not suitable because the constant speed regulation in *TPP* turbine is not identical with maximum output regulation. It is seen that the maximum *TPP* output requires the turbine power to vary in proportion to the square of head. For constant speed regulation the turbine power output varies by a different law in which the exponent of the head is less than two. Hence the turbine regulation for constant speed does not correspond to regulation for maximum output.

(2) Another method is to produce D.C. power, the variable speed turbine being transformed into variable power at a voltage proportional to the cube of turbine speed. This gives turbine efficiency of 84%. However, now-a-days A.C. is used.

(3) The bulb type reversible turbine proposed by Escher Wyss Ltd consists of a horizontal Kaplan runner flanked on both sides by two distributors with speed rings in front of them.

(4) M/S Meyer suggested a double pump-turbine unit with a generator in streamlined casing in front of a turbine. The blade can be adjusted for pumping or turbine operation. In either case one runner acts as distributor.

### 10.22 Advantages of Tidal Power Plants—

(1) *To supply peak loads*—With the establishment of highly efficient thermal plants, there exists a very real problem of peak demand requiring uniform operating conditions for efficient running. It is therefore necessary to install standby capacity to meet peak demand. The need may be met by river power plants but when hydro-power is fully exploited, then tidal power will be useful.

(2) *To compensate the failure of hydro-power*—In India there is no guarantee of hydro-power due to occasional failure of monsoon. *TPP* will be very useful in view of the following advantages.

(3) *To provide constant power*—The most important feature of *TPP* consists in its constant monthly average throughout the year and for longer periods. It thus compensates the fluctuations of river hydro-energy during seasons when both resources are combined. The river hydro-power will compensate the periodical inequality of tidal power. *TPP* does not only give monthly or yearly average constancy but also over any number of years.

(4) *To improve the efficiency of entire power generating system*—*TPP* cannot only be used for meeting the demand created by the shortage in river power due to change in season or draught, and in mineral fuel reserves; but also for improving the efficiency of the entire power generating system i.e., ensuring that the electric plants in the system operate at optimum conditions to suit the load curve.

(5) *To operate the atomic and thermal power plants*—With the aid of tidal power, atomic as well as thermal power plants can be operated under most economical conditions. All these energy resources, used in conjunction, can create a balanced power supply.

If the effect of tidal power on the operation of the system is taken into account, tidal power is invariably cheaper than the thermal (one-third or one-half) of river hydro-energy.

**10.23 Tidal Power Plant Installations**—The construction of 240,000 KW *TPP* supplying 600 million KWhr yearly was started in 1960 in France. Two *TPP*, one at Chausey and the other in Minquiers Island which will supply 25 billions KWhr each, are in the planning stage.

Some *TPP* installations are also given in Table 7.3.

### UNSOLVED PROBLEMS

- 10.1 Write brief notes on recent developments in :
  - (a) Water turbines or hydro-prime movers in general
  - (b) Pelton turbines
  - (c) Francis turbines
  - (d) Kaplan turbines
  - (e) Cavitation
  - (f) Model testes on turbo-machinery.
- 10.2 Why is there a recent trend to use large water turbines ?
- 10.3 How can the rough cost of a turbine unit be calculated ?
- 10.4 Give an idea of physical facilities of modern workshops to manufacture large hydro units.
- 10.5 What are the shipping and handling difficulties of large units ?
- 10.6 On what factors is the choice of the position of best efficiency relative to the normal, operating point, depend ?
- 10.7 What is the value of the maximum efficiency of a water turbine ? Name the type of turbine which could have such a high efficiency. Why cannot this value be achieved in case of other hydro turbines ?
- 10.8 Name the types of steel which are used these days against cavitation.
- 10.9 State the measures adopted during operation for reducing cavitation effects.
- 10.10 State the methods by which the phenomenon of cavitation is studied these days.
- 10.11 What are different types of modern test stands for testing turbine models ? State the approximate cost utilised for carrying turbine model tests.
- 10.12 In which direction has the development of water power resources, taken place during the post World War II period ?
- 10.13 Describe different methods to supply peak load of the power stations. What are the different methods for achieving economical loading of the plant ?
- 10.14 What are pumped-storage plants ? Do they actually supply electric energy to the consumers ?
- 10.15. Classify pumped-storage plants.

- 10.16 What is the purpose of a pumped-storage installation used in conjunction with an extensive electricity distribution system? Explain the working of a typical system, showing how the pumps, the turbines and storage reservoirs are related.

Why is it customary to instal separate pump and turbine units, although a single rotating electrical machine will serve either as motor or as generator? Describe any system you know in which this difficulty is surmounted, and in which, therefore, the rotating hydraulic element will operate either as a pump or as a turbine.

[*AM I Mech E (Lond)*]

- 10.17 What is a reversible turbine-pump? Where do you use such a machine?
- 10.18 Is it economical to instal pump storage plants?
- 10.19 How will Deriaz runner be helpful in increasing the efficiency of pumped storage plants?
- 10.20 Where is it necessary to have underground power stations? Name some such stations.
- 10.21 What are under-water power stations?
- 10.22 Write a note on Tidal Power projects.
- 10.23 Why is tidal power not classified under hydro-power? Why any substantial progress has not been made so far to harness tidal power?
- 10.24 How is proper utilization of tides made?
- 10.25 Explain the tidal theory by which the power could be obtained.
- 10.26 Describe Tidal Power Plant (TPP). What are the advantages of TPP?



## SECTION III

# Pumps



## *Reciprocating Pumps*

**11.1 Pumps**—Pump is a mechanical device to increase the pressure energy of a liquid. In most of the cases pump is used for raising fluids from a lower to a higher level. This is achieved by creating a low pressure at the inlet or suction end and high pressure at the outlet or delivery end of the pump. Due to the low inlet pressure the fluid rises from a depth where it is available and the high outlet pressure forces it up to a height where it is required. Of course, work has to be done by a prime-mover on the pump to enable it to impart energy to the fluid.

A liquid pump is generally placed at a certain height of the liquid surface in the reservoir. The depth from which liquid has to be sucked by a pump is equivalent to its *Suction Head  $H_s$*  (refer Fig 11.1). The surface from where the water is drawn is usually exposed to the atmosphere and theoretical suction head is then equal to atmospheric pressure i.e. 10.3 m of water. In practice, however, it is never more than 8 m partly because of frictional losses and partly because of the fact that under a definite mini-

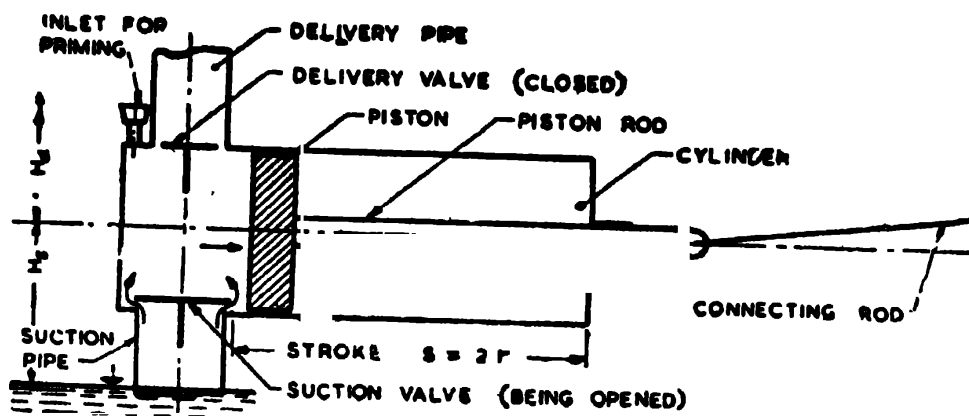
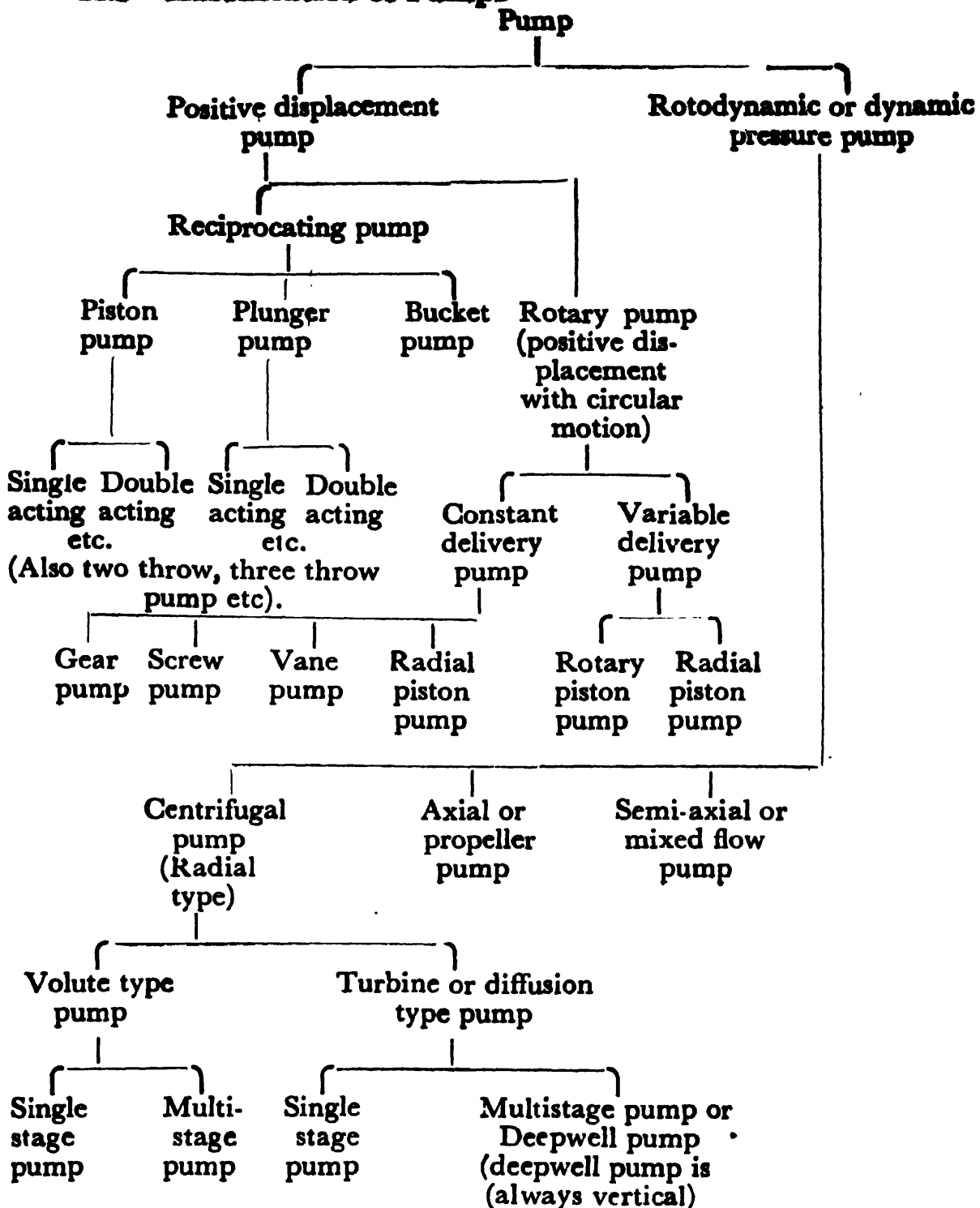


Fig 11.1 Reciprocating Single Acting Piston Pump. (Piston may be directly connected to the connecting rod)

mum absolute pressure, water would evaporate. The *delivery head  $H_d$*  is equivalent to the vertical height to which the liquid can be raised above the centre line of the pump,

## 11.2 Classification of Pumps—



Reciprocating pumps are now-a-days out of date and rotodynamic pumps especially of centrifugal type are replacing them. Reasons for this are given in Art 12.1. The chapter on reciprocating pumps is, therefore, altogether omitted in many modern text books. Here the author has, however, decided in favour of including it.

**11.3 Reciprocating Pumps**—A reciprocating pump consists primarily of a piston or a plunger reciprocating inside a close fitting cylinder, thus performing the suction and delivery strokes. The reciprocating pump is a positive acting type which means it is a displacement pump which creates lift and pressure by displacing liquid with a moving member or

piston. The chamber or cylinder is alternately filled and emptied by forcing and drawing the liquid by mechanical motion. This type is called "positive" inasmuch as the only limitation on pressure which may be developed is the strength of the structural parts. Suction and delivery pipes are connected to the cylinder as shown in Fig 11.1. Each of the two pipes is provided with a non-return valve. The function of the non-return or one way valve is to ensure a unidirectional flow of liquid. Thus the suction pipe valve allows the water only to enter the cylinder while the delivery pipe valve permits only its discharge from the cylinder.

Volume or capacity delivered is constant regardless of pressure, and is varied only by speed changes.

Reciprocating pump generally operates at low speeds and is therefore to be coupled to an electric motor with V-belts.

**Applications**—The reciprocating pump is best suited for relatively small capacities and high heads. In oil drilling operations this type of pump is very common.

The reciprocating pump is used generally for—

Pneumatic pressure systems, feeding small boilers condensate return and light oil pumping.

**11.4 Working Principle**—Movement of the piston or plunger creates a vacuum and atmospheric pressure forces the water up through the suction pipe into the cylinder. Suction pipe and clearance volume of the cylinder are first filled with water to replace the air. This is known as *priming* of the pump. Once the pump has been primed, water follows closely the piston or the plunger on its forward stroke. As a matter of fact, with the movement of piston this water column moves up from the water sump through the suction pipe with the forces of cohesion (i.e., molecules of water of suction pipe attracting the molecules of water in the sump) and the water moves with this piston with the forces of adhesion (i.e. molecules of water in the cylinder attracting the molecules of piston material). In the return or backward stroke water is pushed upwards into the delivery pipe. Delivery pipe valve must be opened before starting the pump.

If  $H_s$  and  $H_d$  be the suction and delivery heads respectively of the pump, then  $(H = H_s + H_d)$  is known as its "static head".

### 11.5 Piston Pumps—

(a) **Single Acting**—It consists of one suction and one delivery pipe simply connected to one cylinder (refer Fig 11.1).

Let  $A$  = the cross sectional area of the piston in  $m^2$

$a$  = the cross-sectional area of the piston rod in  $m^2$

$S$  = the stroke of the piston in m

$N$  = the speed of crank in rpm.

$$\text{Then average rate of flow} = \frac{A \cdot S \cdot N}{60} \text{ m}^3/\text{sec} \quad \dots(11.1)$$

$$\text{Force on piston in forward stroke} = \gamma \cdot H_s \cdot A \quad \text{kg.} \quad \dots(11.2)$$

$$\text{Force on piston in backward stroke} = \gamma \cdot H_d \cdot A \quad \text{kg} \quad \dots(11.3)$$

Neglecting head losses in transmission and at valves, horse power of the pump

$$= \frac{\gamma \cdot Q \cdot H}{75} = \frac{\gamma \cdot (A \cdot S \cdot N)(H_s + H_d)}{4,500} \text{ HP} \quad \dots(11.4)$$

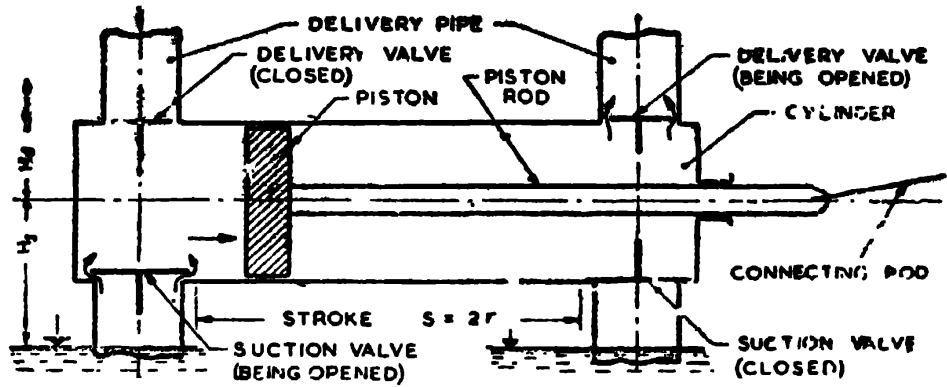


Fig 11.2 Double Acting Single Cylinder Piston Pump

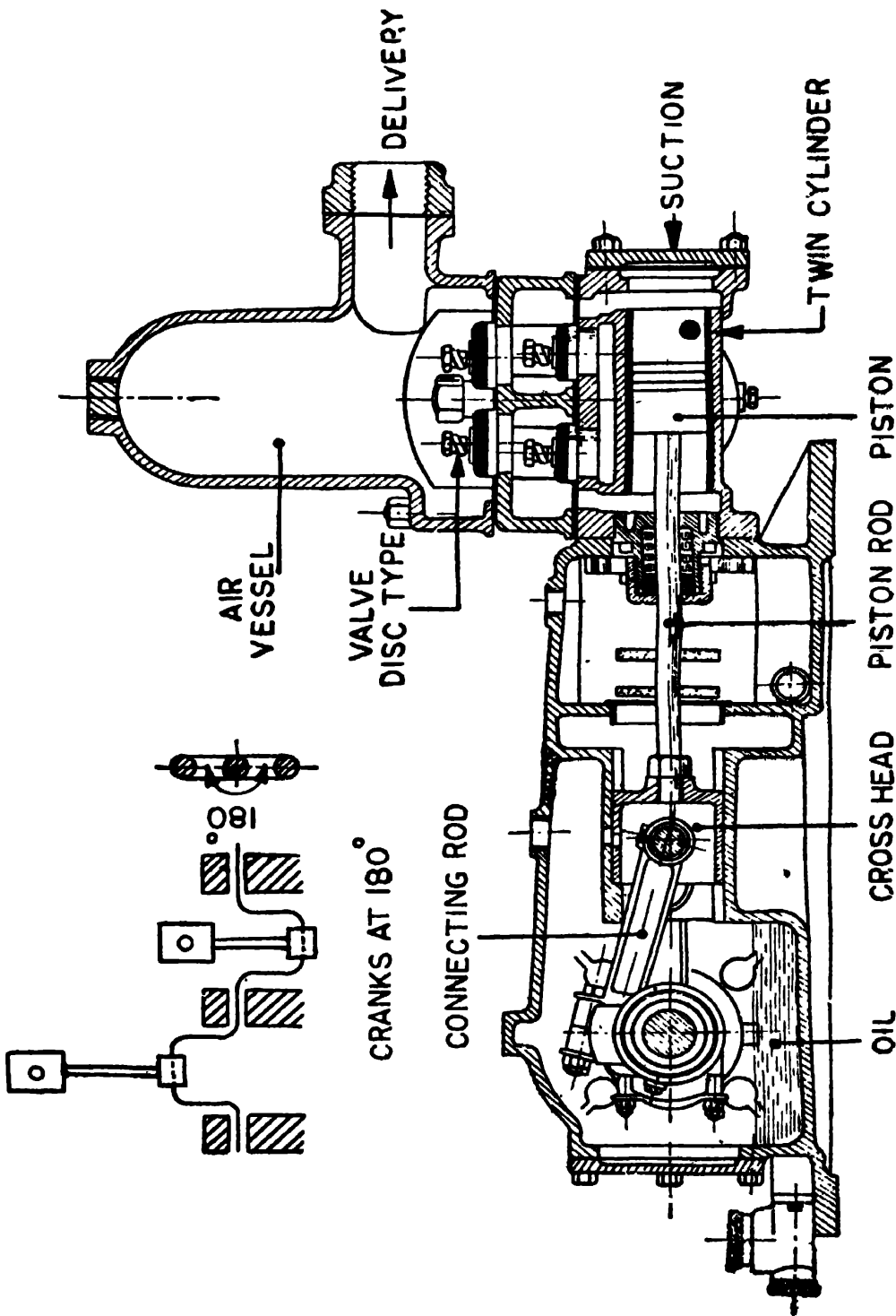


Fig 11.3 (a) Section Through a Low Speed, Heavy Duty Two-Throw Piston Pump  
(Two Cylinder with two Pistons Working with Cranks at 180°)

(b) **Double Acting Single Cylinder Pump**—It has two suction and two delivery pipes connected to one cylinder. Fig 11.2 shows such a slow speed piston pump.

$$Q = A \cdot S \cdot \frac{N}{60} + (A-a) \cdot S \cdot \frac{N}{60}$$

$$= \frac{S \cdot N (2A-a)}{60} \text{ m}^3/\text{sec} \quad \dots(11.5)$$

$$\approx \frac{2A \cdot S \cdot N}{60} \text{ m}^3/\text{sec} \quad \dots(11.5a)$$

Force acting on piston in forward stroke

$$= \gamma \cdot H_s \cdot A + \gamma \cdot H_d \cdot (A-a) \text{ kg} \quad \dots(11.6)$$

Force acting on piston during backward stroke

$$= \gamma \cdot H \cdot (A-a) + \gamma \cdot H_s \cdot A \text{ kg} \quad \dots(11.7)$$

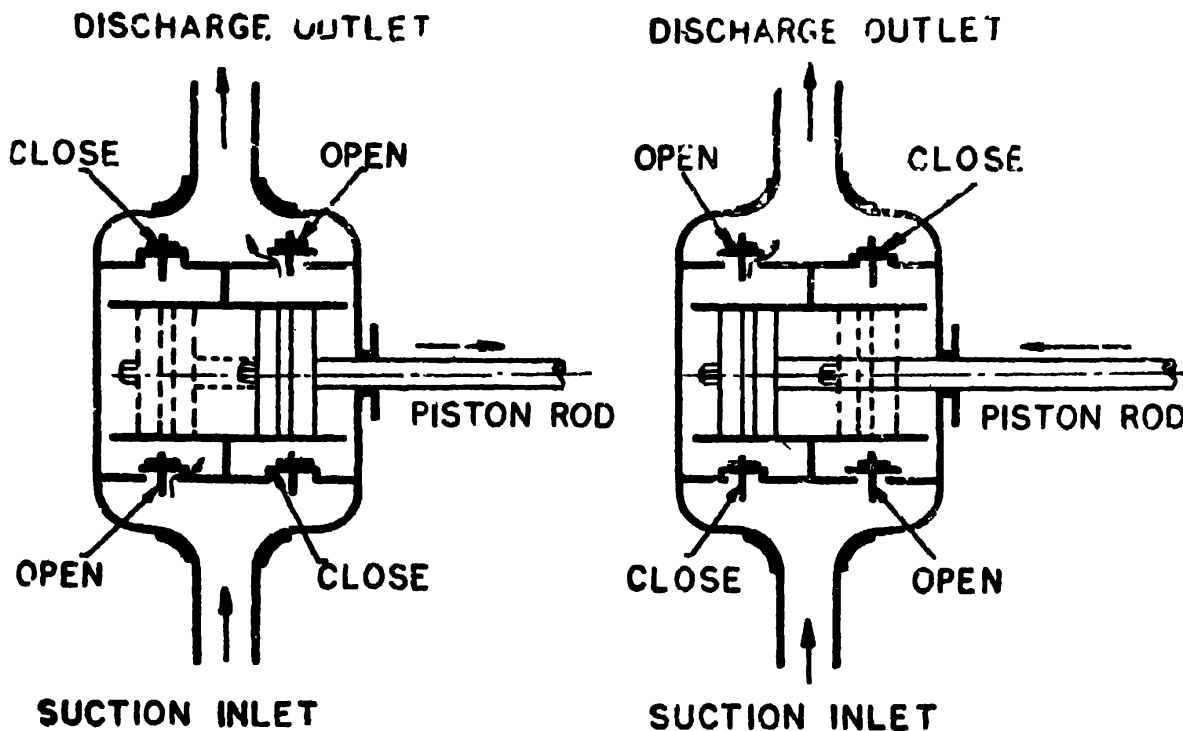


Fig 11.3 (b) Operation of Valves for Two-Throw Reciprocating Pump

(c) **Two-Throw Pump**—It has two cylinders each equipped with one suction and one delivery pipe. The pistons reciprocating in the cylinders are moved with the help of connecting rods fitted with a crank at  $180^\circ$  (refer Fig 11.3a). The operation of valves for such a pump is shown in Fig 11.3 (b).

The rate of flow through two throw pumps is given by Eqn 11.5 (a).

(d) **Three-Throw Pump**—It has three cylinders and three pistons working with three connecting rods fitted to the crank at  $120^\circ$ .

The rate of flow through three-throw pump is

$$Q = 3 A \cdot S \cdot \frac{N}{60} \text{ m}^3/\text{sec}$$

**Problem 11.1** A single acting reciprocating pump has its piston diameter as 15 cm and stroke 25 cm. The piston moves with simple harmonic

motion and makes 50 double strokes per minute. The suction and delivery heads are 5 m and 15 m respectively. Find the force required to work the piston during the suction as well as delivery stroke. Assume the efficiency of the suction and delivery strokes as 60% and 75% respectively. Determine the HP required by the pump.

**Solution**

$$\begin{aligned} D &= 15 \text{ cm} & S &= 25 \text{ cm} & H_s &= 5 \text{ m} \\ H_d &= 15 \text{ m} & \eta_s &= 0.6 & \eta_d &= 0.75 \\ N &= 50 \text{ double stroke per min} = 50 \text{ rpm} \end{aligned}$$

$$\begin{aligned} (a) \text{ Average force for suction} &= \frac{\gamma \cdot H_s \cdot A}{\eta_s} = \frac{1,000 \times 5 \times \frac{\pi}{4} \times 0.15^2}{0.6} \\ &= 147 \text{ kg.} \quad \text{Answer} \end{aligned}$$

$$\begin{aligned} (b) \text{ Average force for delivery} &= \frac{\gamma \cdot H_d \cdot A}{\eta_d} = \frac{1,000 \times 15 \times \frac{\pi}{4} \times 0.15^2}{0.75} \\ &= 374 \text{ kg} \quad \text{Answer} \end{aligned}$$

$$\begin{aligned} \text{HP required by the pump} &= \frac{\text{total force} \times \text{distance moved per sec}}{75} \\ &= \frac{\text{Force (suction + delivery)}}{75} \times \frac{S \cdot N}{60} \\ &= \frac{147 + 374}{75} \times 0.25 \times \frac{50}{60} \\ &= 1.45 \text{ HP} \quad \text{Answer} \end{aligned}$$

**Problem 11.2** A double acting reciprocating pump has a piston of 250 mm diameter and piston rod of 50 mm diameter which is on one side of the piston only. Length of the piston strokes is 380 mm and the speed of the crank moving the piston is 60 rpm. The suction and discharge heads are 5 m and 20 m respectively. Find the force required to work the piston during the 'in' and 'out' strokes. Neglect friction. Determine the quantity of water in litres per min raised by the pump and the HP required.

**Solution**

$$\begin{aligned} D &= 250 \text{ mm} & d &= 50 \text{ mm} & S &= 380 \text{ mm} \\ N &= 60 \text{ rpm} & H_s &= 5 \text{ m} & H_d &= 20 \text{ m} \end{aligned}$$

$$\text{Cross-sectional area of piston } A = \frac{\pi}{4} \times D^2 = \frac{\pi}{4} (0.25)^2 = 0.049 \text{ m}^2$$

$$\text{Cross-sectional area of piston rod } a = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} (0.05)^2 = 0.00196 \text{ m}^2$$

Force required to work the piston :

(a) during 'in' stroke—

$$(i) \text{ for suction} = \gamma \cdot A \cdot H_s = 1,000 \times 0.049 \times 5 = 245 \text{ kg}$$

$$\begin{aligned} (ii) \text{ for delivery} &= \gamma \cdot (A - a) \cdot H_d = 1,000 \times (0.049 - 0.00196) \times 20 \\ &= 940.8 \text{ kg} \end{aligned}$$

$$\therefore \text{ Total force during 'in' stroke} = 245 + 940.8 = 1,185.8 \text{ kg} \quad \text{Answer}$$

(b) Force during 'in' stroke—

$$(i) \text{ for suction} = \gamma \cdot (A-a) \cdot H_s = 1,000 \times (0.049 - 0.00196) \times 5 = 235.2 \text{ kg}$$

$$(ii) \text{ for delivery} = \gamma \cdot A \cdot H_d = 1,000 \times 0.049 \times 20 = 980 \text{ kg}$$

$$\therefore \text{ Total force during 'in' stroke} = 235.2 + 980 = 1,215.2 \text{ kg}$$

*Answer*

$$\text{Discharge during 'in' stroke} = A \cdot S \cdot N = 0.049 \times 0.380 \times 60 \times 1,000 = 1,120 \text{ lit/min}$$

$$\begin{aligned} \text{Discharge during 'out' stroke} &= (A-a) \cdot S \cdot N \\ &= 0.04704 \times 0.380 \times 60 \times 1,000 \\ &= 1,075 \text{ lit/min} \end{aligned}$$

$$\therefore \text{ Total quantity of water raised by the pump} = 1,120 + 1,075 = 2,195 \text{ lit/min}$$

*Answer*

$$\begin{aligned} \text{HP required by the pump} &= \frac{(\gamma \cdot Q \cdot H)}{75} = \frac{2,195 \times (5 + 20)}{60 \times 75} \\ &= 12.2 \text{ HP} \end{aligned}$$

*Answer*

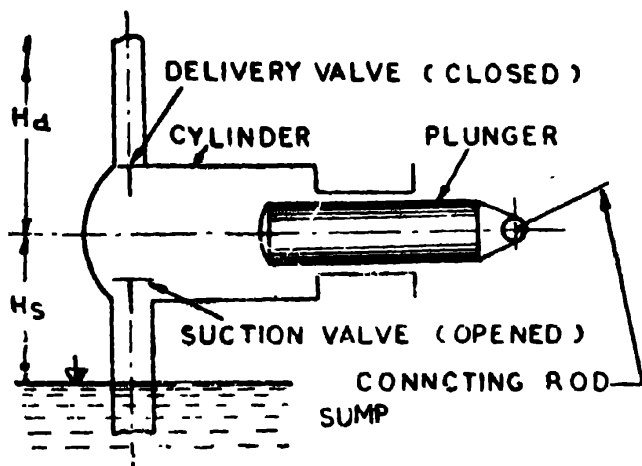


Fig 11.4 Single Acting Plunger Pump

### 11.6 Plunger Pump—

It is just a piston pump with a plunger replacing the piston and its rod. Generally used for rough work, it can build up a very high pressure. It is generally preferred for handling water containing sand. Single acting and double acting plunger pumps are illustrated in Fig 11.4 and 11.5 respectively. Rate of flow, forces acting on plunger and the horse-power of the pump can be easily determined as in the previous cases.

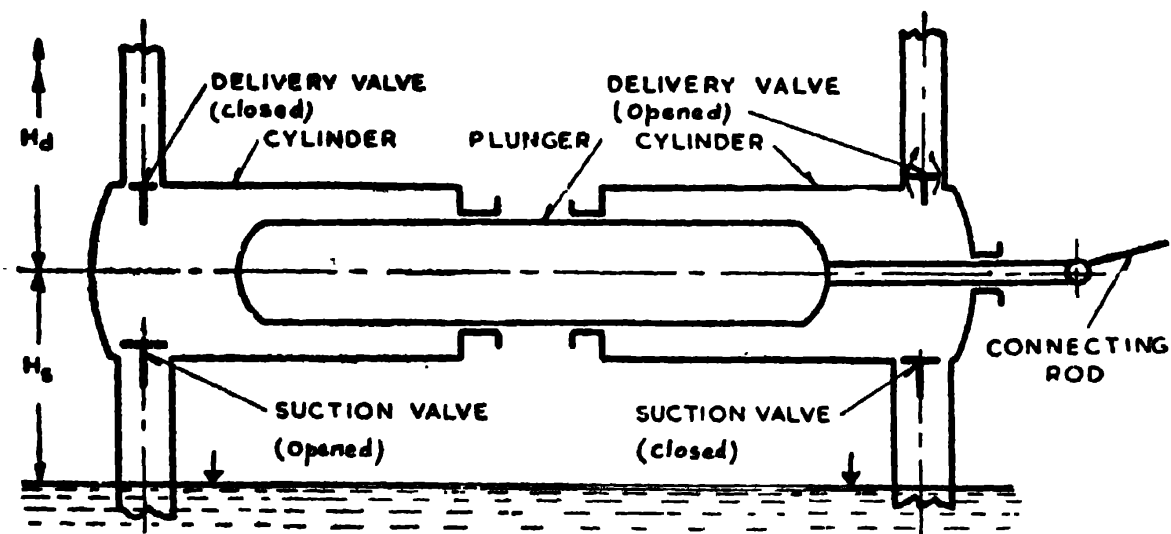


Fig 11.5 Double Acting Single Cylinder Plunger Pump

**11.7 Bucket Pump—**This is a vertical piston pump (refer Fig 11.6). The piston which is provided with a valve is called a bucket. It is essen-

tially a low speed pump. When driven by hand it is popularly known as a *hand pump*.

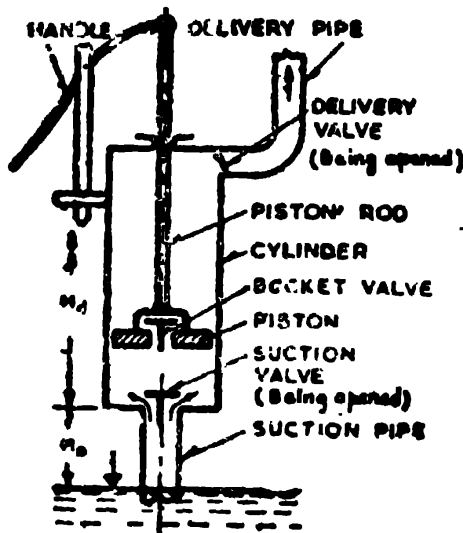


Fig 11.6 (a) Bucket Pump or Hand Pump

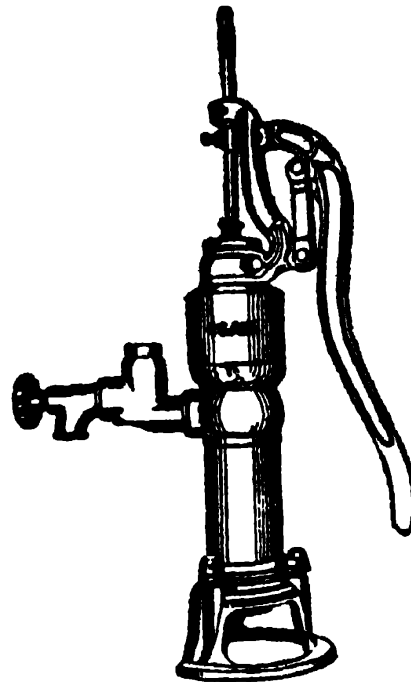


Fig 11.6 (b) Outside View of Bucket or Hand Pump

When the bucket rises, piston valve remains closed. It is the suction-cum-delivery stroke and during this stroke water is drawn into the space under the bucket and simultaneously the water above the bucket is forced into the delivery pipe. When the bucket falls neither delivery nor suction takes place. As the piston valve is open, water flows across the piston from the suction to the delivery side of the cylinder.

Force required to raise the bucket = Force required to produce suction + force required to force up the water (because the suction and the delivery take place at the same time) =  $\gamma H_1 A + \gamma H_2 (A - a)$  kg.

During down stroke the water from the lower side of the piston goes to the top of the piston, but as on the top of piston some volume is occupied by the piston rod, therefore, some force is required to raise this excess water against the delivery head.

$\therefore$  Force required to lower the bucket =  $\gamma H_2 a$  m<sup>3</sup>/sec

Volume discharged during up stroke =  $(A - a) \cdot S$  m<sup>3</sup>/sec

Volume discharged during down stroke =  $a \cdot S$  m<sup>3</sup>/sec

**11.8 Slip and Co efficient of Discharge**—*Slip* is the difference of volume swept through the piston and the actual discharge per stroke.

or  $\text{Slip} = \text{Volume swept/stroke} \text{ minus actual discharged/stroke}$  ... (11.8)

The value of the slip is generally positive. However in practice sometimes the delivery valve opens before the suction stroke is completed, thus delivering a greater volume of water than actually swept by the piston. Hence the slip will be *negative* in such a case.

Co-efficient of discharge  $C_d = \frac{\text{actual discharge/stroke}}{\text{Volume swept/stroke}}$



The value of  $C_d$  is generally less than unity, but in case the slip is negative,  $C_d$  will be more than one.

**Problem 11.3** A single acting reciprocating pump has its piston diameter of 15 cm and stroke of 30 cm. It discharges 300 litres of water per minute at 60 rpm. The suction and delivery heads are 5 metres and 15 metres respectively. Find the theoretical discharge, co-efficient of discharge and the percentage slip of the pump. How much HP will be required to drive the pump if its efficiency is 70%.  
(Nagpur University)

**Solution**

$$D = 15 \text{ cm} = 0.15 \text{ m}; S = 30 \text{ cm} = 0.3 \text{ m};$$

$$N = 60 \text{ rpm}; \eta = 0.7$$

$$Q_{act} = 300 \text{ lit/min} = 5 \text{ lit/sec} = 0.005 \text{ m}^3/\text{sec}$$

$$H = H_s + H_d = 5 + 15 = 20 \text{ m}$$

Cross-sectional area of piston

$$A = \frac{\pi}{4} \times 0.15^2 \text{ m}^2$$

Theoretical discharge of pump

$$\begin{aligned} Q_{th} &= \frac{ASN}{60} \\ &= \frac{\left(\frac{\pi}{4} \times 0.15^2\right) \times 0.3 \times 60}{60} = 0.0053 \text{ m}^3/\text{sec} \\ &= 5.3 \text{ lit/sec} \quad \text{Answer} \end{aligned}$$

Co-efficient of discharge

$$= \frac{Q_{act}}{Q_{th}} = \frac{5}{5.3} = 0.9434 \quad \text{Answer}$$

$$\begin{aligned} \text{Percentage slip} &= \frac{Q_{th} - Q_{act}}{Q_{th}} \times 100 \\ &= \frac{5.3 - 5}{5.3} \times 100 \\ &= 5.66\% \quad \text{Answer} \end{aligned}$$

HP required to drive the pump

$$\begin{aligned} &= \frac{P_{th}}{\eta} = \frac{\gamma Q_{th} H}{75\eta} \\ &= \frac{1000 \times 0.0053 \times 20}{75 \times 0.7} = 2 \text{ HP} \quad \text{Answer} \end{aligned}$$

**11.9 Rate of Delivery**—The reciprocating pumps are run by crank and connecting rod mechanism which gives the motion of piston as simple harmonic. In simple harmonic motion (SHM), the velocity of piston is equal to  $\omega r \sin \theta$  (refer Eqn 11.11), where  $\omega$  is the angular velocity of crank and  $r$  is the radius of crank. The rate of delivery or discharge is equal to the cross-sectional area of the pipe multiplied by the velocity of water. The velocity of water in the pipe is proportional to the velocity of piston, and is equal to  $\omega r \sin \theta \cdot \frac{A}{a}$  where  $A$  and  $a$  are the cross-sectional area of piston and pipe respectively. Thus the rate of

delivery into or out of the pump varies as  $\sin \theta$  where  $\theta$  is the crank angle, and is therefore not uniform.

(a) **Single-Acting Piston or Plunger Pump**—Rate of delivery can be plotted against crank angle. During the first half revolution of the

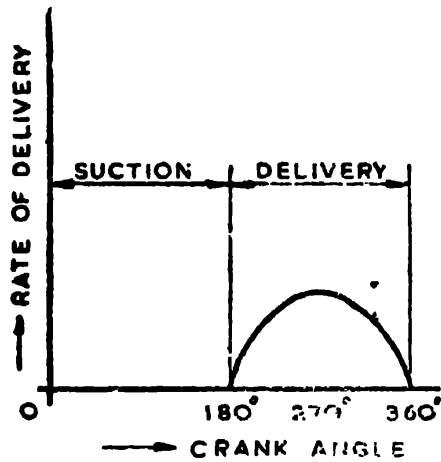


Fig 11.7 Rate of Delivery vs Crank Angle for Single Acting Piston or Plunger Pump

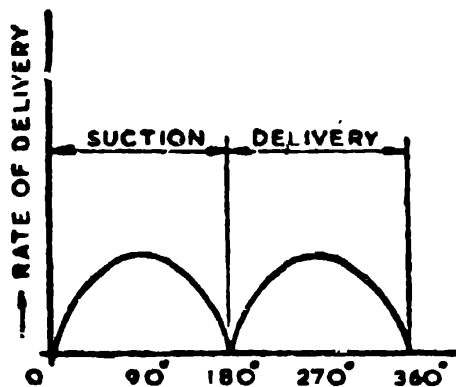


Fig 11.8 Rate of Delivery vs Crank Angle for Double Acting Single Cylinder Piston or Plunger Pump

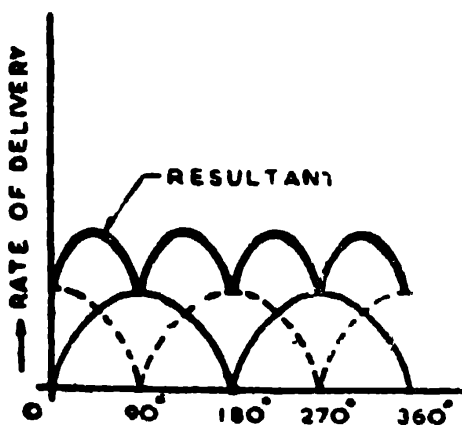


Fig 11.9 Rate of Delivery vs Crank Angle for Double Acting Driven by Two Cranks at Right Angles Piston or Plunger Pump

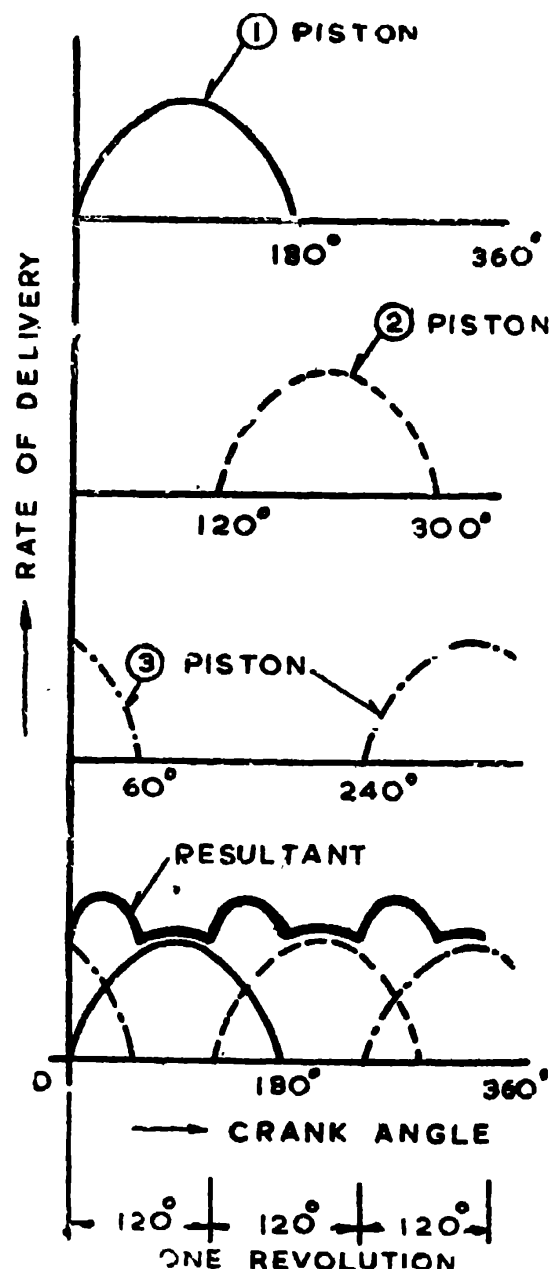


Fig 11.10 Rate of Delivery vs Crank Angle for Three-Throw Pump

crank there is only suction and during the second half only delivery. Afterwards the same cycle is repeated. If rates of flow into and out of the pump are regarded as having positive and negative signs respectively, then since the motion of the piston is simple harmonic, rate of delivery vs crank angle curve will be a sine curve (refer Fig 11.7). The part of the curve on one side of the axis represents suction and that on the other delivery.

Velocity of discharge of water at any instant is proportional to the velocity of the piston or plunger at that instant. Therefore the sine-curve obtained above also represents the velocities of discharge to same scale.

(b) **Double Acting, Single Cylinder Plunger Pump**—Each stroke is a suction cum delivery stroke. Curve of rate of delivery against angle of rotation of crank is therefore the resultant of two sine-curves drawn at a phase difference of  $180^\circ$ , only the delivery being considered (refer Fig 11.8).

(c) **Two-Throw Pump**—Rate of delivery is still variable. To make it somewhat uniform two equal cylinders with pistons connected to the perpendicular cranks of a common crank-shaft are employed. The two-throw pump may be single acting or double acting. For double acting two-throw reciprocating pump, the rate of delivery curve will be the resultant of two similar sine curves drawn at a phase difference of  $90^\circ$  (refer Fig 11.9).

(d) **Three-Throw Pump**—This pump is made by using three equal cylinders with pistons fitted to connecting rods  $120^\circ$  apart from one another

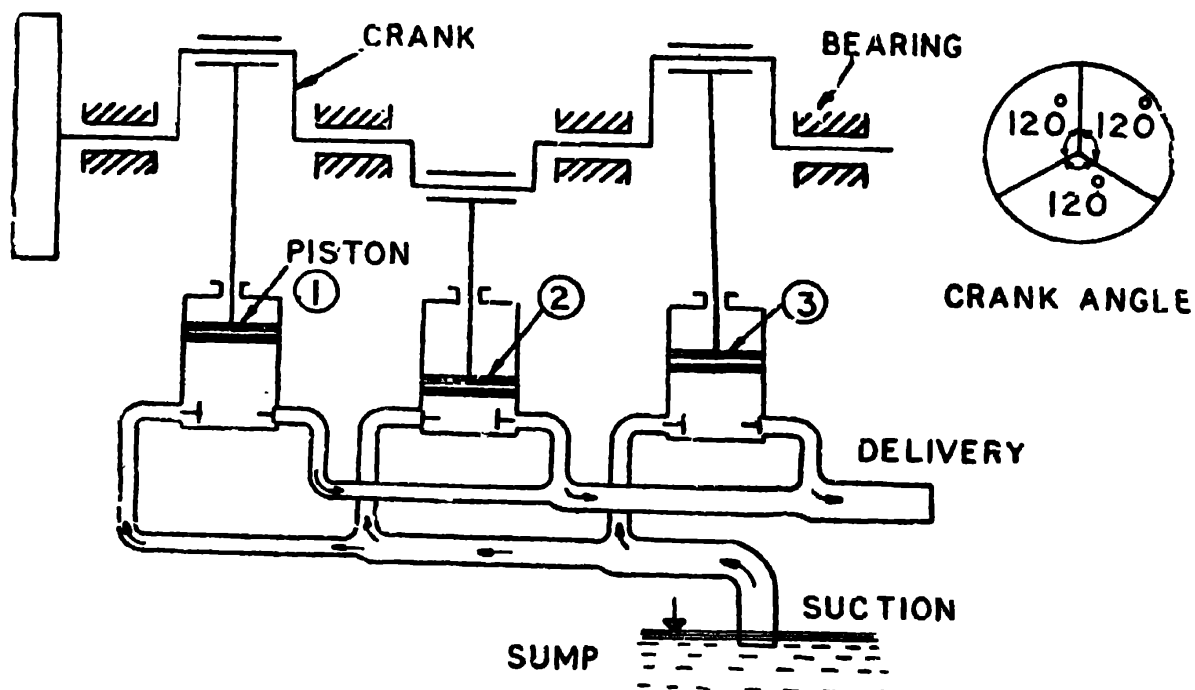


Fig 11.11 Three-Throw Piston Pump

but driven by a common crank shaft. Fig 11.11 shows a three-throw piston pump having three single-acting cylinders. Rate of delivery curve is the resultant of three similar curves at intervals of  $120^\circ$  (refer Fig 11.10). This gives more uniform rate of delivery.

**11.10 Velocity and Acceleration of Water in Reciprocating Pumps**—It has been stated earlier that after the pump has been primed, water follows the piston closely. It is of the utmost importance that there should be no discontinuity of flow i.e., there should be no separation of the flow of water in suction pipe, cylinder or delivery pipe. If at any instant separation takes place, it will result in a sudden change of momentum of the moving water which has been separated from the rest. This causes

an impulsive force which is responsible for the phenomenon of "water hammer" in reciprocating pumps. Pump is liable to fracture under the heavy shocks sustained as a result of this.

To eliminate the cause of water hammer viz., separation on the suction side of pump, driving force must be sufficient to accelerate the mass of water following the piston at the same rate as the piston itself. Assuming that the pressure inside the cylinder is zero when the piston moves forward, total suction pressure is equal to *atmospheric pressure* and it has to work against the following forces :

- (i) Work against gravity equivalent to suction height  $H_s$ ,
  - (ii) Work against inertial forces equivalent to head  $H_{a_s}$ ,
  - (iii) Work against frictional forces equivalent to head  $H_{f_s}$ ,
  - (iv) Work against force required to open the non-return valve  $H_{v_s}$ ,
  - (v) Work against friction in the valve  $H_{v_{f_s}}$ ,
  - (vi) Work against kinetic head due velocity of water in the suction pipe  $\frac{v_s^2}{2g}$ ,
  - (vii) Work against vapour pressure equivalent to head  $H_{vap}$
- $$\therefore H_{atm} = H_s + H_{a_s} + H_{f_s} + H_{v_s} + H_{v_{f_s}} + \frac{v_s^2}{2g} + H_{vap} \quad \dots(11.9)$$

Let  $f_p$  = the acceleration of piston;

$A$  = the cross-sectional area of piston ;

$a_s$  = the cross sectional area of suction pipe.

Then acceleration of water in suction pipe

$$f_s = f_p \cdot \frac{A}{a_s}$$

Accelerating force = mass  $\times$  acceleration

$$F_{a_s} = \frac{\gamma \cdot a_s \cdot L_s}{g} \cdot f_s \text{ kg}$$

where  $L_s$  = the length of suction pipe.

$$\text{Force per unit cross sectional area } p_{a_s} = \frac{\gamma \cdot L_s}{g} \cdot f_s \text{ kg/m}^2$$

$$\text{Head due to this force } H_{a_s} = \frac{L_s}{g} \cdot f_s \text{ m of water} \quad \dots(11.10)$$

**Acceleration of Piston and Water**—Let the crank in Fig 11.12 be at an angular distance  $\theta$  from its zero position at a time  $t$ .

Then the displacement of the position from its zero piston, if the connecting rod is very long,

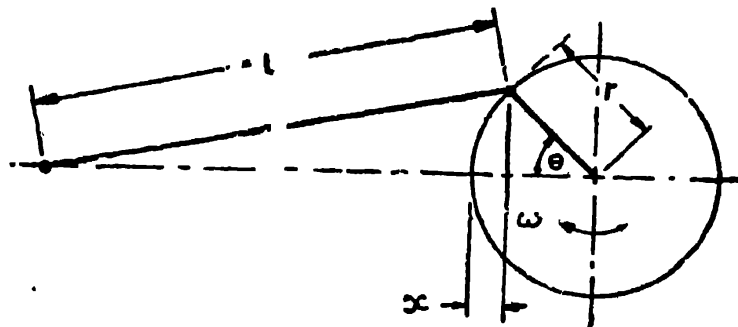


Fig 11.12 Determination of Velocity and Acceleration of Piston

$x = r - r \cos \theta$ , where  $r$  is the radius of the crank,

or  $x = r - r \cos \omega t$  ... (as  $\theta = \omega t$ )

$$\therefore \text{velocity of piston } \frac{dx}{dt} = \omega \cdot r \cdot \sin \theta \quad \dots(11.11)$$

$$\text{and acceleration of piston } \frac{d^2x}{dt^2} = \omega^2 \cdot r \cdot \cos \theta \quad \dots(11.12)$$

$$\text{Now, } f_s = f_p \cdot \frac{A'}{a_s} = \omega^2 \cdot r \cdot \cos \theta \cdot \frac{A}{a_s}$$

$$\text{and } H_{s_s} = \frac{L_s}{g} \cdot f_s = \frac{L_s}{g} \cdot \omega^2 \cdot r \cos \theta \cdot \frac{A}{a_s} \quad \dots(11.13)$$

This is maximum when  $\cos \theta = 1$  or  $\theta = 0^\circ$ , i.e., when piston is at its dead centre.

$$\therefore (H_{s_s})_{\max} = \frac{L_s}{g} \cdot \frac{A}{a_s} \omega^2 \cdot r \quad \dots(11.13 a)$$

To be more accurate, if connecting rod is  $n$  times as long as the crank,

$$H_{s_s} = \frac{L_s}{g} \cdot \omega^2 \cdot r \cdot \cos \theta \cdot \frac{A}{a_s} \cdot \left( \cos \theta \pm \frac{\cos 2\theta}{n} \right)$$

$$\text{and } (H_{s_s})_{\max} = \frac{L_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r \cdot \left( 1 \pm \frac{1}{n} \right) \quad \dots(11.13 b)$$

At the beginning of suction stroke—

$$(H_{s_s})_{\max} = \frac{L_s}{g} \cdot \frac{A}{a_s} \omega^2 \cdot r \left( 1 + \frac{r}{L} \right) \quad \dots(11.14)$$

and at the end of suction stroke—

$$(H_{s_s})_{\max} = \frac{L_s}{g} \cdot \frac{A}{a_s} \omega^2 \cdot r \left( 1 - \frac{r}{L} \right) \quad \dots(11.15)$$

where  $L$  = length of connecting rod.

**11.11 Speed**—The resultant of the seven forces enumerated above against which atmospheric pressure has to drive the water is maximum when the piston is at its dead centre, even though the velocity and consequently the frictional loss is zero at this point. If this maximum value is higher than the atmospheric pressure, separation will occur. The pump should, therefore, be so designed that separation does not occur when the piston is at its dead centre. Of the seven heads listed above,  $H_{vap}$  depends only on temperature and altitude, and  $H_s$  is automatically determined when the site of installation of pump is chosen. But the rest depend upon the length ( $L_s$ ) of the suction tube and speed ( $N$ ) of the crank. In general, the pump should have  $L_s$  a minimum possible under the circumstances, and the speed  $N$  should then be selected so as to avoid separation.

Hence the reciprocating pump are designed for low speed and cannot be coupled directly to modern prime movers or electric motors which operate at high speed.

**Maximum Speed**—The maximum speed of the crank can be determined from Eqn 11.14 at the beginning of suction stroke. Now

$$H_{atm} - (H_s + H_{s_s}) < H_{vap}$$

which shows that at the beginning of suction stroke, the pressure head should be less than the vapour pressure otherwise the separation will occur. The value of  $H_{vap}$  is 2.5 m of water absolute, then

$$10.3 - (H_s + H_{a_s}) < 2.5$$

or  $(H_s + H_{a_s})$  should be less than 7.8 m of water.

Thus when the value of  $H_s + H_{a_s}$  is more than 7.8 m of water vacuum, the vapours are formed and flow will not remain continuous resulting separation or cavitation.

Further suction head  $H_s$  is constant for a particular pump installation, then the only variable is the accelerating head  $H_{a_s}$ , which is required to be kept within certain limits.

$$\begin{aligned} \text{Now } H_{a_s} &= \frac{L_s}{g} \cdot \frac{A}{a_s} \omega^2 r \quad \dots(\text{refer Eqn 11.13 a}) \\ &= \frac{L_s}{g} \cdot \left( \frac{D}{d_s} \right)^2 \cdot \omega^2 r \end{aligned}$$

From this equation the maximum value of  $H_{a_s}$  depends upon the length of suction pipe  $L_s$ , ratio of squares of cylinder diameter to that of suction pipe diameter  $\left( \frac{D}{d_s} \right)^2$ , angular velocity of rotating crank  $\omega$  or crank speed  $N$ , and crank radius. In order to limit  $L_s$  the pump should not be installed away from the sump from which the water has to be drawn. For limiting  $\left( \frac{D}{d_s} \right)^2$  it will be seen that the cylinder bore is not much bigger than the suction pipe diameter. The value of  $r$  is half of stroke  $S$  which is related to cylinder diameter  $D$ . Considering all of  $L_s$ ,  $d_s$ ,  $D$  and  $S$  as constant for a particular pump, the only variable will be its speed  $N$ . Since the value of  $H_{a_s}$  is limited, the speed of reciprocating pump is also restricted. Thus the maximum permissible speed can be found if  $H_{a_s}$  is known.

**Problem 11.4** *The plunger in reciprocating pump moves with simple harmonic motion. The diameter of plunger is 25 cm and stroke 45 cm. The suction pipe is 125 mm in diameter and 12 m long. The suction lift is 3 m. Calculate the speed at which the pump can operate without separation occurring at the beginning of the stroke. There is no air vessel on the suction side. The barometer reads 9.15 m of water. (AMIE)*

**Solution**

$$\begin{aligned} D &= 25 \text{ cm} = 0.25 \text{ m} & S &= 45 \text{ cm} = 0.45 \text{ m} \\ d_s &= 125 \text{ mm} = 0.125 \text{ m} & L_s &= 12 \text{ m} \\ H_s &= 3 \text{ m} & H_{atm} &= 9.15 \text{ m} \end{aligned}$$

Assume that the separation occurs at absolute pressure of 2.5 m of water. Now, at beginning of the suction stroke the velocity is zero, therefore, the friction losses on the suction side,  $H_{fs}$ , are equal to zero. The accelerating head, however, would then be maximum.

$$\therefore H_{atm} = H_s + H_{a_{s_{max}}} + H_{vap}$$

$$\text{or } H_{atm} = H_s + \frac{L_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r + H_{vap}$$

$$\text{or} \quad 9.15 = 3 + \frac{12}{9.81} \times \left( \frac{0.250}{0.125} \right)^2 \times \left( \frac{2\pi N}{60} \right)^2 \times \frac{0.45}{2} + 2.5$$

$$\left( \because r = \frac{S}{2} = \frac{0.45}{2} \right)$$

$$\text{or} \quad 3.65 = \frac{12}{9.81} \times 4 \times \frac{4\pi^2}{3,600} N^2 \times \frac{0.45}{2}$$

$$\therefore N = \sqrt{\frac{9.81 \times 3,600 \times 2 \times 3.65}{12 \times 4 \times 4\pi^2 \times 0.45}}$$

$$\text{or} \quad N = 17.55 \text{ rpm} \quad \text{Answer}$$

This speed is very low. Therefore it is difficult to couple the pump directly with the modern electric motors which generally run at 1,450 rpm (assuming 50 cycles frequency), hence it necessitates an increase in speed.

### 11.12 Indicator Diagrams—

(a) **Theoretical Indicator Diagram**—Fig 11.13 shows the theoretical indicator diagram of a reciprocating pump under ideal conditions. The stroke and the pressure of the piston are represented by  $x$ - and  $y$ -axes on the diagram respectively. The area  $klmn$  of the rectangle represents the work done per revolution for a *single* acting pump. For *double* acting pump, the work done per revolution is equal to twice the area of rectangle  $klmn$ .

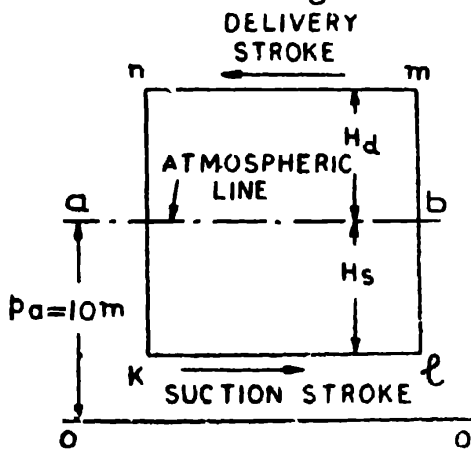


Fig 11.13 Theoretical Indicator Diagram

In the above diagram line  $ab$  represents the atmospheric pressure. The pressure during suction stroke is below atmosphere by an amount equal to the suction head  $H_s$  and therefore the line  $kl$  represents the suction stroke. Similarly the line  $mn$  represents delivery stroke which is above atmospheric pressure by an amount equal to delivery head  $H_d$ .

Work done per revolution for single acting pump—

- (i) during suction stroke = area  $klba$
- (ii) during delivery stroke = area  $abmn$ .

(b) **Effect of Acceleration in Suction Pipe on Indicator Diagram**—The acceleration head  $H_a$  depends upon  $\cos \theta$  as proved in Eqn 11.13, therefore accelerating head curve is a cosine curve. Hence the accelerating head is maximum at the beginning, zero at the centre and maximum (negative) at the end of suction stroke of piston. The indicator diagram represents suction and accelerating heads *versus* suction stroke ( $x = r - r \cos \theta$ ). This makes the shape of the indicator diagram as shown in Fig 11.14, that is at the beginning of the suction stroke the negative pressure becomes high, equal to  $H_s + H_{as}$ , and at the end of the suction stroke the negative pressure is reduced by  $H_{as}$ .

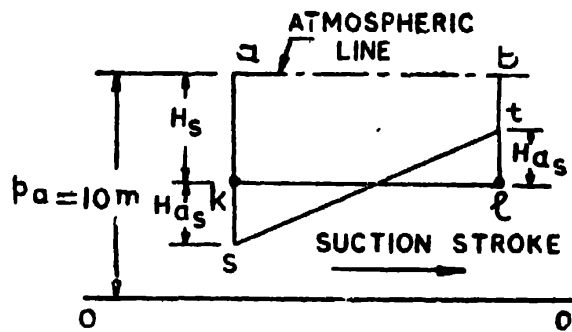


Fig 11.14 Effect of Acceleration in Suction Pipe on Indicator Diagram

The area  $s t b a$  of the diagram remains the same, therefore the work done is unaltered.

It is important to note that point  $s$  should not fall below vapour pressure otherwise the separation will take place.

**(c) Effect of Acceleration in Delivery Pipe on Indicator Diagram**—The pump has to work against the delivery and accelerating heads, therefore the indicator diagram will be modified as shown in Fig 11.15. The acceleration head is added at the beginning and is subtracted at the end of the delivery stroke. The total area  $p q a b$  of the diagram remains same and hence the work done is also not changed.

The maximum pressure head during delivery stroke occurs at point  $p$  which is equal to  $H_d + H_{a_d}$  above atmosphere.

The minimum pressure head in the cylinder  $= H_d - H_{a_d}$  above atmosphere. This is represented by point  $q$  in Fig 11.15.

or Absolute minimum pressure head  $= 10.3 + (H_d - H_{a_d})$

The separation is likely to occur at the end of delivery stroke if this value is less than 2.5 m of water,

or  $10.3 + H_d - H_{a_d} > 2.5 \text{ m}$   $H_{a_d} < 7.8 + H_d$

**(d) Effect of Friction in Pipes on Indicator Diagram**—In practice there is always some loss of head due to pipe friction which can be determined as follows—

In Eqn 11.11 the velocity of piston  $= \omega r \sin \theta$ .

$\therefore$  Velocity of water in the pipe  $v = \frac{A}{a} \omega r \sin \theta$

Loss of head due to friction—

$$H_f = \frac{4fL}{d} \cdot \frac{v^2}{2g} = \frac{4fL}{2gd} \left( \frac{A}{a} \omega r \sin \theta \right)^2 \quad \dots(11.16)$$

At the beginning of stroke  $\theta = 0^\circ$ , then  $H_f = 0$ .

At the mid of stroke  $\theta = 90^\circ$ ,  $\sin \theta = 1$ , then  $H_f$  becomes maximum

$$H_{f_{max}} = \frac{4fL}{2gd} \left( \frac{A}{a} \cdot \omega r \right)^2 \quad \dots(11.16 a)$$

At the end of stroke  $\theta = 180^\circ$ , then  $H_f = 0$ , as the velocity of water is zero.

Considering the above frictional effect in the suction pipe

(i) in the beginning of suction stroke—

Pressure head  $= H_{atm} - (H_s + H_{a_s})$  as  $H_f$  is zero

(ii) at the mid of suction stroke—

Pressure head  $= H_{atm} - (H_s + H_f)$   $H_{a_s}$  is zero

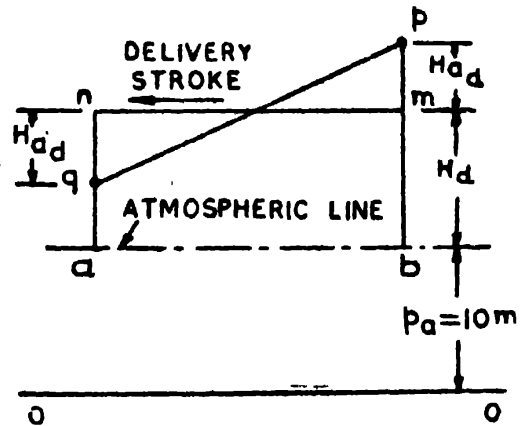


Fig 11.15 Effect of Acceleration in Delivery Pipe on Indication Diagram.



(iii) at the end of suction stroke—

Pressure head =  $H_{atm} - (H_s - H_{as})$  as  $H_f$  is zero

Similarly pressure head at the beginning of delivery stroke

$$= H_{atm} + (H_d + H_{ad})$$

Pressure head at the middle of delivery stroke =  $H_{atm} + (H_d + H_{fd})$

Pressure head at the end of delivery stroke =  $H_{atm} + (H_d - H_{ad})$

Modifying the indicator diagram,

the friction head in suction as well as in delivery pipes will be added to the indicator diagram, such that the inclined lines  $st$  (refer Fig 11.14) and  $pq$  (refer Fig 11.15) will become parabolic curves as shown in Fig 11.16. As the area of the diagram will now increase, the total work done will also increase.

Work done per revolution for single acting pump.

(i) during suction stroke = area  $t u s b a$  ;

(ii) during delivery stroke = area  $a b p v q$  ;

(iii) total during both strokes = area  $t u s p v q$ .

**11.13 Effect of Bent Delivery Pipe on Separation**—The pump may be delivering water with a delivery pipe of shape shown in Fig 11.17a

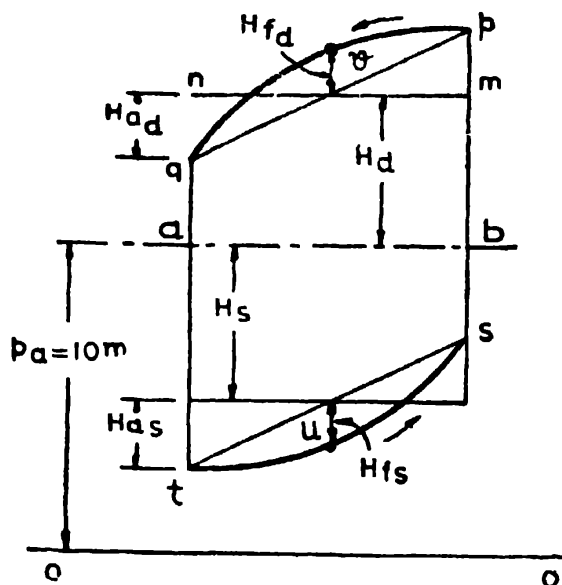


Fig 11.16 Effect of Friction in Pipes on Indicator Diagram

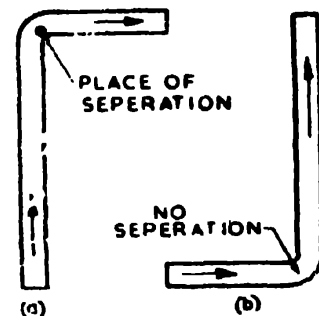


Fig 11.17 Separation in Bent Delivery Pipes (a) First Vertical then Horizontal (c) First Horizontal then Vertical

or Fig 11.17b. In Fig 11.17a the delivery pipe is vertical first and then it is bent to be horizontal. In this case the delivery head will become zero at the bend, after which there is still a long horizontal pipe which will have a considerable value of accelerating head  $H_{ad}$ . Therefore according to the above equation the separation will take place in such a case at the pipe bend. In Fig 11.17b, the pipe is horizontal first and then it becomes vertical. There is no possibility of separation in this case as at the bend, there is still considerable  $H_d$  that is always greater than  $H_{ad}$ .

**Problem 11.5** A single acting reciprocating pump has a plunger diameter of 200 mm and stroke of 300 mm. The suction pipe is 100 mm in

diameter and 8 m long. The water surface in the sump from which the pump draws water is 4 m below the pump cylinder axis. If the pump is working at 30 rpm, find the pressure head on the piston at the beginning, middle and end of the suction stroke. Take  $f = 0.01$ . (AMIE)

**Solution**

$$D = 200 \text{ mm}$$

$$d_s = 100 \text{ mm}$$

$$S = 300 \text{ mm}$$

$$\therefore r = \frac{300}{2} = 150 \text{ mm}$$

$$L_s = 8 \text{ m}$$

$$H_s = 4 \text{ m}$$

$$N = 30 \text{ rpm}$$

$$f = 0.01$$

The atmospheric pressure has to supply :

$$H_{atm} = H_s + H_{a_s} + H_{f_s} + H_{piston} \quad \dots (\text{neglecting kinetic head})$$

(a) Pressure head on the piston at the beginning of suction stroke—

$$H_{f_s} = 0 \quad (\because \text{Velocity is zero})$$

$$\therefore H_{atm} = H_s + \frac{L_s}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r + H_{piston}$$

(assuming connecting rod length to be very long in comparison to crank radius)

$$\therefore 10 = 4 + \frac{8}{9.81} \times \left( \frac{200}{100} \right)^2 \times \left( \frac{2\pi \times 30}{60} \right)^2 \times 0.150 + H_{piston}$$

$$\therefore = 4 + 4.84 + H_{piston}$$

$$\therefore H_{piston} = 10 - (4 + 4.84)$$

$$\text{or} \quad = 1.16 \text{ m of water absolute}$$

$$\text{or} \quad 8.84 \text{ m of water vacuum}$$

$$\text{or} \quad 11.6 \times \frac{1,000}{100 \times 100} = 0.116 \text{ kg/cm}^2 \quad \text{Answer}$$

(b) Pressure head on the piston at the middle of the suction stroke—

$$H_{a_s} = 0 \quad (\because \cos \theta = \cos 90^\circ)$$

$$\therefore H_{atm} = H_s + H_{f_s} + H_{piston} \dots \left( \text{neglecting kinetic head } \frac{v_s^2}{2g} \right)$$

$$= H_s + \frac{4fL_s}{d_s} \cdot \frac{v_s^2}{2g} + H_{piston}$$

$$= H_s + \frac{4fL_s}{d_s} \cdot \frac{\left( \omega \cdot r \cdot \frac{A}{a_s} \right)^2}{2g} + H_{piston}$$

$$\text{or} \quad 10 = 4 + \frac{4 \times 0.01 \times 8}{0.1 \times 2 \times 9.81} \times \left[ \frac{2\pi \times 30}{60} \times 0.15 \times \left( \frac{200}{100} \right)^2 \right]^2 + H_{piston}$$

$$\text{or} \quad 10 = 4 + 0.58 + H_{piston}$$

$$\therefore H_{piston} = 10 - 4.58 = 5.42 \text{ m of water absolute}$$

$$\text{or} \quad 4.58 \text{ m of water vacuum}$$

$$\text{or} \quad 5.42 \times \frac{1,000}{100 \times 100} = 0.542 \text{ kg/cm}^2 \text{ absolute} \quad \text{Answer}$$

(c) Pressure head on the piston at the end of suction stroke—

At the instant when suction valve is just going to be closed and delivery valve has not yet opened, suction head  $H_s$  is still acting on the piston. Also  $\theta = 180^\circ$ .

$$\therefore \cos 180^\circ = -1 \quad \text{i.e., } H_{a_s} \text{ is negative}$$

$$\therefore H_{atm} = H_s + (-H_{a_s}) + H_{f_s} + H_{piston}$$

Further  $H_{f_s} = 0$  (∵ velocity is zero)

$$\therefore H_{atm} = H_s - H_{a_s} + H_{piston}$$

$$\text{or} \quad 10 = 4 - 4.84 + H_{piston} \quad \dots [H_{a_s} \text{ is same as in case of (a)}]$$

$$\text{or} \quad H_{piston} = 10 - 4 + 4.84$$

$$= 10.84 \text{ m of water absolute}$$

i.e., more than atmospheric pressure

or 0.84 m of water gauge

$$\text{or} \quad 10.84 \times \frac{1,000}{100 \times 100} = 1.084 \text{ kg/cm}^2 \text{ absolute Answer}$$

**11.14 Air Vessel—**Air vessel is a closed chamber fitted on the suction as well as on the delivery side, near the pump cylinder, to reduce the accelerating head.

**Functions :**

(a) *Suction side :*

- (1) Reduces the possibility of separation.
- (2) Pump can be run at a higher speed.
- (3) Length of suction pipe below the air vessel can be increased.

(b) *Delivery side—*

- (4) A large amount of power consumed in supplying accelerating head can be saved.
- (5) Constant rate of discharge can be ensured.

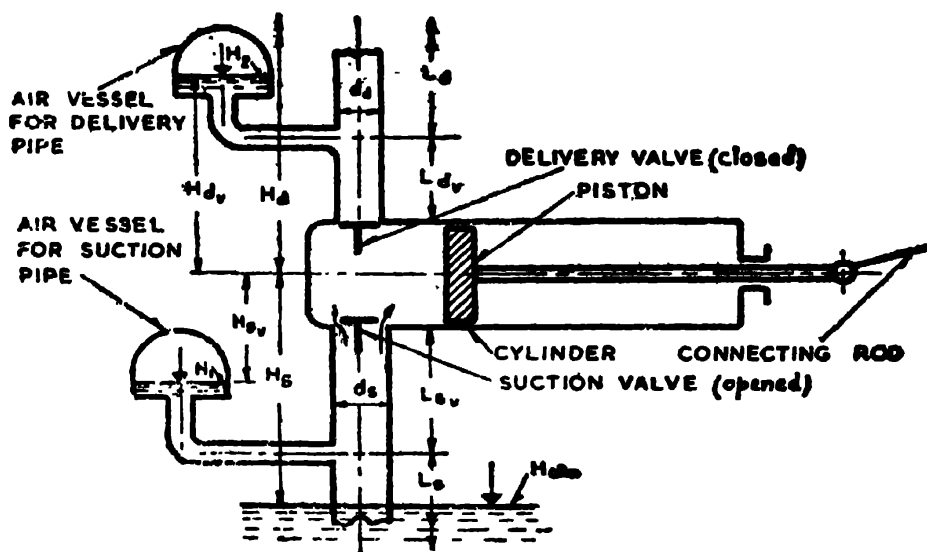


Fig 11.18 Air Vessel on Suction and Delivery Sides

In order to eliminate the possibility of separation, the length of the suction pipe where the fluctuation of acceleration takes place can be effectively reduced by inserting the air vessel in the suction pipe. The air vessel is fitted near the pump cylinder, thus the length of the fluctuating column is considerably reduced, allowing the pump to run at a higher speed without the danger of separation.

In some cases where the pump has to be installed away from the sump, a long suction pipe can be safely used by fitting the air vessel near the pump cylinder.

Air vessel is also fitted on the delivery side to minimise the length of the fluctuating water column. Since the length is usually large, accelerating head produced will be considerable in the absence of an air vessel. Therefore the air vessel will save the power consumed in supplying the accelerating head.

Without air vessel, the rate of discharge varies according to the sine curve (refer Fig 11.7). By fitting the air vessel constant rate of discharge will be ensured.

**Working**—An air vessel in a reciprocating pump acts like a fly wheel of an engine. The top of the vessel contains compressed air which can contract or expand to absorb most of the pressure fluctuations.

Whenever the pressure rises, water in excess of the mean discharge is forced into the air vessel, thereby compressing the air held therein. When the water pressure in pipe falls, the compressed air ejects the excess water out.

The air vessel acts like an intermediate reservoir. On suction side, the water first accumulates here and is then transferred to the cylinder of the pump. On delivery side, the water first goes to the vessel and then flows with a uniform velocity. The column of water which is now fluctuating, is only between the pump cylinder and the air vessels which is very small due to vessels being fitted as near to the pump cylinder as possible.

**11.15 Suction in Pump with Air Vessel**—(refer Fig 11.18)—When an air vessel is used, suction takes place in two steps. First the water flows through the suction pipe into the air vessel and then it is raised from the air vessel to the cylinder.

Considering the portion between air vessel and cylinder, let  $H_{s_1}$  be the static head between the centre line of cylinder and water level in air vessel,  $L_{v_1}$  be the length of the suction pipe between them. Then if  $H_{v_1}$  be the pressure head acting on water surface in the air vessel, it must work against the forces resisting opening of the non-return valve, the vapour pressure and frictional resistance to flow of water. Besides,  $H_{v_1}$  should also accelerate the water.

∴ Head in air vessel = static head + accelerating head + frictional head + valve raising head + valve frictional head + vapour pressure + k.e.

$$\text{or } H_{v_1} = H_{s_1} + H_{a_{s_1}} + H_{f_{s_1}} + H_{v_s} + H_{v_f} + H_{v_{ps}} + \frac{v_{s_1}^2}{2g} \dots (11.17)$$

The acceleration  $f_s$  is maximum when piston is at its dead centre i.e., at the beginning of suction stroke when velocity is zero. Then

$$H_{v_1} = H_{s_s} + (H_{a_{s_s}})_{max} + 0 + H_{v_s} + H_{v_{f_s}} + H_{v_{a_s}}$$

$$\text{or } H_{v_1} = H_{s_s} + \frac{L_{s_s}}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r + H_{v_s} + H_{v_{f_s}} + H_{v_{a_s}} \quad \dots(11.18)$$

Similarly deriving equation for the portion between the sump and the air vessel :

There is no fluctuating head in this portion as the flow is uniform.

$\therefore$  Atmospheric head = static head + head in air vessel + frictional head due to *uniform* flow + kinetic head due to uniform velocity.

$$\text{or } H_{atm} = (H_s - H_{s_s}) + H_{v_1} + \frac{4fL_s}{d_s} \cdot \frac{v_s^2}{2g} + \frac{v_s^2}{2g} \quad \dots(11.19)$$

Therefore adding the two steps (Eqns 11.18 and 11.19), the atmospheric pressure has to work against the following forces :

$$H_{atm} = H_s + \frac{4fL_s}{d_s} \cdot \frac{v_s^2}{2g} + \frac{L_{s_s}}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r + H_{s_s} + H_{v_{f_s}} + H_{v_{a_s}} + \frac{v_s^2}{2g} \quad \dots(11.20)$$

From this equation it is seen that since  $L_{s_s}$  has become smaller because of the introduction of an air vessel,  $H_s$  can, therefore, be increased. Thus, the pump can be placed at a greater height above the available water surface. This is obviously an advantage in many cases.

$\therefore$  Pressure head on the piston at the beginning of suction stroke

$$\begin{aligned} &= H_s + \frac{4fL_s}{d_s} \cdot \frac{v_s^2}{2g} + \frac{L_{s_s}}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r + H_{s_s} + H_{v_{f_s}} + \frac{v_s^2}{2g} \quad \dots(11.21) \\ &= H_s + \frac{4fL_s}{d_s} \cdot \frac{v_s^2}{2g} + \frac{L_{s_s}}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r + \frac{v_s^2}{2g} \\ &\quad \text{(neglecting valve losses)} \quad \dots(11.21) \end{aligned}$$

At the middle of suction stroke,  $\theta = 90^\circ$ , which means  $\cos \theta = 0$  or accelerating head  $H_{a_s}$  is also equal to zero. But the velocity of flow at this point is maximum and is equal to  $\frac{A}{a_s} \cdot \omega \cdot r$ , therefore the frictional head loss  $H_{f_s}$  for the length  $L_s$ , as well as kinetic head due to this variable velocity will have to be considered.

$\therefore$  Pressure head at the middle suction stroke

$$\begin{aligned} &= H_s + H_{f_s} + H_{f_{s_s}} + H_{K.E} \text{ (both)} \\ &= H_s + \frac{4fL_s}{d_s} \cdot \frac{v_s^2}{2g} + \frac{4fL_{s_s}}{2gd_s} \cdot \left( \frac{A}{a_s} \cdot \omega r \right)^2 + \frac{v_{s_s}^2}{2g} + \frac{v_s^2}{2g} \\ &\quad \dots(11.22) \end{aligned}$$

Pressure head at the end of suction stroke

(where  $\theta = 180^\circ$ ,  $\cos \theta = -1$  i.e.  $H_{a_s}$  is negative)

$$\begin{aligned}
 &= H_s + H_{f_s} - (H_{a_s})_{\max} + H_{K.B} \\
 &= H_s + \frac{4fL_s}{d_s} \cdot \frac{v_s^3}{2g} - \frac{L_{a_s}}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r + \frac{v_s^3}{2g} \quad \dots(11.23)
 \end{aligned}$$

**Problem 11.6** The single acting pump in Problem 11.4 is now equipped with an air vessel on the suction side such that length of the suction pipe from the cylinder to the air vessel is 1.5 m and the mean water level in the air vessel is 0.6 m below the centre line of the cylinder. Find the speed at which the pump can operate without separation occurring at the beginning of suction stroke. Assume  $f=0.01$ .

**Solution**

$$\begin{aligned}
 D &= 25 \text{ cm} & S &= 45 \text{ cm} & d_s &= 12.5 \text{ cm} \\
 L_{a_s} &= 1.5 \text{ m} & \therefore L_s &= 10.5 \text{ m} & H_s &= 3 \text{ m} & H_{a_s} &= 0.6 \text{ m}
 \end{aligned}$$

Now, the length of water column where the fluctuation of acceleration takes place, has been decreased from 12 m to 1.5 m. In 10.5 m length of pipe on suction side, the velocity of water is uniform. This velocity is found as follows :

$$\begin{aligned}
 v_s &= \frac{Q}{a_s} = \frac{A \cdot S \cdot \frac{N}{60}}{\frac{\pi}{4} \times d_s^3} = \frac{\frac{\pi}{4} \times (0.25)^2 \times 0.45N}{60 \times \frac{\pi}{4} \times (0.125)^3} \\
 &= 0.03 \text{ N m/sec}
 \end{aligned}$$

As shown in Eqn 11.20 the atmospheric pressure has to supply the accelerating head for the length of water column between the cylinder and the air vessel and the pipe frictional losses for the rest of the length below the air vessel, in addition to the static suction head and valve losses etc.

$\therefore$  Eqn 11.21 for the flow at the beginning of the suction stroke :

$$\begin{aligned}
 H_{atm} &= H_s + (H_{a_s})_{\max} + H_{f_s} + \left[ \frac{4fL_s}{d_s} \times \frac{v_s^3}{2g} \right] + \frac{v_s^3}{2g} \\
 \text{or} \quad 10 &= 3 + \frac{1.5}{9.81} \times \left( \frac{0.25}{0.125} \right)^2 \times \left( \frac{2\pi N}{60} \right)^2 \times \frac{0.45}{2} + 2.5 \\
 &\quad + \frac{4 \times 0.01 \times 10.5}{0.125} \times \frac{(0.03N)^3}{2 \times 9.81} + \frac{(0.03N)^3}{2 \times 9.81} \\
 &= 3 + 2.5 + N^2 (0.001505 + 0.000154 + 0.0000457) \\
 \text{or} \quad N^2 &= \frac{4.5}{0.0017547} = 2,820 \\
 \text{or} \quad N &= 58 \text{ rpm} \quad \text{Answer}
 \end{aligned}$$

Therefore with the air vessel the speed has been increased from 17.55 to 58.

**11.16 Pressure in Cylinder on Delivery Stroke with Air Vessel**—Let the length of delivery pipe between cylinder and air vessel be  $L_{d_1}$  (refer Fig 11.18) and the length of delivery pipe beyond air vessel be  $L_{d_2}$ . By fitting an air vessel on the delivery side fluctuation of pressure takes place between the cylinder and the air vessel and the flow becomes uniform beyond the air vessel. Therefore the velocity of water will also be uniform in length  $L_{d_1}$ .

The frictional head due to the uniform flow in delivery pipe of length  $L_d$  would be

$$H_{f_d} = \frac{4 f L_d}{d_d} \cdot \frac{v_d^3}{2g}$$

where the uniform velocity  $v_d$  in the delivery pipe

$$= \frac{Q}{a_d} = \frac{A \cdot S \cdot N}{a_d \cdot 60} \text{ (for single acting pump).}$$

Due to the uniform velocity, the above frictional head  $H_{f_d}$  would be taken into account when calculating the pressure at the beginning, middle or end of the stroke and it will remain same.

Accelerating head in the delivery pipe between cylinder and air vessel where the fluctuation of pressure takes place :

$$H_{a_d} = \frac{L_{a_v}}{g} \cdot \frac{A}{a_d} \cdot \omega^2 \cdot r \cos \omega t$$

Pressure head on the piston at the beginning of delivery stroke

$$\begin{aligned} &= H_d + H_{f_d} + H_{a_d} + H_{K.E.} \\ &= H_d + \frac{4 f L_d}{d_d} \cdot \frac{v_d^3}{2g} + \frac{L_{a_v}}{g} \cdot \frac{A}{a_d} \cdot \omega^2 \cdot r + \frac{v_d^3}{2g} \dots (11.24) \end{aligned}$$

(N.B. This equation on delivery side is similar to Eqn 11.21a which is derived for suction side.)

This is the pressure when  $\theta = 0$  or  $\cos \theta = 1$  i.e., maximum occurring at the beginning of delivery stroke.

At the middle of the delivery stroke,  $\theta = 90^\circ$  which means  $\cos \theta = 0$  or accelerating head  $H_{a_d}$  is also equal to zero. However, the velocity at this point is maximum and is equal to  $\frac{A}{a_d} \cdot \omega \cdot r$ , therefore the frictional head loss  $H_{f_{d_v}}$  for the length  $L_{d_v}$  as well as kinetic head due to this variable velocity will have to be considered.

$\therefore$  Pressure head at the middle of stroke

$$\begin{aligned} &= H_d + H_{f_d} + H_{f_{d_v}} + H_{K.E.} \text{ (both)} \\ &= H_d + \frac{4 f L_d}{d_d} \cdot \frac{v_d^3}{2g} + \frac{4 f L_{d_v}}{d_d \cdot 2g} \cdot \left( \frac{A}{a_d} \cdot \omega \cdot r \right)^2 + \frac{v_{d_v}^3}{2g} + \frac{v_d^3}{2g} \\ &\dots (11.25) \end{aligned}$$

Pressure head at the end of delivery stroke

$$\begin{aligned} &= H_d + H_{f_d} - (H_{a_d})_{max} + H_{K.E.} \\ &= H_d + \frac{4 f L_d}{d_d} \cdot \frac{v_d^3}{2g} - \frac{L_{a_v}}{g} \cdot \frac{A}{a_d} \cdot \omega^2 \cdot r + \frac{v_d^3}{2g} \dots (11.26) \end{aligned}$$

**11.17 Maximum Speed of the Pump Provided with Air Vessels**—As given in Art 11.12 the maximum speed of the pump is determined by finding the vapour pressure head (or separation head) the value of which should not be greater than 2.5 m of water absolute. Applying

Eqn 11.20, the vapour pressure head at the beginning of suction stroke is given by—

$$H_{rap} = H_{atm} - \left( H_s + H_{f_s} + H_{a_s} + H_{v_s} + H_{v_{fs}} + \frac{v_s^2}{2g} \right)$$

The values of heads required for lifting suction value  $H_s$ , and that for overcoming suction valve friction  $H_{v_s}$ , are comparatively smaller and are therefore neglected.

$$\text{Then } H_{rap} = H_{atm} - \left( H_s + H_{f_s} + H_{a_s} + \frac{v_s^2}{2g} \right) \quad \dots(11.27)$$

The flow and thus the velocity of water in the suction pipe from the sump upto the air vessel is uniform. The friction head  $H_{f_s}$  is determined for this portion.

The acceleration to the velocity of water takes place in the suction pipe between the air vessel and the cylinder. The acceleration head  $H_{a_s}$  is determined for the length  $L_{s_v}$ .

Further the kinetic head  $\frac{v_s^2}{2g}$  is found from the uniform flow.

For uniform flow the velocity of water can be found out as follows—

$$Q = \frac{ASN}{60} \text{ for single acting pump}$$

$$\text{and } v_s = \frac{Q}{a_s} = \frac{1}{60} \cdot \frac{A}{a_s} (2r) \frac{60\omega}{2\pi} = \frac{A}{a_s} \cdot \frac{\omega r}{\pi} \quad \dots(11.28)$$

$$\text{Similarly for double acting pump } v_s = \frac{2A}{a_s} \cdot \frac{\omega r}{\pi} \quad \dots(11.28a)$$

**11.18 Theoretical Power Required to Drive the Pump Fitted with Air Vessels on Suction and Delivery Sides**—In order to determine the theoretical power required to drive the pump, when it is fitted with air vessels both on suction and delivery sides, the maximum pressure head on the piston must be known first. The maximum pressure occurs at the beginning of suction and delivery strokes, (refer Eqns 11.21 a and 11.24).

Adding the heads given by these two equations—

Total head against which the pump has to work

$$= H_s + H_a + \frac{4f \cdot L_s}{d_s} \cdot \frac{v_s^2}{2g} + \frac{4f \cdot L_d}{d_d} \cdot \frac{v_d^2}{2g} + \frac{L_{s_v}}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r + \frac{L_{d_v}}{g} \cdot \frac{A}{a_d} \cdot \omega^2 \cdot r + \frac{v_s^2}{2g} + \frac{v_d^2}{2g}$$

If  $Q$  m<sup>3</sup>/sec of water are required to be raised, the work done per second by the pump

$$= \gamma \cdot Q \cdot \left( H_s + H_a + \frac{4f \cdot L_s}{d_s} \cdot \frac{v_s^2}{2g} + \frac{4f \cdot L_d}{d_d} \cdot \frac{v_d^2}{2g} + \frac{L_{s_v}}{g} \cdot \frac{A}{a_s} \cdot \omega^2 \cdot r + \frac{L_{d_v}}{g} \cdot \frac{A}{a_d} \cdot \omega^2 \cdot r + \frac{v_s^2}{2g} + \frac{v_d^2}{2g} \right) \quad \dots(11.29)$$



As the lengths of pipes  $L_s$  and  $L_d$  are much smaller than  $L_s$  and  $L_d$  respectively, the corresponding accelerating heads  $\frac{L_s}{g} \cdot \frac{A}{a_s} \omega^2 r$  and  $\frac{L_d}{g} \cdot \frac{A}{a_d} \omega^2 r$  can be neglected. Similarly the terms  $\frac{v_s^2}{2g}$  and  $\frac{v_d^2}{2g}$  are also neglected.

$$\begin{aligned} \therefore \text{Theoretical HP required by the pump fitted with air vessels} \\ = \frac{\text{Work done per second}}{75} \\ = \frac{\gamma \cdot Q}{75} \cdot \left( H_s + H_d + \frac{4f \cdot L_s}{d_s} \cdot \frac{v_s^2}{2g} + \frac{4f \cdot L_d}{d_d} \cdot \frac{v_d^2}{2g} \right) \quad \dots (11.29a) \end{aligned}$$

**Problem 11.7** In a double acting reciprocating pump the plunger diameter is 300 mm and delivery pipe is 150 mm in diameter. The lengths of delivery and suction pipes are 450 m and 10 m respectively. An air vessel has been fitted near the pump cylinder on the delivery side in order that the velocity of flow in the delivery pipe is constant. The height of the pump above the level of water in the suction pump is 3 m. The plunger moves with simple harmonic motion and makes 60 strokes per minute. If the stroke is 400 mm and the water is raised through a net head of 75 m, find the diameter of the suction pipe in order to avoid the separation at the beginning of the suction stroke. The separation commences to appear when the pressure in the cylinder falls to 2.5 m of water absolute. Compute the HP of the pump neglecting all losses except due to friction in the delivery pipe. Take  $f = 0.01$ .

### Solution

Double acting pump       $D = 300 \text{ mm}$        $d_d = 150 \text{ mm}$

$L_s = 10 \text{ m}$        $L_d = 450 \text{ m}$        $H_s = 3 \text{ m}$

$S = 400 \text{ mm}$        $\therefore r = \frac{S}{2} = 200 \text{ mm}$        $H = 75 \text{ m}$

$N = 60 \text{ strokes/min} = 30 \text{ rpm}$        $H_{vap} = 2.5 \text{ m}$        $f = 0.01$

Using Eqn 11.9

$$H_{atm} = H_s + H_f + H_{as} + H_{vap} \quad \dots (\text{neglecting kinetic head and valve losses})$$

$$\text{or } 10 = 3 + \frac{4f \cdot L_s}{d_s} \cdot \frac{v_s^2}{2g} + \frac{L_s}{g} \cdot \omega^2 \cdot r \cdot \frac{A}{a_s} \cdot \cos \theta + 2.5$$

At the beginning of suction stroke—

Here the velocity  $v_s$  is zero.  $\theta = 0$ ,  $\therefore \cos \theta = 1$ .

$$\text{or } 10 = 3 + 0 + \frac{L_s}{g} \cdot \omega^2 \cdot r \cdot \frac{A}{a_s} + 2.5$$

$$\begin{aligned} \text{or } \frac{L_s}{g} \cdot \omega^2 \cdot r \cdot \frac{A}{a_s} &= 10 - 5.5 \\ &= 4.5 \end{aligned}$$

$$\text{or } \frac{10}{9.81} \times \left( \frac{2\pi \times 30}{60} \right)^2 \times 0.2 \times \frac{0.3^2}{d_s^2} = 4.5$$

$$d_s = \sqrt{\frac{10 \times 4\pi^2 \times 900 \times 0.2 \times 0.3 \times 0.3}{4.5 \times 9.81 \times 3,600}}$$

$$= \sqrt{0.04} = 0.2 \text{ m or } 200 \text{ mm Answer}$$

The quantity of water flowing in the pump

$$Q = \frac{2 \cdot A \cdot S \cdot N}{60} = 2 \times \left( \frac{\pi}{4} \times 0.3^2 \right) \times 0.4 \times \frac{30}{60} = 0.0283 \text{ m}^3/\text{sec}$$

The velocity in the delivery pipe is uniform—

$$\therefore v_d = \frac{Q}{a_d} = \frac{0.0283}{\frac{\pi}{4} \times 0.15^2} = 1.6 \text{ m/sec}$$

As there is air vessel on the deliver side, there is no accelerating head  $H_{a_d}$ , but the frictional head has to be considered.

$$H_{f_d} = \frac{4f \cdot L_d}{d_d} \cdot \frac{v_d^2}{2g}$$

$$= \frac{4 \times 0.01 \times 450}{0.15} \times \frac{1.6^2}{2 \times 9.81} = 15.7 \text{ m of water}$$

Total head delivered

$$= \text{static head} + \text{frictional head losses} + \text{accelerating heads}$$

$$= (H_s + H_a) + (H_{f_s} + H_{f_d}) + (H_{a_s} + H_{a_d})$$

$$= 75 + (0 + 15.7) + (4.5 + 0) = 95.2 \text{ m of water}$$

$\therefore$  Horsepower required by the pump

$$\frac{\gamma \cdot Q \cdot H}{75} = \frac{1,000 \times 0.0283 \times 95.2}{75} = 36 \text{ HP Answer}$$

**11.19 Work Saved by Air Vessel in Overcoming Pipe Friction**—When a pump is equipped with air vessels, the accelerating heads on both the suction and delivery sides are reduced, which results in the saving of some energy. The length of suction pipe being small the work saved by the air vessel, on this side of the pump, in overcoming the frictional head will also be small, and can be neglected. However, the length of the delivery pipe is large, therefore the work saved is considerable, and it is determined as below.

Considering a single acting pump, the work done per second in overcoming pipe friction on delivery side *without* vessel

$$= \gamma \cdot Q \cdot \left( \frac{2}{3} H_{f_d} \right)$$

(As the motion is simple harmonic, the effective friction head would be  $\frac{2}{3}$  of the total friction head because the curve representing it, is a parabola).

$$= \gamma \cdot Q \cdot \frac{2}{3} \cdot \frac{4f \cdot L_d}{d_d \cdot 2g} \left( \frac{A}{a_d} \cdot \omega \cdot r \right)^2 \quad \dots(11.30)$$

Work done per second in overcoming the pipe friction on delivery side *with* air vessel

$$= \gamma \cdot Q \cdot H_{f_d} = \gamma \cdot Q \cdot \frac{4f \cdot L_d}{a_d} \cdot \frac{v_d^2}{2g}$$

The velocity  $v_d$  in the pipe beyond the air vessel is uniform, and is

$$v_d = \frac{A \cdot S \cdot N}{60 \cdot a_d} = \frac{A}{a_d} \cdot 2r \cdot \frac{\omega}{2\pi}$$

Substituting  $v_d$  in the above equation, work done per second in overcoming the pipe friction on delivery side *with* air vessel

$$= \gamma \cdot Q \cdot \frac{4f \cdot L_d}{d_d \cdot 2g} \left( \frac{A}{a_d} \cdot 2r \cdot \frac{\omega}{2\pi} \right)^2 \quad \dots(11.31)$$

$\therefore$  Work saved by equipping the pump with an air vessel on delivery side.

$$= \gamma \cdot Q \cdot \frac{4f \cdot L_d}{d_d \cdot 2g} \cdot \left( \frac{A}{a_d} \cdot \omega \cdot r \right)^2 \cdot \left( \frac{2}{3} - \frac{1}{\pi^2} \right) \quad \dots(11.32)$$

Percentage of work saved per second

$$= \frac{\left( \frac{2}{3} - \frac{1}{\pi^2} \right)}{\frac{2}{3}} \times 100 = 84.8\%$$

Similarly it may be shown that the same percentage of work can be saved by fitting another vessel on the suction side in the case of a single acting pump.

**Double Acting Pump**—Work done per second in overcoming friction in delivery pipe *without* air vessel

$$= \gamma \cdot Q \cdot \frac{2}{3} \cdot \frac{4f \cdot L_d}{d_d \cdot 2g} \cdot \left( \frac{A}{a_d} \cdot \omega \cdot r \right)^2 \quad \dots(11.33)$$

(same as in case of single acting pump)

Work done per second in overcoming friction in delivery pipe *with* air vessel

$$\begin{aligned} &= \gamma \cdot Q \cdot \frac{4f \cdot L_d}{d_d} \cdot \frac{v_d^2}{2g} \\ &= \gamma \cdot Q \cdot \frac{4f \cdot L_d}{d_d \cdot 2g} \cdot \left( \frac{A}{a_d} \cdot \omega \cdot r \cdot \frac{2}{\pi} \right)^2 \quad \dots(11.34) \\ &\quad \left[ \text{as } v_d = \frac{2A \cdot S \cdot N}{60 \cdot a_d} \right] \end{aligned}$$

$\therefore$  Percentage of work saved per second

$$\frac{\frac{2}{3} - \frac{4}{\pi^2}}{\frac{2}{3}} \times 100 = 39.2\%$$

**Problem 11.8** A single acting piston pump is equipped with an air vessel on the delivery side. The piston moves with a simple harmonic motion. The diameter and stroke of the piston are 300 mm and 600 mm respectively. The delivery pipe is 175 mm in diameter and 60 m long. Determine the HP saved in overcoming friction in the delivery pipe by the air vessel. The pump runs at 120 rpm. Take  $f = 0.01$ .

**Solution**

$$D = 300 \text{ mm} \quad S = 600 \text{ mm} \quad d_a = 175 \text{ mm}$$

$$L_d = 60 \text{ m} \quad N = 120 \text{ rpm} \quad f_d = 0.01$$

Work done per second in overcoming the pipe friction on delivery side without air vessel

$$= \gamma \cdot Q \cdot \frac{2}{3} \cdot H_{f_d} = \gamma \cdot Q \cdot \frac{2}{3} \times \frac{4f \cdot L_d}{d_d \cdot 2g} \left( \frac{A}{a_d} \cdot \omega \cdot r \right)^2$$

(Velocity of water in the delivery pipe without air vessel is proportional to that of the piston)

Work done in overcoming friction in the delivery pipe *with* air vessel

$$= \gamma \cdot Q \cdot H_{f_d} = \gamma \cdot Q \cdot \frac{4f \cdot L_d}{d_d} \cdot \frac{v_d^3}{2g}$$

With air vessel the velocity of water in the delivery pipe is taken to be uniform and equal to  $\frac{Q}{a_d} = \frac{A \cdot S \cdot N}{a_d \cdot 60}$

$$= \frac{A}{a_d} \cdot 2r \cdot \frac{\omega}{2\pi} = \frac{A}{a_d} \cdot \frac{r \cdot \omega}{\pi}$$

∴ Work done against friction with air vessel

$$= \gamma \cdot Q \cdot \frac{4f \cdot L_d}{d_d \cdot 2g} \cdot \left( \frac{A}{a_d} \cdot \frac{r \cdot \omega}{\pi} \right)^2$$

∴ Work saved in overcoming friction in the delivery pipe by fitting the air vessel

$$= \gamma \cdot Q \cdot \frac{4f \cdot L_d}{d_d \cdot 2g} \cdot \left( \frac{A}{a_d} \cdot \omega \cdot r \right)^2 \cdot \left( \frac{2}{3} - \frac{1}{\pi^2} \right) \text{ m·kg/sec}$$

$$= 1,000 \times \left( \frac{\pi}{4} \times 0.3^3 \times 0.6 \times \frac{120}{60} \right) \times \frac{4 \times 0.01 \times 60}{0.175 \times 2 \times 9.81}$$

$$\times \left( \frac{0.3^3}{0.175^3} \times \frac{2\pi \times 120}{60} \times \frac{0.6}{2} \right)^2 \times \left( \frac{2}{3} - \frac{1}{\pi^2} \right)$$

$$= 7,950 \times (0.66 - 0.1017) = 4,430 \text{ m·kg/sec}$$

∴ HP saved by fitting the air vessel

$$= \frac{4,430}{75} = 59 \text{ HP} \quad \text{Answer}$$

### 11.20 Discharge in and out of Air Vessel—

(a) **Delivery Pipe**—(i) *Single-acting Pump*—Velocity of water in the portion of pipe connecting the pump cylinder and the air vessel on the delivery side  $v_d = \frac{A}{a_d} \cdot \omega r \sin \theta$

∴ Discharge in the pipe connecting the cylinder and the air vessel on the delivery side  $= a_d v_d = A \cdot \omega r \sin \theta$

Velocity of water in the portion of delivery pipe beyond air vessel

$$v_d = \frac{A}{a_d} \cdot \frac{\omega r}{\pi} \quad \dots (\text{flow being uniform})$$

$\therefore$  Discharge in the delivery pipe beyond the air vessel

$$= A \cdot \frac{\omega r}{\pi}$$

$\therefore$  Discharge going in the air vessel = discharge from the cylinder  
—discharge beyond air vessel

$$\begin{aligned} &= A \omega r \sin \theta - A \frac{\omega r}{\pi} \\ &= A \omega r \left( \sin \theta - \frac{1}{\pi} \right) \end{aligned} \quad \dots (11.35)$$

(ii) *Double-acting pump*—Discharge in the delivery pipe from the cylinder to the air vessel =  $A \cdot \omega r \sin \theta$  (same as in the case of single acting pump)

Discharge in the delivery pipe beyond the air vessel

$$= 2A \frac{\omega r}{\pi}$$

$\therefore$  Discharge going in the air vessel

$$= A \omega r \left( \sin \theta - \frac{2}{\pi} \right) \quad \dots (11.35a)$$

In case the value of Eqns 11.35 and 11.35a comes to be negative then the discharge is going *out* of air vessel.

(b) **Suction Pipe**—The water with uniform velocity enters the air vessel and from there it is taken to the cylinder.

$\therefore$  Discharge going in the air vessel for single acting pump

= Discharge from the suction pipe into the air vessel—discharge from the air vessel to the cylinder

$$= A \omega r \left( \frac{1}{\pi} - \sin \theta \right) \quad \dots (11.36)$$

Discharge going in the air vessel for double acting pump

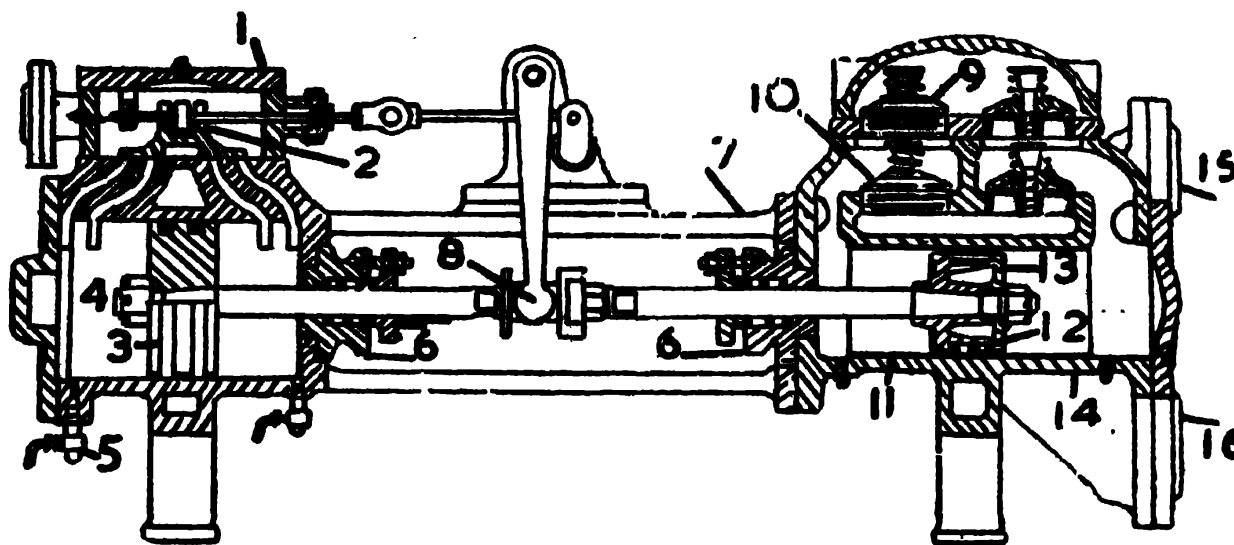
$$= A \omega r \left( \frac{2}{\pi} - \sin \theta \right) \quad \dots (11.36a)$$

If the values of Eqns 11.36 and 11.36 a are negative, the discharge will be going *out* of air vessel.

### 11.21 Other Types of Reciprocating Pumps—

(a) **Direct Acting Steam Pump**—When steam power is available pistons of the pump and the engine can be directly connected to each other (refer Fig 11.19). By this arrangement, crankshaft and flywheel can be eliminated. This pump is mainly used for feeding water to boilers. It may be single or double acting depending upon the pressure required.

**Pump Duty** – The “duty” of the reciprocating pump driven by a steam engine is the number of m-kg of work given out by it for every 1,000,000 kilo calorie supplied to the engine by the boiler.



Steam Engine Side

Pump Side

Fig 11.19 Cross-section of Horizontal Duplex Piston Pump, Manufactured by Worthington Pump and Machinery Co.

1. Steam Chest 2. Side Valve 3. Piston Rings 4. Steam Cylinder  
5. Drain Cock 6. Stuffing Box 7. Cradle 8. Piston Rod 9. Discharge Valve 10. Suction Valve 11. Liner 12. Packing Ring 13. Liquid Piston  
14. Liquid Cylinder 15. Discharge Port 16. Suction Port.

### Pump Duty

$$\gamma \cdot Q \cdot H \times 1,000,000$$

— Wt. of steam used in kg/sec  $\times$  total heat of 1 kg of steam supplied  
...(11.37)

(b) **Differential Pump** : Single acting piston pump is equipped with a pipe bend connecting the piston rod-end of the cylinder to the delivery pipe (refer Fig 11.20). Cross sectional area of piston rod is generally made half that of the piston itself. The thick piston rod and the actual piston are known as differential and main pistons respectively.

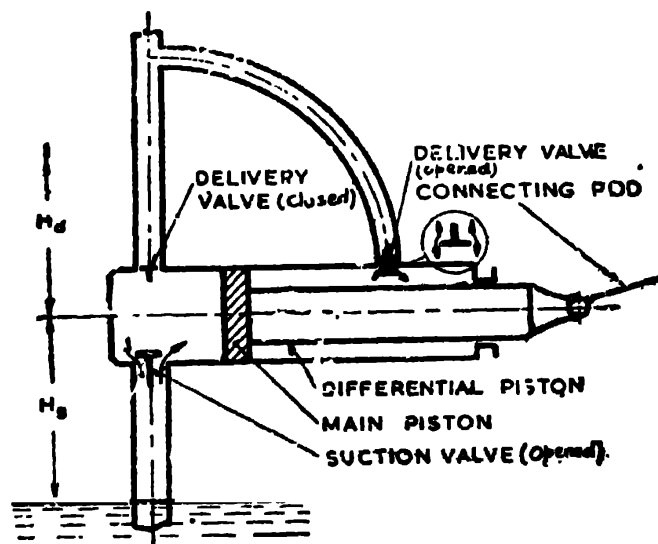


Fig 11.20 Differential Pump

When the piston moves forward in the delivery stroke, only half of the water in the cylinder is delivered, the rest being transferred to the other side of the piston rod. This water is delivered when the piston moves backward in the suction stroke. Thus delivery takes place at a more uniform rate.

**11.22 Theory of Working of Air Vessel\***—Let  $v_p$  be the velocity of piston which is moving with S.H.M., assuming a long connecting rod such that the ratio of connecting rod length to crank radius is large. The piston velocity at any instant will be given by

$$v_p = v_{p \max} \cdot \sin \theta \quad \dots(\text{refer Fig 11.21})$$

where

$$v_{p \max} = \text{maximum piston velocity}$$

$$\theta = \text{crank angle}$$

Let

$$v_{p \text{ mean}} = \text{mean velocity of piston} \\ = \text{mean velocity of charge}$$

*For Single-acting pump—*

$$\text{Then } v_{p \text{ mean}} = \frac{\int_0^\pi v_p \cdot d\theta}{2\pi} = \frac{v_{p \max} \int_0^\pi \sin \theta \cdot d\theta}{2\pi}$$

$$\therefore \frac{v_{p \text{ mean}}}{v_{p \max}} = \frac{\int_0^\pi \sin \theta \cdot d\theta}{2\pi} = \frac{-(\cos \pi - \cos 0^\circ)}{2\pi} \\ = \frac{-(-1 - 1)}{2\pi} = \frac{1}{\pi} \quad \dots(11.38)$$

Similarly for double-acting pump, it can be proved that the value

$$\frac{v_{p \text{ mean}}}{v_{p \max}} = \frac{2}{\pi}.$$

Considering the delivery side of the pump, Fig 11.21 shows rate of delivery *vs* crank angle for a single-acting pump. This is sine-curve. The value of mean piston velocity  $v_{p \text{ mean}}$  is shown by a horizontal line which cuts the sine-curve at points A and B, when the crank angles are  $\theta_1$  and  $\theta_2$  respectively. Starting from crank angle 0 to  $\theta_1$ , the piston velocity is less than the piston mean velocity, therefore, water is being discharged out in the delivery pipe from the air vessel placed on the delivery side. This means that the point A, volume of water in the air vessel is minimum and

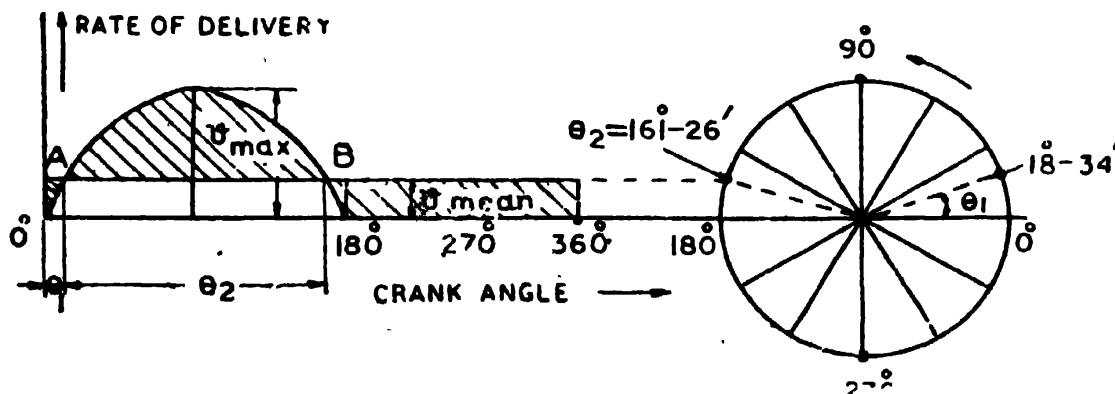


Fig 11.21 Theory of Working of Air Vessel (Rate Delivery Curve for Single Acting Pump)

\* Art 11.22 to 11 25 are helpful in designing of reciprocating pumps.

the volume of air at  $A$  is maximum. From  $A$  to  $B$ , the discharge from the cylinder to the air vessel is more than its mean value, therefore at point  $B$ , the volume of air in the air vessel is minimum.

Let  $V_{max}$  = maximum volume of air in the air vessel ;

$V_{min}$  = minimum volume of air in the air vessel ;

then  $V_{max} - V_{min}$  = fluctuating volume of air in the air vessel.

If  $V_{mean}$  = mean volume of air in the air vessel,

then  $\frac{V_{max} - V_{min}}{V_{mean}}$  = **co-efficient of fluctuation** =  $\delta$ , ... (11.39)

also  $V_{max} - V_{min}$  = maximum volume of water—mean volume of water delivered by the piston

$$= \int_0^T \left[ A \left( v_{p \max} \cdot \sin \theta \right) dt - A \left( v_{p \text{ mean}} \right) dt \right]$$

where

$A$  = cross-section area of piston ;

$T$  = time taken by the piston to move from  $A$  to  $B$

$$= \int_0^T \left[ A \left( v_{p \max} \sin \theta \right) dt - A \left( \frac{v_{p \max}}{\pi} \right) dt \right]$$

(for a single acting pump)

If  $\omega$  = angular velocity of crank ;

then  $\theta = \omega \cdot t$

and  $d\theta = \omega \cdot dt$  or  $dt = \frac{d\theta}{\omega}$

$$\begin{aligned} \therefore V_{max} - V_{min} &= \frac{A \cdot v_{p \max}}{\omega} \int_{\theta_1}^{\theta_2} \left( \sin \theta \cdot d\theta - \frac{1}{\pi} d\theta \right) \\ &= \frac{A \cdot v_{p \max}}{\omega} \left( -\cos \theta - \frac{\theta}{\pi} \right)_{\theta_1}^{\theta_2} \\ &= \frac{A \cdot v_{p \max}}{\omega} \left\{ \left( -\cos \theta_2 - \frac{\theta_2}{\pi} \right) - \left( -\cos \theta_1 - \frac{\theta_1}{\pi} \right) \right\} \\ &= \frac{A \cdot v_{p \max}}{\omega} \left( \cos \theta_1 - \cos \theta_2 - \frac{\theta_2 - \theta_1}{\pi} \right) \end{aligned}$$

For a single-acting pump,

$$v_{p \max} \cdot \sin \theta = v_{p \text{ mean}} \quad \text{at points } A \text{ and } B$$

$$\therefore v_{p \max} \cdot \sin \theta = \frac{v_{p \max}}{\pi}$$

$$\text{or} \quad \sin \theta = \frac{1}{\pi} = 0.3183$$

$$\therefore \theta_1 = 18^\circ - 34' \quad \text{or } 0.3246 \text{ radians}$$

...(refer Fig 11.21)

$$\begin{aligned} \text{and} \quad \theta_2 &= 180 - (18^\circ - 34') \\ &= 161^\circ - 26' \quad \text{or } (\pi - 0.3246) \text{ radians} \end{aligned}$$



Substituting the values  $\theta_1$  and  $\theta_2$ ,

$$\begin{aligned}
 V_{max} - V_{min} &= \frac{A \cdot v_{p \max}}{\omega} \times \\
 &\left[ \cos(18^\circ - 34') - \cos(161^\circ - 24') - \frac{\pi - 0.3246 - 0.3246}{\pi} \right] \\
 &= \frac{A \cdot v_{p \max}}{\omega} \left( 0.9478 + 0.9478 - \frac{\pi - 0.6492}{\pi} \right) \\
 &= \frac{A \cdot v_{p \max}}{\omega} \times 1.1022 \quad \dots[11.40]
 \end{aligned}$$

Now volume of stroke of piston =  $A \cdot S$

$$= A \cdot v_{p \text{ mean}} \times \text{time for one revolution of crank}$$

$$= A \cdot \frac{v_{p \max}}{\pi} \times \frac{2\pi}{\omega} \quad (\text{or single acting pump})$$

$$= \frac{2A \cdot v_{p \max}}{\omega}$$

$$\therefore \frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}} = \frac{\frac{A}{\omega} \times v_{p \max} \times 1.1022}{2 \frac{A}{\omega} \cdot v_{p \max}} = 0.55 \quad \dots(11.41)$$

Similarly it can be shown that for a double-acting single cylinder pump or two throw pump

$$\frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}} = 0.21 \quad \dots(11.41a)$$

and for a three throw pump,

$$\frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}} = 0.009$$

This is the proportion of total water discharge per revolution which enters and leaves the air vessel per cycle.

**11.23 Volume of Air Vessel**—The fluctuations of volumes of air and water cause the variations of pressure which should be within the limits. As the air is in contact with water surface, the change of volume of air in the air vessel takes place according to Boyle's Law i.e., at constant temperature,

$$\text{or} \quad p \cdot V = \text{constant}$$

$$\begin{aligned}
 \therefore p_{min} \cdot V_{max} &= p_{max} \cdot V_{min} \\
 &= p_{mean} \cdot V_{mean} = \text{constant}
 \end{aligned}$$

where  $p_{mean}$  = mean air pressure in air vessel.

$$\therefore p_{min} = \frac{p_{mean} \cdot V_{mean}}{V_{max}} \text{ and } p_{max} = \frac{p_{mean} \cdot V_{mean}}{V_{min}}$$

$$\text{Now } \delta_f, \text{ the co-efficient of fluctuation} = \frac{p_{max} - p_{min}}{p_{mean}}$$

$$\therefore \delta_f = \frac{\frac{p_{mean} \cdot V_{mean}}{V_{min}} - \frac{p_{mean} \cdot V_{mean}}{V_{max}}}{p_{mean}}$$

$$= \frac{V_{mean} \cdot (V_{max} - V_{min})}{V_{max} \cdot V_{min}}$$

Approximately  $V_{max} \cdot U_{min} = V_{mean}^2$

$$\therefore \delta_f = \frac{V_{max} - V_{min}}{V_{mean}}$$

This has already been defined under Eqn 11.39

$$\therefore \delta_f = \frac{p_{max} - p_{min}}{p_{mean}} = \frac{V_{max} - V_{min}}{V_{mean}} \quad \dots(11.42)$$

$\therefore$  Mean volume of air vessel can be written as

$$\begin{aligned} V_{mean} &= \frac{V_{mean}}{V_{max} - V_{min}} \times \frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}} \times \text{Piston Stroke Volume} \\ &= \frac{1}{\delta_f} \times \frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}} \times \text{Piston Stroke Volume} \end{aligned} \quad \dots(11.43)$$

The volumes of the delivery and suction air vessels are about 40 to 60 times and 20 to 30 times respectively the volume of water entering it per cycle, these proportions increasing with the rotational speed and the length of the respective pipes.

**Effect of Connecting Rod Length**—In deriving Eqn 11.43, a large length of connecting rod was assumed, such that the movement of the piston was simple harmonic. With a finite length of connecting rod the value  $\frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}}$  increases. Table 11.1 shows such variations.

TABLE 11.1

**Effect of Connecting Rod Length on  $\frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}}$**

| $\frac{r}{l}$                                           | 0    | $\frac{1}{5}$ | $\frac{1}{4}$ | $\frac{1}{3}$ |
|---------------------------------------------------------|------|---------------|---------------|---------------|
| $\frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}}$ | 0.21 | 0.25          | 0.27          | 0.30          |

**Problem 11.9** Determine the size of air vessel for a differential pump if the permissible fluctuation of pressure is 3%.

**Solution**

The differential pump acts like a single acting pump on suction stroke (refer Art 11.21 & Fig 11.20), therefore the suction air vessel should be designed accordingly.

∴ Suction air vessel mean volume

$$\begin{aligned}
 &= \frac{1}{8} \times \frac{V_{max} - V_{min}}{\text{Piston Stroke Volume}} \times \text{Piston Stroke Volume} \\
 &= \frac{100}{3} \times 0.55 \times \text{Piston stroke Volume} \\
 &= 18.3 \times \text{Piston Stroke Volume} \quad \text{Answer}
 \end{aligned}$$

On delivery side, the differential pump acts like a double acting pump,

∴ Mean volume of delivery air vessel

$$\begin{aligned}
 &= \frac{100}{3} \times 0.21 \times \text{Piston Stroke Volume} \\
 &= 7 \times \text{Piston Stroke Volume} \quad \text{Answer}
 \end{aligned}$$

**11.24 Resonance in Reciprocating Pumps**—It is seen that in delivery pipe, connected to the air vessel and running away from it, the time period of free oscillation of water equals the time period of impulse of the pump. Such a phenomenon is known as *resonance*, which if it occurs gives rise to a very high pressure. The pipe may not be made to withstand such a high pressure and it may burst.

*Time period of impulse of pump*—If the speed of a pump is  $n$  rpm, there are  $n$  impulses in a minute for a single-acting pump. Then the time period of the impulse of the pump is  $\frac{1}{n}$  minute or  $\frac{60}{n}$  sec for a single acting pump. For a double-acting pump, the time period of one impulse is  $\frac{30}{n}$  sec.

*Time period of free oscillation of water*—In Fig 11.22 water column in delivery pipe is held in equilibrium by the pressure of air in the air vessel,

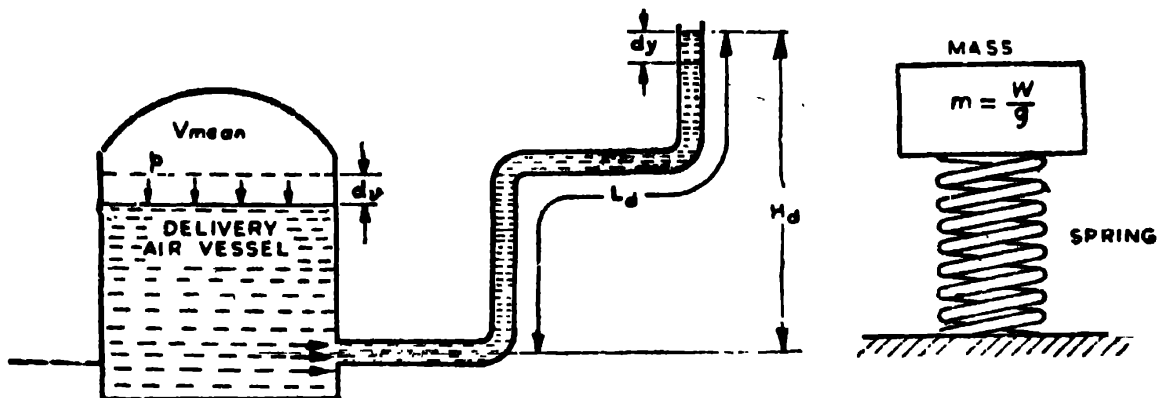


Fig 11.22 Determination of Time-Period of Free Oscillation of Water in Delivery Pipe

such that it acts like a spring carrying a mass, shown in the figure. There will be a time period of oscillation of water in the delivery pipe, analogous to that of the mass placed on the spring.

For a mass  $m$  supported by a spring of stiffness  $k$  kg per metre length, the time period is given by—

$$T = 2\pi \sqrt{\frac{m}{k}} \quad \text{seconds} \quad \dots(11.44)$$

where  $k = \frac{\text{Force}}{\text{deflection}} = \frac{dP}{dy}$

In an air vessel, the mass moved is the mass of water in the delivery pipe (refer Fig 11.22)

$$= \frac{\gamma \cdot a_d \cdot L_d}{g}$$

Force acting at the base of this water column  $= p \cdot a_d$

where  $p$  = pressure of air at equilibrium position.

Let  $V_{mean}$  = volume of air in the air vessel at equilibrium position

then  $p \cdot V_{mean} = \text{constant}$  ... (Boyles' Law)

If water column in the delivery pipe is depressed by  $dy$ , the pressure in the air vessel will rise by  $dp$  and at the same time the volume is diminished by  $dV$

$$\therefore (p + dp)(V_{mean} - dV) = \text{constant}$$

$$\text{or } p \cdot V_{mean} + dp \cdot V_{mean} - p dV = \text{constant}$$

(neglecting  $dp \cdot dV$ , being infinitesimal of higher order)

$$\text{or } dp \cdot V_{mean} = p \cdot dV \quad \dots (p \cdot V_{mean} = \text{constant})$$

$$\text{or } \frac{dp}{dV} = \frac{p}{V_{mean}} \quad \dots (11.45)$$

Additional force acting on the base of water column due to depression  $= a_d \cdot dp$

$$\begin{aligned} \therefore \text{Stiffness } k &= \frac{a_d \cdot dp}{dy} = \frac{a_d \cdot a_d \cdot dp}{a_d \cdot dy} = \frac{a_d \cdot a_d \cdot dp}{dV} \\ &= a_d \cdot \frac{a_d \cdot p}{V_{mean}} \quad \dots (11.46) \end{aligned}$$

$\therefore$  time period of free oscillation of water

$$\begin{aligned} &= 2\pi \sqrt{\frac{m}{k}} \\ &= 2\pi \sqrt{\frac{\frac{\gamma \cdot a_d \cdot L_d}{g}}{\frac{a_d \cdot a_d \cdot p}{V_{mean}}}} = 2\pi \sqrt{\frac{\gamma \cdot L_d \cdot V_{mean}}{g \cdot a_d \cdot p}} \\ &= 2\pi \sqrt{\frac{L_d \cdot V_{mean}}{g \cdot a_d \cdot H_d}} \quad \left( \because H_d = \frac{p}{\gamma} \right) \quad \dots (11.47) \end{aligned}$$

$\therefore$  To avoid resonance

$$\frac{60}{n} \neq 2\pi \sqrt{\frac{L_d \cdot V_{mean}}{g \cdot a_d \cdot H_d}} \quad \dots (11.48)$$

In case of resonance, the values of either  $a_d$  or  $V_{mean}$  are altered. From Eqn 11.47 it is seen that time period of free oscillation of water

depends on factor  $\frac{L_d}{H_d}$  which varies, to a large extent, according to the kind of service, the pump has to perform

**Practical data for  $\frac{L_d}{H_d}$**

|                      |                          |
|----------------------|--------------------------|
| For Water works pump | = 100 approx             |
| Water reservoir pump | = 10 approx              |
| Water storage pump   | = 1 approx               |
| Boiler feed pump     | = $\frac{1}{10}$ approx  |
| Press pump           | = $\frac{1}{100}$ approx |

If for a pump the value  $\frac{L_d}{H_d}$  is large, the time period of free oscillation is also large, which means there is little chance of resonance in water works and receiver pumps. However there is every likelihood of resonance in the case of the boiler feed and press pumps, therefore, they are not equipped with any air vessel on the delivery side. The air vessel for such pumps are not needed as the length of the delivery pipe will be very small.

That the boiler feed pump as well as the press pump are not provided with air vessels can be explained from the consideration of accelerating head. When no air vessel is fitted, the pressure developed in the pump has also to supply the accelerating head of water arising out of the fluctuation of speed of whole of water column. In case of boiler feed pump or press pump, the accelerating head on the delivery side is just a fraction of the total delivery head. Hence no air vessel is required on the delivery side of such pumps.

**11.25 Design of Valves for Reciprocating Pumps**—The main dimension of the valve to be designed is its lift. The opening resistance of the valve should also be determined which is very important in case of suction valve, because if the opening resistance is more, the suction valve may not open.

Let Piston area =  $a_p$

$\therefore$  Piston velocity =  $\omega \cdot r \cdot \sin \theta = v_p \cdot \sin \theta$

Water pushed forward by the piston =  $a_p \cdot v_p \cdot \sin \theta$

This water comes out of the opening is shown in Fig 11.23. The valve being raised, the water discharges out.

Let area of opening =  $a_1 = \frac{\pi}{4} d_1^2$

Velocity of water in the opening =  $v_1$

$\therefore$  Discharge through the opening =  $a_1 \cdot v_1$

and discharge through the valve =  $C_d \cdot \pi d \cdot h \cdot v$

where  $C_d$  = the co-efficient of discharge ;

$d$  = the diameter of valve ;

$h$  = the valve lift or valve displacement (refer Fig. 11.25) ;

$v$  = the velocity of water discharging through the valve.

Water pushed forward by the piston = water coming out of opening  
= water discharging out of the valve.

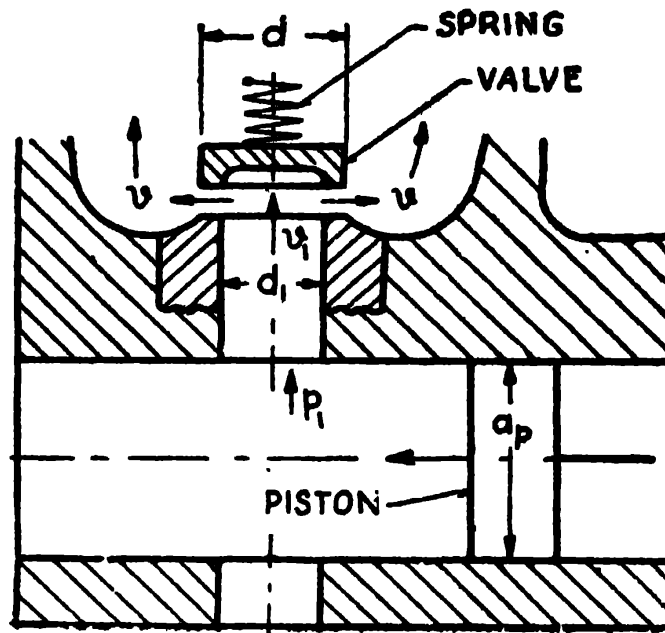


Fig 11.23 Valve of Reciprocating Pump Being Opened

$$\text{or } a_p \cdot v_p \cdot \sin \theta = a_1 \cdot v_1 = C_d \cdot \pi \cdot d \cdot h \cdot v \quad \dots(11.49)$$

$$h = \frac{a_p \cdot v_p \sin \theta}{C_d \cdot \pi \cdot d \cdot v} \quad \dots(11.50)$$

$$\text{Put } \frac{a_p \cdot v_p}{C_d \cdot \pi \cdot d \cdot v} = k$$

$$\text{Then valve displacement } h = k \sin \theta \quad \dots(11.51)$$

$$\text{Valve velocity} = \frac{dh}{dt} = k \cdot \omega \cdot \cos \theta \quad \dots(11.52)$$

$$\text{and valve acceleration} = \frac{d^2h}{dt^2} = -k \cdot \omega^2 \cdot \sin \theta \quad \dots(11.53)$$

Tabulating the values of displacement, velocity and acceleration of valve and piston—

|        | Displacement          | Velocity                           | Acceleration                          |
|--------|-----------------------|------------------------------------|---------------------------------------|
| Valve  | $k \cdot \sin \theta$ | $k \cdot \omega \cdot \cos \theta$ | $-k \cdot \omega^2 \cdot \sin \theta$ |
| Piston | $r (1 - \cos \theta)$ | $r \cdot \omega \sin \theta$       | $r \cdot \omega^2 \cos \theta$        |

It is seen (refer Fig 11.24) that velocities of valve and piston differ by a phase angle of  $90^\circ$ , i.e.,

when  $\theta = 0$ , velocity of valve is maximum and velocity of piston is zero.

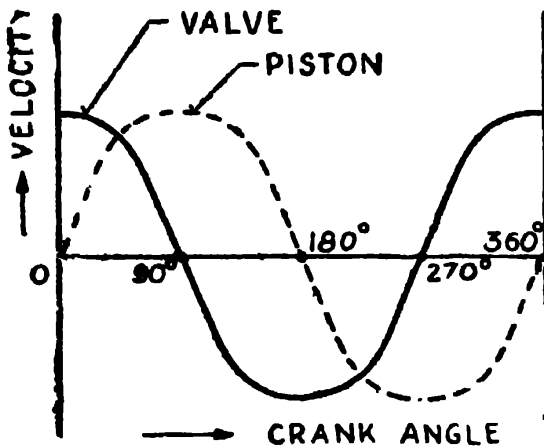


Fig 11.24 Velocity of Valve and Piston vs Crank Angle

In practice whole of the water pushed forward by the piston is not delivered through the valve when the valve is moving upward as there will always be some space beneath it. The water delivered by the piston will fill up this space first and the remaining water goes out of the valve periphery.

Let  $v' =$  valve velocity,

then quantity of water beneath the valve  $= a \cdot v' = \frac{\pi}{4} d^2 \cdot v'$

$$\text{Hence } a_p \cdot v_p \sin \theta = a \cdot v' + C_d \cdot \pi \cdot d \cdot h \cdot v \quad \dots(11.54)$$

This equation holds good when the valve is being lifted. When the valve is being lowered, the discharge equation will be

$$C_d \cdot \pi \cdot d \cdot h \cdot v = a_p \cdot v_p \cdot \sin \theta + a \cdot v' \quad \dots(11.55)$$

Both the discharge equations can be written as—

$$C_d \cdot \pi \cdot d \cdot h \cdot v = a_p \cdot v_p \cdot \sin \theta \pm a \cdot v' \quad \dots(11.56)$$

+ve sign is for the valve being raised,

and -ve sign is for the valve being lowered.

This is known as *Westphal's Law*.

$$\text{or } h = \frac{a_p \cdot v_p \cdot \sin \theta}{C_d \cdot \pi \cdot d \cdot v} \pm \frac{a \cdot v'}{C_d \cdot \pi \cdot d \cdot v} \quad \dots(11.57)$$

$$\text{Now } v' = \frac{dh}{dt} = k \cdot \omega \cdot \cos \theta \quad \dots(\text{refer Eqn 11.52})$$

$$\begin{aligned} \therefore h &= \frac{a_p \cdot v_p \cdot \sin \theta}{C_d \cdot \pi \cdot d \cdot v} \pm \frac{a \cdot k \cdot \omega \cdot \cos \theta}{C_d \cdot \pi \cdot d \cdot v} \\ &= \frac{a_p \cdot \omega \cdot r \sin \theta}{C_d \cdot \pi \cdot d \cdot v} \pm \frac{a \cdot \left( \frac{a_p \cdot \omega \cdot r}{C_d \cdot \pi \cdot d \cdot v} \right) \cdot \omega \cdot \cos \theta}{C_d \cdot \pi \cdot d \cdot v} \\ &= \frac{a_p \cdot \omega \cdot r}{C_d \cdot \pi \cdot d \cdot v} \left( \sin \theta \pm \frac{a \cdot \omega \cdot \cos \theta}{C_d \cdot \pi \cdot d \cdot v} \right) \quad \dots(11.57 a) \end{aligned}$$

$$\text{and } v = \frac{dh}{dt} = \frac{a_p \cdot \omega^2 \cdot r}{C_d \cdot \pi \cdot d \cdot v} \left( \cos \theta \mp \frac{a \cdot \omega \cdot \sin \theta}{C_d \cdot \pi \cdot d \cdot v} \right) \quad \dots(11.58)$$

From Eqn 11.57a, it is clear that when  $\theta = 0$ ,  $h$  is not equal to zero. Let this value of displacement be  $h_0$

$$\begin{aligned} \text{then } h_0 &= \frac{a_p \cdot \omega \cdot r}{C_d \cdot \pi \cdot d \cdot v} \left( 0 \pm \frac{a \cdot \omega}{C_d \cdot \pi \cdot d \cdot v} \right) \\ &= \frac{a_p \cdot a \cdot \omega^2 \cdot r}{(C_d \cdot \pi \cdot d \cdot v)^2} \quad \dots(11.59) \end{aligned}$$

Thus this amount of valve remains open, when the piston is at its dead centre. Now the piston starts its suction stroke, with which the delivery valve is sucked back on its seat. This gives rise to a phenomenon known as *Pounding of Valve*, which should never be allowed to occur. In order to avoid pounding of valve, the valve must be closed before it is sucked by the piston on the return stroke. It is possible by providing the valve with a spring. The stiffness of the spring depends upon the pump speed. The greater the speed of the pump, the stiffer the spring will be.

For the **suction valve**, which opens by the atmospheric pressure, the stiffness of the spring should not be such that it may not open.

**Practical data :**

$$h_o = \frac{1}{250} d$$

$$N \cdot h_{max} = 16$$

where  $d$  = the diameter of valve ;  
 $N$  = the pump speed in rpm ;  
 $h_{max}$  = the maximum valve lift

Referring to Fig 11.23

$$\frac{\pi}{4} d_1^2 \cdot v_1 = C_d \cdot \pi \cdot d \cdot h \cdot v$$

Let  $d_1 = d$

then  $h = \frac{d}{4} \times \frac{v_1}{v}$  (approx)

**Valve Seats** (refer Fig 11.25) are either flat or conical. The flat seats are preferred. Disadvantages of conical seats are as follows—

- (1) The actual opening is less than lift,
- (2) The valve guide must be loose, because it is difficult to attain the coincidence of axes of the valve and the seat.

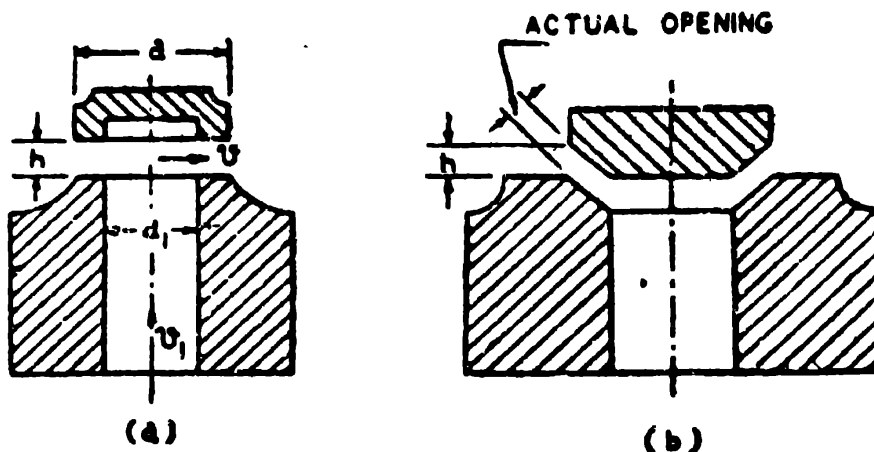


Fig 11.25 Types of Valve Seats

(a) Flat Valve Seat

(b) Conical Valve Seat

**Opening Resistance of Valve**—Before the valve begins to open some resistance is encountered. This is known as opening resistance of valve. On the top of delivery valve, the pressure acting is  $p$  kg/cm<sup>2</sup> which is due to delivery head  $H_d$ . In addition to the pressure, there is spring load 2.5 kg. Let the pressure below the delivery valve be  $p_1$  (refer Fig 11.23).



Then force trying to open the valve =  $a_1 \cdot p_1$

Opposing force =  $a \cdot p + s + m \cdot f_1 + w'$

where  $m = \text{the mass of valve} = \frac{w}{g}$  ;

$w = \text{the weight of valve}$  ;

$w' = \text{the weight of valve in water}$  ;

$f_1 = \text{the acceleration of valve}$ .

Force trying to open the valve = opposing force

i.e.,  $a_1 \cdot p_1 = a \cdot p + s + m f_1 + w'$

or  $a_1 \cdot p_1 - a_1 \cdot p = a \cdot p - a_1 \cdot p + s + m f_1 + w'$   
(subtracting  $a_1 \cdot p$  from both sides)

or  $a_1 \cdot (p_1 - p) = (a - a_1) p + s + m \cdot f_1 + w'$

or  $p_1 - p = \frac{a - a_1}{a_1} p + \frac{s + w'}{a_1} + \frac{m f_1}{a_1}$

or  $\frac{p_1 - p}{\gamma} = \frac{a - a_1}{a} \cdot \frac{p}{\gamma} + \frac{s + w'}{a_1 \cdot \gamma} + \frac{m \cdot f_1}{a_1 \cdot \gamma} \quad \dots(11.60)$

$\frac{p_1 - p}{\gamma} = \text{opening resistance of valve, which is very important to calculate in case of suction valve. Ordinarily it should not be more than 2.5 m of water absolute, otherwise the valve may not open.}$

$\frac{s + w'}{a_1 \cdot \gamma} = 0.4 \text{ to } 1.2 \text{ m of water.}$

**Problem 11.10** *A single acting, single cylinder reciprocating pump delivers water to a height of 30 m. The crank, having a length of 15 m, makes 40 rpm. The diameter of the pump cylinder is 15 m. The delivery valve has a diameter of 112 mm and the opening in the cylinder to the delivery valve has a diameter of 100 mm. Assume simple harmonic motion for the piston. Determine the opening resistance of the delivery valve.*

Take  $\frac{s + w'}{a_1 \cdot \gamma} = 0.75 \text{ m of water}$

**Solution**

$$H_s = 30 \text{ m} = \frac{p}{\gamma}, \quad r = 0.15 \text{ m} \quad N = 40 \text{ rpm}$$

$$D = 15 \text{ cm} \quad d = 112 \text{ mm} \quad d_1 = 100 \text{ mm}$$

Maximum acceleration of piston,  $f_p = \omega^2 \cdot r \cdot \left(1 + \frac{1}{n}\right)$   
(assuming  $n$  as infinity)

$$f_p = \omega^2 \cdot r = \left(\frac{2\pi \times 40}{60}\right)^2 \times 0.15 = 2.63 \text{ m/sec}^2$$

Acceleration of water through the delivery opening

$$f_1 = \frac{a_p}{a_1} \cdot f_p = \frac{D^2}{d_1^2} \cdot f_p = \left(\frac{150}{100}\right)^2 \times 2.63 = 5.92 \text{ m/sec}^2$$

$$a_1 = \frac{\pi}{4} d_1^2 = \frac{\pi}{4} \times (0.1)^2 = 0.00785 \text{ m}^2$$

Now  $\frac{s+w'}{a_1 \cdot \gamma} = 0.75 \text{ m of water}$

$$\begin{aligned} \therefore s+w' &= 0.75 \times a_1 \cdot \gamma \\ &= 0.75 \times 0.00785 \times 1,000 \\ &= 5.89 \text{ kg} \end{aligned}$$

Out of this value, take  $w$ , the weight of valve less than 2.5 kg

$\therefore$  Opening resistance of valve

$$\begin{aligned} \frac{p_1 - p}{\gamma} &= \frac{a - a_1}{a_1} \cdot \frac{p}{\gamma} + \frac{s+w'}{a_1 \cdot \gamma} + \frac{m \cdot f_1}{a_1 \cdot \gamma} \\ &= \frac{0.112^2 - 0.1^2}{0.1^2} \times 30 + 0.75 + \frac{2.5}{9.81} \times \frac{5.92}{0.00785 \times 1,000} \\ &= 7.86 + 0.75 + 0.192 \\ &= 8.802 \text{ m of water} \quad \text{Answer} \end{aligned}$$

### UNSOLVED PROBLEMS

- 11.1 Define pump. Explain how the liquid is raised by a pump.
- 11.2 What is "suction head" and what is "delivery head" as applied to pumps?
- 11.3 How would you classify the pumps?
- 11.4 Explain working principle of a reciprocating pump with the help of a line sketch, naming all the main parts.
- 11.5 Explain the difference between single-acting and double-acting reciprocating pumps with the help of line diagrams.
- 11.6 What is a plunger pump? Where would you recommend to use the same?
- 11.7 Explain the work principle of bucket pump. Where is it employed?
- 11.8 Define slip, negative slip and co-efficient of discharge of a reciprocating pump.
- 11.9 Explain with the help of delivery curves, how the resultant rate of discharge can be made uniform in different types of reciprocating pumps.
- 11.10 What is a three-throw pump? Explain its working.
- 11.11 Why is the suction height of a pump limited? On what factors does it depend?
- 11.12 Define "separation" in a reciprocating pump.
- 11.13 Explain how separation of flow is caused in piston pumps. What preventive measures are usually taken to reduce the same appreciably?  
(Roorkee University)

- 11.14 Why is the speed of reciprocating pump lower than that of centrifugal type? On what factors does the speed of the reciprocating pump depend? (AMIE)
- 11.15 How is the maximum speed of a reciprocating pump determined? Explain it when the pump is provided with air vessel and when it is not provided with air vessel.
- 11.16 Describe with the help of indicator diagrams how the acceleration and friction in suction and delivery pipes effect the work done by a reciprocating pump.
- 11.17 In a reciprocating pump the velocity of water in suction pipe during the suction stroke varies between zero and  $v$ , the displacement being SHM. Prove that the mean friction head through the stroke is  $\frac{4flv^2}{3gD}$  where  $l, D, f$  are length, diameter and frictional coefficient of pipe respectively. (UPSC)
- 11.18 Explain the use of air vessels on suction and delivery pipes of a reciprocating pump. (AMIE)
- 11.19 Explain the functions and working of air vessels on—  
(a) the suction side ;  
(b) the delivery side of a reciprocating pump.
- 11.20 What is the volume of air vessels fitted on suction and on delivery pipes?
- 11.21 How is the discharge going in or coming out of the air vessel calculated?
- 11.22 What is “resonance” in reciprocating pumps? How is it avoided?
- 11.23 Explain the working of a direct-acting steam pump. Where it is employed? Define its “duty”.
- 11.24 Explain the principle of working of a differential pump. Explain how the rate of discharge can be made uniform with the help of such a pump.

### Numericals

**Note :** In the following problems atmospheric or barometric pressure is taken as 10 m of water and vapour pressure of water as 2.5 m unless otherwise stated.

- 11.25 Determine the HP required to drive a double acting piston pump when piston diameter is 20 cm piston stroke 40 cm and rpm of crank 25. The suction and delivery heights are respectively 5 m and 25 m. Assume suitable values for different efficiencies. (9.31 HP, assume efficiencies as 0.6 and 0.75 for suction and delivery sides) (Jadavpur University)
- 11.26 In a bucket pump, the diameters of the bucket and the pump-rod are 300 mm and 62.5 mm respectively. The pump stroke is 1 m. The pump works under a mean suction head of 6 m and a mean delivery head of 12 m of water. Taking into account the influence of the pump-rod, determine the pull on the rod during the up stroke. Also calculate the volume of water discharged during (a) one up stroke and (b) one down stroke of the pump.  
[1250.64 kg ; (a) 0.0705 m<sup>3</sup> (b) 0.00306 m<sup>3</sup>]

- 11.27 In a single acting pump the cylinder has a diameter of 150 mm and a stroke 300 mm. The water is to be raised to a height of 20 m when the pump is running at 40 rpm. Determine the theoretical discharge and theoretical horsepower.  
If the actual discharge of the pump is 3.5 lps, find the co-efficient of discharge and percentage slip.  
(0.212 m<sup>3</sup>/min ; 0.930 HP, 0.99, 0.945%)
- 11.28 A three-throw pump having rams 30 cm diameter by 60 cm stroke is required to lift 5,000 lit/min against a static head of 116 m. The friction loss in the suction pipe is estimated at 1.20 m and in the delivery pipe at 17 m. The pipe velocity is 1 m/sec. The overall efficiency of the pump is 90%, and the slip is 2%.  
Calculate the speed at which the pump should run and the HP required to drive it.  
(40 rpm ; 165 HP) (Panjab University\*)
- 11.29 It is desired to compare the performance of the following three types of reciprocating pump :  
(a) Single-cylinder single-acting pump without air vessel ;  
(b) Same as (a), but with air vessel on delivery side ;  
(c) Three-throw or triplex pump without air vessel.  
In each case the crankshaft speed is 75 rpm, the diameter of the delivery is 15 cm and the mean discharge is 630 lit/min. Show by means of graphs, as nearly to scale as possible, the relationship between time, and the instantaneous velocity in the delivery pipe, for each of the three pumps.  
From the shape of the graphs, what can you deduce concerning the relative water hammer or inertia pressure in the delivery systems of the three pumps ?  
(AMI Mech E—Lond)
- 11.30 A single acting reciprocating pump has a cylinder bore of 15 cm and stroke of 22.5 cm. The suction pipe is 11.25 cm in diameter and 20 m long. Assuming no air vessel is fitted on the suction side of pump, find the maximum permissible suction lift if the pump speed is 30 rpm.  
(3.5 m) (Delhi University)
- 11.31 A single acting pump having plunger diameter 20 cm, stroke 30 cm is placed with its centre line 4 m above the level of water in the suction tank. The suction pipe is 7.5 cm in diameter and 5 m long. If separation occurs when the absolute pressure head is 2.40 m of water, find the maximum speed of the pump to avoid separation at the commencement of the suction stroke. Assume normal barometric pressure as 10.4 m of water and simple harmonic motion of the plunger.  
(25.8 rpm) (AMI Mech E)
- 11.32 The diameter of a plunger and stroke of a reciprocating pump are 30 cm and 60 cm respectively. On the suction side the pump is connected to a pipe 22.5 cm in diameter and 24 m long. The suction lift is 4.0 m. Determine the maximum speed at which it can operate without separation occurring at the beginning of the stroke. Assume the motion as SHM and take the effective height of the barometer as 8.4 m of water.  
(12.5 rpm)

- 11.33 What factors tend to limit the speed of a reciprocating pump ?

The following data refer to a reciprocating double acting force pump :

|                     |                      |
|---------------------|----------------------|
| Diameter of plunger | = 100 mm             |
| Delivery            | = 3,600 litres/hr    |
| Suction pipe        | = 50 mm and 9 m long |
| Suction head        | = 5.5 m              |

Determine the rpm of the pump if the cavitation occurs at an absolute pressure of 2.45 m of water.

(26.7 rpm) (AICTE)

- 11.34 A single acting horizontal reciprocating plunger pump has a suction pipe 62.5 mm diameter and 5.5 m long. The sump is 2 m below the cylinder axis, and the pump plunger is 150 mm diameter and of 250 mm stroke. If the pump is working at 25 rpm, find the absolute pressure head in kg/cm<sup>2</sup> at the beginning and at the end of suction stroke. If separation occurs at 0.775 kg/cm<sup>2</sup> below atmospheric pressure state whether the separation would occur at the beginning or at the end of the suction stroke. Take  $f = 0.01$  and barometric pressure as 10.38 m of water.

(0.565 kg/cm<sup>2</sup> absolute ; 1.115 kg/cm<sup>2</sup>, no separation at all)

(Rajputana University)

- 11.35 A single acting reciprocating pump runs at 60 rpm. The diameter of piston is 12.5 cm and a stroke of 30 cm. The centre of the pump is 3.5 m above the water surface of sump. The suction pipe diameter is 10 cm and its length is 4 m. Find the pressure heads at the commencement and at the end of suction stroke. What conditions on the delivery side will cause negative slip ?

(3.1 m and 10.5 m of water *absolute* ; Negative slip will occur if the pressure on the delivery side is less than 10.5 m of water absolute)

(Poona University)

- 11.36 A single-acting piston pump with no air chamber, has a piston diameter of 225 mm, stroke 450 mm, and a connecting rod 900 mm long. The suction head is 4.5 m, diameter of suction pipe 150 mm and its length is 10 m. If the opening resistance of the valve is equivalent to 0.8 m of water and the pump is employed to raise water at 25°C, what would be the maximum permissible speed without separation? The vapour pressure at 25°C may be taken as 35 kg/m<sup>2</sup>

(24.0 rpm) (Roorkee University)

- 11.37 It is desired to obtain a discharge of 45 litres per sec from a two cylinder plunger pump, the delivery running through a pipe 25 cm diameter and 150 m long. The constant static head in the flow is 96 m. No air vessel is provided. If the length of the stroke is twice the plunger diameter, what should be the diameter ? What would be the maximum pressure in the pipe neglecting the elasticity of water and pipe walls ? The pump is to run at 42 rpm.

(27.35 cm. ; 202.47 m of water) (Bombay University)

- 11.38 A plunger is fitted in a vertical pipe full of water. Its lower end is submerged in a suction sump. The plunger is drawn up with an acceleration of 1.8 m/sec<sup>2</sup>. Find out the maximum height above

the level in the sump at which plunger can work without separation. Separation occurs at a pressure of 2.5 m of water absolute. Barometer height of water may be taken equal to 9.8 m

(4 m)

(*Madras University*)

- 11.39 Explain the term separation or cavitation as applied to reciprocating pumps.

The diameter of a plunger of reciprocating pump is 200 mm and stroke 300 mm. Pump is to be driven with SHM at 60 rpm and it draws water from a sump 4 m below pump centre line. Find the least diameter of the suction pipe which is 5 m long in order to prevent separation at this speed.

Assume normal barometric pressure and that separation commences when the absolute head becomes 2.5 m of water.

(185 mm)

(*UPSC*)

- 11.40 A single cylinder, single acting reciprocating pump of 23 cm and 38 cm stroke runs at 18 rpm under suction and delivery heads of 3 m and 1.5 m respectively. The lengths of a suction and delivery pipes are 15 m and 300 m respectively. Find the diameter of suction pipe if the diameter of delivery pipe is 15 cm. There is no air vessel in the suction pipe. Assume the minimum pressure in the suction pipe as 7.5 m of water below atmosphere. Find the theoretical HP required to drive the pump. Take  $f = 0.008$ .

(11 cm ; 3.63 HP)

(*BHU*)

- 11.41 A single acting reciprocating pump has a plunger of diameter 250 mm and stroke 450 mm, the delivery pipe is to be 100 mm diameter and water is lifted to a tank whose level is 15 m above the pump and 30 m horizontally from it. If separation takes place at 2.35 m of water absolute, find the speed of the pump in rpm at which separation would occur in the delivery pipe if no air vessel was fitted for the cases :—

(a) If the pipe was vertical from pump and then horizontal upto the tank.

(b) If the pipe ran horizontally from the pump and then vertically to tank.

Assume barometric pressure as 10.3 m of water and SHM of the plunger.

[(a) 13.42 rpm, (b) 18.05 rpm]

(*AMI Mech E*)

- 11.42 A double-acting single cylinder reciprocating pump of 19 cm bore and 38 cm stroke runs at 36 double strokes per minute, suction head 3.6 m and discharge head 30 m. The length of the suction pipe is 9 m and of the discharge pipe is 60 m and the diameter of each pipe is 10 cm. Large air vessels are provided 3 m away from the pump on the suction side and 6 m away on the discharge side, both measured along the pipe lines.  $f = 0.008$ . Neglecting entrance and exit losses for the pipes, estimate for the beginning of the stroke—

(a) Heads in the two ends of the cylinder.

(b) Load on the piston rod neglecting the size of the piston rod and assuming simple harmonic motion.

- (a) 3.3837 m of water absolute ; 48.8833 m of water absolute,  
(b) 1,290 kg] (Panjab University)

- 11.43 In a double-acting reciprocating pump the suction lift is 3.6 m length of suction 6 m, diameter of suction pipe 10 cm, diameter of pump plunger 15 cm, length or stroke 45 cm, head lost in friction in suction is  $0.025 \frac{lv^3}{2gd}$ . A very large air vessel is fitted on the suction side close to the pump. Assuming simple harmonic motion, height of water barometer 9.6 m and separation 1.8 m, find the maximum safe speed of the pump.  
(170 rpm) (Panjab University)
- 11.44 A single acting reciprocating pump has a cylinder bore of 20 cm and stroke 30 cm. The delivery pipe of the pump has 15 cm diameter and a larger air vessel is fitted on the delivery side. The crank revolves at 30 rpm, piston having SHM. Find the discharge going in and coming out of the air vessel when the crank makes  $60^\circ$ ,  $120^\circ$  and  $180^\circ$  with inner dead centre. Find the crank angles at which there is no flow to or from the air vessel.
- 11.45 A double acting reciprocating pump runs at 120 rpm having a suction pipe 10 cm diameter and fitted with air vessel. The diameter of the cylinder is 15 cm and stroke 45 cm. The piston has SHM. Find the discharge into or from the air vessel when the crank makes  $30^\circ$ ,  $90^\circ$  and  $120^\circ$  with inner dead centre. Calculate the crank angle at which there is no flow to or from the air vessel.  
(AMIE)
- 11.46 A single acting reciprocating pump has a plunger 375 mm diameter and stroke 60 cm. The delivery pipe is 150 mm diameter and 90 m long, the frictional co-efficient being 0.008. Find the HP lost in friction in the delivery pipe when the pump runs at 50 rpm ;  
(a) when a large air vessel is fitted near the pump outlet,  
(b) assuming simple harmonic motion of the plunger and no air vessel.  
(c) Find the HP saved by installing the air vessel.  
[(a) 6.98 HP ; (b) 49.2 HP ; (c) 42.22 HP]  
(AMI Mech E)
- 11.47 A single acting reciprocating pump has a piston diameter of 25 cm and stroke of 45 cm. The piston has SHM and  $f = 0.01$ . An air vessel is fitted on the delivery side. The delivery pipe is 50 m long and has a diameter of 11 cm. The pump speed is 60 rpm. Determine the HP saved if  
(a) the air vessel is large and it is fitted just near the cylinder ;  
(b) the air vessel is large and it is fitted 2 m away from the cylinder.  
(7.7 HP ; 7.4 HP)
- 11.48 The following data refers to a double acting reciprocating pump :  
Plunger dia = 16 cm      Stroke length = 48 cm  
Dia of each of suction and delivery pipe = 10 cm  
Suction pipe length = 8 m      Delivery pipe length = 8 m  
Speed = 60 rpm for both pipes = 0.008

Assuming SHM of plunger, calculate the work saved in fitting an air vessel 2 m away from the cylinder on both suction and delivery pipes. (AMIE)

- 11.49 A differential pump running at 40 rpm has to deliver  $0.45 \text{ m}^3/\text{sec}$ . Determine the diameters of the main and the differential pistons if the stroke is twice the diameter of the main piston.

- 11.50 Determine the crank angle at which the valve of piston pump actually closes. Dimensions of the pump and valve are as follows—

Piston diameter 15 cm, stroke 30 cm, valve diameter 10 cm, valve lift (max) 1.25 cm, rpm of crank driving piston is 30. Assume contraction co-efficient of the water stream coming out of the valve slit as 0.64 and valve seat flat.

Prove any formula used.

(Jadavpur University)

- 11.51 Determine the size of an air chamber for a double-acting piston pump of piston diameter 15 cm and stroke 30 cm when piston is driven from a crank making 35 rpm. The pressure variation should not exceed 3 percent. Test if resonance takes place or not when the delivery pipe is 30 m long and 10 cm diameter, the delivery head being 20 m.

(Jadavpur University)



## *Centrifugal Pumps*

**12.1 Comparison with Reciprocating Pumps**—During the last 40 years the centrifugal pump has been rapidly superseding the reciprocating and other types of pumping equipment. Today it is the most popular pump for most of the jobs. The advantages of centrifugal pump over the reciprocating type can be summarised as hereunder :

| <i>Centrifugal Pump</i>                                                                                                                                             | <i>Reciprocating Pump</i>                                                                                                               |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| (i) Smooth and even flow.                                                                                                                                           | Pulsating flow.                                                                                                                         |
| (ii) Low initial cost.                                                                                                                                              | Initial cost as high as four times that of centrifugal pump.                                                                            |
| (iii) Compact, occupies less floor space.                                                                                                                           | Occupies 6 to 8 times the space required for horizontal centrifugal pump.                                                               |
| Vertical type requires even lesser space.                                                                                                                           |                                                                                                                                         |
| (iv) Gross weight is small.                                                                                                                                         | Gross weight is considerable.                                                                                                           |
| (v) Installation is easy.                                                                                                                                           | Installation is more difficult than centrifugal pump.                                                                                   |
| (vi) Efficiency of low head pumps is high.                                                                                                                          | Efficiency of low head pumps may be as low as 40% chiefly because of relatively higher energy losses viz. frictional, valve losses etc. |
| (vii) Construction is simplified by elimination of many parts such as non return valves, glands, air vessel etc. Therefore less number of spare parts are required. | Complicated construction. Therefore a number of spare parts are necessary.                                                              |

| <i>Centrifugal Pump</i>                                                                                                       | <i>Reciprocating Pump</i>                                                          |
|-------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------|
| (viii) Low maintenance cost. Periodical check up is sufficient.                                                               | Maintenance charges are high because parts like valves require constant attention. |
| (ix) High speed. Can be coupled directly through flanged coupling to electric motors or steam turbines.                       | Low speed due to separation difficulties. Belt drive indispensable.                |
| (x) Uniform torque.                                                                                                           | Torque not uniform.                                                                |
| (xi) Easy handling of highly viscous fluids such as oils, muddy and sewage water, paper pulp, sugar molasses, chemicals, etc. | Valves and glands cause trouble when required to transmit viscous fluids.          |

The *advantages* of the reciprocating pump over the centrifugal type are as follows :

- (1) The reciprocating pump can build up very high pressures, upto  $700 \text{ kg/cm}^2$  or even more,
- and (2) The efficiency of reciprocating pump is more than that of centrifugal pump to handle small discharge at a high head. It may be as high as 90%.

**12.2 Historical Development of Centrifugal Pump**—The National Art Museum, Paris has an oldest centrifugal pump which has wooden impeller with double curvature vanes. This is said to belong to fifth century and was found in a copper mine in San Domingos (Portugal) on 31 July 1772.

Leonardo Da Vinci (1452—1519) gave the idea of lifting water by centrifugal forces, through his original sketches. French physicist Denis PAPIN (1647—1714) was the first to describe the centrifugal pump scientifically in 1687. He built the pump in 1705 which had impeller with blades and a volute. At that time reciprocating pumps were very popular. John Skeys got a propeller pump patent in 1785. The first centrifugal pump in the USA was built by the Massachusetts pump factory. W.H.N. Johson (USA) built the first three-stage pump in 1846. In 1849 James Stuart Gwynne (UK) built multistage centrifugal pump. At that time John George Appold used backward curved vanes for such pumps. James Thomson (1822—1892) developed in the UK the guide vanes in 1850. Osborne Reynolds built a pump in 1875 using diffusion vanes. Then Mather & Platt UK started manufacturing such pump in 1893. W.J. Johnson in America was the first man to introduce the multistage pump by putting the impellers in series. In 1890 Sulzer Brothers (Switzerland) started manufacturing pumps after systematic and scientific investigation, with the result the design of mixed flow and axial flow pumps were evolved.

Thus centrifugal pump of today is made by 250 years old evolution. It has now attained a high degree of perfection. It is widely used as it can be coupled directly to electric motors, steam turbines etc.

**12.3 Definitions**—The centrifugal pump is a contrivance to raise liquids from a lower to a higher level by creating the required pressure with the help of centrifugal action. In general it can be defined as a machine which increases the pressure energy of a fluid, as a pump may not be used to lift water at all, but just to boost the pressure in a pipe line. Whirling motion is imparted to the liquid by means of backward curved blades mounted on a wheel known as the *impeller*. As already explained in Art 2.3, liquid enters the impeller at its centre technically known as the *eye* of the pump and discharges into the casing surrounding the impeller (refer Fig. 12.1). The pressure head developed by centrifugal action is entirely due to the velocity imparted to the liquid by the rotating impeller and not due to any displacement or impact.

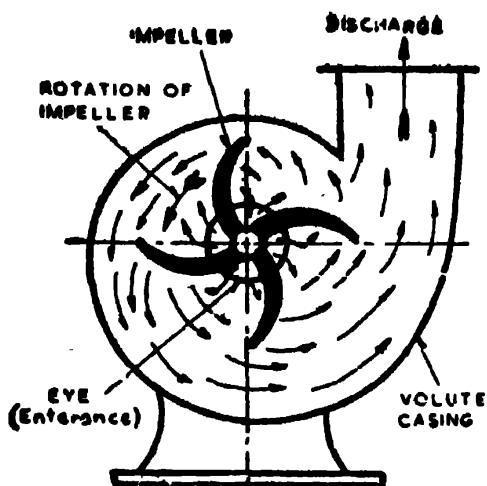


Fig 12.1 Centrifugal Pump Principle

The pump is usually named after the type or its casing (volute or diffusion). Diffusion casing is equipped with vanes and its design is adopted from the Francis turbine. The pump provided with it is therefore called a *turbine pump*. Vertical turbine pumps which are particularly suited for pumping water from deep wells are often called *deepwell pumps*.

**12.4 Principle and Operation**—The basic principle on which a centrifugal pump functions has been explained earlier (refer Art 2.7). The first step in the operation of a pump is *priming* that is, the suction pipe and casing are filled with water so that no airpocket is left. Now the revolution of the pump impeller inside a casing full of water produces a forced vortex which is responsible for imparting a centrifugal head to the water. Rotation of impeller effects a reduction of pressure at the centre. This causes the water in the suction pipe to rush into the eye. The speed of the pump should be high enough to produce centrifugal head sufficient to initiate discharge against the delivery head.

Mechanical action of the pump is to impart a velocity to the water. A water particle with a given velocity will rise to the same vertical height through which any particle should fall freely under gravity in order to attain the same velocity starting from rest. The required relation therefore is :

$$v = \sqrt{2gH} \quad \text{or} \quad H = \frac{v^2}{2g}$$

Thus if the outlet velocity of water in a pump is  $v$ , the pump can theoretically deliver against a head of  $\frac{v^2}{2g}$ .

**12.5 Classification of Centrifugal Pumps**—Centrifugal pumps possess the following characteristic features on the basis of which they can be classified.

- (i) Working head ;
- (ii) Type of casing ;
- (iii) Number of impellers per shaft ;
- (iv) Relative direction of flow through impeller ;
- (v) Number of entrances to the impeller ;
- (vi) Disposition of shaft ;
- (vii) Liquid handled ;
- (viii) Specific speed ; and
- (ix) Non dimensional factor  $K$ .

First is a commercial classification of pumps from the point of view of their utility but (ii) to (vii) are all practical consideration each governing an important constructional feature of the pump. The last two are purely theoretical aspects and provide probably the soundest basis for absolute classification.

**12.6 Working Head** – It is the head at which water is delivered by the pump.

According to the range of working head, pumps may be divided broadly in three categories.

(a) **Low Lift Centrifugal Pumps** are meant to work against heads upto 15 m. Impeller is surrounded by a volute and there are no guide vanes. The shaft is generally horizontal and water may enter the impeller from one or both sides depending upon the quantity of water to be delivered.

(b) **Medium Lift Centrifugal Pumps** are used to build up heads as high as 40 m. They are generally provided with guide vanes. Water may enter from one or both sides depending on the quantity to be pumped.

(c) **High Lift Centrifugal Pumps** are employed to deliver liquids at heads above 40 m. High lift pumps are generally multistage pumps because a single impeller cannot easily build up such a high pressure. They may be horizontal or vertical, the latter being used in deep wells.

**12.7 Type of Casing**—Pump casing should be so designed as to minimise the loss of kinetic head through eddy formation etc. Efficiency of the pump largely depends on the type of casing. In general, the casings are of two types and the pump is named after the casing it uses.

(a) **Volute pump**—(refer Fig 12.2, 12.3 and 12.4) : It has a volute casing into which the impeller discharges water at a high velocity. Volute is of a spiral form and the cross sectional area of the moving stream gradually increases from the tongue towards the delivery pipe. The cross-sectional area at any point is therefore proportional to the quantity of water flow-

ing across that section and therefore the mean velocity remains constant ; the stream-lines may be assumed to be continuous. Thus the losses of kinetic head which would occur if simply a circular casing were employed are avoided.

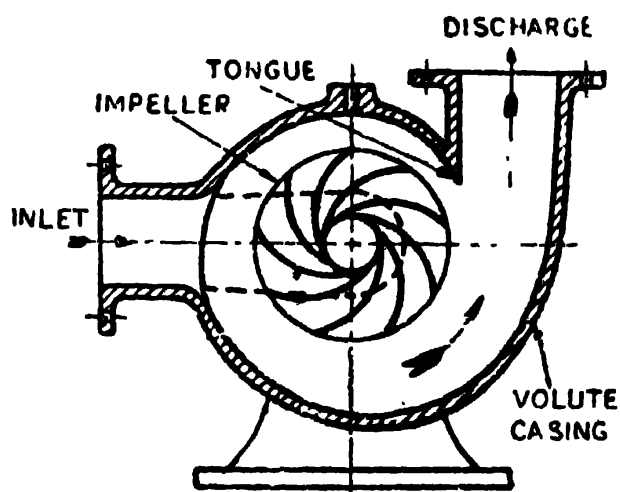


Fig 12.2 Volute Pump

- 1 Casing
- 2 Suction cover and branch
- 9 Bearing cover driving end
- 10 Bearing cover gland end
- 11 Stuffing Box
- 13 Impeller
- 14 Casing sealing rings
- 24 Screwed plug for priming
- 25 Drain plug
- 26 Plug for pressure gauge
- 27 Plug for vacuum gauge
- 31 Impeller locking nut
- 39 Pump shaft
- 42 Ball type thrust and journal bearing
- 46 Drip pipe
- 74 Locking washer
- 91 Gland Packing
- 166 Bearing Pedestal
- 328 Felt ring
- 335 End cover
- A Oil Filling and air release plug
- B Oil Level Indicator
- F Oil drain

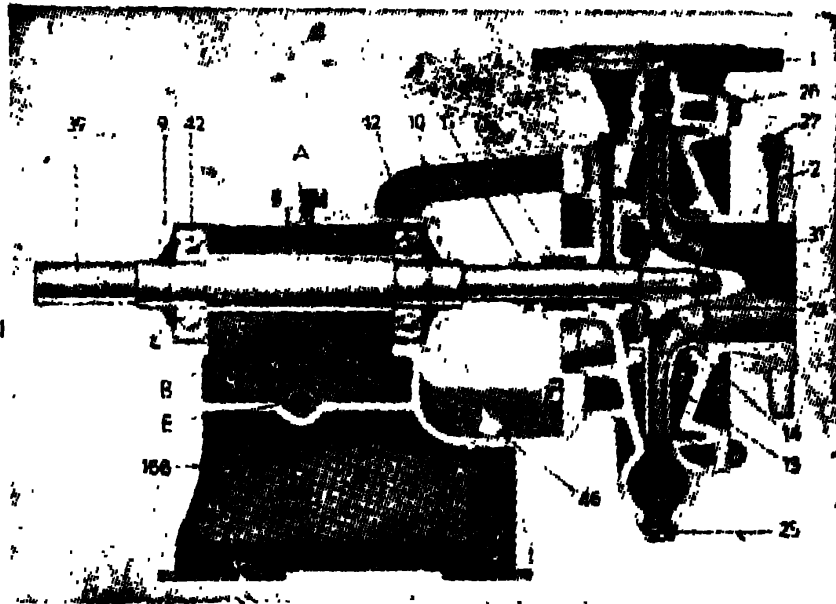


Fig 12.3 Single Stage Centrifugal Volute Pump with Ball Bearings. Manufactured by K. B. Bremen (West Germany)

The functions of a volute casing can be summarised as follows :

(i) To collect water from the periphery of the impeller and to transmit it to the delivery pipe at a constant velocity. As the flow progresses from the impeller towards the opening into the delivery pipe, more and more water is added to the stream. In order that the velocity of water in the casing may not increase, the cross-sectional area of the casing is gradually increased, so that the extra area can accommodate the water added at each point of the casing.

(ii) To eliminate the loss of head, by making the casing of spiral or volute form. As the velocity of water leaving the impeller equals the velocity of flow in the volute, loss of head due to change of velocity can be eliminated.

(iii) To increase evidently the efficiency of the pump by eliminating the loss of head due to change in velocity of flow in the volute.

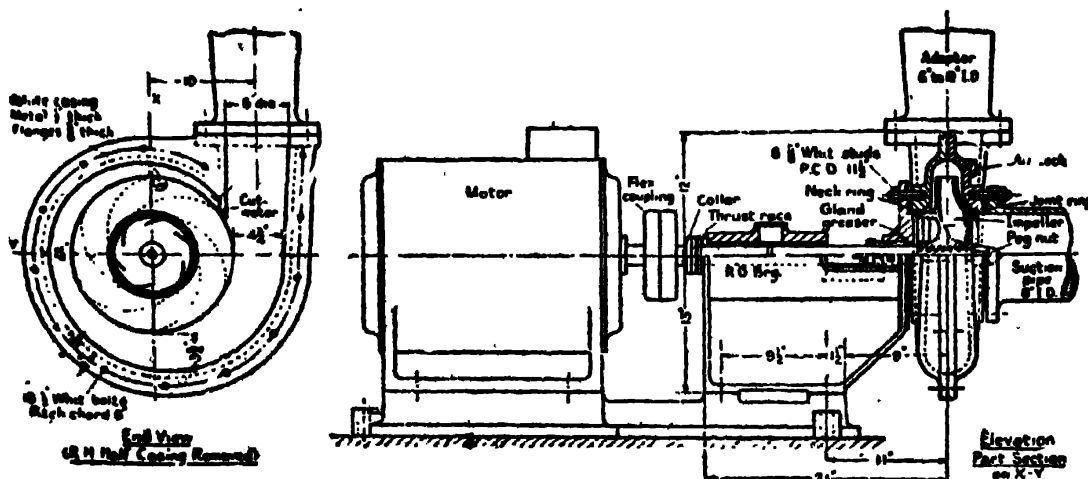


Fig 12.4 Section through a Single Stage Centrifugal Volute Pump

A subsequent improvement of this design is the provision of an annular space between the volute and the impeller (refer Fig 12.2, 12.3 and 12.4). The space which acts as a diffuser is known as *Vortex* or *Whirlpool chamber*. A free vortex is formed and as the water moves radially away from centre, velocity of whirl decreases thus building up pressure at the cost of velocity.

Almost all volute pumps are single stage type with the horizontal shafts irrespective of the shape of impeller.

(b) **Turbine Pump or Diffusion Pump** (refer Fig 12.5)—Impeller surrounded by a guide wheel consisting of a number of stationary vanes or diffusers providing outlets with cross-section gradually enlarging towards the periphery. Water emerging from the impeller flows past the guide vanes and as the section across flow increases, velocity falls and pressure is built up. Angle of guide vanes at the entrance should coincide with the direction of absolute velocity of water at impeller outlet.

This arrangement is employed in all multi-stage pumps. Single stage pumps, however, have less expensive volute casing.

Diffusion pumps may be either horizontal or vertical shaft type. The vertical type occupies very little space and suitable for installation in deep wells. It is often called a deep-well pump. They may also be used in narrow wells and mines etc.

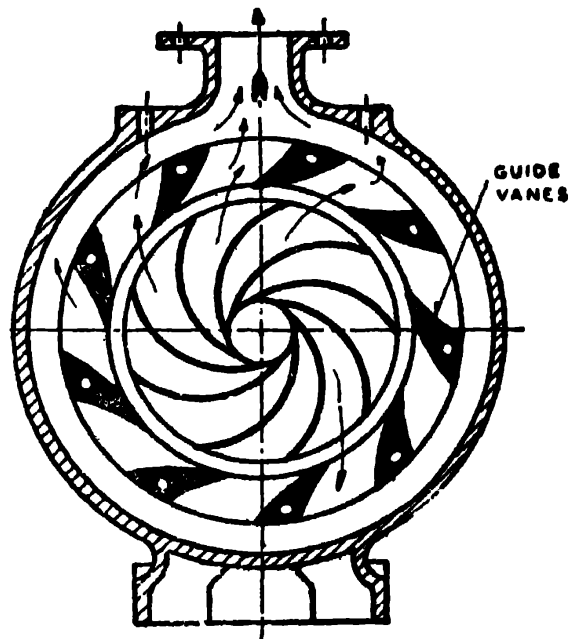


Fig 12.5 Turbine Pump or Diffusion Pump

## 12.8 Number of Impellers—

(a) **Single Stage Centrifugal Pump**—It has one impeller keyed to the shaft (refer Fig 12.6 a). This is generally horizontal but can be vertical also. It is usually a low lift pump.

(b) **Multi stage Centrifugal Pump**—It has two or more impellers keyed to a single shaft [refer Fig 12.6 (b)] and enclosed in the same casing. Pressure is built up in steps. The impellers are surrounded by guide vanes and the water is led through a by-pass channel from the outlet of one stage to the entrance of the next until it is finally discharged into a wide chamber from where it is pushed on to the delivery pipe.

These pumps are used essentially for high working heads and the number of stages depends on the head required. The author has tested pumps having as many as fourteen stages but ordinarily not more than ten are employed. Until a few years ago, the arrangement used to generate high delivery heads was to operate a number of single stage pumps in series but the modern multi-stage pump has completely replaced these unsatisfactory arrangements.

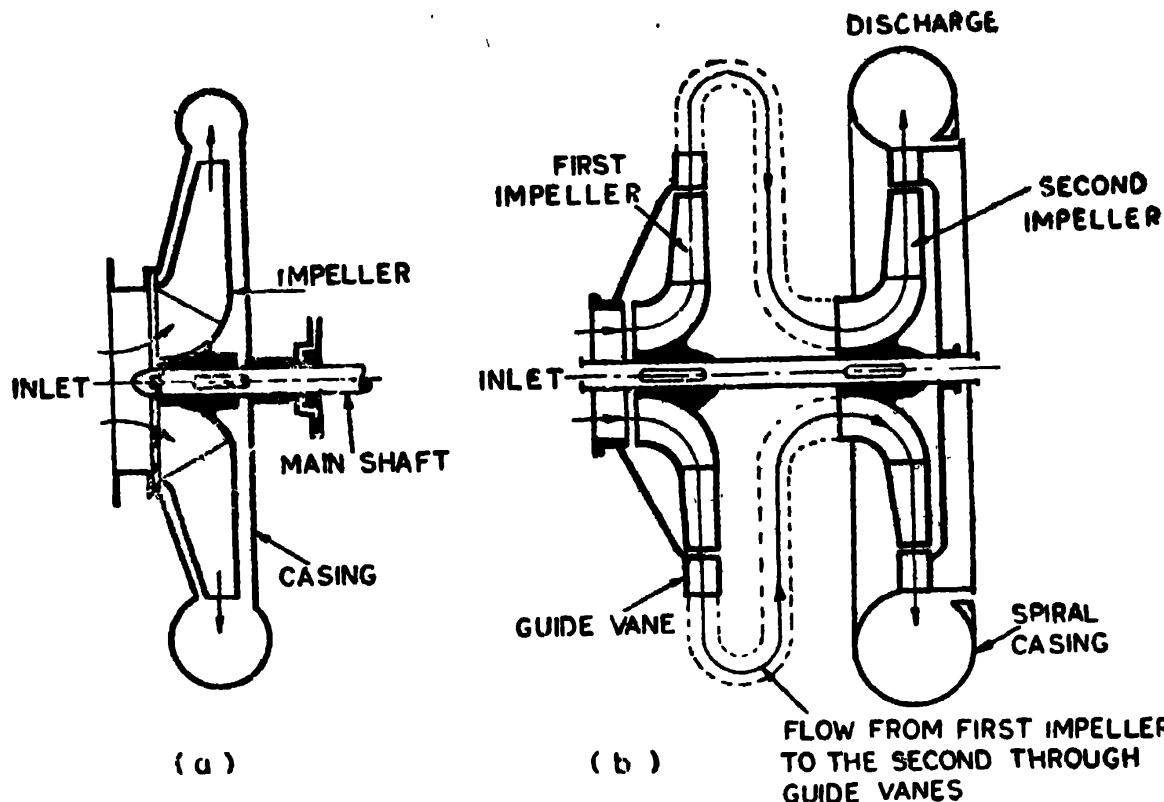


Fig 12.6 (a) Single Stage Centrifugal Pump

Fig 12.6 (b) Multi stage (Two Stage) Centrifugal Pump

**12.9 Relative Direction of Flow through Impeller** (refer Fig 12.7)—Theoretically any reaction turbine can also be used as a pump and therefore the classification of pumps from the point of view of direction of flow is similar to that of turbines (refer Art 5.13). But there are no inward flow pumps primarily because of difficulty in starting them.

(a) **Radial Flow Pump**—(refer Fig 12.7 a) Ordinarily all centrifugal pumps are manufactured with radial flow impellers.

(b) **Mixed Flow Pump**—As will be seen later (refer Art 13.5) the mixed flow impeller is just a modification of radial flow type enabling it to pump a large quantity of water. Flow through the impeller is a combination of radial and axial flows and the impeller resembles the propeller of a ship. Some mixed flow impellers look like a screw and are known as *screw* impellers [refer Fig. 12.7 (b).] In older designs a large quantity of water was delivered by running several pumps in parallel but this arrangement is now obsolete. Mixed flow pumps are mostly used for irrigation purposes where a large quantity of water at a low head is required.

(b) **Axial Flow Pump**—Though axial flow pump is a roto-dynamic pump it is hardly justifiable to call it a centrifugal pump because centrifugal force is not called into play for the generation of pressure. Pressure is developed by flow of liquid over blades of aerofoil section just as the wings of an aeroplane produce the lift. The action is just the converse of a propeller turbine. If Kaplan turbine runner (refer Fig 7.26) is used as an impeller, the pump may be called a *Kaplan Pump*. It will have adjustable blades.

Axial flow pumps are designed to deliver very large quantities of water at comparatively low heads. They are ideally suited for irrigation purposes.

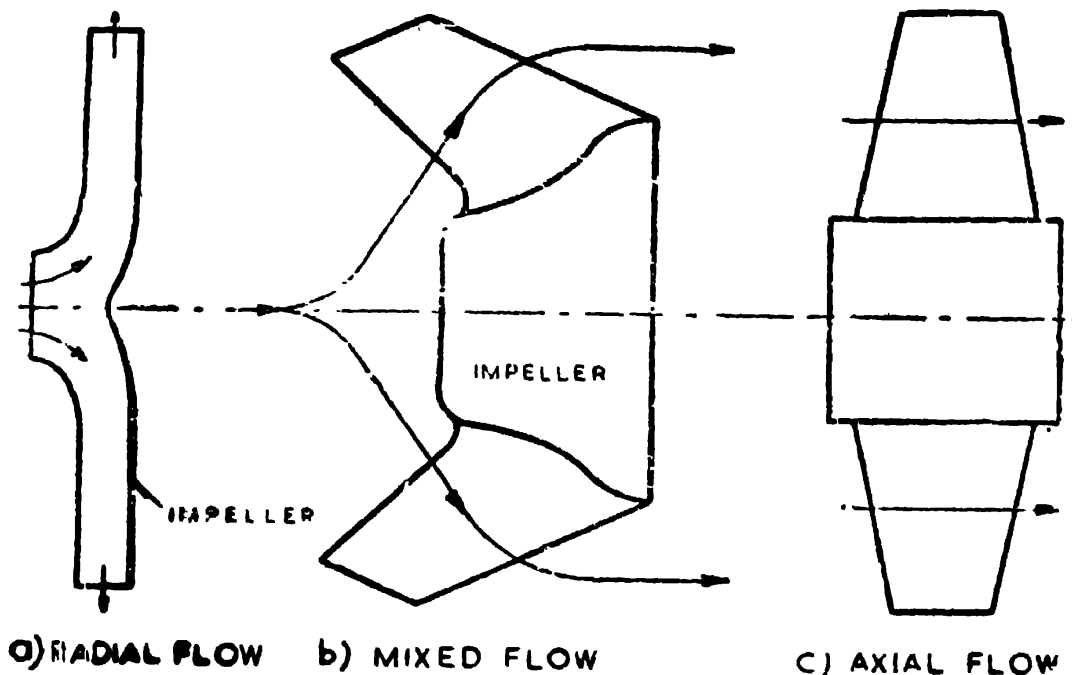


Fig 12.7 Classification of Centrifugal Pump Considering the Direction of Flow Through the Impeller

**12.10 Number of Entrances to the Impeller**—The pump may be single entry or double entry type. In the single entry or *single suction* pump, water is admitted from a suction pipe on one side of the impeller (refer Fig. 12.6 a).

Double entry pumps or *double suction* pumps (refer Fig 12.8) admit water from both sides of the impeller. It is suitable for pumping large quantities of liquid. The arrangement has the added advantage that axial thrust is neutralised.

**12.11 Disposition of Shaft**—The shaft may be disposed horizontally or vertically. Generally centrifugal pumps are designed with horizontal shafts. Vertical disposition of shaft effects an economy in space occupied and is therefore suitable for deepwells and mines etc. They are also used for irrigation purposes.

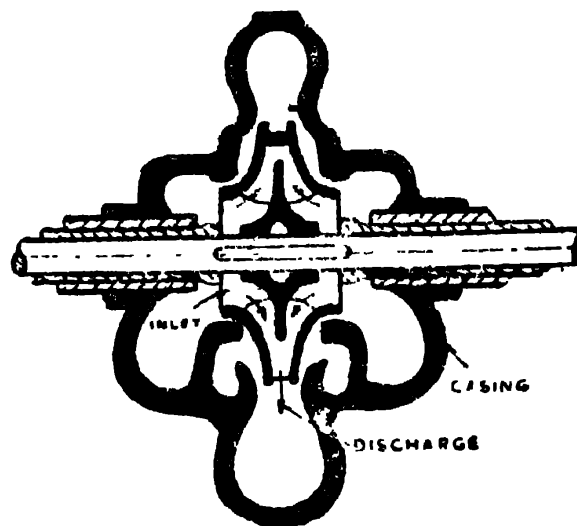


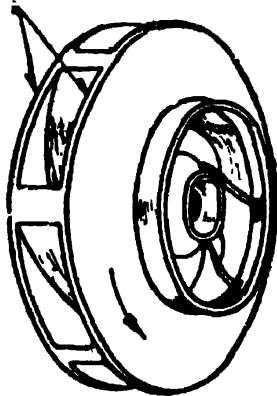
Fig 12.8 Double Suction Pump

**12.12 Liquid Handled**—Depending on the type and viscosity of liquid to be pumped, the pump may have a closed or open impeller (refer Fig 12.9) Each of these types may have ferrous, non ferrous or stone-coated impeller to resist chemical attack of liquid being pumped.



(a) **Closed Impeller Pump**—(refer Fig 12.9a)—An ordinary centrifugal pump is equipped with a closed impeller in which the vanes are covered with shrouds on both sides. This type is meant to handle non-viscous liquid such as ordinary water, hot water, hot oil and chemicals like acids etc. Material of the impeller should be selected according to the chemical properties of liquid used. For hot water at temperatures exceeding  $150^{\circ}\text{C}$ . cast steel impeller is recommended. Non-ferrous impellers are used for chemicals which are liable to corrode a ferrous surface. For pumping acids, the impeller and all inside surfaces in contact with the liquid should be coated with stone.

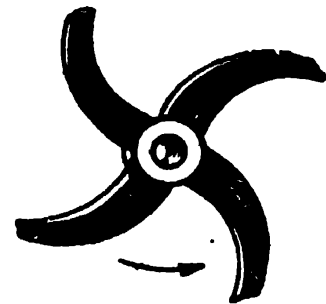
*Shrouds on both sides*



(a) CLOSED TYPE IMPELLER



(b) SEMI-OPEN TYPE IMPELLER



(c) OPEN TYPE IMPELLER

Fig 12.9 Different Types of Impellers

(b) **Semi Open Impeller Pump or Non-Clog Pump** (refer Fig 12.9b)—The impeller is provided with shroud on one side only. This pump is used for viscous liquids such as sewage water, paper pulp, sugar molasses etc. In order to minimise the chances of impeller getting clogged, the number of vanes is reduced and their height is increased.

Sewage pumps are made without any protuberances around which rags could wrap and catch. They are always of the single suction type, as with the double suction pumps the shaft extends through the eyes of the impeller, thereby forming an easy place for rags to catch and wrap and thus clogging the pump.

The non-clog pumps for paper stock and other similar material have open type impellers with entrance blades especially designed to prevent separation of stock and water.

Choice of material for manufacture of impeller is influenced by the chemical nature of liquid to be handled. Non-clog pumps must be heavily constructed mechanically in order to give satisfactory service. The number of blades for impeller is small, using not more than two.

(c) **Open Impeller pump** (refer Fig 12.9c)—The impeller is not provided with any shroud. Such pumps are used in dredgers and elsewhere for handling mixtures of water, sand, pebbles and clay, in which the solid contents may be as high as one part in four. The impeller has very rough duty to perform. It is generally made of forged steel. Its life depends upon the material handled, may be 40 or 50 hours or in some cases from 500 to 1,000 hours.

**12.13 Specific Speed**—As in the case of turbines, specific speed is a sound basis for a technical classification of centrifugal pumps. Specific speed is the only characteristic index or distinguishing feature of a pump when several impellers can be used for the same head and capacity. The performance and dimensional proportions of pumps having the same specific speed will be the same even though their outside diameters and actual operating speeds may be different.

Specific speed is defined as the speed of a geometrically similar pump when delivering one m<sup>3</sup>/sec against a head of one metre.

Then specific speed

$$N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}} \quad \dots(\text{refer Eqn 3.11})$$

where  $N_s$  = actual speed of pump in rpm,  
 $Q$  = quantity flowing in m<sup>3</sup>/sec,  
 $H$  = delivery head (total or manometric) in m.

The values of  $Q$  and  $H$  to be substituted in the equation for the purpose of calculating the specific speed are those corresponding to the maximum *efficiency* of the pump at its normal working speed.

It may be noted that the value of  $H$  to be used in the equation for  $N_s$  of a multi-stage pump is obtained by dividing the actual head developed by the number of stages. Similarly the value of  $Q$  for a double suction pump is taken as half the actual value for the purpose of this calculation.

**12.14 Non-dimensional Factor  $K_s$** —The non-dimensional factor  $K_s$  (mentioned in Art 4.19) for the classification of water turbine can be used for this purpose also,

$$K_s = \frac{Q \cdot N^3}{v^5}$$

where  $Q$  = quantity discharged in m<sup>3</sup>/sec  
 $N$  = rpm  
 $v = \sqrt{2g H_{\text{mano}}}$  in m/sec  
 $H_{\text{mano}}$  = total head the pump has to work against, in m.

**Relation between  $N_s$  and  $K_s$  for Centrifugal Pumps :**

$$\begin{aligned} N_s &= \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}} = \frac{N \cdot \sqrt{Q}}{\left(\frac{v^2}{2g}\right)^{\frac{3}{4}}} \\ &= (2g)^{\frac{3}{4}} \cdot \frac{N \cdot \sqrt{Q}}{v^{\frac{3}{2}}} = (2 \times 9.81)^{\frac{3}{4}} \cdot \frac{N \cdot \sqrt{Q}}{v^{\frac{3}{2}}} \end{aligned}$$

$$\text{or } N_s^2 = \frac{(2 \times 9.81)^{\frac{3}{2}} \cdot N^2 \cdot Q}{v^3} = (2 \times 9.81)^{\frac{3}{2}} \cdot K_s$$

$$\text{or } K_s = \left\{ \frac{N_s}{(2 \times 9.81)^{\frac{3}{4}}} \right\}^2 = \left( \frac{N_s}{9.32} \right)^2$$

**Representing Classification of Centrifugal Pump in Tabular Form :**

TABLE 12.1

|   | Type of Impeller                     | $N_s$      | $K_s$        | $\frac{D_2}{D_1}$ |
|---|--------------------------------------|------------|--------------|-------------------|
| a | Slow Speed radial flow runner        | 10 to 30   | 1.15 to 10   | 3.5—2.0           |
| b | Normal speed radial flow runner      | 30 to 50   | 10 to 30     | 2.0—1.5           |
| c | High speed radial flow runner        | 50 to 80   | 30 to 75     | 1.5—1.3           |
| d | Mixed flow runner (screw runner)     | 80 to 160  | 75 to 300    | 1.2—1.1           |
| e | Axial flow runner (Propeller runner) | 110 to 500 | 140 to 2,900 | 1                 |

TABLE 12.2

|                |           |     |        |     |
|----------------|-----------|-----|--------|-----|
| $N_s$          | 20        | ... | ...    | 300 |
| $H$            | 80        | ... | ...    | 8 m |
| Type of casing | diffusion |     | volute |     |

**Problem 12.1** A six stage centrifugal pump delivers 120 lit/sec against a net pressure rise of 51 kg/cm. Determine its specific speed if it rotates at 1,450 rpm. What type of impeller would be selected for the pump?

**Solution**

$$\text{No. of stages} = 6$$

$$Q = 120 \text{ lit/sec} = 0.12 \text{ m}^3/\text{sec}$$

$$p = 51 \text{ kg/cm}^2 \quad N = 1,450 \text{ rpm}$$

$$H = \frac{p}{\gamma} = \frac{51 \times 100 \times 100}{1,000} = 510 \text{ m of water}$$

$$\text{Head developed per impeller} = \frac{510}{6} = 85 \text{ m of water}$$

$$\therefore \text{Specific speed of the pump } N_s = \frac{N \times \sqrt{Q}}{H^{\frac{3}{4}}}$$

$$\text{or} \quad N_s = \frac{1,450 \times \sqrt{0.12}}{85^{\frac{3}{4}}} = \frac{1450 \times 0.346}{9.2 \times 3.02}$$

$$= 18 \text{ Units} \quad \text{Answer}$$

A radial impeller will suit this specific speed.

**Problem 12.2** Find the specific speed of a double suction centrifugal pump delivering  $1.5 \text{ m}^3/\text{sec}$  against a net head of  $15 \text{ m}$ , when running at  $725 \text{ rpm}$ . State the type of impeller to be employed for this pump.

**Solution**

$$Q = 1.5 \text{ m}^3/\text{sec} \quad H = 15 \text{ m} \quad N = 725 \text{ rpm}$$

Considering only one half of the impeller,  $Q$  is  $0.75 \text{ m}^3/\text{sec}$

$$\text{Specific speed } N_s = \frac{N \times \sqrt{Q}}{H} = \frac{725 \times \sqrt{0.75}}{15} = 82.4 \quad \text{Answer}$$

The type of the impeller to be used in mixed flow screw type.

#### 12.14 Evolution of Axial Flow Impeller from the Radial Type

**Type**—It is interesting to note that the recent development of axial flow impellers has been considerably facilitated with the knowledge of specific speed. The study of specific speed is of great academic interest and it is a valuable aid in the design of pumps.

Table 12.1 as well as Fig 12.10 show the development of impellers with regard to their specific speeds and non dimensional factors. This can be compared with "Evolution of Kaplan Turbine Runner" discussed in Art 7.15 and Fig 7.15 and 7.26.

**12.16 Layout, Accessories and Starting of Centrifugal Pump**—A typical layout of a centrifugal pumping installation is shown in Fig 12.11. The main accessories are—

- (a) Strainer and foot valves, (b) Suction pipe with fittings,
- (c) Delivery valve or regulating valve, (d) Delivery pipe with fittings

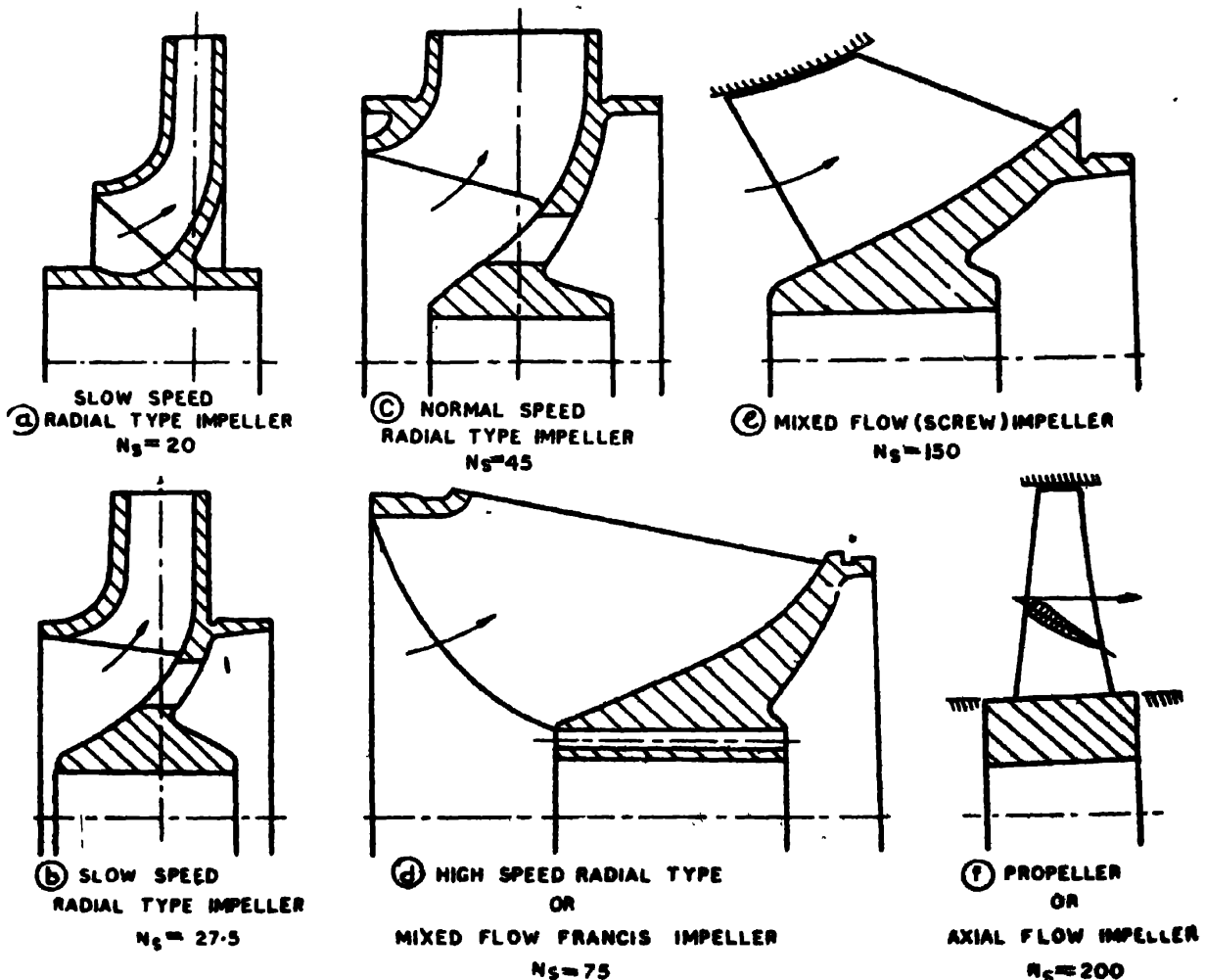


Fig 12.10 Evolution of Axial Flow Impeller from Radial Flow Type

(a) **Strainer and Foot Valve** is fitted at the end of the suction pipe and is submerged in water in such a way that it is only a few cm above the bottom of water to be pumped. The water from the sump or well first enters the strainer which is meant to keep the floating bodies, such as leaves, wooden pieces and other kind of rubbish, away from the pump. In the absence of strainer the foreign material will pass through the pump and clog it, thus hindering its working. With the strainer is cast a foot valve of non-return or oneway type, opening upwards. The water will pass through the foot valve upwards and it will not allow the water to move downwards.

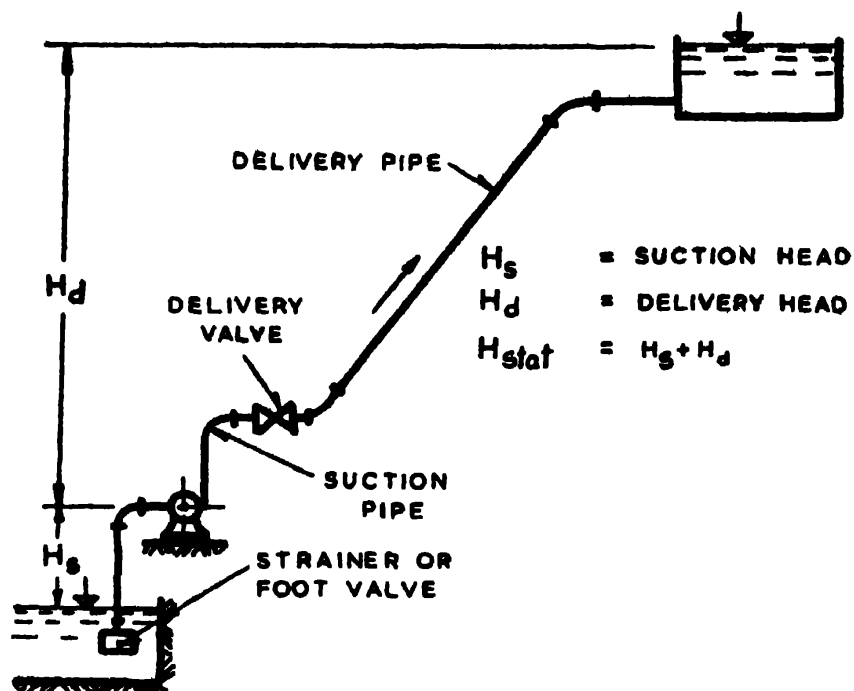


Fig 12.11 Layout of a Pump

(b) **Suction Pipe with Fittings**—The suction type is laid in such a way that it rises all its way to the pump *upwards*. In case the suction pipe first rises and then drops, thus making a loop, the air will collect in that portion and will hinder the satisfactory working of the pump. Great care must be taken to ensure that the pipe fittings are all air-tight. The fittings on the suction side may consist of different bends, but no regulating valve. The details of suction lift are given in Art 12.39. The diameter of suction pump (refer Art 12.24) is slightly less than that of delivery pipe (refer Fig 12.24)).

(c) **Delivery Valve or Regulating Valve**—As the water comes out of the pump, it must pass through a regulating valve. The regulating valve is of sluice type. It must be closed when the pump is being started and has built up its pressure. If the delivery valve is closed and the pump is working and delivery water at its full pressure, the delivery pipe or pump will not burst as the impeller will just be churning the water in the casing. The delivery valve is also essential to vary the discharge at the time of needs. It should be closed before the pump has stopped working, otherwise the full delivery pressure will be transmitted to the suction pipe which may be harmful.

(d) **Delivery Pipe and Fittings**—The water is delivered to a place or reservoir through the delivery pipe having a number of fittings mentioned

in Table 12.3 and Fig 12.17. The pipe diameter depends upon the discharge to be handled by the pump and can be calculated with the velocity of flow. Fig 12.26 gives the curve representing pipe diameters versus discharge, drawn from experimental data.

**Starting of Centrifugal Pump**—As given in Art 12.40, 'Priming of Pumps', it is necessary that before the pump is started, its casing together with impeller, and the suction pipe must be filled with water in order to remove the air, gas or vapour in that region. The pump will not generate its pressure, if there is any air left on the suction side. Whenever there is any difficulty in the satisfactory running of a pump, it is mainly due to presence of air in the piping or pump. The removal of air by filling the pump with water is known as *priming*. Hence priming is necessary to start the pump.

The correct layout pumping installation is very important for the satisfactory and successful working of the centrifugal pump. The layout should be such that no air pocket is formed in the piping. Great care must be taken on the suction side as explained above. The details of defects and remedies of centrifugal pump are given in Art. 12.49.

**12.17 Head of a Pump**—The term "head" of a pump stands for the following heads :

- (i) Static Head ;
- (ii) Manometric Head ;
- (iii) Total, Gross or Effective Head.

(i) **Static Head**—It is the sum of the suction and delivery heads (refer Fig 12.11). Suction head is the vertical height of the centre line of the pump shaft above the surface of a available liquid i.e., the lower surface from which it is being raised. Delivery head is the vertical height measured from the centre line of main pump shaft to where the liquid is delivered. The static head is also known as *geodetic head*.

If  $H_s$  and  $H_d$  be suction and delivery heads respectively, the static head,

$$H_{stat} = H_s + H_d \quad \dots(12.2)$$

(ii) **Manometric Head** is the head measured across the pump inlet and outlet flanges. It expresses the increase in *pressure energy* per unit weight of liquid handled by the impeller.

$$H_{mano} = \frac{p_d - p_s}{\gamma} + h_g \quad \dots(12.3)$$

$$= H_{mano(d)} - H_{mano(s)} + h_g \quad \dots(12.3a)$$

Thus the manometric head is the difference between the delivery (positive) and suction (negative) pressure heads (i.e. difference between the readings shown by the monometers or gauges) *plus* the vertical distances  $h_g$  between the pressure tappings for the suction and delivery gauge.

The manometric head includes all losses against which the pump has to work except the kinetic head.

(iii) **Total, Gross or Effective Head**—This is the actual head against which the pump has to work. It is equal to the static head plus

all the head losses occurring in flow before, through, and after the impeller.

It is equal to the increase in *total* energy of the liquid between the inlet and outlet.

$$H = \frac{p_d - p_s}{\gamma} + h_g + \frac{v_d^2 - v_s^2}{2g}$$

where

$H$  = total or effective head in metres of liquid column ;

$v_d$  = velocity of water in delivery pipe

$v_s$  = velocity of water in suction pipe.

Thus the difference of total head and manometric head is the difference of kinetic heads between the delivery and suction. This difference is comparatively much less, therefore for practical purposes the manometric head is equal to the total head.

If  $H_L$  be the total loss of head, then the manometric head,

$$H_{\text{mano}} = H_{\text{tot}} + H_L \quad \dots(12.4)$$

If  $h$  be the gauge pressure of available liquid,

$$H_{\text{real}} = H_{\text{apparent}} - h$$

The above heads are independent of the density of the liquid being raised. A centrifugal pump running at a particular speed will raise water, oil or mercury to the same height. But the power required will be different. Also, the pressure generated in kg/cm<sup>2</sup> will be different in each case. It is, therefore, convenient to express the head in metres of liquid column.

If the pump raises liquid from a source which is already subjected to some pressure, then the equivalent of this gauge pressure in metres of liquid column should be subtracted to get the real total head. On the other hand if a pump takes liquid from vacuum, the equivalent of vacuum in metres should be added to get the actual total head.

**12.18 Power**—The horsepower required to drive the pump shaft is given by

$$P = \frac{\gamma \cdot Q \cdot H_{\text{mano}}}{75 \eta_{\text{overall}}} \text{ HP} \quad \dots(12.5)$$

where  $\gamma$  = the specific weight of liquid raised in kg/m<sup>3</sup> ;

$Q$  = the discharge in m<sup>3</sup>/sec ;

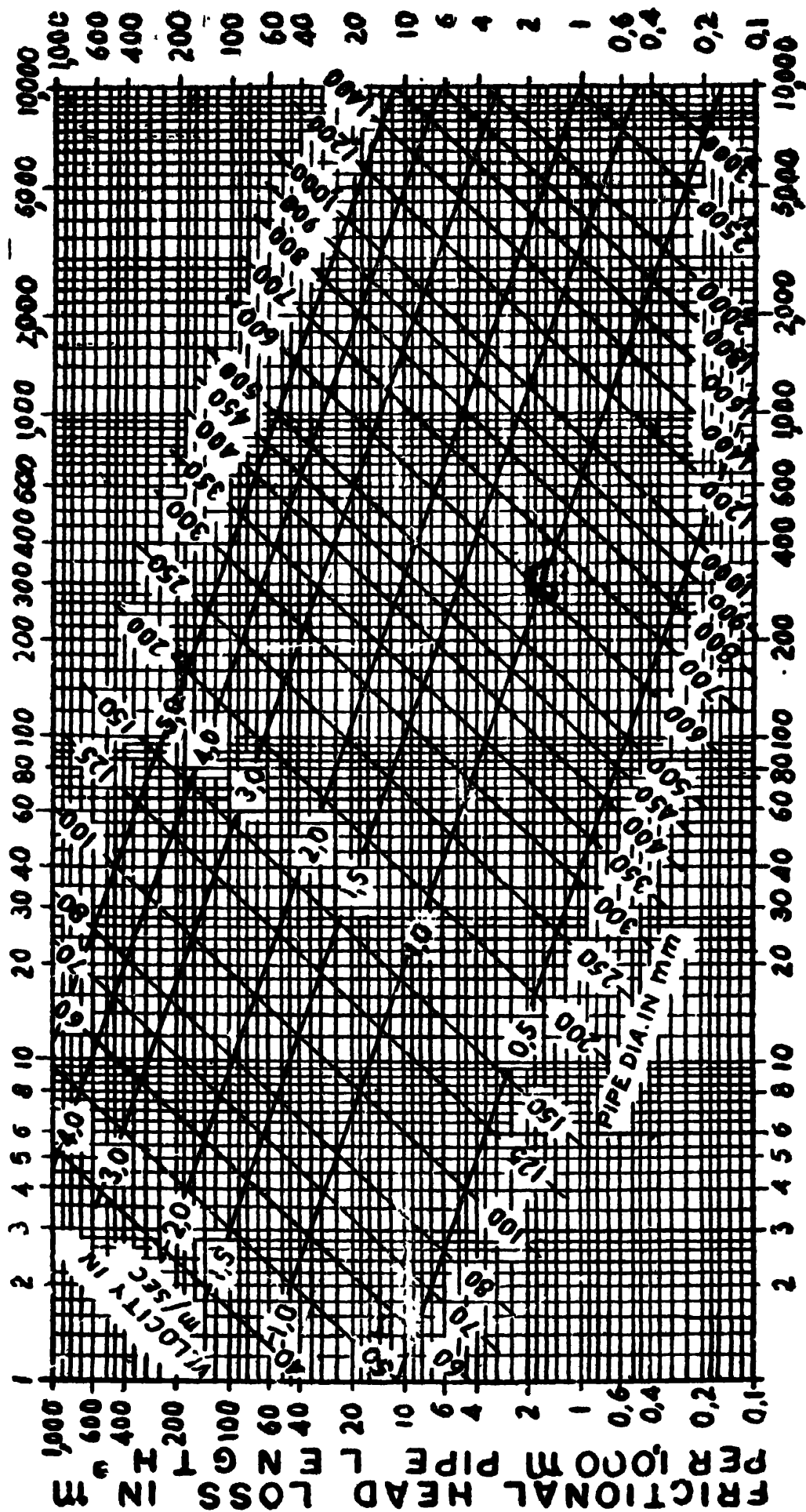
$H_{\text{mano}}$  = the manometric or gross head in m ;

$\eta_{\text{overall}}$  = the overall efficiency of the pump.

**12.19 Efficiency**—**Overall Efficiency** is the ratio of the power supplied by the pump to the power delivered to the pump shaft.

$$\eta_{\text{overall}} = \frac{\text{Output}}{\text{Input}} = \frac{\gamma \cdot Q \cdot H_{\text{mano}}}{75 \cdot (\text{BHP} - \text{HP lost in coupling mechanism})} \quad \dots(12.6)$$

where BHP is the brake horse power of the electric motor or primemover which drives the pump.



## DISCHARGE IN LIT SEC

Fig 12.12 Loss of Head in Pipes due to Friction. Curves obtained from Sulzer Brothers Ltd. Switzerland



The static efficiency of a pump also deserves mention because it accounts for all hydraulic losses in the pipelines.

$$\text{Then } \eta_{\text{stat}} = \frac{\gamma \cdot Q \cdot H_{\text{stat}}}{75 \cdot (\text{BHP})} \quad \dots(12.7)$$

**12.20 Loss of Head in Pipe Lines and Pipe Fittings**—All head losses from the foot valve up to the outlet of the delivery pipe are given by the following equation

$$H_L = H_{L_s} + H_{L_d} + \Sigma H_{L_f} + \frac{v_s^2}{2g} \quad \dots(12.8)$$

[All losses of head are measured in metres of liquid column]  
where  $H_L$  = the total loss of head in the pipes and fittings ;

$H_{L_s}$  = the frictional loss in suction pipe ;

$$= \frac{4fL_s}{d_s} \cdot \frac{v_s^2}{2g}$$

$H_{L_d}$  = the frictional loss in delivery pipe ;

$$= \frac{4fL_d}{d_d} \cdot \frac{v_d^2}{2g}$$

$\Sigma H_{L_f}$  = the sum of head losses in pipe fittings ;

$\frac{v_s^2}{2g}$  = the loss of kinetic head due to velocity of water at exit.

Pipe fittings referred to above include the strainer, the foot valve, all pipe bends, all regulating and non-return valves and all changes of cross-section.

**12.21 Determination of Loss of Head in Pipe Lines and Pipe Fittings**—Such losses may be classified as follows :

- (a) Loss of head in pipes ;
- (b) Loss due to enlargement to cross section of pipe ;
- (c) Loss due to contraction of cross section of pipe ;
- (d) Loss due to bends in pipe ;
- (e) Loss due to pipe fittings such as valves, joints, elbows, etc.

It is, in general, not easy to determine these losses analytically. Therefore, empirical relations obtained from experiments are used. Often it is convenient to represent the experimental results graphically.

(a) *Loss of head in pipes* is given by the equation  $\frac{4 \cdot f \cdot l}{d} \cdot \frac{v^2}{2g}$ , where

$f$ , the frictional factor, is a function of Reynolds' number  $R_s$ . In order to avoid long calculations every time, loss of head in pipelines is generally given by tables or graphs which are prepared from the experimental results. Fig 12.12 shows the frictional head loss in metres, for each of pipe length, drawn against discharge in litres/sec. for different pipe diameters in mm.

(b) *Loss of head due to a sudden enlargement* of the cross-section of pipe is generally given by the expression  $\frac{(v_1 - v_2)^2}{2g}$  which was analytically derived by Carnot on certain assumptions. Here  $v_1$  and  $v_2$  are the velocities of flow through the pipe before and after enlargement respectively.

Loss of head due to gradual enlargement of cross-section is given by

$$H_L = \frac{k(v_1 - v_2)^2}{2g} \quad \dots(12.9)$$

The factor  $k$  depends on the cone angle and a curve of  $k$  versus cone angle, is shown in Fig 12.13.

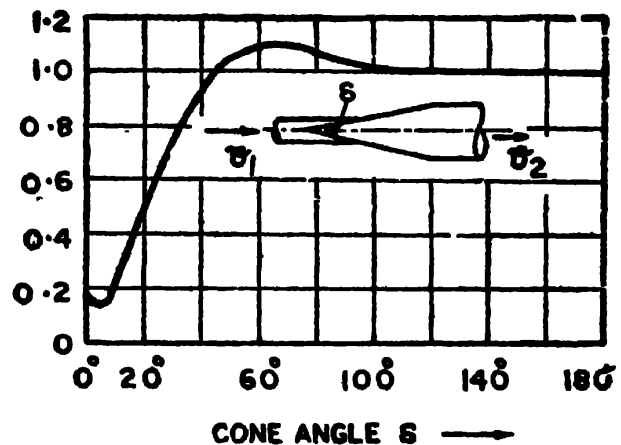


Fig 12.13 Loss of Head due to Gradual Enlargement  
 $k = f(\delta)$   
(Experiments by A.H. Gibson)

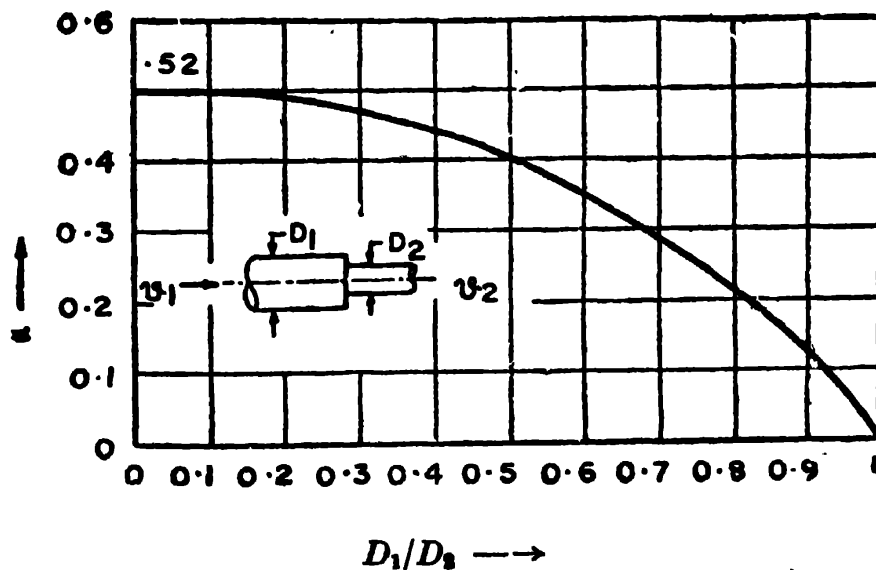


Fig 12.14 Losses of Head due to Sudden Contraction

(c) *Loss of head due to sudden contraction* is given by

$$H_L = k \cdot \frac{v_2^2}{2g} \quad \dots(12.10)$$

where  $v_2$  is the velocity after contraction.

$$k = f\left(\frac{D_1}{D_2}\right)$$

The value of co-efficient  $k$  can be determined from the curve (refer Fig 12.14) showing

$$k = f\left(\frac{D_1}{D_2}\right)$$

(d) *Loss of head due to a 90° bend* is given by

$$H_L = k \cdot \frac{v^2}{2g} \quad \dots(12.11)$$

The co-efficient  $k$  depends on the ratio  $\phi \left( = \frac{D_m}{D} \right)$  and Reynolds' Number  $R_e$ . The relationship is graphically represented in Fig 12.15.

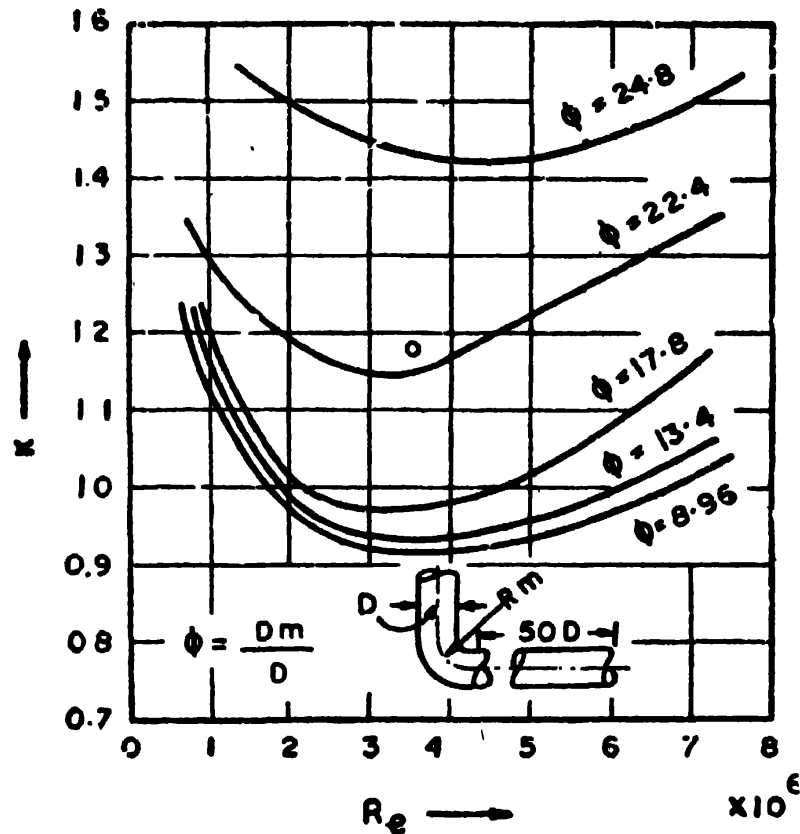


Fig 12.15 Loss of Head due to 90° Bend

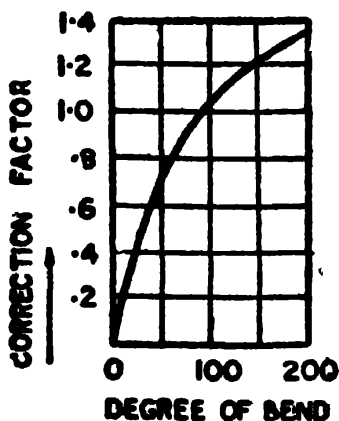


Fig 12.16 Degree of Bend

For bends at angles other than 90°, the same expression may be used with a correction factor which can be obtained from the curve shown in Fig 12.16.

(e) *Loss of head in pipe fittings* can be expressed as a fraction of the total kinetic head, thus

$$H_L = k \cdot \frac{v^3}{2g} \quad \dots(12.11a)$$

The value of the factor  $k$  is obtained from the experimental results.

Table 12.3 shows frictional head loss of different kinds of pipe fittings given in terms of length of straight pipe having the same diameter as that of the pipe fitting.

TABLE 12.3

**Frictional Head Loss of Valves and Pipe-Fittings**  
Length of Straight Pipe giving Equivalent Resistance to Flow

| Size of Pipe in cm | 90° Standard Elbow | 90° Medium Radius Elbow | 90° Long Radius Elbow | 45° Elbow | Tee | Return Bend | Gate Valve Open | Globe Valve Open | Angle Valve Open | Swing Check |
|--------------------|--------------------|-------------------------|-----------------------|-----------|-----|-------------|-----------------|------------------|------------------|-------------|
| 1.25               | 1.5                | 1.4                     | 1.1                   | 0.77      | 3.4 | 3.8         | 0.35            | 16               | 8.4              | 3.3         |
| 2.0                | 2.1                | 1.8                     | 1.4                   | 1.0       | 4.5 | 5.0         | 0.47            | 22               | 12               | 5.0         |
| 2.5                | 2.7                | 2.3                     | 1.7                   | 1.3       | 5.8 | 6.1         | 0.6             | 27               | 15               | 6.7         |
| 3.0                | 3.7                | 3.0                     | 2.4                   | 1.6       | 7.8 | 8.5         | 0.8             | 37               | 18               | 8.3         |
| 4                  | 4.3                | 3.6                     | 2.8                   | 2.0       | 9.0 | 10          | 0.95            | 44               | 22               | 10          |
| 5                  | 5.5                | 4.6                     | 3.5                   | 2.5       | 11  | 13          | 1.2             | 57               | 28               | 13          |
| 6                  | 6.5                | 5.4                     | 4.2                   | 3.0       | 14  | 15          | 1.4             | 66               | 33               | 17          |
| 7                  | 8.1                | 6.8                     | 5.1                   | 3.8       | 17  | 18          | 1.7             | 85               | 42               | 20          |
| 8.5                | 9.5                | 8.0                     | 6.0                   | 4.4       | 19  | 21          | 2.0             | 99               | 50               | 23          |
| 10                 | 11                 | 9.1                     | 7.0                   | 5.0       | 22  | 24          | 2.3             | 110              | 58               | 27          |
| 11.25              | 12                 | 10                      | 7.9                   | 5.6       | 24  | 27          | 2.6             | 130              | 61               | 30          |
| 12.5               | 14                 | 12                      | 8.9                   | 6.1       | 27  | 31          | 2.9             | 140              | 70               | 33          |
| 15                 | 16                 | 14                      | 11                    | 7.7       | 33  | 37          | 3.5             | 160              | 83               | 40          |
| 20                 | 21                 | 18                      | 14                    | 10        | 43  | 49          | 4.5             | 220              | 110              | 53          |
| 25                 | 26                 | 22                      | 17                    | 13        | 56  | 61          | 5.7             | 290              | 140              | 67          |
| 30                 | 32                 | 26                      | 20                    | 15        | 66  | 73          | 6.7             | 340              | 170              | 80          |
| 35                 | 36                 | 31                      | 23                    | 17        | 76  | 85          | 8               | 390              | 190              | 93          |
| 40                 | 42                 | 35                      | 27                    | 19        | 87  | 100         | 9               | 430              | 220              | 107         |
| 45                 | 46                 | 40                      | 30                    | 21        | 100 | 110         | 10.2            | 500              | 250              | 120         |
| 50                 | 52                 | 43                      | 34                    | 23        | 110 | 120         | 12              | 560              | 280              | 134         |
| 55                 | 58                 | 50                      | 37                    | 25        | 130 | 140         | 13              | 610              | 310              | 147         |
| 60                 | 63                 | 53                      | 40                    | 28        | 140 | 150         | 14              | 680              | 340              | 160         |
| 75                 | 79                 | 68                      | 50                    | 35        | 165 | 190         | 17              | 860              | 420              | 200         |
| 90                 | 94                 | 79                      | 60                    | 43        | 200 | 220         | 20              | 1000             | 500              | 240         |
| 105                | 120                | 95                      | 72                    | 50        | 240 | 260         | 23              | 1200             | 600              | 280         |
| 120                | 135                | 110                     | 82                    | 58        | 275 | 300         | 26              | 1400             | 680              | 320         |

(From "Engineering Data on Flow of Fluid in Pipes"—Crane Co.)  
Converted to Metric Units

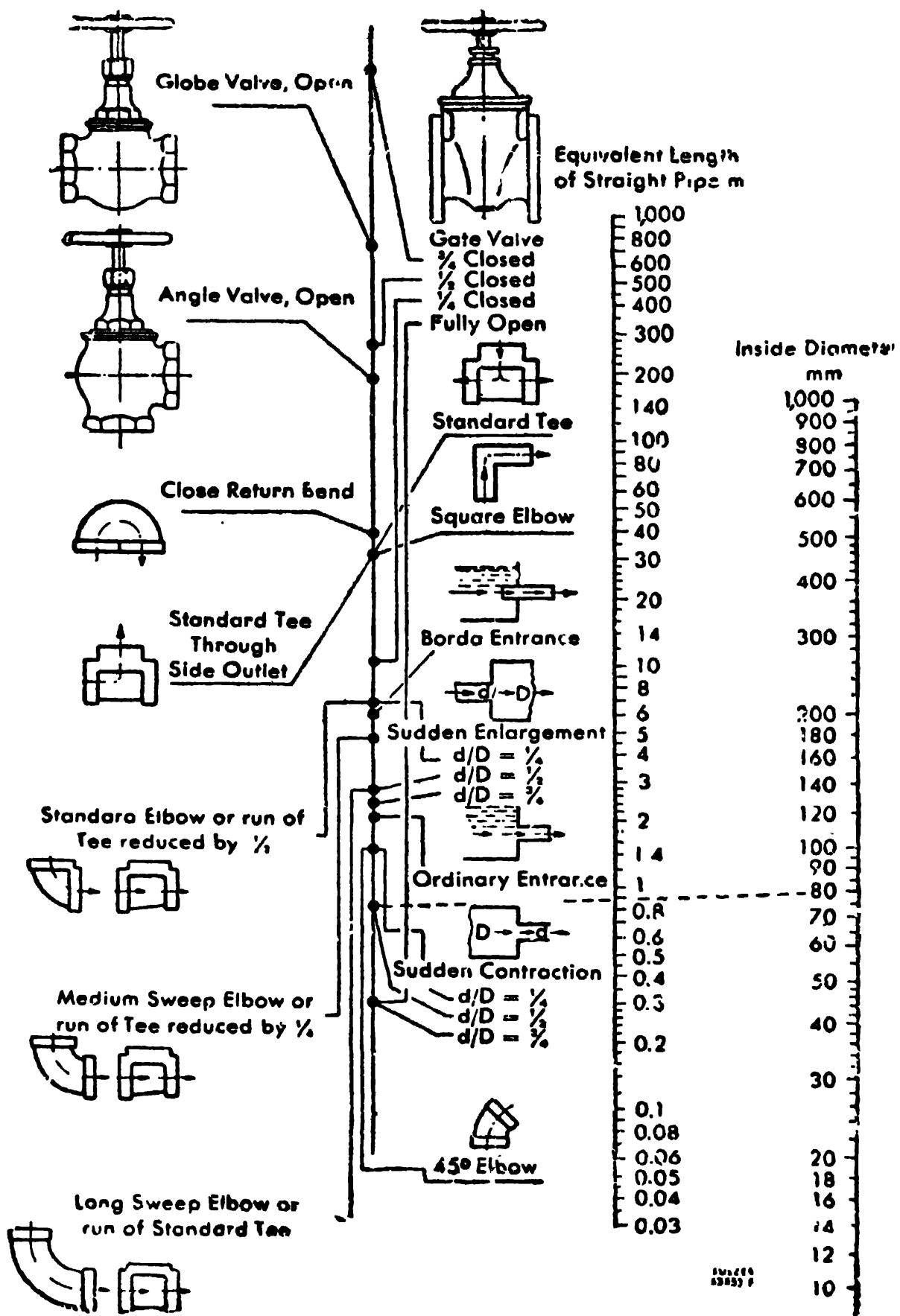


Fig 12.17 Approximation of Head Losses due to Friction in Pipe Fittings—From Crane Co., USA

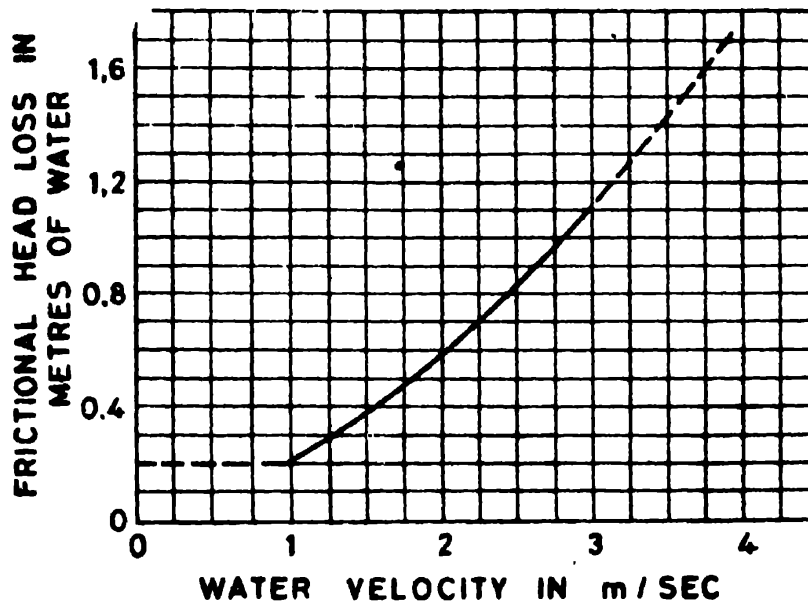


Fig 12.18 Frictional Head Loss in Strainer with Foot Valve vs. Water Velocity (upto 300 m in diameter ; Sulzer Brothers Ltd. Switzerland)

**Problem 12.3** A centrifugal pump is installed to supply water from a reservoir to a tank which is at a vertical height of 12 m above it. The diameter of the suction pipe is 300 mm and it is 6 m long. At the end of the suction pipe which extends under the water level in the reservoir, are fitted a foot valve and strainer. There is one 90° bend on the side of suction. The discharge pipe is 250 mm diameter and 135 m long. It is fitted with a gate valve and two medium radius 90° elbows. Find the HP of the pump to discharge 85 lit/sec of water considering all losses of head in pipe fittings. The overall efficiency of pump is 70%. Find also the size of driving motor.

Assume—

Viscosity of water  $\mu = 117 \times 10^{-6} \text{ kg sec/m}^2$

Specific weight of water  $\gamma = 1,000 \text{ kg/m}^3$

$$\text{Loss of head due to friction in pipes } H_f = \frac{4 f L}{d} \cdot \frac{v^2}{2g}$$

$$4 f = 0.0032 + \frac{0.21}{R_s^{0.237}}$$

where  $R_s$  is the Reynolds' number.

**Solution**

$$\text{Lift} = 12 \text{ m}$$

$$Q = 85 \text{ lit/sec} = 0.085 \text{ m}^3/\text{sec}$$

$$d = 300 \text{ mm}$$

$$\eta_{\text{overall}} = 70\% \quad L_s = 6 \text{ m}$$

$$\text{Density } \rho = \frac{\gamma}{g} = \frac{1,000}{9.81}$$

$$\mu = 117 \times 10^{-6} \text{ kg sec m}^{-2}$$

$$d_d = 250 \text{ mm} \quad L_d = 135 \text{ mm} = 102 \text{ kg sec}^2 \text{m}^{-4}$$

$$\text{Kinematic viscosity } \nu = \frac{\mu}{\rho} = \frac{117 \times 10^{-6}}{102} = 1.146 \times 10^{-6} \text{ m}^2 \text{sec}^{-1}$$

$$\text{Discharge} \quad Q = \frac{\pi}{4} d_s^2 \cdot v_s = \frac{\pi}{4} d_d^2 \cdot v_d$$

Velocity of water in suction pipe

$$v_s = \frac{Q}{\frac{\pi}{4} d_s^2} = \frac{0.085}{0.785 \times 0.3^2} = 1.2 \text{ m/sec}$$

Velocity of water in delivery pipe

$$v_d = \frac{Q}{\frac{\pi}{4} d_d^2} = \frac{0.085}{0.785 \times 0.25^2} = 1.735 \text{ m/sec}$$

$$R_s \text{ for suction pipe} = \frac{v_s \cdot d_s}{\nu} = \frac{1.2 \times 0.3}{1.146 \times 10^{-6}} = 314,000$$

$$R_d \text{ for delivery pipe} = \frac{v_d \cdot d_d}{\nu} = \frac{1.735 \times 0.25}{1.146 \times 10^{-6}} = 348,000$$

$$\text{Frictional factor } 4f = 0.0032 + \frac{0.221}{R_s^{0.237}}$$

$$4f \text{ for suction pipe} = 0.0032 + \frac{0.221}{314,000^{0.237}} = 0.0143$$

$$4f \text{ for discharge pipe} = 0.0032 + \frac{0.221}{348,000^{0.237}} = 0.0138$$

$$H_f = \frac{4f_s \cdot L_s}{d_s} \cdot \frac{v_s^2}{2g} = \frac{0.0143 \times 6}{0.3} \times \frac{1.2^2}{19.62} = 0.321 \text{ m of water}$$

$$H_{fd} = \frac{4f_d \cdot L_d}{d_d} \cdot \frac{v_d^2}{2g} = \frac{0.0138 \times 135}{0.25} \times \frac{1.735^2}{19.62} = 1.14 \text{ m of water}$$

Loss of head in the pipe fitting is obtained from the curves given in Figs 12.13 to 12.18 and in Table 12.3.

- (a) Foot valve and the strainer on suction side = 0.28 m,
- (b) 90° bend on the suction side = 0.035 m,
- (c) Gate valve on the delivery side = 0.268 m,
- (d) Two 90° medium radius elbows = 0.073 m,

Total loss = 0.656 m of water

$$\begin{aligned} \text{Total losses } H_L &= H_{L_s} + H_{L_d} + \Sigma H_L + \frac{v_d^2}{2g} \\ &= 0.021 + 1.14 + 0.656 + \frac{1.735^2}{19.62} \\ &= 0.021 + 1.14 + 0.656 + 0.153 \\ &= 1.97 \text{ m of water} \end{aligned}$$

$$\begin{aligned} \text{Total head } H_{mano} &= H_s + H_d + H_L \\ &= H_{st} + H_L = 12 + 1.97 \\ &= 13.97 \text{ m of water} \end{aligned}$$

$$\text{HP of pump} = \frac{\gamma \cdot Q \cdot H_m}{75 \cdot \eta_{\text{overall}}} = \frac{1,000 \times 0.085 \times 13.97}{75 \times 0.7} = 22.6 \text{ HP} \quad \text{Answer}$$

$$\begin{aligned} \text{Capacity of driving motor} &= 22.6 + 10\% \text{ of } 22.6 \\ &= 25 \text{ HP} \quad \text{Answer} \end{aligned}$$

**Problem 12.4** A centrifugal pump draws water at ordinary temperature through a 15 m long 150 mm diameter G.I. suction pipe which is fitted with one 90° bend. The maximum capacity of the pump is 75 lit/sec. Determine the static suction head. Assume the maximum vacuum as 5.5 m of water. Loss in the bend may be taken to be 8 cm of water.  $f = 0.006$ .

**Solution**

$$L_s = 15 \text{ m} \quad H_{vac} = 5.5 \text{ m} \quad d_s = 150 \text{ mm} \quad f = 0.006$$

$$Q = 75 \text{ lit/sec} = 0.075 \text{ m}^3/\text{sec}$$

$$v_s = \frac{Q}{a_s} = \frac{Q}{\frac{\pi}{4} d_s^2} = \frac{0.075}{0.785 \times 0.15^2} = 4.25 \text{ m/sec}$$

$$\begin{aligned} Hf_s &= \frac{4f \cdot L_s}{d_s} \cdot \frac{v_s^2}{2g} = \frac{4 \times 0.006 \times 15 \times 4.25^2}{0.15 \times 2 \times 9.81} \\ &= 2.21 \text{ m of water} \end{aligned}$$

$$\text{Loss of head due to } 90^\circ\text{-bend} = 0.08 \text{ m}$$

$$\therefore \text{Total loss of head } H_{f, \text{total}} = 2.21 + 0.08 = 2.29 \text{ m of water}$$

$$\text{Kinetic head loss} = \frac{v_s^2}{2g} = \frac{4.25^2}{19.62} = 0.92 \text{ m of water}$$

$$\text{Now } H_{vac} = H_s + H_{f_s} + \frac{v_s^2}{2g}$$

$$\text{or } 5.5 = H_s + 2.29 + 0.92$$

$$H_s = 5.5 - 3.21$$

$$= 2.29 \text{ m} \quad \text{Answer}$$

## THEORY OF CENTRIFUGAL PUMP

**12.22 Fundamental Equation of Centrifugal Pump**—Let points on the liquid's path at suction inlet, impeller inlet, impeller outlet and casing outlet be denoted by  $i$ , 1, 2 and  $d$  respectively (refer Fig 12.19). The equation of flow between any two consecutive points can be obtained by applying the Bernoulli's Theorem.

(a) Thus for flow from ( $i$ ) to (1) i.e., through the stationary suction pipe, since  $v_i$  and  $v_1$  represent absolute velocities of water.

$$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_i^2}{2g} + \frac{p_i}{\gamma} + z_i - H_{L(i-1)} \quad \dots \text{ (I)}$$

(b) From (1) to (2) i.e., through the

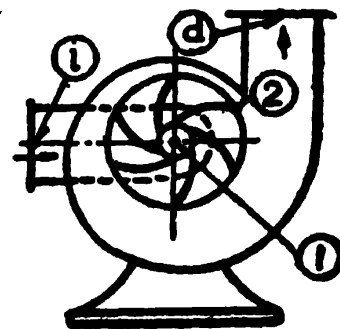


Fig 12.19 Liquid Flow Through a Centrifugal Pump



movable impeller, since  $w_1$  and  $w_2$  represent relative velocities of water, the equation of flow is

$$\frac{w_2^2}{2g} - \frac{u_2^2}{2g} + \frac{p_2}{\gamma} + z_2 = \frac{w_1^2}{2g} - \frac{u_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - H_{L(1-2)} \dots (II)$$

(refer Appendix 4)

(c) Flow from (2) to (d), i.e., through the stationary casing inside which the motion of water is absolute, is given by

$$\frac{v_d^2}{2g} + \frac{p_d}{\gamma} + z_d = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 - H_{L(2-d)} \dots (III)$$

Now, adding the equations (I), (II) and (III) and re-arranging

$$\begin{aligned} & \frac{v_2^2 - v_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} + \frac{u_2^2 - u_1^2}{2g} \\ &= \left\{ \left( \frac{v_d^2}{2g} + \frac{p_d}{\gamma} + z_d \right) - \left( \frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 \right) \right\} + (H_{L1-2} + H_{L2-3} + H_{L3-d}) \end{aligned}$$

The first term on the right hand side is, by definition, the gross manometric head of the pump :

$$H_{mano} = \left( \frac{v_d^2}{2g} + \frac{p_d}{\gamma} + z_d \right) - \left( \frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 \right)$$

And the second term stands for the total pump losses due to the fluid resistance, inside the pump only i.e.

$$\Delta H_{mano} = H_{L1-2} + H_{L2-3} + H_{L3-d}$$

$$\therefore \frac{v_2^2 - v_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} + \frac{u_1^2 - u_2^2}{2g} = H_{mano} + \Delta H_{mano}$$

This is known as the *fundamental equation of centrifugal pump*.

Considering the losses of head in the pump, its efficiency.

$$\eta_{mano} = \frac{H_{mano}}{H_{mano} + \Delta H_{mano}} \dots (12.12)$$

is known as *Manometric Efficiency*.

$$\text{or } H_{mano} + \Delta H_{mano} = \frac{H_{mano}}{\eta_{mano}}$$

whence,

$$\frac{v_2^2 - v_1^2}{2g} + \frac{w_1^2 - w_2^2}{2g} + \frac{u_2^2 - u_1^2}{2g} = \frac{H_{mano}}{\eta_{mano}} \dots (12.13)$$

The above equation can be simplified by substituting for  $u_1$  and  $w_2$  from the velocity triangles at inlet and outlet (refer Fig 12.20).

Thus :

$$w_1^2 = u_1^2 + v_1^2 - 2u_1v_1 \cos \alpha_1$$

$$\text{and } w_2^2 = u_2^2 + v_2^2 - 2u_2v_2 \cos \alpha_2$$

$$\therefore w_1^2 - w_2^2 = u_1^2 - u_2^2 + v_1^2 - v_2^2 - 2u_1v_1 \cos \alpha_1 + 2u_2v_2 \cos \alpha_2$$

Substituting this in the fundamental equation,

$$\begin{aligned} \frac{H_{mano}}{\eta_{mano}} &= \frac{v_2^2 - v_1^2}{2g} + \frac{u_2^2 - u_1^2}{2g} \\ &+ \frac{u_1^2 - u_2^2 + v_1^2 - v_2^2 - 2u_1v_1 \cos \alpha_1 + 2u_2v_2 \cos \alpha_2}{2g} \end{aligned}$$

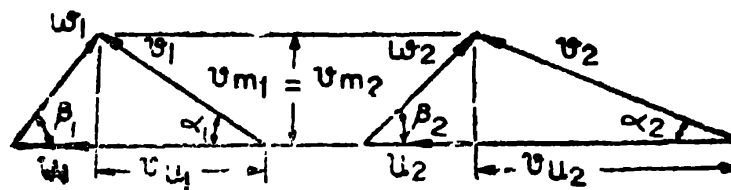
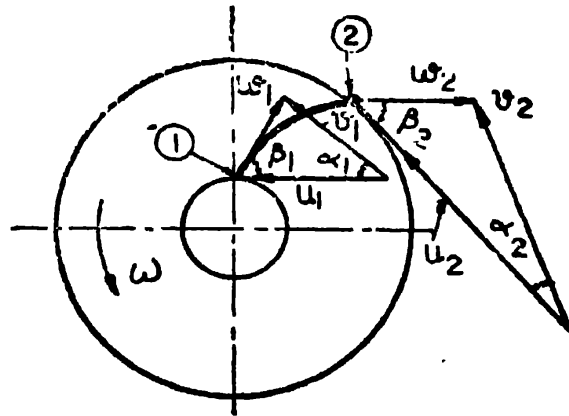


Fig 12.20 Velocity Triangles of a Centrifugal Pump

$$= \frac{2u_2 v_2 \cos \alpha_2 - 2u_1 v_1 \cos \alpha_1}{2g}$$

$$\text{or } \frac{H_{mano}}{\eta_{mano}} = \frac{u_2 v_2 \cos \alpha_2 - u_1 v_1 \cos \alpha_1}{g} \quad \dots (12.13a)$$

Generally  $\alpha_1 = 90^\circ$  and  $\cos \alpha_1 = 0$ , then, neglecting prerotation\*,

$$\text{or } \frac{H_{mano}}{\eta_{mano}} = \frac{u_2 v_2 \cos \alpha_2}{g} = \frac{u_2 \cdot v_{u2}}{g} \quad \dots (12.14)$$

$$\text{or } H_{mano} = \eta_{mano} \times \frac{\text{Peripheral speed at outlet} \times \text{velocity of whirl at outlet}}{g}$$

This is the form in which the fundamental equation is used in practice.

### 12.23 Work Done and Manometric Efficiency —

Work done/sec by the pump impeller is given by Eqn 1.34 (a) i.e.

$$P = \frac{\gamma \cdot Q}{g} (u_1 \cdot v_{u1} - u_2 \cdot v_{u2})$$

The suffices (1) and (2) used in this equation will hold true if point 1 denotes the pressure side and point 2 denotes the suction side of the pump. However if point 1 denotes inlet and point 2 the outlet of the pump impeller then the above equation will be written as

$$P = \frac{\gamma \cdot Q}{g} (u_2 \cdot v_{u2} - u_1 \cdot v_{u1}) \quad \dots (12.15)$$

$$\text{or Work done per kg of water} = \frac{u_2 \cdot v_{u2} - u_1 \cdot v_{u1}}{g}$$

\*The water approaching the impeller eye should acquire prerotation in the direction of impeller rotation. Prerotation in impellers corresponds to running in the direction of the train's motion before boarding a moving train.

Since  $\alpha_1 = 90^\circ$ , i.e. radial entrance,  
 $\therefore \cos \alpha_1 = 0$ .

$$\therefore \text{Work done per kg of water} = \frac{u_2 \cdot v_{u_2}}{g} \quad \dots (12.16)$$

(refer also Eqn 1.37)

This is the energy supplied to the fluid by the impeller per kg per sec. But the energy supplied to the fluid by the impeller is equal to the head generated, provided there is no loss inside the pump.

i.e. Head generated by the pump = Difference between the total energy of fluid at inlet and outlet of the pump.  
 $= \text{Manometric head } (H_{\text{mano}})$

Thus 
$$\frac{u_2 \cdot v_{u_2}}{g} = H_{\text{mano}} \quad \dots \text{if there is no internal loss of the pump})$$

In practice there are *always* some head losses inside the pump as described in Art 12.22.

$$\therefore \frac{u_2 \cdot v_{u_2}}{g} = H_{\text{mano}} + \text{internal losses in the pump}$$

$$= H_{\text{mano}} + \Delta H_{\text{mano}}$$

$$\text{Manometric Efficiency} = \frac{H_{\text{mano}}}{H_{\text{mano}} + \Delta H_{\text{mano}}} = \frac{H_{\text{mano}}}{\frac{u_2 \cdot v_{u_2}}{g}}$$

$$\text{i.e. Manometric Efficiency} = \frac{\text{manometric head}}{\text{energy given to the fluid/kg/sec}}$$

or 
$$\eta_{\text{mano}} = \frac{H_{\text{mano}}}{\frac{u_2 \cdot v_{u_2}}{g}}$$

but 
$$H_{\text{mano}} = H_{\text{static}} + H_f + \frac{v_d^2}{2g}$$

(Considering also the exit velocity head, which is generally neglected).

$$\eta_{\text{mano}} = \frac{H_{\text{static}} + H_f + \frac{v_d^2}{2g}}{\frac{u_2 \cdot v_{u_2}}{g}} \quad \dots (12.17)$$

**12.24 Pressure Rise in Pump Impeller and Manometric Head**—Applying Bernoulli's Theorem between inlet and outlet edges of impeller (refer Fig 12.20).

Energy at inlet = energy at outlet—useful work done by impeller

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} - \frac{u_2 \cdot v_{u_2}}{g}$$

$\therefore$  Pressure rise between outlet and inlet edges of impeller

$$\frac{p_2 - p_1}{\gamma} = \frac{v_1^2}{2g} - \frac{v_2^2}{2g} + \frac{u_2 \cdot v_{u_2}}{g}$$

## CENTRIFUGAL PUMPS

Now  $v_{u_2} = u_2 - v_{m_2} \cot \beta_2$   
(refer outlet velocity triangle Fig 12.20)

and 
$$\begin{aligned} v_2^2 &= (v_{m_2})^2 + (v_{u_2})^2 \\ &= (v_{m_2})^2 + (u_2 - v_{m_2} \cot \beta_2)^2 \\ &= v_{m_2}^2 (1 + \cot^2 \beta_2) + u_2^2 - 2u_2 \cdot v_{m_2} \cot \beta_2 \end{aligned}$$

$$\therefore \frac{p_2 - p_1}{\gamma} = \frac{v_1^2}{2g} - \frac{1}{2g} \left\{ v_{m_2}^2 (1 + \cot^2 \beta_2) + u_2^2 - 2u_2 \cdot v_{m_2} \cot \beta_2 \right\} + \frac{u_2}{g} (u_2 - v_{m_2} \cot \beta_2)$$

but  $1 + \cot^2 \beta_2 = \operatorname{cosec}^2 \beta_2$

$$\therefore \frac{p_2 - p_1}{\gamma} = \frac{1}{2g} (v_1^2 - v_{m_2}^2 \operatorname{cosec}^2 \beta_2 - u_2^2 + 2u_2 \cdot v_{m_2} \cot \beta_2 + 2u_2^2 - 2u_2 \cdot v_{m_2} \cot \beta_2)$$

or Pressure rise  $= \frac{1}{2g} (v_1^2 + u_2^2 - v_{m_2}^2 \operatorname{cosec}^2 \beta_2) \quad \dots(12.18)$

Manometric head  $H_{mano} = \eta_{mano} \cdot \frac{u_2 \cdot v_{u_2}}{g}$  (refer Eqn 12.14)

$$= \eta_{mano} \cdot \frac{u_2 (u_2 - v_{m_2} \cot \beta_2)}{g} \quad \dots(12.19)$$

**Problem 12.5** A centrifugal fan has to deliver  $4.5 \text{ m}^3/\text{sec}$ , when running at 750 rpm. The diameter of the impeller at inlet is 53 cm and at outlet is 76 cm. It may be assumed that the air enters radially with a speed of 15 m/sec. The vanes are set backwards at outlet at  $70^\circ$  to the tangent, and the width at outlet is 10 cm. The volute casing gives at 30% recovery of the outlet velocity head. The losses in the impeller may be taken as equivalent to 25% of the outlet velocity head. Blade thickness effects may be neglected. Determine the manometric efficiency and the pressure at the discharge.

(Panjab University)

### Solution

$$\begin{aligned} Q &= 4.5 \text{ m}^3/\text{sec} & N &= 750 \text{ rpm} & D_1 &= 53 \text{ cm} \\ D_2 &= 76 \text{ cm} & v_1 &= 15 \text{ m/sec} & \beta_2 &= 70^\circ \\ B_2 &= 10 \text{ cm} \end{aligned}$$

Peripheral velocity at inlet  $u_1 = \frac{\pi D_1 N}{60}$

$$= \pi \times 0.53 \times \frac{750}{60} = 20.8 \text{ m/sec}$$

Peripheral velocity at outlet  $u_2 = \frac{\pi D_2 N}{60}$

$$= \pi \times 0.76 \times \frac{750}{60} = 29.9 \text{ m/sec}$$

Discharge at outlet  $Q = \text{velocity of flow at outlet} \times \text{area of flow at outlet}$ .

or  $Q = v_{m_2} \cdot \pi D_2 B_2$

$$\therefore v_{m_2} = \frac{Q}{\pi D_2 B_2} = \frac{4.5}{\pi \times 0.76 \times 0.1} = 18.9 \text{ m/sec}$$

From outlet velocity triangle (refer Fig 12.20)

$$\tan \beta_2 = \frac{v_{m_2}}{u_2 - v_{u_2}}$$

or  $\tan 70^\circ = \frac{18.9}{29.9 - v_{u_2}}$

$$29.9 - v_{u_2} = \frac{18.9}{2.747} = 6.89$$

$$\therefore v_{u_2} = 29.9 - 6.89 = 23 \text{ m/sec}$$

$$\begin{aligned} \therefore \text{Absolute velocity at exit } v_2 &= \sqrt{v_{m_2}^2 + v_{u_2}^2} \\ &= \sqrt{18.9^2 + 23^2} = \sqrt{359 + 530} \\ &= \sqrt{889} = 29.8 \text{ m/sec} \end{aligned}$$

Applying Bernoulli's theorem at inlet and outlet of impeller—

$$\frac{p_1}{\gamma} + \frac{v_1^2}{2g} = \frac{p_2}{\gamma} + \frac{v_2^2}{2g} - \frac{v_{u_2} \cdot u_2}{g}$$

(Assuming  $\alpha_1 = 90^\circ$ )

or  $v_{u_1} = 0$

or 
$$\begin{aligned} \frac{p_2}{\gamma} - \frac{p_1}{\gamma} &= \frac{v_1^2 - v_2^2}{2g} + \frac{v_{u_2} \cdot u_2}{g} \\ &= \frac{15^2 - 29.8^2}{19.62} + \frac{23 \times 29.9}{9.81} \\ &= \frac{225 - 880}{19.62} + 70 = -33.5 + 70 \\ &= +36.5 \text{ m of air} \end{aligned}$$

$$\text{Velocity head at outlet} = \frac{v_2^2}{2g} = \frac{29.8^2}{19.62} = 45.2 \text{ m of air}$$

$$\begin{aligned} \text{Head recovered in volute casing} &= 30\% \text{ of } \frac{v_2^2}{2g} \\ &= 0.3 \times 45.2 = 13.56 \text{ m of air} \end{aligned}$$

$$\begin{aligned} \text{Head lost in impeller} &= 25\% \text{ of } \frac{v_2^2}{2g} \\ &= 0.25 \times 45.2 = 11.3 \text{ m of air} \end{aligned}$$

$$\begin{aligned} \text{Net pressure rise} &= \frac{p_2 - p_1}{\gamma} + \text{head recovered in casing} - \text{head lost in impeller} \\ &= 36.5 + 13.56 - 11.3 = 38.76 \text{ m of air} \end{aligned}$$

$$\eta_{mano} = \frac{\text{Net pressure rise}}{\text{Head of delivered per kg of air}} = \frac{38.76}{\frac{v_{u_2} \cdot u_2}{g}}$$

$$= \frac{38.76}{70} = 0.554 \quad \text{Answer}$$

**12.25 Minimum Starting Speed of Centrifugal Pump**—When the pump is started, there will not be any flow of water until the pressure rise in the impeller is large enough to overcome the gross or manometric head.

Centrifugal head or pressure head caused by the centrifugal force on rotating water when the impeller is rotating, but there is *no flow* (refer Eqn 2.2)

$$= \frac{u_2^2 - u_1^2}{2g}$$

Flow will commence only if—

$$\frac{u_2^2 - u_1^2}{2g} \geq H_{mano}$$

or 
$$\frac{u_2^2 - u_1^2}{2g} \geq \eta_{mano} \frac{u_2 \cdot v_{u_2}}{g} \quad \dots(12.20)$$

Theoretically 
$$\frac{u_2^2 - u_1^2}{2g} \geq \frac{u_2 \cdot v_{u_2}}{g} \quad \dots(12.20a)$$

(Refer solved Problem 2.5).

## 12.26 Efficiencies of Centrifugal Pump—

(a) **Overall Efficiency**—As already discussed in Art 12.19, the overall efficiency of a pump is

$$\eta_{overall} = \frac{\text{Fluid or water horsepower output}}{\text{horsepower input to pump shaft}}$$

$$= \frac{\text{WHP}}{\text{SHP}} \quad \dots(12.21)$$

This is also known as *gross efficiency* or *actual efficiency*.

The shaft horsepower (SHP) of a centrifugal pump is required to supply the following powers, i.e.

$$P_{shaft} = P_{input \text{ to impeller}} + P_{leakage} + P_{mechanical \text{ loss}} \quad \dots(12.22)$$

where,  $P_{shaft} = \text{SHP} = (\text{BHP of driving unit}) - (\text{HP lost in coupling})$

$P_{input \text{ to impeller}} = \text{energy given to impeller per kg per sec}$

$$= \frac{u_2 \cdot v_{u_2}}{g} \text{ per kg of water}$$

$$= \text{WHP} + P_{hyd} \quad \dots(12.23)$$

$$\text{WHP} = \frac{\gamma \cdot Q \cdot H_{mano}}{75}$$

$$P_{hyd} = \frac{\gamma \cdot Q \cdot (\Delta H_{mano})}{75}$$

= Power required to overcome head losses due to

- (i) Circulatory or secondary flow
- (ii) Frictions of volute and impeller
- (iii) Turbulence.

$P_{leakage}$  = Power required to overcome leakage

This is the power required to pump through the impeller the additional amount of water which leaks and is not delivered. The leakage water is that which slips back from the pressure side to the suction side of impeller. It may also include the water which is used for balancing purposes. (refer Fig 12.27).

$P_{mach loss}$  = Power required to overcome all mechanical losses, which are—

- (i) Disc friction loss (refer also Art 7.29d)
- (ii) Bearings and glands losses.

(b) **Mechanical Efficiency** is the ratio of the power delivered by the impeller to the fluid, to the horsepower input to the pump shaft, i.e.

$$\eta_{mech} = \frac{\gamma \left( \frac{Q + \Delta Q}{75} \right) \frac{u_2 \cdot v_{u_2}}{g}}{SHP} \quad \dots(12.24)$$

$$= \frac{(SHP) - P_{mach loss}}{SHP} \quad \dots(12.24a)$$

(c) **Volumetric Efficiency**—

$$\eta_Q = \frac{Q}{Q + \Delta Q} \quad \dots(12.25)$$

where,

$Q$  = discharge delivered by the pump

$\Delta Q$  = amount of leakage

(d) **Manometric Efficiency**—As already discussed

$$\begin{aligned} \eta_{mano} &= \frac{\text{actual measured head or gross lift}}{\text{head imparted to fluid by impeller}} \\ &= \frac{H_{static} + H_f + \frac{v_d^2}{2g}}{\frac{u_2 \cdot v_{u_2}}{g}} \\ &= \frac{H_{mano}}{\frac{u_2 \cdot v_{u_2}}{g}} = \frac{\gamma \left( \frac{Q + \Delta Q}{75} \right) \cdot H_{mano}}{SHP - P_{mach loss}} \quad \dots(12.26) \end{aligned}$$

This is also known as *hydraulic efficiency*.

From the above Eqns 12.21 to 12.26

$$\eta_{overall} = \eta_{mech} \cdot \eta_Q \cdot \eta_{mano} \quad \dots(12.27)$$

**12.27 Ideal, Virtual and Manometric Heads of Centrifugal Pump**—The fundamental equation (refer Eqn 12.14) of a centrifugal pump is derived as—

$$\frac{H_{mano}}{\eta_{mano}} = \frac{u_2 \cdot v_{u_2}}{g}$$

where  $H_{mano}$  = the actual measured head by pressure gauges on suction and delivery sides

and  $\frac{u_2 \cdot v_{u_2}}{g}$  = the theoretical head considering infinite number of blades as *ideal or Euler head*

The ideal or Euler head is obtained if there is no hydraulic loss (head as well as volumetric losses), i.e., the flow is frictionless and non-turbulent, and the impeller has *infinite* number of blades.

This head is then denoted by  $H_{th\infty}$

$$\text{i.e., } H_{th\infty} = \frac{u_2 \cdot v_{u_2}}{g} \quad \dots(12.28)$$

**Influence of Number of Blades**—The ideal head (refer Eqn 12.14 and 12.28) is developed considering infinite number of blades in the impeller through which the water has to pass. In practice the impeller must have only a finite number of blades, in which case the velocity of whirl ( $v_{u_2}$ ) is reduced owing to the secondary or circulatory flow effects in the impeller. The new value of velocity of whirl is given by

$$v'_{u_2} = v_{u_2} - \Delta v_{u_2}$$

The head developed, taking  $v'_{u_2}$  into consideration, will be known as theoretical head considering *finite* number of blades or *virtual head*. It is denoted by  $H_{th}$ .

$$\text{Thus } H_{th} = \frac{u_2 \cdot v'_{u_2}}{g} = \frac{u_2(v_{u_2} - \Delta v_{u_2})}{g} \quad \dots(12.29)$$

The virtual head  $H_{th} = \epsilon H_{th\infty}$  where  $\epsilon$  is a Greek letter and known as *Correction Factor* for finite number of blades in an impeller.

$$\epsilon = \frac{H_{th}}{H_{th\infty}} = \frac{v_{u_2} - \Delta v_{u_2}}{v_{u_2}} = 1 - \frac{\Delta v_{u_2}}{v_{u_2}} \quad \dots(12.30)$$

$\epsilon$  is sometimes known as vane or blade efficiency and denoted by  $\eta_{blade}$ .

The following table showing the relation between  $\epsilon$  and  $z$  the number of blades, is due to Pfleiderer.

TABLE 12.4  
Correction Factor  $\epsilon$  vs  $z$

| $z$        | 1    | 2   | 4     | 6     | 10   | 20   | $\infty$ |
|------------|------|-----|-------|-------|------|------|----------|
| $\epsilon$ | 0.25 | 0.4 | 0.572 | 0.666 | 0.77 | 0.87 | 1.0      |

It should be seen that the reduction in head from  $H_{th\infty}$  to  $H_{th}$  does not represent a loss but a discrepancy, for secondary flow effects, which is not taken into account in the basic assumptions.

Manometric head ( $H_{mano}$ ) or the actual head developed by the pump is still smaller than virtual head  $H_{th}$  because there are friction losses in suction passages, interior of impeller and discharge volute. Over and



above there is head loss due to turbulence. Fig 12.21 shows the head curves drawn against discharge.

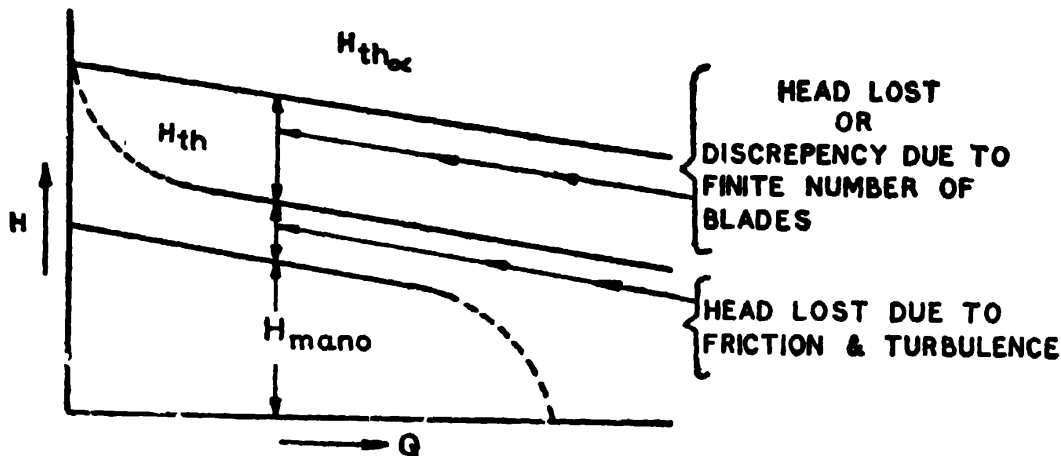


Fig 12.21 Ideal, Virtual and Manometric Heads vs Discharge

**12.28 Overall Head Coefficient or Speed Ratio**—From the fundamental equation 12.13a derived above,

$$\frac{1}{\eta_{mano}} = \frac{u_2 \cdot v_2 \cos \alpha_2 - u_1 \cdot v_1 \cos \alpha_1}{g H_{mano}}$$

$$= \frac{2(u_2 \cdot v_{u_2} - u_1 \cdot v_{u_1})}{2g H_{mano}}$$

Introducing non-dimensional velocity co-efficients\*

$$K_{u_2} = \frac{u_2}{\sqrt{2g H_{mano}}}, K_{v_2} = \frac{v_{u_2}}{\sqrt{2g H_{mano}}} \text{ etc.}$$

$$\frac{1}{\eta_{mano}} = \frac{2(K_{u_2} \cdot K_{v_{u_2}} - K_{u_1} \cdot K_{v_{u_1}})}{1}$$

$$\text{or } \eta_{mano} = \frac{1}{2(K_{u_2} \cdot K_{v_{u_2}} - K_{u_1} \cdot K_{v_{u_1}})} \quad \dots(12.31)$$

$$\text{If } \alpha_1 = 90^\circ \quad K_{v_{u_1}} = 0$$

$$\text{and } \eta_{mano} = \frac{1}{2K_{u_2} \cdot K_{v_{u_2}}}$$

Since  $\eta_{mano} < 1$ , the product  $K_{u_2} \cdot K_{v_{u_2}} > \frac{1}{2}$

If  $K_{u_2} \approx 1$  (See Table 12.5), then  $K_{v_{u_2}} \approx 0.5$ .

TABLE 12.5  
Practical Data for Speed Ratio  $K_{u_2}$

|                           | Small Pumps      | Large Pumps       |
|---------------------------|------------------|-------------------|
| Pumps without guide vanes | $K_{u_2} = 1.06$ | $K_{u_2} = 0.95$  |
| Pump with guide vanes     | $K_{u_2} = 1.01$ | $K_{u_2} = 0.963$ |

\*The factor is  $K_{u_2}$  known as the "speed ratio" as in the case of turbines.

**Problem 12.6** A diffusion type centrifugal pump has a suction lift of 1.5 m and the delivery tank is 13.5 m above the pump. The velocity of water in the delivery pipe is 1.5 m/sec. The radial velocity of flow through the wheel is 3 m/sec and the tangent to the vane at exit from the wheel makes an angle of  $120^\circ$  with the direction of motion. Assuming that the water enters radially and neglecting friction and other losses, find

- the velocity of wheel at exit ;
  - the pressure head at exit from the wheel ;
  - the velocity head at exit from the wheel
- and (d) the desirable direction of the fixed guide vanes. (AMIE)

**Solution**

$$H_s = 1.5 \text{ m}$$

$$\beta_2 = 180^\circ - 120^\circ = 60^\circ$$

$$H_d = 13.5 \text{ m}$$

$$\alpha_1 = 90^\circ (\because \text{water enters radially})$$

$$v_d = 1.5 \text{ m/sec}$$

$$v_{m_2} = v_{m_1} = 3 \text{ m/sec}$$

$$H_{mano} = H_{static} = H_s + H_d = 1.5 + 13.5 = 15 \text{ m}$$

(neglecting all friction losses)

$$\text{or } H = H_{mano} = 15 \text{ m}$$

$$\tan \beta_2 = \frac{v_{m_2}}{u_2 - v_{u_2}} \text{ (refer Fig 12.20)}$$

$$\text{or } \tan 60^\circ = \frac{3}{u_2 - v_{u_2}}$$

$$(u_2 - v_{u_2}) = \frac{3}{1.732} = 1.732 \text{ m/sec}$$

$$\text{Work done per kg of water} = \frac{v_{u_2} \cdot u_2}{g} = H$$

Assume the efficiency to be 100% i.e., neglecting all losses)

$$\text{or } v_{u_2} \cdot u_2 = 15 \times 9.81 = 147.2$$

Substituting for  $v_{u_2}$

$$(u_2 - 1.732) u_2 = 147.2 \quad \text{or } u_2^2 - 1.732 u_2 - 147.2 = 0$$

$$\text{whence } u_2 = 13.066 \text{ m/sec}$$

$$v_{u_2} = \frac{147.2}{u_2} = \frac{147.2}{13.066} = 11.3 \text{ m/sec}$$

$$\text{and } v_2 = \sqrt{v_{m_2}^2 + v_{u_2}^2} = \sqrt{3^2 + 11.3^2}$$

$$= 11.7 \text{ m/sec} \quad \text{Answer}$$

$$\therefore \text{Velocity head at exit} = \frac{v_2^2}{2g} = \frac{11.7^2}{2 \times 9.81} = 6.97 \text{ m} \quad \text{Answer}$$

$$\text{Now } \frac{v_2^2}{2g} + \frac{p_2}{\gamma} = H_d$$

$$\therefore \text{Pressure head at the exit of the wheel} = \frac{p_2}{\gamma} = H_d - \frac{v_2^2}{2g}$$

$$= 13.5 - 6.97$$

$$= 6.53 \text{ m} \quad \text{Answer}$$

Desirable direction of fixed guide vane is  $\alpha_2$

where  $v_2 \cos \alpha_2 = v_{u_2}$

$$\therefore \cos \alpha_2 = \frac{v_{u_2}}{v_2} = \frac{11.3}{11.7} = 0.965$$

$$\text{or } \alpha_2 = 15^\circ - 12' \quad \text{Answer}$$

**Problem 12.7** A centrifugal pump lifts water under a static head of 36 m of water of which 4 m is suction lift. Suction and delivery pipes are both 150 mm in diameter. The head loss in suction pipe is 1.8 m and in delivery pipe 7 m. The impeller is 380 mm in diameter and 25 mm wide at mouth and revolves at 1,200 rpm. Its exit blade angle is  $35^\circ$ . If the manometric efficiency of the pump is 82% determine the discharge and the pressure at the suction and delivery branches of the pump. (AMIE)

**Solution**

$$H_{\text{static}} = 36 \text{ m} \quad D_2 = 380 \text{ mm} \quad H_s = 4 \text{ m} \quad b_2 = 25 \text{ mm}$$

$$H_d = 36 - 4 = 32 \text{ m} \quad N = 1,200 \text{ rpm} \quad H_{L_s} = 1.8 \text{ m}$$

$$H_{L_d} = 7 \text{ m} \quad d_s = d_d = 150 \text{ mm} \quad \eta_{\text{mano}} = 82\%$$

$$\text{Total head to be supplied by the pump } H_{\text{mano}} = H_{\text{stat}} + \Sigma H_L \\ = 36 + 1.8 + 7 = 44.8 \text{ m of water}$$

$$\text{Peripheral velocity of wheel at outlet } u_2 = \frac{\pi \cdot D_2 \cdot N}{60}$$

$$u_2 = \frac{\pi \times 0.38 \times 1,200}{60} = 23.9 \text{ m/sec}$$

$$\text{Assume flow at the inlet to be radial, } \therefore \alpha_1 = 90^\circ$$

$$\text{Work done per kg of water} = \frac{v_{u_2} \cdot u_2}{g}$$

$$\text{Manometric efficiency} = \frac{H_{\text{mano}}}{\frac{v_{u_2} \cdot u_2}{g}}$$

$$\text{or } 0.82 = \frac{44.8}{\frac{v_{u_2} \times 23.9}{9.81}}$$

$$\text{or } v_{u_2} = \frac{44.8 \times 9.81}{0.82 \times 23.9} = 22.4 \text{ m/sec}$$

$$\text{Now } \tan \beta = \frac{v_{m_2}}{u_2 - v_{u_2}} \quad (\text{refer Fig 12.20})$$

$$\tan 35^\circ = \frac{v_{m_2}}{23.9 - 22.4}$$

$$\text{or } v_{m_2} = 0.7002 \times 1.5 = 1.05 \text{ m/sec}$$

$$\text{and } Q = (\pi \cdot D_2 \cdot b_2) \cdot v_{m_2} \\ = \pi \times 0.380 \times 0.0205 \times 1.05 \\ = 0.0314 \text{ m}^3/\text{sec} \quad \text{Answer}$$

$$\begin{aligned}\text{Velocity in suction or delivery pipe } v_d = v_s &= \frac{Q}{a_s} = \frac{0.0314}{\frac{\pi}{4} \times 0.15^2} \\ &= 1.78 \text{ m/sec}\end{aligned}$$

$$\text{Velocity head} = \frac{1.78^2}{19.62} = 0.161 \text{ m of water}$$

$$\begin{aligned}\text{Total effective pressure on the delivery side} &= H_d + H_{L_d} + \frac{v_d^2}{2g} \\ &= 32 + 7 + 0.161 = 39.161 \text{ m of water}\end{aligned}$$

$$\text{or } 39.161 \times \frac{1,000}{100 \times 100} = 3.9161 \text{ kg/cm}^2 \quad \text{Answer}$$

$$\begin{aligned}\text{Pressure on the suction side} &= H_s + H_{L_s} + \frac{v_s^2}{2g} = 4 + 1.8 + 0.161 \\ &= 5.961 \text{ m of water vacuum}\end{aligned}$$

$$\text{or } 10 - 5.961 = 4.039 \text{ m of water absolute}$$

$$\text{or } 4.039 \times \frac{1,000}{100 \times 100} = 0.4039 \text{ kg/cm}^2 \text{ absolute} \quad \text{Answer}$$

**Problem 12.8** A radial, single stage, double suction, centrifugal pump is manufactured for the following data :

$$\begin{array}{lll} Q = 75 \text{ lit/sec} & D_1 = 100 \text{ mm} & D_2 = 290 \text{ mm} \\ H = 30 \text{ m} & N = 1,750 \text{ rpm} & b_1 = 25 \text{ mm per side} \\ \eta_{\text{overall}} = 55\% & b_2 = 23 \text{ mm in total} & \alpha_1 = 90^\circ \\ \text{Leakage losses} = 2.25 \text{ lit/sec} & \text{Mechanical losses} = 1.41 \text{ HP} & \\ \text{Contraction factor due to vane thickness} = 0.87 & \beta_2 = 27^\circ & \end{array}$$

Determine —

- the inlet vane angle  $\beta_1$  ;
  - the angle at which the water leaves the wheel  $\alpha_2$  ;
  - the speed ratio  $K_{u_2}$  ;
  - the absolute velocity of water leaving impeller  $v_2$  ;
  - the manometric efficiency ;
- and (f) the volumetric and mechanical efficiencies.

**Solution**

Total quantity of water handled by the pump

$$\begin{aligned}Q_{\text{total}} &= Q_{\text{delivered}} + Q_{\text{leaked}} = 75 + 2.25 \\ &= 77.25 \text{ lit/sec}\end{aligned}$$

$$Q \text{ per side} = \frac{77.25}{2} = 38.625 \text{ lit/sec}$$

(a) Peripheral speed at inlet

$$u_1 = \frac{\pi \cdot D_1 \cdot N}{60} = \frac{\pi \times 0.1 \times 1,750}{60} = 9.15 \text{ m/sec}$$

$$\begin{aligned}\text{Area of flow at inlet} &= \pi \cdot D_1 \cdot b_1 \times \text{contraction factor} \\ &= \pi \times 0.1 \times 0.025 \times 0.87 = 0.00683 \text{ m}^2\end{aligned}$$

$\therefore$  Velocity of flow at inlet

$$v_{m_1} = \frac{Q}{\text{area of flow}} = \frac{38.625 \times 10^{-3}}{0.00683} = 5.66 \text{ m/sec}$$

Now  $\alpha_1 = 90^\circ$

$\therefore$  From inlet velocity triangle (refer Fig 12.20)

$$\tan \beta_1 = \frac{v_{m_1}}{u_1} = \frac{5.66}{9.15} = 0.62$$

or  $\beta_1 = \tan^{-1} 0.62 = 31^\circ - 48' \quad \text{Answer}$

(b) Area of flow at outlet

$$= \pi \cdot D_2 \cdot b_2 \times \text{contraction factor}$$

where  $b_2 = 23 \times \frac{1}{2} = 11.5 \text{ mm for one side}$

$$\therefore \text{Area} = \pi \times 0.29 \times 0.0115 \times 0.87 = 0.0091 \text{ m}^2$$

$\therefore$  Velocity of flow at outlet

$$v_{m_2} = \frac{Q}{\text{area of flow}} = \frac{38.625 \times 10^{-3}}{0.0091} = 4.25 \text{ m/sec}$$

Peripheral speed at outlet

$$u_2 = \frac{\pi \cdot D_2 \cdot N}{60} = \frac{\pi \times 0.29 \times 1,750}{60} = 26.55 \text{ m/sec}$$

Now  $\beta_2 = 27^\circ$

$\therefore$  From outlet velocity triangle (refer Fig 12.20)

$$\tan \beta_2 = \frac{v_{m_2}}{u_2 - v_{u_2}}$$

$$\text{or} \quad \tan 27^\circ = \frac{4.25}{26.55 - v_{u_2}}$$

$$\text{or} \quad 26.55 - v_{u_2} = 8.34$$

$$\text{or} \quad v_{u_2} = 18.21 \text{ m/sec}$$

$$\text{Further,} \quad \tan \alpha_2 = \frac{v_{m_2}}{v_{u_2}} = \frac{4.25}{18.21}$$

$$\therefore \alpha_2 = 13^\circ - 8' \quad \text{Answer}$$

$$(c) \text{ Speed ratio } K_{u_2} = \frac{u_2}{\sqrt{2g H_{mano}}} = \frac{26.55}{4.43 \times \sqrt{30}} = 1.0905$$

*Answer*

(d) Absolute velocity of water leaving the impeller

$$v_2 = \frac{v_{u_2}}{\cos \alpha_2} = \frac{18.21}{0.971} = 18.75 \text{ m/sec} \quad \text{Answer}$$

(e) Manometric efficiency—

$$\frac{H_{mano}}{\eta_{mano}} = \frac{u_2 \cdot v_{u_2}}{g} \quad (\text{refer Eqn 12.14})$$

$$\text{or} \quad \eta_{mano} = \frac{g \cdot H_{mano}}{u_2 \cdot v_{u_2}} = \frac{9.81 \times 30}{26.55 \times 18.21} = 0.61$$

or 61% Answer

$$(f) \text{ Volumetric efficiency } \eta_Q = \frac{Q}{Q_{total}} = \frac{75}{77.25} = 0.971$$

or 97.1 Answer

$$\text{Water HP} = \frac{\gamma \cdot Q \cdot H_{mano}}{75} = \frac{1,000 \times 75 \times 10^{-3} \times 30}{2 \times 75} = 15 \text{ HP}$$

$$\text{Shaft horsepower SHP} = \frac{\text{Water HP}}{\eta_{overall}} = \frac{15}{0.55} = 27.27 \text{ HP}$$

$$\eta_{mech} = \frac{SHP - P_{mech loss}}{SHP} = \frac{27.27 - 1.41}{27.27} = \frac{25.86}{27.27} = 0.949 \quad \text{or} \quad 94.9\% \quad \text{Answer}$$

**12.29 Note on Fundamental Equation**—A little consideration of the various factors involved the fundamental equation would suggest that if for a particular pump the outlet peripheral speed  $u_2$  and the outlet relative velocity of water  $w_2$  are kept constant, the velocity of whirl  $v_{u_2}$  would vary with angle  $\alpha_2$ . The smaller the angle  $\alpha_2$  the larger would be  $v_{u_2}$ , thus increasing the total head  $H_{mano}$ .

However, the best practical value of  $\alpha_2$  is about  $20^\circ$ . Further, if  $w_2$  is higher,  $v_2$  will also be correspondingly higher for a constant value of  $u_2$  and the head  $H_{mano}$  will be more. But a higher  $w_2$  also implies higher frictional loss and therefore it is not available to raise the head in this manner.

**Practical Data :**

$$\alpha_1 = 90^\circ \quad \beta_1 = 15^\circ \text{ to } 30^\circ \quad \beta_2 = 20^\circ \text{ to } 40^\circ$$

$$\frac{\tan \alpha_2}{\tan \beta_2} = 0.6 \quad v_2 = 0.6 \text{ to } 0.8 \text{ times } u_2$$

**12.30 Breadth of Impeller**—Let  $Q$  be the quantity in  $\text{m}^3/\text{sec}$  water flowing through the impeller. Then, from the equation of continuity.

$$\begin{aligned} Q &= \pi D_1 \cdot B_1 \cdot v_{m_1} \\ &= \pi D_2 \cdot B_2 \cdot v_{m_2} \\ &= \pi \cdot D \cdot B \cdot v_m \end{aligned}$$

where  $D$  and  $B$  are the diameter and breadth of the impeller at points indicated by the suffices (refer Fig 12.22), and  $v_{m_1}$  and  $v_{m_2}$  are the velocities of flow at inlet and outlet respectively.

From the above equations, in which the thickness of the vanes has not been considered, approximate breadth of the impeller at inlet and outlet can be worked out as hereunder :

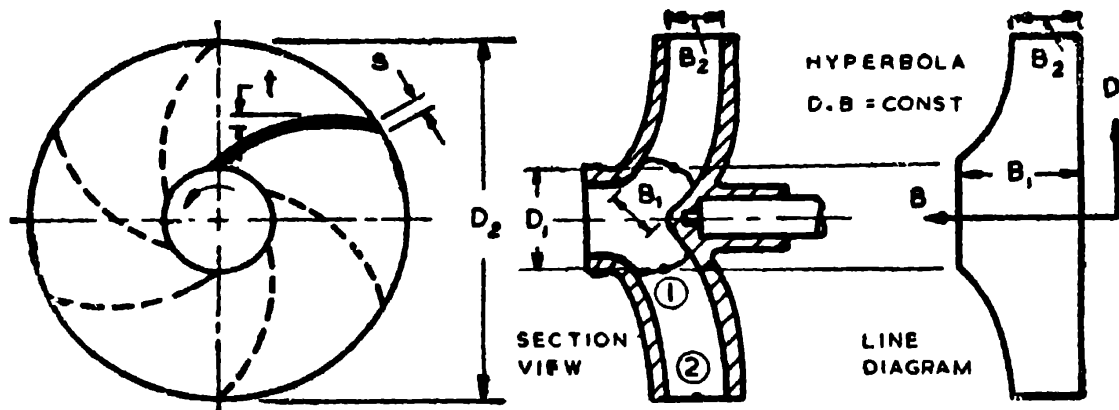


Fig 12.22 Breadth and Diameter of Impeller

$$B_1 = \frac{Q}{\pi D_1 \cdot v_{m1}} \quad \dots (12.33)$$

and  $B_2 = \frac{Q}{\pi D_2 \cdot v_{m2}} \quad \dots (12.34)$

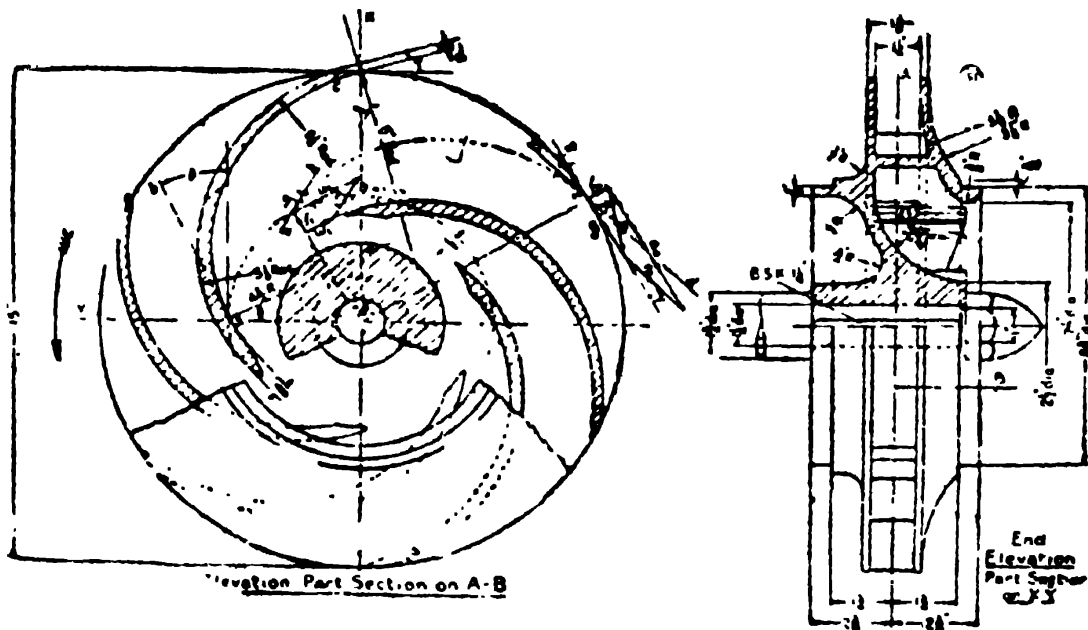


Fig 12.23 Section through an Impeller of a Centrifugal Pump

The velocities of flow at inlet and outlet are generally taken to be equal

$$\begin{aligned} \text{i.e.} \quad v_{m1} &= v_{m2} \\ \therefore D_1 \cdot B_1 &= D_2 \cdot B_2 \\ \text{or} \quad D \cdot B &= \text{constant} \quad \dots (12.35) \end{aligned}$$

A curve  $B$  vs  $D$  will therefore, be a hyperbola (refer Fig 12.22). This is, generally, theoretical curvature of impeller blades.

For a more accurate computation, vane thickness should be considered and if  $z$  be the number of blades.

$$\begin{aligned} Q &= (\pi D_1 - z \cdot s_1) \cdot B_1 \cdot v_{m1} \\ &= (\pi D_2 - z \cdot s_2) \cdot B_2 \cdot v_{m2} \\ &= (\pi D - z \cdot s) B \cdot v_m \quad \dots (12.3) \end{aligned}$$

Hence  $s$  is thickness of vane measured along the circle, and if  $t$  be the actual metal thickness,

$$s = \frac{t}{\sin \beta}$$

**12.31 Different Shapes of Blades**—The blade of an impeller may have one of the following different shapes :

|       | Shape                  | $\alpha_1$      | $\beta_1$         | $\alpha_2$        | $\beta_2$         | $\eta\%$     |
|-------|------------------------|-----------------|-------------------|-------------------|-------------------|--------------|
| (i)   | Blades bent backward   | $\frac{\pi}{2}$ | $< \frac{\pi}{2}$ | $< \frac{\pi}{2}$ | $< \frac{\pi}{2}$ | 85 to 90     |
| (ii)  | Straight blades        | $\frac{\pi}{2}$ | $< \frac{\pi}{2}$ | $< \frac{\pi}{2}$ | $< \frac{\pi}{2}$ | $\approx 80$ |
| (iii) | Blades ending radially | $\frac{\pi}{2}$ | $< \frac{\pi}{2}$ | $< \frac{\pi}{2}$ | $\frac{\pi}{2}$   | 80 to 85     |
| (iv)  | Blades bent forward    | $\frac{\pi}{2}$ | $< \frac{\pi}{2}$ | $\frac{\pi}{2}$   | $> \frac{\pi}{2}$ | $\approx 75$ |

In order to have a high efficiency, the centrifugal pumps are generally provided with impellers having their blades bent backwards. Straight blades are used for small pumps where economy is the main consideration. Blades ending radially are better than straight blades but cannot compete effectively with the first type although they have the advantage of being cheap. Blades bent forward yield a very low head efficiency and are, therefore, not used.

**12.32 Number of Blades of Impeller**—If the number of blades of impeller is infinite, then only the ideal head is developed by the impeller. (refer Art 12.27 and Table 12.4), but in practice it is not practicable, because the greater the number of blades, the more will be the area of obstruction which means the frictional losses will be greater and the passage between the blades will be choked by undesirable material (*viz.* rubbish) passing through the impeller.

The number of blades for radial impeller is taken as 6 to 12. If  $H-Q$  curve for a centrifugal pump is drawn (refer Fig 12.24 and Fig 15.27) by carrying out the tests for a number of impellers using different number of blades, but the impeller size, speed and blade angle being identical, it will be seen that the curve will be flat if the number of blades is more and it is dropping when the number of blades is less (refer Fig 12.24)

### 12.33 Curvature and Proportions of Blades—

#### (i) Blades Bent Backwards :

With reference to Fig 12.25. if  $M_2$  be the centre of curvature of the impeller blade section as shown, then from triangle  $M_1M_2P_1$ ,

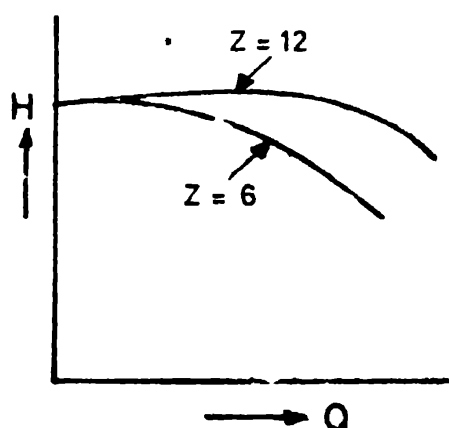


Fig 12.24 Effect of Number of Blades on  $H-Q$  curve



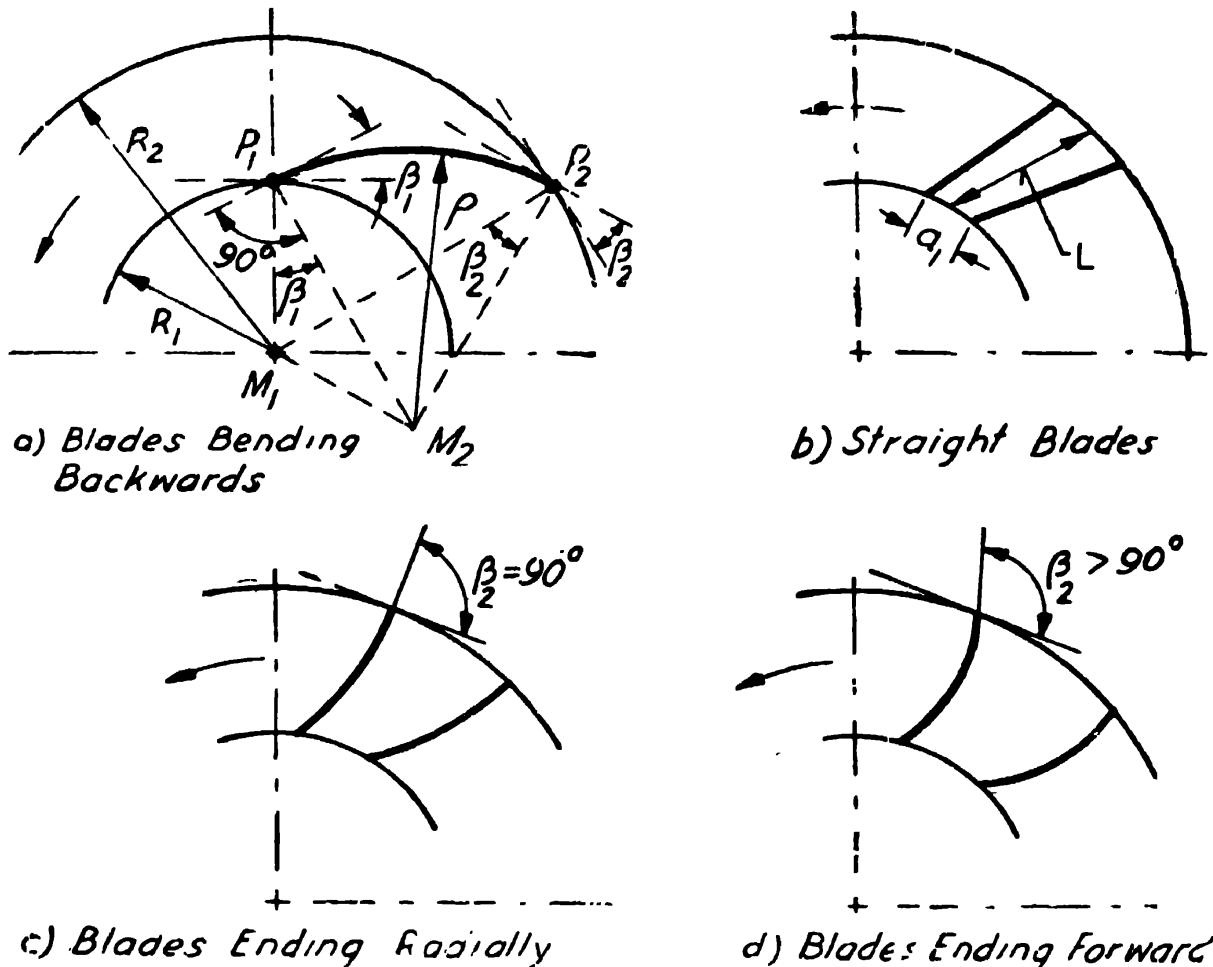


Fig 12.25 Determination of Blade Curvature

$$(M_1M_2)^2 = \rho^2 + R_1^2 - 2\rho R_1 \cos \beta_1$$

and from  $\triangle M_1M_2P_2$ ,

$$(M_1M_2)^2 = \rho^2 + R_2^2 - 2\rho R_2 \cos \beta_2$$

Solving for  $\rho$

$$R_1^2 + \rho^2 - 2R_1 \rho \cos \beta_1 = R_2^2 + \rho^2 - 2R_2 \rho \cos \beta_2$$

or 
$$R_1^2 - 2R_1 \rho \cos \beta_1 = R_2^2 - 2R_2 \rho \cos \beta_2$$

$$\therefore \rho = \frac{R_2^2 - R_1^2}{2(R_2 \cos \beta_2 - R_1 \cos \beta_1)} \quad \dots(12.37)$$

### (ii) Straight Blades

Radius of curvature  $\rho = \infty$

$$\therefore R_2 \cos \beta_2 - R_1 \cos \beta_1 = 0$$

$$\frac{R_1}{R_2} = \frac{\cos \beta_2}{\cos \beta_1} = v \text{ (say)} \quad \dots(12.38)$$

Generally, the ratio  $v$  varies from  $\frac{1}{2}$  to  $\frac{1}{3}$

also  $L = 3 a_1 \text{ to } 5 a_1$

(refer Fig 12.25 b)

### (iii) Blades Ending Radially :

Here  $\beta_2 = \frac{\pi}{2}, \therefore \cos \beta_2 = 0$

and 
$$\rho = \frac{R_2 - R_1^2}{-2R_1 \cos \beta_1} \quad \dots(12.39)$$

The negative sign shows reversal of curvature i.e., from convex forward to concave forward.

(iv) **Blades Bent Forward :**

$$\beta_2 > \frac{\pi}{2} \quad \therefore \cos \beta_2 < 0$$

and 
$$\rho = \frac{R_2^2 - R_1^2}{2(R_2 \cos \beta_2 - R_1 \cos \beta_1)} \quad \dots(12.40)$$

Since  $\cos \beta_2$  is negative, the two terms in the denominator are additive and  $\rho$  is small. Also, the negative sign indicates concave curvature forward.

### 12.34 Diameter of Impeller (refer Fig 12.22)—

(i) **Outside Diameter of Impeller—**

$$u_2 = \frac{\pi \cdot D_2 \cdot N}{60}$$

also 
$$u_2 = K_{u_2} \cdot \sqrt{2gH}$$

$$\therefore D_2 = \frac{60}{\pi \cdot N} \cdot K_{u_2} \cdot \sqrt{2g} \sqrt{H} \text{ metres} \quad \dots(12.41b)$$

If  $K_{u_2} = 1,$

$$D_2 = \frac{60}{\pi} \times \sqrt{2 \times 9.81} \cdot \frac{\sqrt{H}}{N} = \frac{84.6 \sqrt{H}}{N} \text{ metres} \quad \dots(12.41c)$$

where  $H$  is in metres.

These equations are also used to determine the head which a pump can develop if  $D_2$  and  $N$  are known, specially, in practice, as a check for an existing pump.

(ii) **Inlet Diameter :**

The inlet diameter  $D_1$  is  $\frac{2}{3}$  to  $\frac{1}{3}$  of  $D_2$  depending upon specific speed  $N_s$  or total head  $H_{mano}$

TABLE 12.6

Practical Values of  $\frac{D_1}{D_2}$

(See also Table 12.1)

| $H_{mano}$ in metres | 10            | 50            | 100           |
|----------------------|---------------|---------------|---------------|
| $\frac{D_1}{D_2}$    | $\frac{2}{3}$ | $\frac{1}{2}$ | $\frac{1}{3}$ |

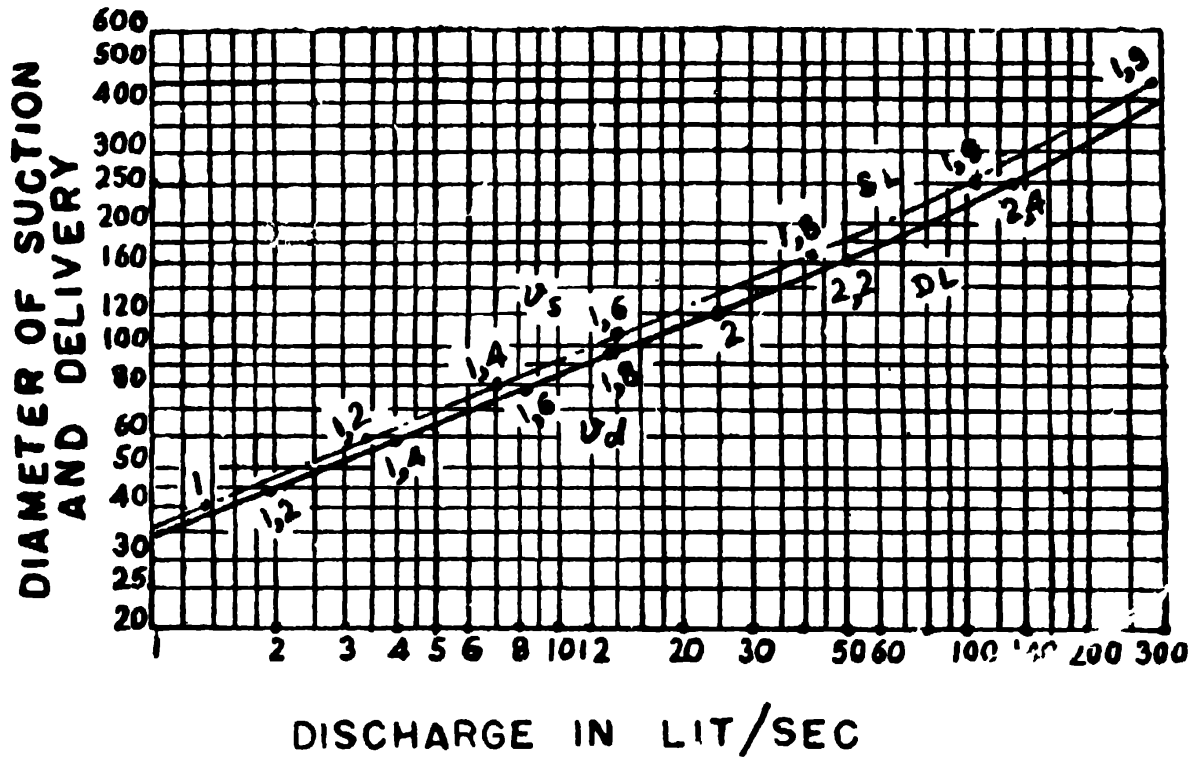


Fig 12.26 Pipe Diameter vs Discharge

### 12.35 Pipe Diameters—

(1) **Suction Pipe :** The amount of water to be pumped is given by

$$Q = \frac{\pi}{4} \cdot d_s^2 \cdot v_s \text{ where } d_s = \text{diameter of suction pipe}$$

$v_s$  = velocity of water in suction pipe

(Generally  $v_s$  is 1.5 to 3 m/sec)

$$\therefore d_s = \sqrt{\frac{Q}{\frac{\pi}{4} \cdot v_s}} \quad \dots(12.42)$$

Fig 12.26 and 12.27 show  $d_s$  and  $d_d$  vs  $Q$  drawn from experimental data.

(ii) **Delivery Pipe :**

$$Q = \frac{\pi}{4} \cdot d_d^2 \cdot v_d, \text{ where } d_d = \text{diameter of delivery pipe ;}$$

$v_d$  = velocity of water in delivery pipe.

(Generally  $v_d$  is 1.5 to 4 m/sec)

$$\therefore d_d = \sqrt{\frac{Q}{\frac{\pi}{4} \cdot v_d}} \quad \dots(12.43)$$

Generally  $v_d$  is equal to or slightly higher than  $v_s$ . Refer also Fig 12.26 and 12.27 for practical values of  $d_d$ .

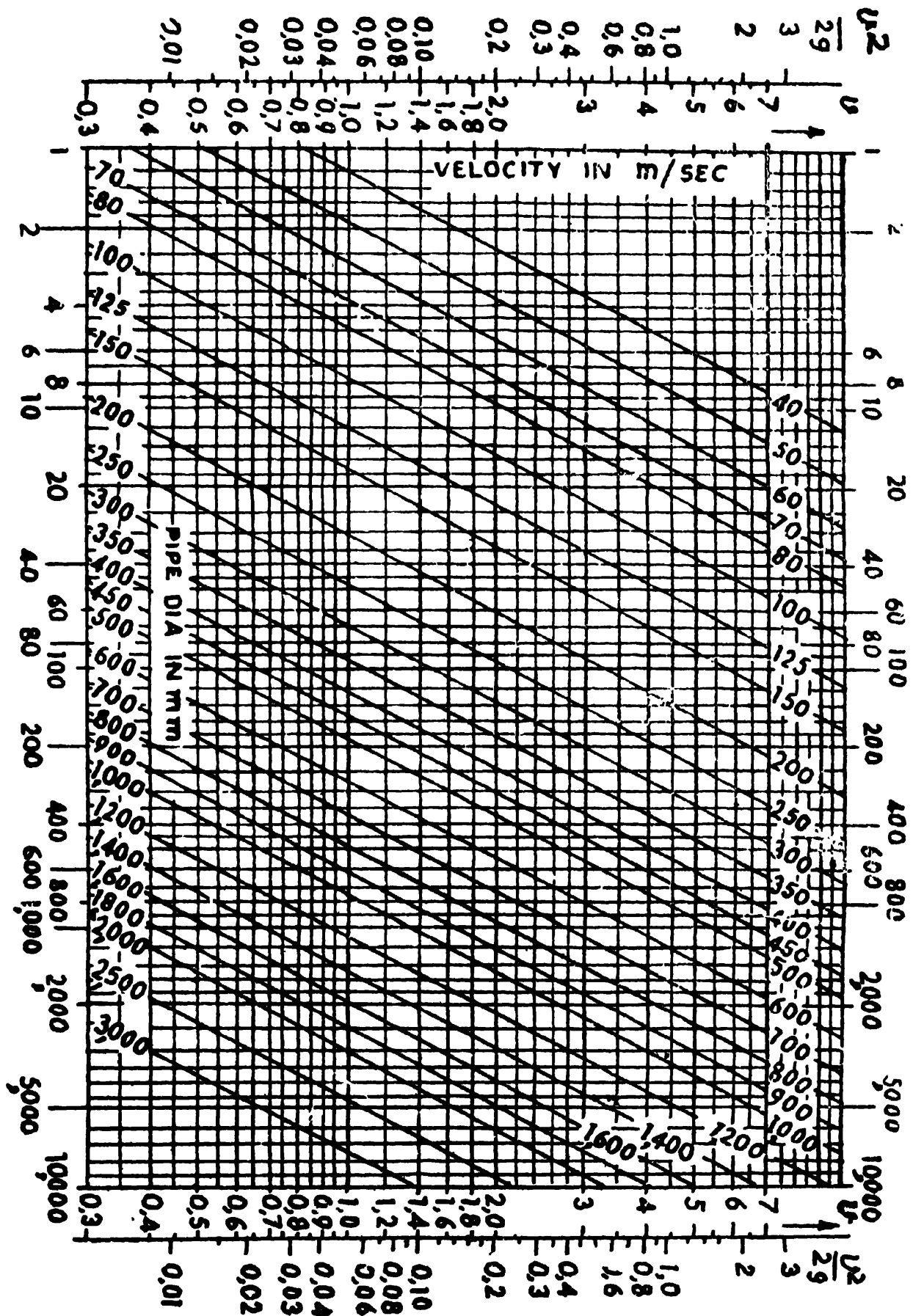


Fig 12.27 Determination of Water Velocity or Pipe Diameters For Given Discharge

### 12.36 Pump Casing—

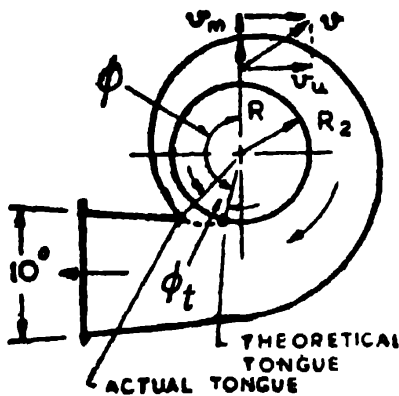


Fig 12.28 Volute Casing

(i) **Volute Casing** : A volute is shown in Fig 12.28. The point where the volute begins is known as theoretical tongue or cut-water. But in practice, the actual tongue is at an angular distance  $\phi_t$  from this point. This angle  $\phi_t$  is known as the tongue-angle and is approximately equal to  $\alpha_2$ . The volute is so designed that flow out of the impeller is uniform all round its periphery. If the total rate of flow be  $Q$ , then the quantity of water flowing across a section of the volute at an angle  $\phi^\circ$  from the theoretical tongue is  $\frac{\phi}{360} \cdot Q$ .

The volute curve is of spiral form. The outlet of volute may be extended, as shown, in order to lower the outlet velocity of water down to a value required to carry the given discharge through the delivery pipe. However, the angle of divergence for this part should not exceed  $10^\circ$  so as to avoid boundary layer effect and cavitation.

**Shape of Volute** : According to Pfleiderer, to ensure spiral flow inside the volute,

$$R \cdot v_u = C$$

where  $v_u$  is the velocity of whirl at any radius  $R$  (refer Fig 12.28) and  $C$  is a constant. The constant  $C$  can be evaluated from known values of  $R$  and  $v_u$  at outlet of impeller.

If  $b$  the breadth at a radius  $R$  at some section at an angle  $\phi$  from the theoretical tongue (refer Fig 12.28 and 12.29) then rate of flow through an elementary strip of thickness  $dR$  is

$$dQ_\phi = dA \cdot v_u$$

But 
$$v_u = \frac{C}{R}$$

$\therefore dQ_\phi = b \cdot dR \cdot \frac{C}{R}$

$$Q_\phi = \int_{R_2}^R dQ_\phi = C \int_{R_2}^R \frac{b \cdot dR}{R} \quad \dots(12.44)$$

This must be equal to  $\frac{\phi}{360} \cdot Q$

The external side of the spiral casing may be of any shape. Generally it is circular or elliptical. Elliptical form can also be obtained by equalising areas  $A_1$  and  $A_2$  show in Fig 12.29. The angle of volute  $\theta$  is generally  $60^\circ$ .

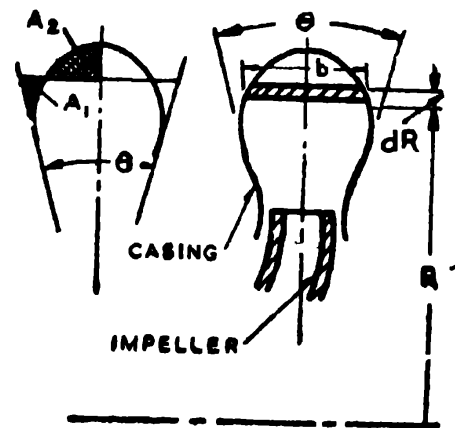


Fig 12.29 Shape of Casing

...(refer Eqn 2.8)

(ii) **Diffusion Casing:** As has already been explained in Art 12.7(b) this casing contains a number of vanes which provide passages for liquid to flow. The cross-sectional area of any passage between two vanes must gradually enlarge in order to reduce the velocity of water emerging from the impeller (refer Fig 12.5) so that pressure head can be built up at the expense of kinetic energy. The angle of divergence for the enlarging section must not exceed  $10^\circ$ . The outlet area should be so chosen as to reduce the velocity of water to a final value just sufficient to carry the discharge through the delivery pipe. A sufficient number of vanes should be employed to ensure uniformly guided flow of water.

**12.37 Axial Thrust in Centrifugal Pumps—**Axial thrust is a force casting parallel to the axis of the pump shaft, caused due to the following reasons—

(a) The water while passing through the impeller is rotating with a forced vortex, but that outside the shrouding (refer Fig 2.8 and 2.9) is in a state of comparative rest. This causes a differential static thrust acting parallel to the axis of pump shaft and towards the impeller inlet.

(b) Liquid enters the pump axially and is then deflected from its original path to a radial direction. The dynamic action of liquid causes a force to act on the pump in the direction of flow at inlet. The magnitude of this force, measured by the change in momentum per second, is

$$\frac{\gamma \cdot Q}{g} \cdot u_1$$

To enable the pump to withstand the thrust, the following methods may be employed.

(a) *For small pumps :*

(i) Providing a thrust ball bearing in this direction of axial thrust shown in Fig 12.30 a.

(ii) Inserting a cast iron ring in the casing which should fit in with a similar ring cast integral with the impeller as shown in Fig 12.30 (b).

(b) *For large pumps :*

Where the axial thrust is heavy.

(i) Use of double suction impeller (refer Fig 12.30c).

Suction on two sides of the impeller neutralises the thrust. But this method can be employed only for single stage pump.

(ii) Provision of relieving holes (refer Fig 12.30d.)

Relieving holes are provided in the impeller to allow suction pressure to act on both sides.

(iii) A balance plate fitted at the end of the pump shaft (refer Fig 12.30e).

(iv) Balance Piston—This is employed for large pumps (refer Fig 12.30 f).

(c) *For multi-stage pump :*

The number of impellers are made generally even. This will facilitate to arrange the inlets of the half of the impellers in the opposite direction as shown in Fig 12.30g. These will balance the axial thrust produced by the other half numbers of the impellers.

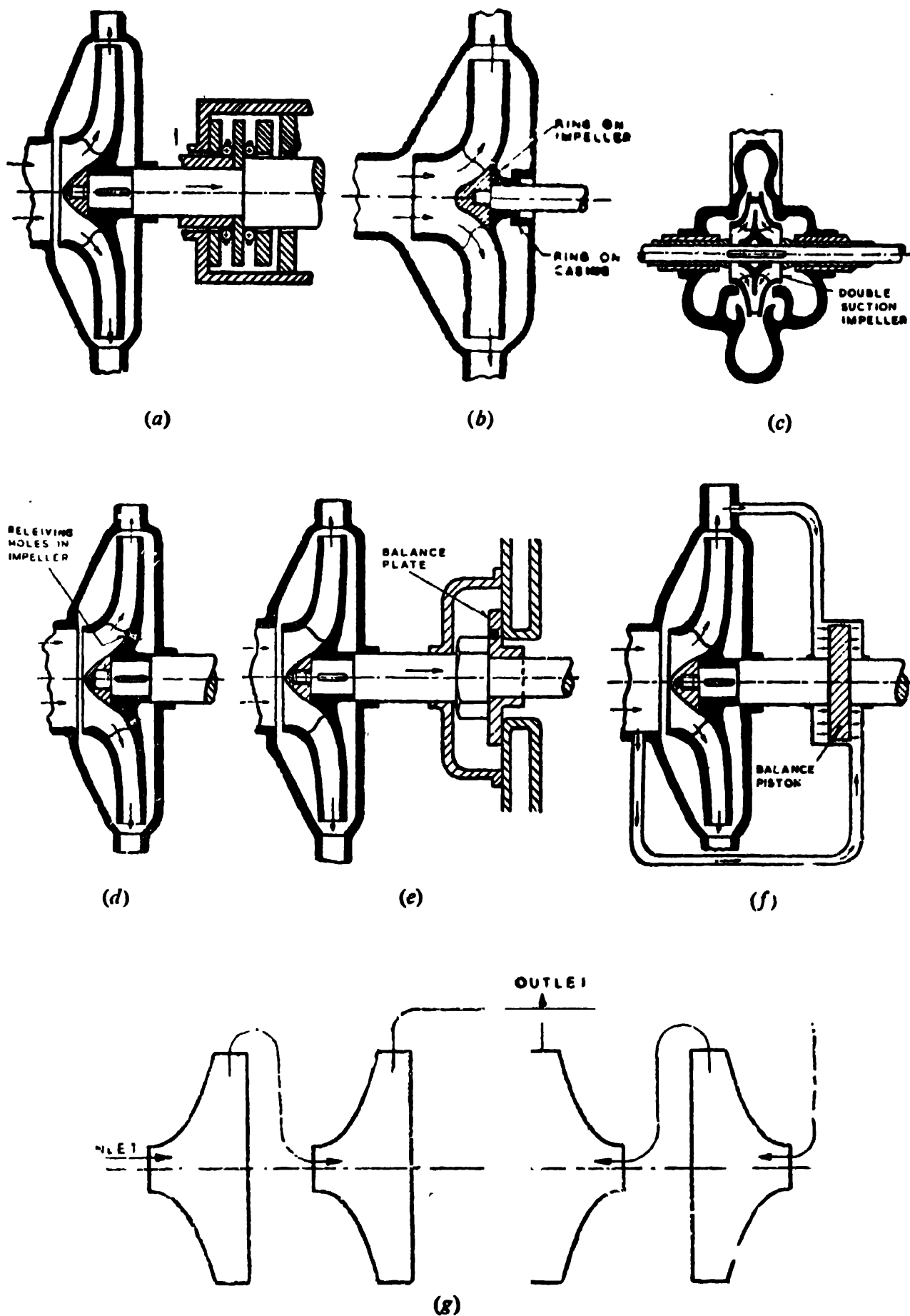


Fig 12.30 Balancing of Axial Thrust in Centrifugal Pumps

**12.38 Variation of Speed and Diameter**—Even after a pump has been manufactured and used, its head or capacity or both may have to be changed to suit a new set of conditions. This can be accomplished by varying the speed of pump or by changing the diameter of the impeller. It should be borne in mind that this step is to be taken only when a minor change in head or capacity is required, otherwise it is recommended that a new pump be manufactured with altogether different dimensions.

(a) **Effect of Variation of Speed on Discharge, Head and Power :**

$$\text{Peripheral velocity } u = \frac{\pi \cdot D \cdot N}{60}$$

$$\therefore u \propto N$$

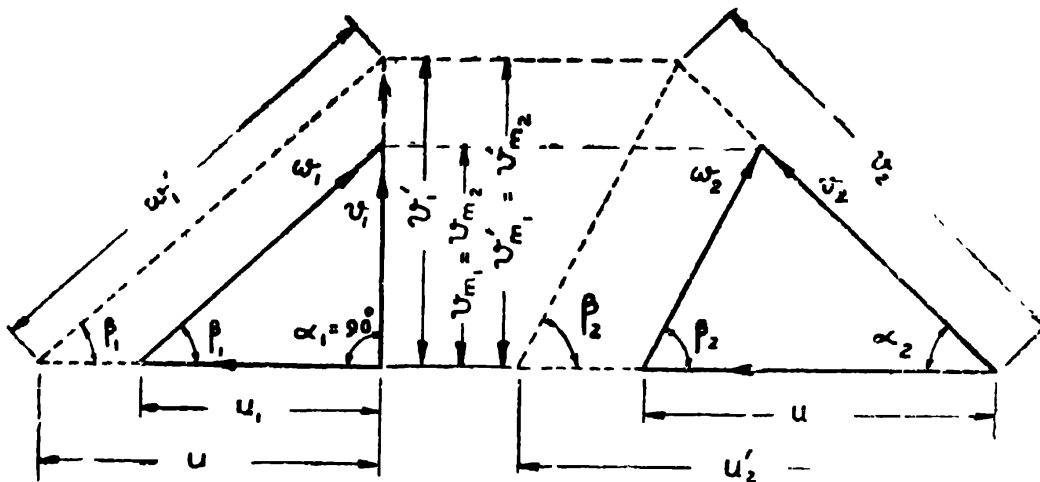


Fig 12.31 Effect of Variation of Speed on Velocity Triangles of a Centrifugal Pump

From the velocity triangles shown in Fig 12.31 it is evident that since the vane angles remain constant, if  $u$  changes to  $u'$ , then

$$w \propto u$$

$$w \propto N$$

But the discharge,  $Q = \text{area across flow multiplied by } w$

$$\therefore Q \propto w$$

$$\text{i.e. } Q \propto N$$

Thus discharge varies directly as speed.

Further, from the fundamental equation (refer Eqn 12.1) of pumps it is obvious that head depends on squares of velocities  $u$ ,  $w$ , and  $v$ . From Fig 12.31, it is seen that these quantities are proportional to speed  $N$ .

Summing up

$$\frac{Q}{Q'} = \frac{N}{N'} \quad \dots(12.45)$$

$$\frac{H}{H'} = \left( \frac{N}{N'} \right)^2 \quad \dots(12.46)$$

and

$$\frac{HP}{HP'} = \left( \frac{N}{N'} \right)^3 \quad \dots(12.47)$$



Hydraulic losses vary more or less uniformly with the hydraulic horsepower. But mechanical losses are relatively small at higher speeds. Therefore, considering the brake horsepower, the total efficiency of the pump will increase slightly with an increase in speed.

**Problem 12.9** A centrifugal pump delivers 20 lit/sec of water against a head of 12 m and requires 6 HP when running at 1,450 rpm. Find the discharge, head and horsepower required if it has to run at 1,800 rpm.

**Solution**

$$N = 1,450 \text{ rpm}$$

$$Q = 20 \text{ lit/sec}$$

$$H = 12 \text{ m}$$

$$N' = 1,800 \text{ rpm}$$

$$HP = 6 \text{ HP}$$

Now

$$\frac{Q}{Q'} = \frac{N}{N'} \quad \therefore Q' = \frac{N'}{N} \times Q = \left( \frac{1,800}{1,450} \right) \times 20$$

$$= 24.8 \text{ lit/sec} \quad \text{Answer}$$

Further,

$$\frac{H}{H'} = \left( \frac{N}{N'} \right)^2 \quad \therefore H' = \left( \frac{N'}{N} \right)^2 \times H = \left( \frac{1,800}{1,450} \right)^2 \times 12$$

$$= 18.45 \text{ m} \quad \text{Answer}$$

Also,

$$\frac{HP}{HP'} = \left( \frac{N}{N'} \right)^3 \quad \therefore HP' = \left( \frac{N'}{N} \right)^3 \cdot HP = \left( \frac{1,800}{1,450} \right)^3 \times 6$$

$$= 11.5 \text{ HP} \quad \text{Answer}$$

(b) **Effect of Alterations of Diameter on Discharge, Head and Power**—It has been mentioned earlier that the capacity of a pump can be changed by altering either its speed or the diameter of its impeller. The former is generally not possible because the speed of driving motor is fixed. Therefore, the diameter of the impeller has to be enlarged or reduced according as  $H$  or  $Q$  is to be increased or decreased. It may be effected by fixing rings to the shrouds of impeller (refer Fig 12.32) to increase the outside diameter and extending the impeller blades to the required size. The effect of alteration of  $D$  is twofold.

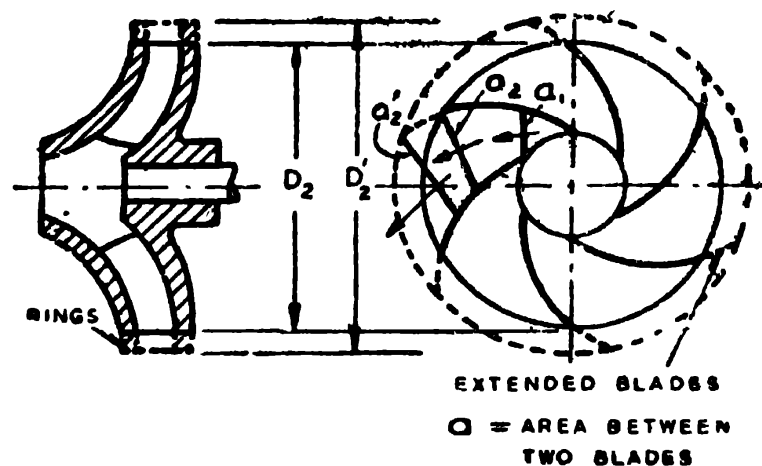


Fig 12.32 Effect of Increasing the Outlet Diameter of an Impeller

First, it changes  $u$  without changing  $N$ .

i.e.  $u \propto D$

$\therefore w \propto D$

Second, with reference to Fig 12.32, area across flow,

$$a \propto D$$

But  $Q = w \cdot a$

$\therefore Q \propto D^3$

Also, since  $H$  depends on  $u^2 \cdot w^2$  etc and the latter are proportional to  $D^2$ ,

$$H \propto D^2$$

Now, power,  $HP \propto Q \cdot H$

$\therefore HP \propto D^4$

Summing up

$$\frac{Q}{Q'} = \left( \frac{D}{D'} \right)^3 \quad \dots(12.48)$$

$$\frac{H}{H'} = \left( \frac{D}{D'} \right)^2 \quad \dots(12.49)$$

and  $\frac{HP}{HP'} = \left( \frac{D}{D'} \right)^4 \quad \dots(12.50)$

The efficiency  $\eta$  increases slightly with size.

If, together with the diameter  $D$  the height of impeller passage and dimensions of other parts are proportionately altered,

Then  $a \propto D^2$

$\therefore Q \propto D^3$

and  $HP \propto D^5$

**Problem 12.10** A single centrifugal pump is built to give a discharge of  $Q$  when working against a manometric head of 17 m. On test it was found that the head actually generated was 18.5 m for the designed discharge  $Q$ . If it is required to reduce the original diameter of 30 cm without reducing the speed of the impeller, compute the reduction necessary.

(Panjab University)

**Solution**

$$\text{Head generated by pump } H = \frac{u^2}{2g} = \frac{\left( \frac{\pi \cdot D \cdot N}{60} \right)^2}{2g}$$

or  $H \propto D^2$

$\therefore \frac{H}{H'} = \left( \frac{D}{D'} \right)^2$

$H = 18.5 \text{ m} \quad H' = 17 \text{ m} \quad D = 30 \text{ cm}$

$\therefore D' = D \left( \frac{H'}{H} \right)^{\frac{1}{2}} = 30 \times \left( \frac{17}{18.5} \right)^{\frac{1}{2}}$

$= 28.8 \text{ cm}$

Answer

(c) **Relation between  $Q$ ,  $H$ ,  $HP$ ,  $N$  and  $D$** —If the diameter and other dimensions of impeller are proportionately altered and the speed is also changeable, then the general relation between these quantities can be derived by combining the results obtained above.

$$\text{Hence } Q \propto D^3 \cdot N \quad \dots(12.51)$$

$$H \propto D^2 \cdot N^2 \quad \dots(12.52)$$

$$HP \propto D^5 \cdot N^3 \quad \dots(12.53)$$

These relations are approximate because leakage, windage and bearing losses have been neglected.

**12.39 Model Pumps**—It has been mentioned in Chapter 9 that water turbines are always manufactured according to given specifications and not on mass scale. Almost all turbines are large units, therefore turbine models are prepared to predict the performance of the actual size machines before they are actually manufactured. In case of pumps, mostly they are manufactured on mass scale. In some cases large-sized pumps are to be installed. In such cases mass production cannot be resorted to, therefore, similar to water turbines, large units of pumps are also not manufactured without making and testing their models because it would be possible to make any alteration in design in time which would otherwise be difficult to carry out in the prototype machines.

The model should have complete geometrical similarity with the prototype, not only in pump proper but in intake and discharge conduits also. The speed of the model can be determined on the assumption that the model and the prototype have the same specific speeds. The same conditions of similarity as explained in Art 9.2 for turbines would apply to the pumps also. The speed and discharge for the model pump can be determined from Eqns 9.10 and 9.11 respectively.

$$\text{i.e.} \quad \frac{N_m}{N_a} = \frac{D_a}{D_m} \times \sqrt{\frac{H_m}{H_a}} \quad (\text{refer Eqn 9.10})$$

$$\text{and} \quad \frac{Q_m}{Q_a} = \left( \frac{D_m}{D_a} \right)^2 \sqrt{\frac{H_m}{H_a}} \quad (\text{refer Eqn 9.11})$$

where suffices 'a' and 'm' indicate actual-size and model pump respectively.

**Problem 12.11** To predict the performance of a large centrifugal pump, its model, having the following specification, was made :

$$P = 24 \text{ HP} \quad H = 8 \text{ m} \quad N = 925 \text{ rpm}$$

Diameter of model pump impeller is 9 times smaller than that of the prototype. The prototype pump has to work against a head of 30 m. Find its working speed and HP required to drive it. Determine also the rate of flow of both the pumps. (UPSC)

**Solution**

$$P_m = 24 \text{ HP}, \quad H_m = 8 \text{ m}, \quad N_m = 625 \text{ rpm},$$

$$D_a = 9 D_m, \quad H_a = 80 \text{ m}, \quad \text{Scale } \frac{D_a}{D_m} = 9.$$

$$(a) \text{ For similar pumps, specific flow is same,} \quad (\text{refer Eqn 9.3})$$

$$\text{i.e.,} \quad (Q_{11})_a = (Q_{11})_m$$

$$\text{or} \quad \frac{Q_a}{D_a^2 \cdot \sqrt{H_a}} = \frac{Q_m}{D_m^2 \cdot \sqrt{H_m}}$$

$$\text{Now} \quad P = \frac{\gamma \cdot Q \cdot H}{75}$$

$$\therefore Q = \frac{75 \cdot P}{\gamma \cdot H} \text{ m}^3/\text{sec}$$

or  $Q_m = \frac{75 \times 24}{1,000 \times 8} = 0.225 \text{ m}^3/\text{sec} \quad \text{Answer}$

and  $Q_a = Q_m \left( \frac{D_a}{D_m} \right)^2 \sqrt{\frac{H_a}{H_m}}$

$$= 0.225 \times 9^2 \times \sqrt{\frac{30}{8}} = 35.2 \text{ m}^3/\text{sec} \quad \text{Answer}$$

(b) For similar pumps the specific speed is same,

$$\text{i.e., } \frac{N_a \cdot \sqrt{Q_a}}{H_a^{\frac{3}{4}}} = \frac{N_m \cdot \sqrt{Q_m}}{H_m^{\frac{3}{4}}} \quad (\text{refer Eqn 9.1 and 3.11})$$

or  $N_a = N_m \cdot \sqrt{\frac{Q_m}{Q_a}} \cdot \left( \frac{H_a}{H_m} \right)^{\frac{3}{4}}$

$$= 625 \times \left( \frac{0.225}{35.2} \right)^{\frac{1}{2}} \times \left( \frac{30}{8} \right)^{\frac{3}{4}}$$

$$= 134 \text{ rpm} \quad \text{Answer}$$

(c) For similar pumps the specific power is same,

$$\text{i.e., } \frac{P_a}{D_a^3 \cdot H_a^{\frac{3}{2}}} = \frac{P_m}{D_m^3 \cdot H_m^{\frac{3}{2}}} \quad (\text{refer Eqn 9.12})$$

$$\therefore P_a = P_m \left( \frac{D_a}{D_m} \right)^3 \left( \frac{H_a}{H_m} \right)^{\frac{3}{2}} = 24 \times 9^2 \times \left( \frac{30}{8} \right)^{\frac{3}{2}}$$

$$= 14,100 \text{ HP} \quad \text{Answer}$$

#### 12.40 Priming of Pump—

**Priming**—Before starting a pump, its impeller and suction pipe have to be filled with water in order to remove any air, gas or vapour from the waterways of the pump. If a centrifugal pump is not primed before starting, air pockets inside the impeller may give rise to vortices and cause discontinuity of flow. The wearing rings may rub and seize causing serious damage if the pump is allowed to run dry. It is also essential that packing be lubricated by liquid leaking past it.

Originally, priming is done by pouring water through a funnel, displaced air being allowed to escape through air vents. When a pump is being primed or stopped, the delivery valve should be kept closed.

Necessity of priming is the main disadvantage of a centrifugal pump. To overcome this difficulty, the following methods are employed in practice :

- (a) the pump is installed below the suction water level ;
- (b) the pump is equipped with one of the priming or self-priming devices given in the next article.

#### Priming Devices—

(a) *Pouring Water*—Water is poured in the pump through priming funnel. Air vent is opened to provide exit to the air. It is closed after the priming is over.

(b) *Connection with City Water Main*—The pump may be connected with the city water main which can be opened to fill the impeller and the suction pipe in order to prime the pump.

(c) *Priming Chamber*—In small pumps a priming chamber may be used on the delivery side of the impeller. When the pump is stopped, some water is stored in the tank and this can be used to fill the impeller and suction line before restarting. Normally, the capacity of the tank is about three times the volume of the suction pipe.

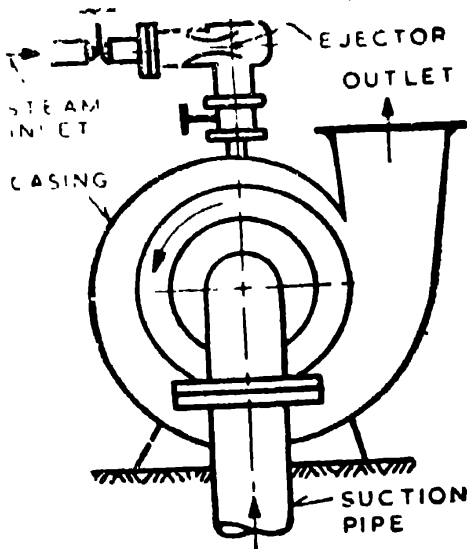


Fig 12.33 Priming of Centrifugal Pump

(d) *Vacuum Producing Devices*—With such devices the suction line and pump are exhausted of all air so that atmospheric pressure at the sump, forces the water up into the pump. An ejector using high pressure water, steam or compressed air is employed to create vacuum at the top of the casing (refer Fig 12.33) so that water is sucked into the suction pipe and the impeller.

**12.41 Self-Priming Devices**—The following are the two types :

(1) **By Formation of Air-Water Mixture**—The pump is made to exhaust

the air of the impeller and the suction pipe. The density of the air is about 800 times smaller than that of water, therefore, the head generated by the revolving impeller is 800 times less. Now if air is mixed with water, the head generated by the impeller will increase depending upon the density of the mixture.

Fig 12.34 shows an open impeller mounted in the casing such that there is a small clearance between the blades and the side walls and also

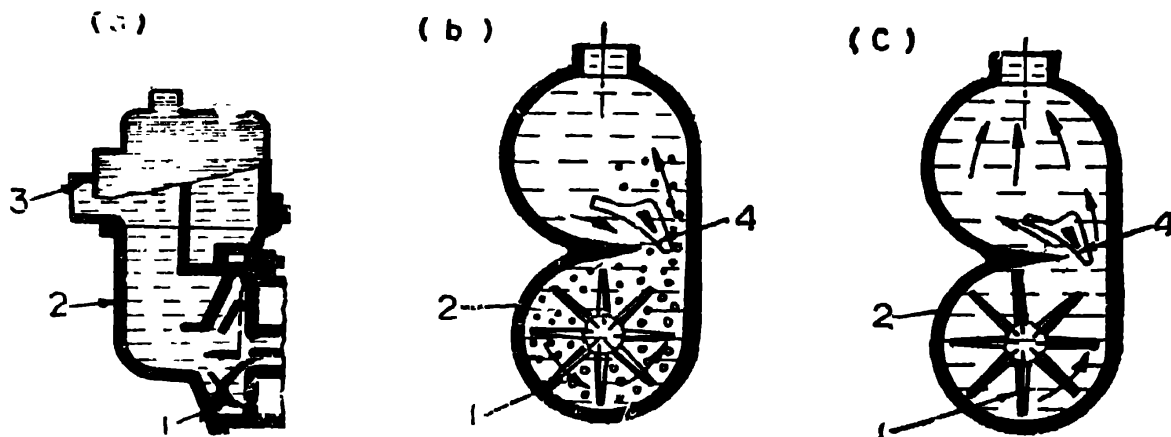


Fig 12.34 Principle of Operation of Self-Priming of Centrifugal Pump  
1. Impeller 2. Casing 3. Suction Branch 4. Fixed Guide Vane

between the blade tips and the circumference of the casing. In order to start for the first time, the casing is filled with water upto a level where the suction pipe begins. The suction pipe remains full of air. When the pump starts working, it draws air from the suction pipe and a mixture of water and air is formed in the casing which is discharged by the impeller into a widened portion known as separator. Because of a large cross sectional area of separator, the water velocity decreases and the air is separated from the mixture and escapes through the delivery pipe which should open to atmosphere at its exit. The water free of air, comes back to the impeller from the separator through the passage made by the fixed vane as shown in

Fig 12.34 (b). The impeller goes on revolving and the water is drawn up the suction pipe gradually till whole of the air is evacuated from it by the formation of air-water mixture and the air separating out. The pump will then start functioning normally.

(2) **By Providing Air or Vacuum Pump**—The pump impeller and its suction pipe is primed by an air pump or vacuum pump, run by the same pump shaft and fitted on the suction side of pump. This air pump runs in parallel with the centrifugal pump, exhaust air from the suction pipe and discharges into the open air or in delivery pipe if it is open from its exit end. After the pump is primed, the air pump discharges the water back into the suction pipe. The air pumps are generally of two types—water ring pump and side channel pump.

**12.42 Cavitation in Pump**—The phenomenon of cavitation has already been explained in the treatment of turbines (refer Art 6.17). When the liquid is flowing in the pump, it is possible that the pressure at any part of the pump may fall below the vapour pressure, then the liquid will vapourise and the flow will no longer remain continuous. The vapourisation of the liquid will appear in the form of bubbles released in the low pressure region of the pump. These bubbles are carried along with water stream and when these pass through a region of high pressure, these collapse suddenly. When the bubbles collapse on a metallic surface such as tips of impeller blades, the cavities are formed. Successive bubble-collapsing at the same metallic surface produces pitting since penetration in the grain boundaries take place. Once the pitting takes place, the liquid rushes to fill the pits causing mechanical destruction and the liquid hits the blades with such a great force that it damages the impeller. The phenomenon of formation of cavities by the vapourisation of liquid is known as *cavitation*. A great noise is experienced due to cavitation leading to vibration of the pumping set.

Since the cavitation occurs when the pressure falls below atmospheric, the trouble is experienced mainly at the impeller vane inlet due to high suction lift which must be brought within limits.

**12.43 Suction Lift or Suction Head**—Ordinarily a pump lifts liquid from the sump, and atmospheric pressure is acting on its liquid surface which is responsible for forcing the liquid up the suction pipe. Theoretically the maximum suction lift possible at sea level is the atmospheric head which is 10.3 m of water, but in practice the suction lift reduces to about 4.5 m of water because of several factors which are summed up as follows :

(1) The atmospheric pressure at sea level is 10.3 m of water. Since the pump is not always installed at sea level, the value of atmospheric pressure is less than 10.3 m. The permissible atmospheric pressure  $H_{atm}$  at a certain attitude can be found from Table 6.4

(2) The pump cannot produce a pressure below vapour pressure of the liquid, then the limiting available pressure will be equal to permissible atmospheric pressure *minus* the vapour pressure  $H_{vap}$  of the liquid at the prevailing temperature.

(3)  $H_f$  the head lost due to friction in the suction pipe and its fittings as well as due to turbulence between the surface of liquid in the sump and the pump suction flange.

(4)  $\frac{v_s^2}{2g}$  the kinetic head due to velocity of water in the suction pipe or dynamic suction lift.

$$\therefore H_{atm} = H_s + H_{vap} + H_f + \frac{v_s^2}{2g}$$

or Suction lift  $H_s = H_{atm} - \left( H_{vap} + H_f + \frac{v_s^2}{2g} \right)$

So long as the suction lift is kept below this value, the output ( $Q$ ) and efficiency ( $\eta$ ) of the pump are unaffected. As soon as the maximum lift exceeds the limiting value, the output would fall abruptly. It is only the output which is affected but there is every possibility of having the following troubles in the working of the pump :

1. With the drop of output the efficiency will also be much less.
2. Seizing of the pump may take place.
3. The pump may become unprimed.
4. The pump is subjected to cavitation and air binding, making the operation very noisy ; the wear becomes rapid.

The suction lift is limited by the following factors :

1. Viscosity : Centrifugal pump should not be used for liquids of viscosity higher than 1,500 Say-Bolt Universal Seconds if the output is 150 lit/sec.
2. Temperature (refer Table 7.3).
3. Total lift.
4. Losses in pipe fittings.
5. High velocity of water entering the pump.
6. Mechanical defects such as leaking of joints, thick blades of impeller and blunt or incorrect shape of blades of impeller or that of the guides or volute because of which there is a possibility of formation of eddies.

**12.44 Net Positive Suction Head NPSH** is defined as the net head in metres of liquid required to force the liquid into the pump through the suction pipe. It is equal to the barometric head *minus* the sum of static suction head, vapour pressure head, friction head loss and kinetic head. For details of NPSH the reader may refer to the book on "Centrifugal Pump and Blowars" by Prof. A.H. Church and the author. The value of NPSH can be altered to suit the conditions by installing the pump near the pump and changes made in the pipe system. The factors effecting the NPSH are diameter of suction pipe  $d_s$ , inlet and outlet diameters of impeller  $d_1$  and  $d_2$ , shape and number of impeller vanes, area of flow between vanes and shaft diameter.

**12.45 Horsepower of Driving Motor**—Centrifugal pumps are generally coupled to 3 phase AC electric motors. Direct current motors are recommended where duty of the pump changes periodically. Horsepower of a driving motor should exceed the horsepower calculated for the pump by the following percentages :

1. For a pump requiring up to 2 horsepower add about 50%
2. For a pump requiring from 2 to 5 horsepower add about 30%
3. For a pump requiring from 5 to 10 horsepower add about 20%
4. For a pump requiring from 10 to 20 horsepower add about 15%
5. For a pump requiring more than 20 horsepower add about 10%.

The percentages stated above are valid for pumps handling water or similar liquids. For special duty pumps, the percentages will differ according to the nature of the liquid to be pumped.

Electric motors below  $\frac{1}{2}$  HP are not employed for driving pumps. For 50 cycles frequent the speed of an induction type motor may be about 1,450 or 2,900 rpm.

**12.46 Multi-stage Pump**—If a pump is required to deliver against a very high head which cannot be built up in a single impeller, a number of impellers are connected in series, so that discharge from one goes to the inlet of the next and so on (refer Fig 12.6b). All impellers are identical and keyed to the same shaft. Theoretically, at every stage the head should be raised by the same amount. The last diffuser discharges into the delivery pipe (refer Fig 12.35).

Particular attention should be paid to the design of a multi-stage pump shaft. It must not run at its critical speeds. This is particularly important in case of boiler feed-pumps.

**Problem 12.12** A centrifugal pump is required to deliver 70 lit/sec of water at room temperature against a head of 100 m, when running at 1,450 rpm. Find the number of stages for best efficiency.

**Solution**

$$Q = 70 \text{ lit/sec} \quad H = 100 \text{ m} \quad N = 1,450 \text{ rpm}$$

$$\therefore \text{Specific speed } N_s = \frac{N \times \sqrt{Q}}{H^{\frac{3}{4}}}$$

For single stage pump with the given specifications

$$\begin{aligned} N_s &= \frac{1,450 \times \sqrt{70 \times 10^{-3}}}{100^{\frac{3}{4}} \times 10^{\frac{1}{4}}} \\ &= 12.17 \text{ units} \end{aligned}$$

Pumps having a specific speed less than 12 are generally not recommended. In fact, the efficiency of a pump falls considerably if the specific speed is less than 20. This is because the impeller becomes disproportionate, the diameter being too large relative to the width. It results in leakage and higher disc friction and fluid friction losses owing to narrow passage for the fluid. It is, therefore, advisable to use impellers of small diameters and consequently high specific speed. This will reduce the disc friction losses which vary with the radius. The efficiency will rise and the cost will fall.

To select a suitable number of stages consider

$$(i) \text{ a three-stage pump } N_s = \frac{1,450 \times \sqrt{70 \times 10^{-3}}}{\left(\frac{100}{3}\right)^{\frac{3}{4}}} = 27.75 \text{ units}$$

$$(ii) \text{ a five stage pump } N_s = \frac{1,450 \times \sqrt{70 \times 10^{-3}}}{\left(\frac{100}{5}\right)^{\frac{3}{4}}} = 40.6 \text{ units}$$

It is known from experience and the available data that 70 lit/sec pump is most efficient and economical if  $N_s = 40$ . Therefore, five stages may be recommended.

**Problem 12.13** A two stage centrifugal pump is required for a fire engine for a duty of 60 lit/sec at a head of 75 m. If the overall efficiency is 75% and specific speed per stage about 40, find

- (a) the running speed in rpm,  
and (b) the BHP of driving engine.

If the actual manometric head developed is 65% of the theoretical head, assuming no slip, the outlet angle of the blades  $30^\circ$  and radial flow velocity at



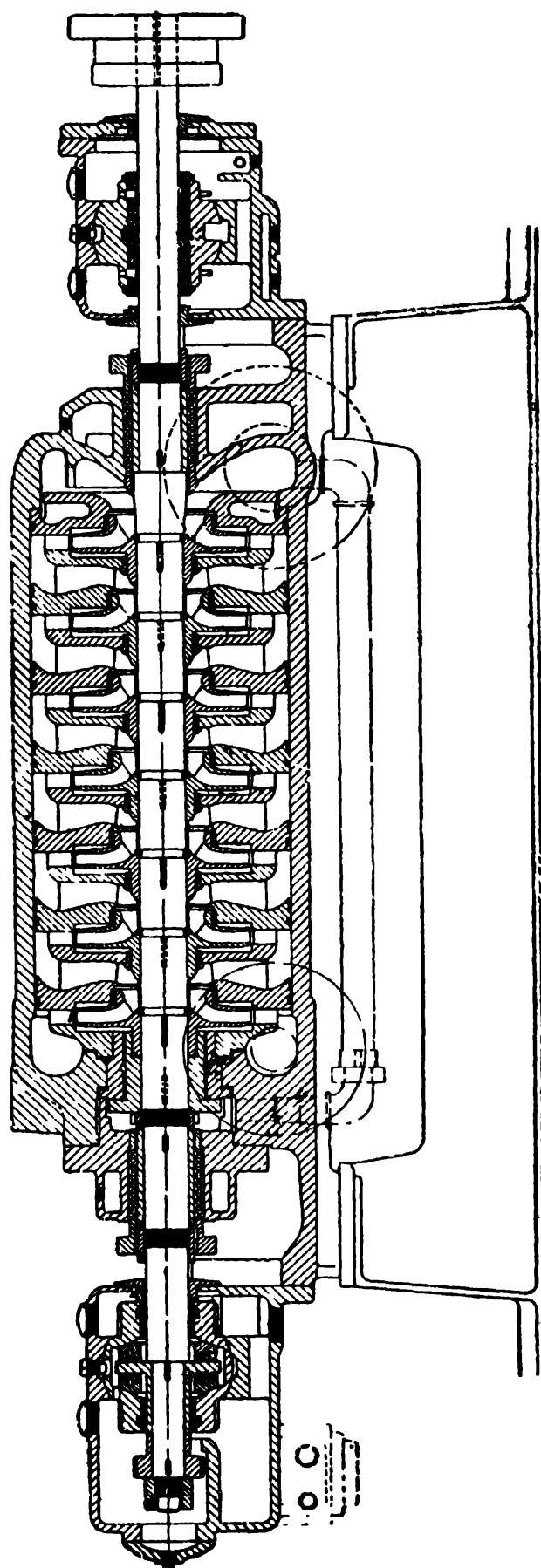


Fig 12.35 Multistage Centrifugal for Pump for Boiler Feed Purposes, 7-stages (Manufactured by Ingersoll-Rand)

exit 0.15 times the tipped speed at exit, find the diameter of the impellers. Indicate the advantage of the two stage pump over the single stage for this duty. (UPSC)

**Solution**

(i)  $Q = 60 \text{ lit/sec} = 60 \times 10^{-3} \text{ m}^3/\text{sec}$

$H = 75 \text{ m} \quad \therefore H \text{ per stage} = \frac{75}{2} = 37.5 \text{ m}$

$N_s = 40 \text{ per stage} \quad \eta_{\text{overall}} = 0.75$

(a)  $N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}}$

$\therefore N = \frac{N_s \cdot H^{\frac{3}{4}}}{\sqrt{Q}} = \frac{40 \times 37.5^{\frac{3}{4}}}{(60 \times 10^{-3})^{\frac{1}{2}}} = \frac{40 \times 6.14 \times 2.48}{2.45 \times 10^{-1}} = 2,485 \text{ rpm} \quad \text{Answer}$

(b)  $Q = 60 \times 10^{-3} \text{ m}^3/\text{sec}$

$\therefore \text{BHP of the driving engine} = \frac{\gamma \cdot Q \cdot H}{75 \cdot \eta_{\text{overall}}}$   
 $= \frac{1,000 \times 60 \times 10^{-3} \times 75}{75 \times 0.75} = 80.8 \text{ HP} \quad \text{Answer}$

(ii)  $H_{\text{mano}} = 0.65 H_{\text{th}}$

$\beta_2 = 30^\circ$

$v_{m_2} = 0.15 u_2$

From outlet velocity triangle (refer Fig 12.20)

or  $\tan \beta_2 = \frac{v_{m_2}}{u_2 - v_{u_2}} \quad \text{or} \quad \tan 30^\circ = \frac{0.15 u_2}{u_2 - v_{u_2}}$

or  $0.5774 = \frac{0.15 u_2}{u_2 - v_{u_2}} \quad \therefore \frac{u}{v_2 - v_{u_2}} = \frac{0.5774}{0.15} = 3.85 \quad \dots (1)$

Assuming flow at entrance to be radial and  $\alpha_1 = 90^\circ$ , fundamental equation of pump would be :

$$\frac{H_{\text{mano}}}{\eta_{\text{mano}}} = \frac{u_2 \cdot v_{u_2}}{g}$$

where  $\eta_{\text{mano}}$  manometric efficiency of pump is 65%

$\therefore \frac{37.5}{0.65} = \frac{u_2 \cdot v_{u_2}}{9.81} \quad \text{Or} \quad u_2 \cdot v_{u_2} = \frac{37.5 \times 9.81}{0.65} = 566$

or  $v_{u_2} = \frac{566}{u_2} \quad \dots (2)$

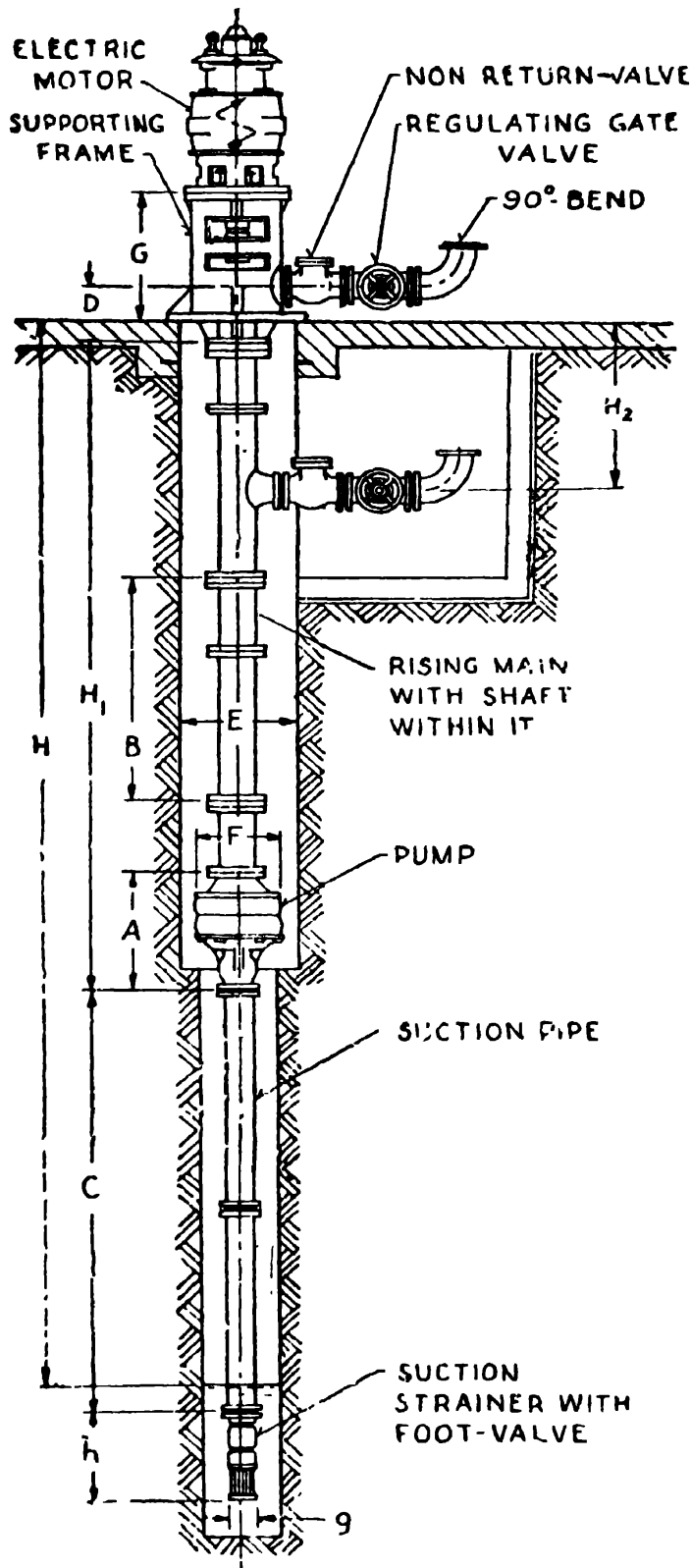
Substituting for  $v_{u_2}$  in equation (1),

$\frac{u_2}{u_2 - \frac{566}{u_2}} = 3.85 \quad \text{or} \quad \frac{u_2^2}{u_2^2 - 566} = 3.85 \quad \text{or} \quad u_2 = 27.67 \text{ m/sec}$

Further  $u_2 = \frac{\pi \cdot D_2 \cdot N}{60} \quad \text{or} \quad 27.67 = \frac{\pi \times D_2 \times 2,485}{60}$

or  $D_2 = 0.2125 \text{ m} \quad \text{or} \quad 21.25 \text{ cm} \quad \text{Answer}$

$\therefore$  Diameter of impeller is 21.25 cm as all impellers in a multistage pump are similar.



A—Length of Bowl, B—Length of Main Pipe, C—Length of Suction Pipe, D—Head above Ground Level, E—Inside Diameter of Column Pipe, F—Outside Diameter of Bowl, G—Height of Supporting Frame, H—Head below Ground Level,  $H_1$ —Water Level while Pumping  $H_2$ —Depth of Delivery Connection below Ground Level,  $h$ —Strainer Length,  $g$ —Dia Strainer.

Fig 12.36 Deep-well or Vertical Turbine Pump

### 12.47 Deepwell Pump or Vertical Turbine Pump (refer Figs 12.36 and 12.37).

This is generally multi-stage pump, the number of stages depending upon the head required. The impellers are assembled in a group at the lower end of the pump column and shaft. All the impellers and at least three metres of suction pipe, with a strainer at the end, are placed below the water level. That is why such pumps are known as deepwell or borehole pumps. Water is conducted to the surface through the rising main pipe or column pipe, as it is sometimes called, which connects the impellers with the outlet. The impellers are keyed to a vertical pump shaft which is further coupled to a line shaft enclosed in a cover pipe with bronze bearings, placed at suitable intervals along the shaft, to prevent vibration and whip. The shaft is also aligned with these bearings. The bearings may be lubricated by oil introduced at the top of the cover pipe, or by water. When there is no cover pipe, water naturally passing through the column pipe lubricates the bearings which are then made from rubber. At the upper end of the line shaft is fitted a thrust bearings which carries the weight of all impellers and the shaft, and balances the thrust or impellers, caused by pressure head.

The pump is generally driven by direct coupled electric motor of vertical type, placed at the top of the line shaft, usually on ground level. The motor is properly protected against water. Sometimes, the pump is driven by a prime mover, generally a

diesel engine. In such cases a belt drive is employed for small and medium-sized pumps and toothed gearing for larger sizes.

The impeller used in a turbine pump is closed or semi open. Open impellers are seldom used. The impeller normally rotates with the blades bending backwards. If the direction of rotation is reversed, the discharge will fall, and the line shaft will be unscrewed at the upper end.

**Bypassing in Turbine Pumps**—When water leaves out from the discharge to the suction side of the impeller, the phenomenon is known as bypassing. Its effect is to lower the discharge, and hence the efficiency of the pump is reduced. Since the ordinary volute pump is horizontal, the tendency to bypass is less and it is further prevented by replacelable wear rings. Corrosion of impeller and guide vanes caused by impure water increases by passing and friction, thereby lowering the efficiency of the pump. Impellers and guide vanes should therefore be made of corrosion resistant metals. Erosion is the wear of impeller caused by sand particles in water. Besides corrosion and erosion, cavitation (resulting from a high velocity of flow) is another phenomenon detrimental to the metal.

**12.48 Special Purpose Pumps**—Special purpose centrifugal pumps are designed either to handle liquids with extra ordinary chemical or physical properties *e.g.* abnormal density or viscosity, or special duties *e.g.* fire-extinguishing. This last one is just an ordinary high head multi-stage pump.

The impellers of special purpose pumps have to be made of a special material or covered with a protective coating when required to handle chemicals. For instance, if a pumps is meant to raise acids, the impeller and whole inside surface or pipes, casing etc., should be coated with some acid-resisting substance *i.e.* stone.

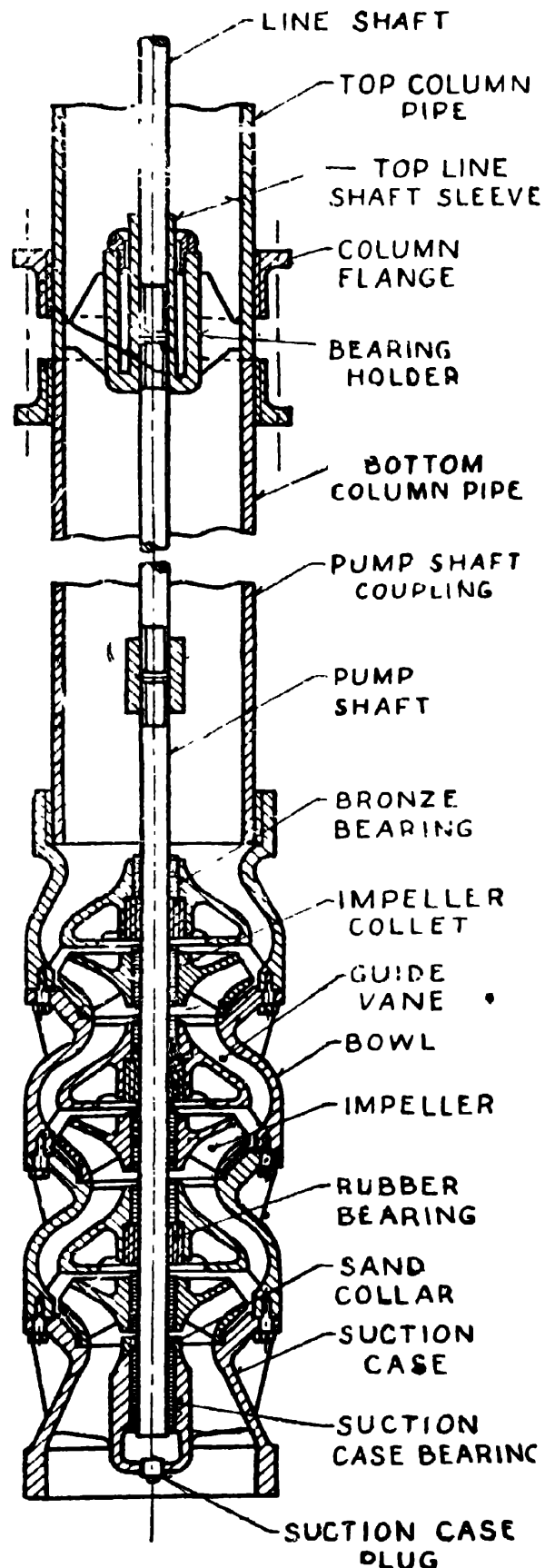


Fig 12.37 Longitudinal Section through a Three-Stage Bore Hole Centrifugal Pump with Shaft Guided in the Rising Main

**12.49 Pumps for Liquids of Special Densities :**

$$p = \gamma \cdot H$$

$\therefore$  Pressure generated  $\propto$  Density of liquid ( $H$  constant)

Also 
$$HP = \frac{\gamma \cdot Q \cdot H}{75} \text{ HP}$$

$\therefore$  Power consumed  $\propto$  Density of liquid ( $Q$  and  $H$  constant)

or 
$$HP = \frac{p \cdot Q}{75} \text{ HP} \quad \dots (12.54)$$

This equation shows that power required to deliver a certain quantity of liquid at a given pressure is independent of the density.

**Problem 12.14** A centrifugal pump handles a mixture of sand and water whose specific gravity is 1.08. The quantity of mixture to be pumped is 300 lit/sec against a total head of 12 m. Find the HP required by the pump if its overall efficiency is 75%. Find also the pressure in kg/cm<sup>2</sup> developed by the pump.

**Solution**

Specific gravity of the mixture = 1.08

$\therefore$  Specific weight =  $1,000 \times 1.08 \text{ kg/cm}^3$

$H = 12 \text{ m}$   $Q = 300 \text{ lit/sec}$   $\eta = 0.75$

Power required for the pump =  $\frac{\gamma \cdot Q \cdot H}{75 \times \eta} \text{ HP}$

$$1.08 \times 1,000 \times \frac{300}{10^3} \times 12 \times \frac{1}{75 \times 0.75} = 69 \text{ HP} \quad \text{Answer}$$

$$\begin{aligned} \text{Pressure developed in the pump, } p &= \gamma \cdot H = 1.08 \times 1,000 \times \frac{12}{100 \times 100} \\ &= 1.296 \text{ kg/cm}^2 \quad \text{Answer} \end{aligned}$$

**Problem 12.15** A centrifugal pump is required to handle 50 lit/sec of brine against a net pressure of 4 kg/cm<sup>2</sup>. Determine the head in metres of liquid and the HP required by the pump if its overall efficiency is 62%. Take the specific gravity of brine as 1.19.

**Solution**

$Q = 50 \text{ lit/sec}$

$p = 4 \text{ kg/cm}^2$

Sp gr = 1.19

$\eta = 62\%$

$p = 4 \times 10^4 \text{ kg/m}^2$

$\gamma = 1.19 \times 10^3 \text{ kg/m}^3$

$$H = \frac{p}{\gamma} = \frac{4 \times 10^4}{1.19 \times 10^3} = 33.6 \text{ m of brine} \quad \text{Answer}$$

$$\begin{aligned} \text{HP required by the pump} &= \frac{\gamma \cdot Q \cdot H}{75 \times \eta} \\ &= \frac{1.19 \times 1,000 \times 50 \times 10^{-3} \times 33.6}{75 \times 0.62} \\ &= 43 \text{ HP} \quad \text{Answer} \end{aligned}$$

**Problem 12.16** A centrifugal pump is required to handle 50 lit/sec of gasoline against a net pressure of 4 kg/cm<sup>2</sup>. Determine the head in m of liquid and the HP required by the pump, if its overall efficiency is 62%. Take the specific gravity of gasoline as 0.7.

**Solution**

$$Q = 50 \text{ lit/sec} \quad p = 4 \text{ kg/cm}^2 = 4 \times 10^4 \text{ kg/m}^2$$

$$\text{Sp gr} = 0.7 \quad \gamma = 0.7 \times 10^3 \text{ kg/m}^3 \quad \eta = 62\%$$

$$H = \frac{p}{\gamma} = \frac{4 \times 10^4}{0.7 \times 10^3} = 57.14 \text{ m of gasoline} \quad \text{Answer}$$

$$\begin{aligned} \text{HP required by the pump} &= \frac{\gamma \cdot Q \cdot H}{75 \times \eta} \\ &= \frac{0.7 \times 1,000 \times 50 \times 10^{-3} \times 57.14}{75 \times 0.62} \\ &= 43 \text{ HP} \quad \text{Answer} \end{aligned}$$

It is evident from the two preceding examples that the power required by the pump for a given discharge is independent of the specific weight of the liquid so long as the net pressure  $p$  generated is the same. The head in metre of liquid column, however, depends on the specific gravity.

**12.50 Essential data Required in Selection of Centrifugal Pumps—**

1. Number of units required.

2. Nature of the liquid to be pumped.

Is the liquid :

- (a) Fresh or salt water, acid or alkali, oil, gasoline, slurry, or paper stock.
- (b) Cold or hot and if not, at what temperature? What is the vapour pressure of the liquid at the pumping temperature?
- (c) What is the specific gravity of the liquid?
- (d) Is it viscous or non-viscous? If viscous, what is the viscosity?
- (e) Clear and free from suspended foreign matter or dirty and gritty? If the latter, what is the size and nature of the solids and are they abrasives? If the liquid is of a pulpy nature, what is the consistency expressed either in percentage or in its per cubic metre of liquid? What is the suspended material?
- (f) What is the chemical analysis, PH value etc.? What are the expected variation of this analysis? If corrosive, what has been the past experience, both with successful materials and with unsatisfactory materials?

3. Capacity—

What is the required capacity as well as the minimum and maximum amount of liquid the pump will ever be called upon to deliver?

**4. Suction Conditions—**

Is there :

- (a) A suction lift ? and how much ?
- (b) Or a suction head ? and how much ?
- (c) What are the length and diameter of the suction pipe ?
- (d) No. of bends and elbows in suction pipe.

**5. Discharge Conditions—**

- (a) What is the delivery head ? Is it constant or variable ?
- (b) What is the friction head ?
- (c) What is the maximum discharge pressure against which the pump must deliver the liquid ?
- (d) What is the length and diameter of delivery pipe ?

**6. Total Head—**

What will be total head ?

**7. Service —**

- (a) Is the service continuous or intermittent ? And for what duration ?
- (b) Is the pump to be installed in a horizontal or a vertical position ?  
If the latter ;
  - (i) In a wet pit ?
  - (ii) In a dry pit ?

**8. Power —**

- (a) What type of power is available to drive the pump and what are the characteristics of the power ? i.e., direction or rotation, speed of prime-mover and the horse-power required.
- (b) If the pump is to be driven by electric motor, state the type of current DC or AC (single phase or three phase), frequency in cycles per second, voltage and preferred speed in rpm.

9. What space, weight or transportation limitations are involved ?

**10. Location of installation—**

- (a) Geographical Location.
  - (b) Elevation above sea-level
  - (c) Indoor or outdoor installation.
11. Are there any special requirements or marked preferences with respect to the design, construction or performance of the pump ?

**What is a proposal ?**

Most manufacturers combine their recommendation and bid into a single *Document Called a Proposal*.

The usual proposal contains the following information.

Pump model no., class, type, construction details and materials, type of drive for which the pump is designed, performance curve, unit, weight, price etc.

Enclosed with a drawing of the pump and the catalogue.

To evaluate a proposal, it is necessary to review all the steps made in choosing a pump from a given set of hydraulic conditions. The steps are as under :

- (a) Sketch layout.
- (b) Capacity determination.

Job conditions fix the capacity required. For example, the maximum flow from the exhaust of a turbine, along with steam conditions determines the max. amount of cooling water needed at a given temperature. Following points are taken into consideration :

- (a) *Temperature* : Since liquid density changes with temperature therefore, it is of major importance to specify the temperature.
- (b) *Determining Liquid Flow* : Flow requirements of the system must be known. Minimum and maximum demands should be calculated.

Tables are provided for industrial water and steam requirement and the water amount gives roughly the capacity of the pump.

1. *Demand Variation* : Variations in the amount of liquid delivered generally influence the size of a pump. It gives the maximum output.

Excessive operation of the pump at capacities greater than the normal may lead to premature maintenance difficulties, and hence in such cases, use of two pumps instead of one are advisable.

2. *Capacity Choice* : Generally taking into account the possibility of failure of one pump, two pumps each taking the full demand are installed but when total demand is so low that it is impossible to choose two pumps which will operate in their most efficient ranges, a single unit may have to be substituted because two pumps otherwise would have higher initial cost and operating costs.

3. *Total Head*.

4. *Study Liquid Conditions*.

(a) *Viscosity*. Viscous liquids affect the performance of centrifugal pumps in the following manner :

- (i) The pump develops a lower head than when handling water.
- (ii) Pump capacity is reduced when moderate or high viscosity liquids are handled .
- (iii) Horsepower input required is higher.

*Volatile liquids* The principle problem encountered in pumping volatile liquids is that of net suction head (NPSH). The system must be such as to prevent vapourisation of the liquid in the suction pipe.

*Vapour Pressure* : When the pump suction pressure is so high that the packing will not function properly, a bleed off bushing can be fitted at the inlet end of the box.

*Chemicals* : (i) Corroding action,

5. *Choose class and type—*

Power Input = ...



*Horizontal Versus Vertical Pumps—*

Vertical pumps have advantages in the form of

- (i) Floor space required is less
- (ii) Required NPSH
- (iii) Priming and flexibility.

Horizontal pumps are used where corrosion, abrasion and maintenance are the main considerations.

**UNSOLVED PROBLEMS**

- 12.1 What are the advantages of a centrifugal pump over a reciprocating type ?  
(Jadavpur University)
- 12.2 Explain with sketches the working of a single-stage centrifugal pump.
- 12.3 Name the heads under which the centrifugal pumps are classified.
- 12.4 What is the difference between low lift, medium lift and high lift centrifugal pumps ?
- 12.5 Show by sketches the difference between the volute and the diffusion pump.
- 12.6 What are the functions of a volute casing of a centrifugal pump.  
(Jadavpur University)
- 12.7 What is the difference between single-stage and multi-stage pumps ?
- 12.8 What is a multi-stage centrifugal pump ? How does it function ?
- 12.9 What is the difference between radial, axial and mixed flow pumps ? Describe with the help of line diagrams.
- 12.10 What is the difference between axial flow pump and Kaplan pump ?
- 12.11 What is the difference between single suction and double suction pump ? State the advantages of double suction over single suction type ?
- 12.12 State the difference between closed, semi-closed and open impeller of a centrifugal pump.
- 12.13 What kind of impeller would you select for a pump required to handle paper pulp or molasses ?  
(AMIE)
- 12.14 Define specific speed of a centrifugal pump and derive an equation for the same.
- 12.15 What value of head and discharge will be used in determining the specific speed of multi-stage and double suction pump respectively ?
- 12.16 How does the specific speed of a turbine differ from that of a centrifugal pump ?
- 12.17 What is non dimensional factor  $K_s$  in case of centrifugal pumps ? State the difference between  $K_s$  and  $N_s$  for centrifugal pumps ?
- 12.18 State the specific speed of a radial flow, mixed flow, (Francis type), mixed flow (Screw type) and centrifugal pumps.
- 12.19. Show by means of different sketches how the ordinary centrifugal type of pump impeller is developed to an axial flow type. Write the approximate ranges of specific speed of the different shapes of impeller you draw.  
(AMIE)

- 12.20 Define static, manometric and gross head of a centrifugal pump. Indicate these heads in a line diagram.
- 12.21 Define overall efficiency of a pump.
- 12.22 State the different type of head losses which occur outside the pump itself. How will you determine such losses?
- 12.23 Define manometric efficiency of a centrifugal pump?
- 12.24 What are the different efficiencies of a centrifugal pump?
- 12.25 Differentiate between ideal, virtual and manometric head. State the influence of number of blades on virtual head.
- 12.26 Explain with reasons the advantages of setting back the vane angle at exit in the case of a centrifugal pump. (AMIE)
- 12.27 What is effect of number of impeller blades on the  $H-Q$  curve?
- 12.28 What is an axial thrust in centrifugal pump? State its causes. Describe a few methods to withstand such a thrust for (a) small pumps, (b) large pumps and (c) multi-stage pumps.
- 12.29 State the effect of variation of speed on discharge, head and power.
- 12.30 State the effect of change of diameter of impeller on discharge, head and power of a centrifugal pump.
- 12.31 What are model pumps? Under what circumstances are they required to be prepared?
- 12.32 What is meant by "Priming" of a pump? Describe some priming devices.
- 12.33 What are the two major self priming devices for centrifugal pumps? Explain one of them in detail.
- 12.34 Describe the cavitation phenomenon in centrifugal pump.
- 12.35 What is meant by Net Positive Suction Head (NPSH)?
- 12.36 How does the location of the centrifugal pump effect its suction head?
- 12.37 Why can the suction height of a pump not exceed certain limit? (AMIE)
- 12.38 How would you determine the horsepower of a motor if the horsepower of a pump is given?
- 12.39 Draw the line diagram of a multi stage pump giving the names of its main parts. How will you balance the axial thrust by fixing the impeller in the opposite directions?
- 12.40 What is a deep-well pump? What is its practical use?
- 12.41 Draw sectional view of a multiple stage deep-well pump.
- 12.42 Prove that the discharge of a centrifugal pump is independent of specific weight of the liquid it handles, so long as the net intensity of pressure generated is the same.
- 12.43 Describe main defects which one experiences while running a centrifugal pump. What are their remedies?
- 12.44 State the essential data, required in the selection of centrifugal pumps.

12.45 How do you make a 'Proposal' for a pumping installation ?

### Numericals

12.46 A multi-stage centrifugal pump raises 75 lit/sec of water against a total pressure of 40 kg/cm<sup>2</sup>. The pump shaft rotates at 725 rpm. Determine the type of impeller if the pump has four stages.

(14.9 rpm ; Slow speed radial flow type) (*Jadavpur University*)

12.47 A multi-stage pump running at 1,500 rpm delivers 1,200 lit/sec of water against a head of 35 m. A similar multi-stage pump has a specific speed of 30 rpm, find the number of stages.

(15 Impellers in parallel) (*UPSC*)

12.48 The following duties are to be performed by roto-dynamic pumps driven by electric synchronous motors—

(a) 15 m<sup>3</sup>/sec of water against 2 m head ;

(b) 0.75 m<sup>3</sup>/sec of oil (relative density = 0.8) against 0.8 kg/cm<sup>2</sup> pressure ;

(c) 0.12 m<sup>3</sup>/sec of water against 85 m head. Designs of pumps are available of which the specific speeds are 20, 115, 210 and 600. Which design and speed should be used for each duty ?

(250 rpm for  $N_s = 600$ , 600 rpm for  $N_s = 115$  ;  
1,500 rpm for  $N_s = 20$ )

12.49 A centrifugal pump having a overall efficiency of 75% delivers 1,820 litres of water per minute to a height of 18 m through a pipe of 10 cm diameter and 90 metres length. If  $f = 0.012$ , calculate the HP to drive the pump. (10 HP) (*AMIE - Civil*)

12.50 A pump is used to deliver 200 lit/sec of oil to a large storage tank through a 20 cm, clean steel pipe 120 m long. The elevation of surface of oil in the storage tank is 6 m above the delivery end of the pump. The pipe line includes a gate valve, 3 short radius elbows, and the outlet is submerged. Sp gr 0.87 and kinematic viscosity =  $4 \times 10^{-3}$  cm<sup>2</sup>/sec. Find pressure in kg/sq cm<sup>2</sup> at the delivery end of the pump.

Use the following values of  $k$  in  $h_f = k \cdot \frac{v^3}{2g}$  for the losses in valves :—

Gate valve = 0.19, short radius elbows = 0.9.

Determine the value of  $f$  from the relation  $4f = 0.0032 + \frac{C \cdot 221}{R_o^{0.837}}$

(2.55 kg/cm<sup>2</sup>)

12.51 In order to determine the efficiency of a centrifugal pump, the following observations were made :—

Pressure gauge reading on suction side = 2.5 m

Pressure gauge reading on delivery side = 125 m of water

Total water raised by the pump = 250 lit/sec

Total input to the pump = 580 HP

Find the efficiency of the pump.

(73.3%) (*Roorkee University*)

12.52 A centrifugal pump draws water from a sump through a vertical

15 cm pipe. The pump has a horizontal discharge pipe 10 cm diameter which is 3.5 m above water level in the sump. While pumping 35 lit/sec, gauges near the pump at entrance and discharge read  $-0.35 \text{ kg/cm}^2$  and  $+1.8 \text{ kg/cm}^2$  respectively. The discharge gauge is 0.5 m above the suction gauge. Determine the horsepower output of the pump. (10.82 HP)

- 12.53 A single stage centrifugal pump has an impeller of 250 mm diameter which rotates at 1,800 rpm and lifts 60 lit/sec to 25 m with an efficiency of 70%. Obtain the number of stages and diameter of each impeller of a similar multi-stage pump to lift 75 lit/sec to 175 m at 1,500 rpm.

(8 stages ; 280 mm dia) (*Panjab University*)

- 12.54 The impeller of a centrifugal pump has an outer diameter of 35 cm and runs at 1,000 rpm. The blades are bent at  $150^\circ$  to the direction of motion at discharge and the radial velocity of flow is constant through the impeller at 2.5 m/sec. The measured head across the pump is 20 m and the frictional resistance of the pump is estimated to be 2 m. Find the efficiency according to velocity triangles. (76.4%) (*London University*)

- 12.55 A centrifugal pump impeller runs at 950 rpm. Its external and internal diameters are 500 mm and 250 mm. The vanes are set back at an angle of  $35^\circ$  to the outer rim. If the radial velocity of water through the impeller be maintained constant as 2 m/sec, find the angle of the vanes at inlet, the velocity and direction of water at outlet and the work done by the impeller per kg of water.

( $9^\circ-9'$  ; 22 m/sec ;  $5^\circ-12'$  ; 55.6 kg-m)

- 12.56 A centrifugal pump impeller has an outside diameter of 200 mm and rotates at 2,900 rpm. Determine the head generated if the vanes are curved backward at  $25^\circ$  and the velocity of flow which is constant throughout the wheel is 3 m/sec. Assume hydraulic efficiency as 75%

Determine also the HP required to turn the impeller if the breadth of the wheel at outlet is 15 mm. Neglect the effect of vane thickness and mechanical friction and leakage losses.

(55.4 m ; 20.82 HP) (*Jadavpur University*)

- 12.57 A centrifugal pump of 1.2 m diameter runs at 200 rpm and pumps 1,880 lit/sec, the average lift being 6 m. The angle which the vanes make at exit with the tangent to the impeller is  $26^\circ$ , and the radial velocity of flow is 2.5 m/sec. Determine the useful horsepower and the efficiency. Find also the least speed to start pumping against a head of 6 m, the inner diameter of the impeller being 0.6 m. (150 HP ; 62.7% ; 200 rpm) (*Panjab University*)

- 12.58 The outside diameter of the vanes of a centrifugal pump is 50 cm. The vanes are curved back to make an angle of  $40^\circ$  with the periphery. The pump has no volute and is needed for a lift of 10 m. Calculate the proper speed and the hydraulic efficiency. By how much would the lift be increased if a whirlpool chamber of  $40^\circ$  were added and the pump runs at the calculated speed ?

Velocity of flow may be taken as  $= \frac{1}{8} \sqrt{2gH}$

(535 rpm ; 58.8% ; 5.4 m)

- 12.59 A centrifugal pump having impeller diameter 105 cm delivers 1,320 litres of water per minute at a total manometric head of 65 m when running at 750 rpm, the measured shaft HP being 1,370. The impeller vanes are backward curved and make an angle of  $20^\circ$  to the tangent and the effective circumferential area at outlet is  $0.35 \text{ m}^2$ .

Assuming leakage loss through clearance rings 3% of the discharge, external losses including disc friction, bearing and gland friction 50 HP, determine--

- the theoretical head which could be developed if the net HP given to the water flowing through the impeller was converted to head without hydraulic losses ;
  - the theoretical head assuming an infinite number of impeller vanes ;
- the hydraulic efficiency and the overall efficiency.

Explain why the head obtained from (a) is less than that from (b) and state what design factors influence this ratio.

- (a) 73.5 m of water ; (b) 128.2 m of water ; head obtained in (a) is less than (b), because the impeller has got a number of blades in (a) ; (c)  $\eta_{\text{overall}} = 83.5\%$ ,  $\eta_{\text{hyd}} = 86.8\%$   
(AMI Mech E—Lond)

- 12.60 A centrifugal pump runs at 750 rpm and the difference in surface levels at suction and delivery, together with the pipe friction losses is 15 m. The relative velocity of water at impeller exit is inclined backwards at  $45^\circ$  to the tangent. The velocity of flow through the impeller, and in suction and delivery pipes is 2 m/sec. A manometric efficiency of 77% is expected and the water enters the impeller without whirl.

Find the diameter of the impeller and the fraction of kinetic energy at discharge from the impeller which must be recovered as pressure if other losses are neglected.

If the impeller diameter at inlet is 0.4 times the diameter at outlet, and entry is through one side only, find the quantity of water the pump can discharge for these conditions, and the axial width of the impeller at inlet and exit. Find also the vane angle at inlet. If, when running at this speed with the discharge valve closed, 2 HP is required to drive, find the probable HP required at the given discharge, and the overall efficiency.

(378 mm ; 0.445 ; 34.4 lit/sec ; 37 mm ; 14.8 mm ;  $19^\circ$  ; 10.82 HP ; 63.5%)

- 12.61 Following information is given about a 950 rpm centrifugal pump having 600 mm outside dia.

Radial velocity (constant) = 5.5 m/sec

$$H_t = \frac{1}{g} \cdot u_2 \cdot v_2 \quad \dots(1)$$

$$H_a = \frac{1}{g} u_2 \cdot v_2 \quad \dots(2)$$

$$H_t = H_a (1+p) \quad \dots(3)$$

where equations (1) and (2) represent respectively the ideal and actual work on the runner vanes per kg of water  $v_2$  and  $v_2'$  are the ideal and actual whirl components and  $u_2$  the tangential velocity and  $p$  is a factor relating the difference between the actual and ideal conditions. If the energy received per kg of water is

15 m ( $H$ ) and the hydraulic efficiency  $\frac{H}{H_a} = 0.86$ , construct the

ideal and actual outlet velocity triangles and find thereby the outlet blade angle and ideal and the actual whirl components. Value of  $p$  can be taken as 0.3.

(5.78 m/sec, actual ; 7.45 m/sec, ideal ;  $13^\circ 48'$ )

(Panjab University)

- 12.62 A centrifugal pump which runs at 1,000 rpm is installed so that its centre is 2m above the water surface in the sump. It delivers water to a point 15 m above its centre. The friction loss in the suction pipe is  $0.18Q^2$  and that in the delivery pipe is  $0.17Q^2$  where  $Q$  is measured in  $\text{m}^3/\text{sec}$ . The outside diameter of impeller is 30 cm and its width at exit is 1.5 cm. The blades occupy 5% of the circumference and are backward curved at  $30^\circ$  to the tangent. The flow is radial at inlet and the radial component of velocity remains unchanged through the impeller. Assuming that 50% of the velocity head of the water leaving the impeller is converted to pressure head in the volute and that friction and shock losses in the pump, the velocity heads in the suction and delivery pipes are all negligible. Calculate the discharge and the manometric efficiency of the pump.

(5  $\text{m}^3/\text{sec}$  ; 66.2%)

- 12.63 A centrifugal pump develops an effective head of 15 m, its impeller exit diameter is 0.25 m delivering  $0.06 \text{ m}^3/\text{sec}$ . The blades of the impeller are backward curved making an angle of  $30^\circ$  with the tangent at outlet. The effective width at output of impeller is 25 mm. Diffuser vanes are fitted recovering 2.4 m of head. It may be assumed that hydraulic losses in the impeller amount to 1.8 m, that the radial velocity remains constant and that there is negligible velocity of whirl at inlet. Calculate the speed, the hydraulic efficiency and the best angle for the entrance to the diffuser blades and the head loss in the diffuser blades. Assume that the velocity in the suction and delivery pipes is 3.7 m/sec.

- 12.64 A new pumping station is to be installed to supply water to a town with 13,000 inhabitants. The town is situated at a height of 2,000 m above mean sea level. The water consumption per head is 180 litres a day. Half of the total consumption is to be supplied in 8 hours. The total static head is 60 m and the head losses in the suction and the delivery sides are 34 m. Select a suitable pump for the purpose and find the HP of the driving motor for it. Assuming a suitable motor speed, determine the specific speed of the pump impeller, as well as its inlet and outlet diameter. Assume the pump and motor efficiencies.

(Water HP = 51, with pump efficiency of 75% motor HP = 72, 5-stage pump,  $N_s = 32.2$  with 1,450 rpm, impeller diameters 258 mm and 129 mm with speed ratio of 1.02) (AMIE)

- 12.65 A single-stage centrifugal pump was designed to give a discharge against a manometric head of 25 m with an impeller dia of 375 mm. After manufacture the pump was tested and it was found that when the amplified discharge was flowing through the pump, the head generated was 20 m. Now the shaft speed cannot be altered as the pump is coupled to an induction motor having a speed of 1,450 rpm.

Suggest the method of getting the design head. If certain corrections are necessary, calculate them.

(Dia of impeller should be increased from 375 mm to 420 mm)  
(Punjab University)

- 12.66 A centrifugal pump was manufactured in order to couple directly to a 15 HP electric motor running at 1,450 rpm, delivering 40 lit/sec against a total head of 20 m. However a customer likes to use diesel engine in place of motor. The speed of the engine is 1,000 rpm and the pump is to be coupled directly to the engine shaft. Find the probable discharge and the head developed by the pump. Also specify the HP of the engine you would employ.

(27.6 lit/sec ; 9.53 m ; 4.91 HP, 5 HP engine)

(Jadavpur University)

- 12.67 It is proposed to build a three-stage centrifugal pump to handle 70 lit/sec of water at a speed of 900 rpm and under a total manometric head of 70 m. The vanes are to be radial at inlet and are to be curved backward at exit at an angle of  $45^\circ$ . Assuming a manometric efficiency of 84% and a mechanical efficiency of 75% and considering that vane thickness accounts for 8% of the circumferential area, determine the probable external diameter and width of each impeller and the HP input necessary. The velocity of flow may be assumed constant at 3 m/sec.

(18 mm, 89 HP) (Roorkee University)

- 12.68 A turbine pump has six impellers of 25 cm diameter with vanes curved backwards at exit at an angle of  $45^\circ$  to the tangent. At full capacity the velocity of the water relative to the vanes as it leaves the impellers is 2.5 m/sec. Calculate the total head against which the pump will deliver water when it is running at a speed of 750 rpm, assuming that 50 per cent of the kinetic energy of the water is converted into pressure energy in the diffusers.

(50 m of water) [AMI Mech E (Lond)]

A multi-stage boiler feed pump is required to pump 110,000 kg of water per hour against a pressure difference of  $30 \text{ kg/cm}^2$  when running at a speed of 2,900 rpm. The density of pre-heated feed water is  $960 \text{ kg/m}^3$ .

If all the impellers are identical and the specific speed per stage is not to be less than 20, find—

(a) the least number of stages and the head  $h$  per stage.

(b) the diameters of the impellers assuming the peripheral velocity  $= 0.96\sqrt{2gh}$

(c) the shaft HP required to drive the pump, assuming an overall efficiency of 78%.

[(a) 75 m per stage ; (b) 242 mm ; (c) 157 HP] (Delhi University)

- 12.70 A multistage centrifugal pump is to be designed to deliver 750 lit/sec of water against a manometric head of 60 m. There are to be four equal impellers keyed to the same shaft which has a speed of 350 rpm. The vanes are curved back so that the direction of relative velocity of discharge makes an angle of  $120^\circ$  with the direction of the corresponding peripheral velocity, and the impeller is surrounded by guides. Assume that the water enters the vane passages in a radial direction, that the velocity of flow through the impeller is 0.27 of the peripheral velocity, and that the losses in the pump amount to one-third of the velocity head at discharge from the impeller. Find—

- (a) the outer dimensions of the impeller,
- (b) the manometric efficiency,
- (c) the angle of guides.

$[D_2 = 780 \text{ mm} ; B_2 = 80 \text{ mm} ; 85.4\% ; 17^\circ - 45']$

(London University)

- 12.71 A multi-stage centrifugal pump delivers 240 kg of water per sec. against a manometric head of 60 m. The pump consists of four equal impellers keyed to the same shaft, which rotates at 300 r.p.m. The impellers are fitted with vanes which are curved back so that the direction of the relative velocity at discharge makes an angle of  $135^\circ$  with the direction of the corresponding peripheral velocity and the impeller is surrounded by guides. Assume that water enters the vane passage radially, that the velocity of flow through the impeller at outlet is 0.3 times the outlet peripheral velocity and the losses in the pump amount to  $\frac{1}{4}$  times the velocity head at discharge from impeller. Calculate

- (a) Manometric efficiency ; (b) Angle of guides ; and

- (c) Outer dimensions of the impeller.

(AMIE)

- 12.72 A centrifugal pump running at 750 rpm discharges water at a rate of 56 lit/sec against a head of 15 m. Find the discharge and the head pumped if the same pump is running at a speed of 1,500 rpm.

(112 lit/sec ; 60 m) (Delhi University)

- 12.73 A centrifugal pump 20 cm in diameter was found to be most efficient when delivering 5,600 lit/sec of water at 1,200 rpm, against a head of 6 m. A similar pump is required to deliver 1,400 lit/sec at 700 rpm. Calculate the diameter of second pump and head delivered by it.

(AMIE)

- 12.74 It is required to predict the performance of a large centrifugal pump from that of a scale model one-quarter the diameter. The model absorbs 20 HP when pumping under the test head of 6 m at its best speed of 400 rpm. The head for the full scale pump is 20 m. At what speed should it be driven, what power will be required to drive it and what will be the ratio of the quantities discharged by the prototype pump and the model ?

(AMI Mech E—Lond)

- 12.75 A centrifugal pump is required to lift  $0.7 \text{ m}^3/\text{sec}$  of water against a head of 5 metres. A model is made which delivers  $0.028 \text{ m}^3/\text{sec}$ , the efficiency being the maximum in both cases. Find the specific



speed and estimate the speed and diameter of the impeller of the prototype pump if the diameter of the impeller of the model pump is 22.5 cm.

- 12.76 A centrifugal pump draws oil of specific gravity 0.78 at a temperature of 200°C from a closed tank in which the pressure is 6 kg/cm<sup>2</sup> gauge. The vapour pressure of oil at the above temperature is 6.4 kg/cm<sup>2</sup> absolute. The pump is located at 4 m above the oil level in the tank. If the pump installed at an elevation of mean sea level, find the available suction head. Assume the frictional losses in the suction pipe equal to 1 m of oil.  
(3.8 m of oil). (AMIE)
- 12.77 A fluid is to be pumped 800 m, through a pipe 200 mm diameter. Determine the HP required to pump one tonne per minute, when the kinematic viscosity is 20 cm<sup>2</sup>/sec and the density of fluid is 915 kg/m<sup>3</sup>.  
(66.67 HP) (Roorkee University)
- 12.78 In a long pipe-line carrying crude oil, it is usual to instal several pumping stations spaced at intervals along the line, rather than to rely on a single station at the pipe inlet. Why is this? What are the advantages of multiple stations? Illustrate your answer by sketching comparative hydraulic gradients.

A horizontal pipe-line 380 km long and 600 mm diameter is to carry 210,000 barrels per day of crude oil of S.G. 0.89. Its viscosity is such that in the formula  $p = \frac{4fl}{d} \times \text{velocity head}$ , the value of the frictional co-efficient  $f$  is 0.007. It is stipulated that at each pumping station, the pressure difference generated is not to exceed 40 kg/cm<sup>2</sup>. How many identical stations would be needed, and what should be the power delivered to the oil in each one?

(Note : 1 barrel of oil = 0.159 m<sup>3</sup>)

(4 stations ; 1,960 HP or 2,000 HP) (AMI Mech E, Lond)

## Some More Pumps and Water Lifting Devices

### 13.1 Propeller Pumps or Axial Flow Pumps (refer Fig 13.1)

As already explained in Art 12.9 c the propeller or axial flow pumps are specially designed for large capacities and low heads, generally under twelve metres. This pump is often called propeller pump because its impeller somewhat resembles a marine propeller. The axial flow impeller has evolved from the radial type (refer Art 12.14 and Fig 12.10). The radial flow pump (i.e., the ordinary centrifugal pump) develops the liquid head by centrifugal action as distinguished from the hydrodynamic "lifting" (refer Art 10.21) produced by the vanes of an axial flow or propeller pump. The specific speed of the impeller of a propeller pump is 110 to 500 units (refer Table 12.1). Axial flow pumps are generally designed to operate with the shaft vertical, mainly for the sake of compactness.

Propeller or axial pumps are the converse of propeller or Kaplan turbines (refer Chapter 7). When the pump is required to operate with a high percentage variation in head, the efficiency of fixed blade impeller is low at part overloads. To overcome this difficulty, adjustable blade impeller has been developed. Pumps so equipped are sometimes called *Kaplan* pumps. (refer Fig 13.2) The discharge of a Kaplan pump can be regulated by adjusting the position of impeller blades (refer Fig 13.3).

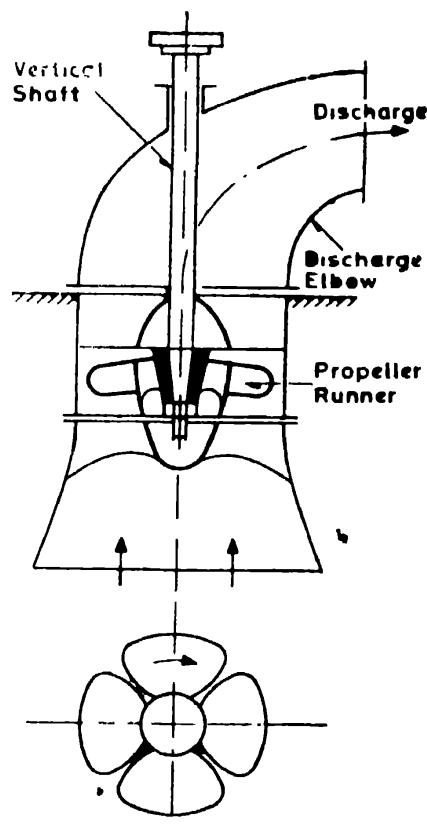


Fig 13.1 Section Through a Propeller Pump, Vertical Closed Type

Manufactured by  
Sulzer Bros Ltd., Switzerland.

**Problem 13.1** Find the main dimensions of a propeller pump which is required to deliver 210 lit/sec of water against an average head of 2.75 m.

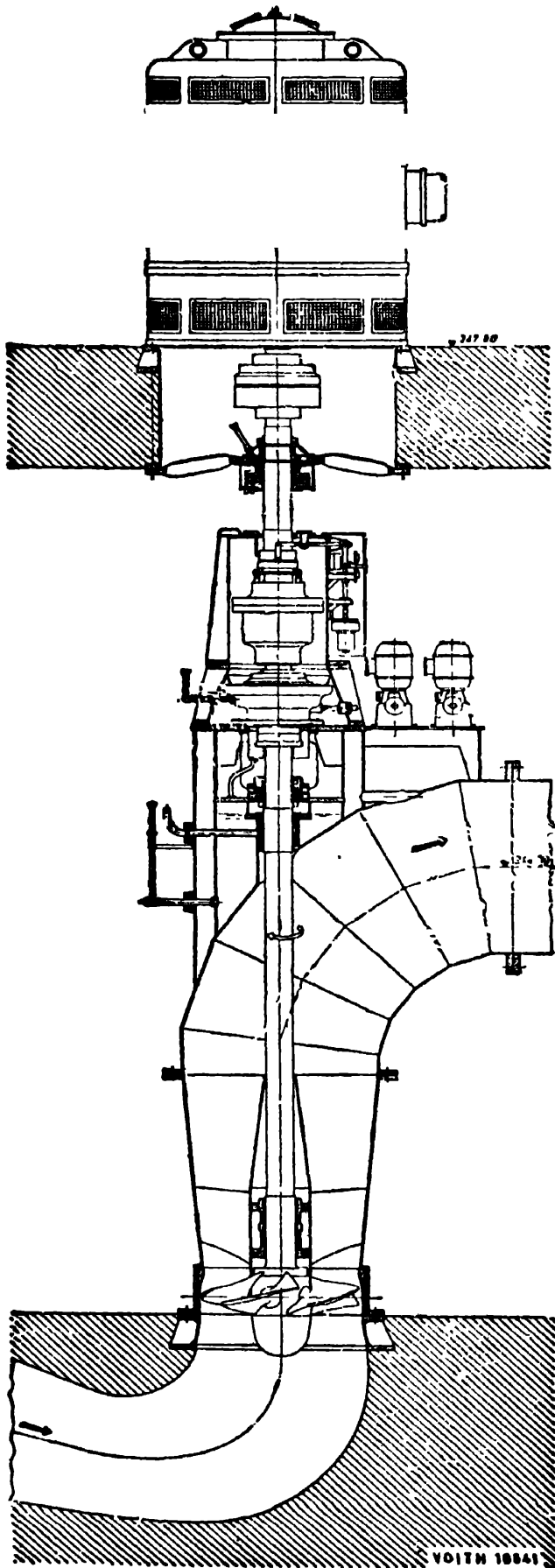


Fig 13.2 Kaplan Pump  
 $H_{\text{mano}} = 7.5 \text{ m}$ ;  $Q = 3.5 \text{ m}^3/\text{sec}$ ;  $P = 425 \text{ HP}$   
 $N = 580 \text{ rpm}$   
 Manufacturers : Voith, Heidenheim

Assume specific speed of the pump as 200 units, speed ratio 2.1 and flow ratio 0.45.

### Solution

$$Q = 210 \text{ lit/sec}; H = 2.75 \text{ m};$$

$$N_s = 200 \text{ units};$$

$$K_{u_s} = 2.1; K_{v_{m_s}} = 0.45$$

Specific speed of the pump is

$$\text{given by } N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}}$$

$$\text{or } N = \frac{200 \times 2.75^{\frac{3}{4}}}{\sqrt{\frac{210}{1,000}}}$$

$$= \frac{200 \times 1.655 \times 1.288}{0.457}$$

$$= 935 \text{ rpm}$$

In order to be able to couple the pump with an induction type electric motor, adopt a speed of **960 rpm** Answer

Peripheral velocity of impeller at outlet

$$u_2 = K_{u_s} \sqrt{2gH}$$

$$= 2.1 \sqrt{2 \times 9.81 \times 2.75}$$

$$= 15.4 \text{ m/sec}$$

$$\text{But } u_2 = \frac{\pi \cdot D_2 \cdot N}{60}$$

$\therefore$  Impeller outlet diameter  $D_2$

$$= \frac{60 \cdot u_2}{\pi \cdot N} = \frac{60 \times 15.4}{\pi \times 960}$$

$$= 0.305 \text{ m or } 305 \text{ mm}$$

Impeller inlet diameter  $= D_1$

$$= D_2 = 305 \text{ mm} \text{ Answer}$$

Velocity of flow  $v_{m_s}$

$$= K_{v_{m_s}} \sqrt{2gH}$$

$$= 0.45 \sqrt{19.62 \times 2.75}$$

$$= 3.305 \text{ m/sec}$$

$$Q = 210 \times 10^{-3} = 0.210 \text{ m}^3/\text{sec}$$

Assume 4 blades, each 10 mm thick,

$$v_{m_s} = \frac{Q}{\text{area of flow}}$$

$$= \frac{Q}{\frac{\pi}{4} [D_2^2 - n \cdot D_b^2]}$$

$$\text{where } n = \frac{\text{diameter of boss}}{\text{outlet diameter of impeller}} = \frac{d}{D_2}$$

$$\therefore 3.305 = \frac{0.210}{\frac{\pi}{4} \times 0.305^2 \times (1 - n^2)}$$

$$\text{or } n^2 = 1 - \frac{0.210 \times 4}{3.505 \times \pi \times 0.305^2} = 1 - 0.878 = 0.122$$

$$\text{or } n = 0.35$$

$$\therefore \text{Diameter of boss} = 0.35 \times 305 = 107 \text{ mm} \quad \text{Answer}$$

Horsepower required to drive the pcmp

$$= \frac{\gamma \cdot Q \cdot H}{75 \times \eta_t} = \frac{1,000 \times 0.210 \times 2.75}{75 \times 0.695} = 11.1 \text{ HP} \quad \text{Answer}$$

(assuming pump overall efficiency as 69.5%)

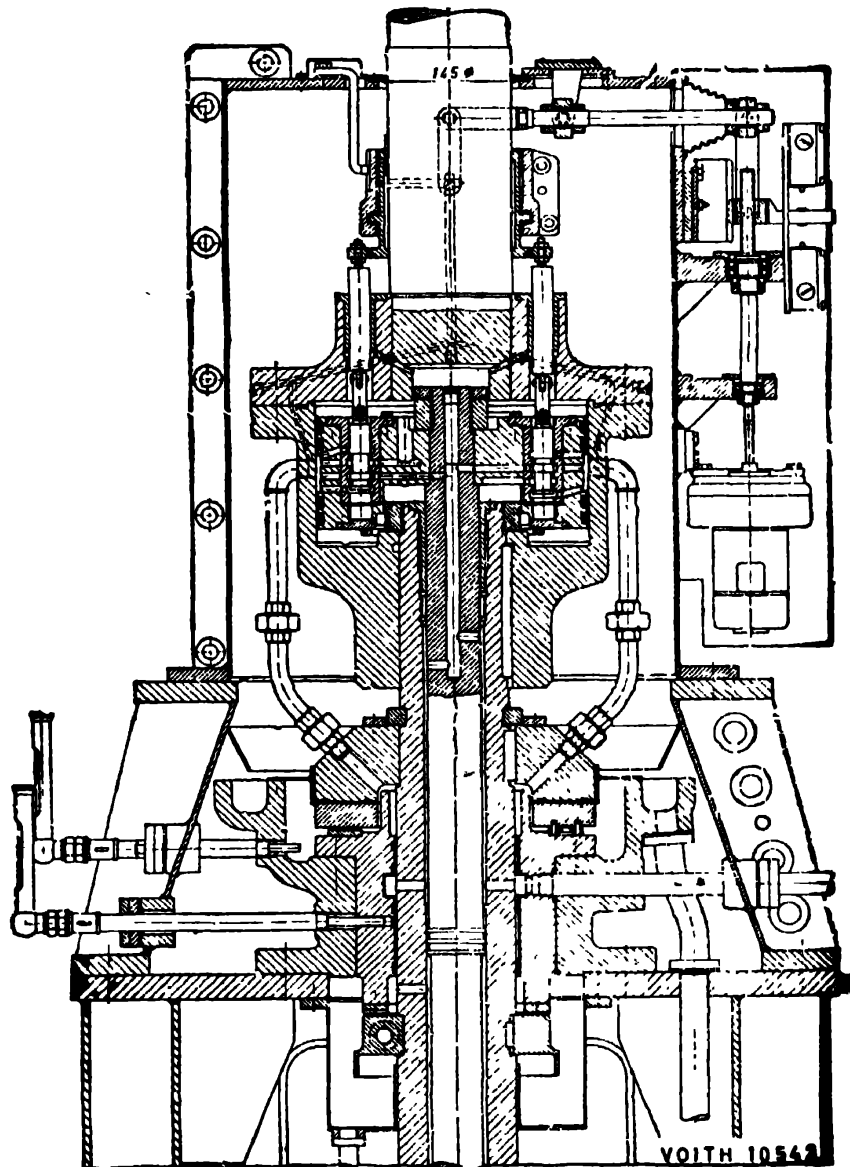


Fig 13.3 Details of Voith Kaplan Pump Impeller Vanes Rotating Mechanism

**13.2 Cavitation in Propeller Pumps**—Propeller or axial flow pumps have a limited suction lift, about one metre of liquid. The impeller is therefore located within the suction limit, very close to the pump inlet.

Unlike radial pumps the axial flow pump cannot be operated throughout the range of performance curve. Operation must be confined to the range of maximum efficiency otherwise it becomes noisy, and there is danger of cavitation (refer Art 7.17). Cavitation results from the same cause in turbines as well as in pumps and Thoma's co-efficient  $\sigma = \frac{H_b - H_s}{H}$  derived for turbines is also applicable to pumps. Modern trend, however, is to introduce another criterion besides, Thoma's co-efficient  $\sigma$ , for pumps.

Net positive suction head = excess of absolute suction head over vapour pressure

Symbolically:  $H_{sv} = H_b - H_s$

Then 
$$\sigma_{crit} = \frac{H_{sv}}{H} \quad \dots(13.1)$$

Specific speed of a centrifugal pump is given by

$$N_s = \frac{N \cdot \sqrt{Q}}{H^{\frac{3}{4}}}$$

$$H = \frac{N^{\frac{4}{3}}}{N_s^{\frac{4}{3}}} \cdot Q^{\frac{2}{3}} = \frac{\text{const}}{N_s^{\frac{4}{3}}} \quad \dots(13.2)$$

Substituting this value of  $H$  in (13.1)

$$\sigma_{crit} = \frac{H_{sv}}{\text{const}} \cdot N_s^{\frac{4}{3}} \quad \dots(13.3)$$

$\therefore$  For two pumps

$$\frac{(\sigma_{crit})_2}{(\sigma_{crit})_1} = \left( \frac{N_{s2}}{N_{s1}} \right)^{\frac{4}{3}} \quad \dots(13.4)$$

Relationship of  $\sigma$  and  $N_s$  can be known empirically. Thus

$$\sigma = K \cdot N_s^{\frac{4}{3}} \quad \dots(13.5)$$

where the co-efficient  $K$  depends upon the type of pump and hydraulic efficiency.

H.H. Anderson gave the following empirical solution for single suction pump—

$$\sigma = \frac{8.8 \times 10^{-4}}{\eta_Q^3} \cdot N_s^{\frac{4}{3}} \quad \dots(13.6)$$

Wislicenus and Karassik gave the value of  $K$  in Eqn 13.5 as  $12.2 \times 10^{-4}$  for single suction pumps and  $7.7 \times 10^{-4}$  for double suction pumps.

Wislicenus\* introduced another cavitation criterion, known as Suction specific speed or cavitation specific speed,

$$s = \frac{N \cdot \sqrt{Q}}{H_{sv}^{\frac{3}{4}}} \quad \text{where } H_{sv} = \sigma_{crit} \cdot H \quad \dots(13.7)$$

\*Trans, ASME Vol 61, 1939, p. 17.

The relation between the factors  $s$ ,  $N_s$  and  $\sigma_{crit}$  can be derived from equations (13.3) and (13.7)

$$\frac{N_s}{s} = \sigma_{crit} \quad \dots(13.8)$$

Stepanoff † gave the value of  $s$  as 167.5 for single suction pumps and 237 for double suction pumps. The maximum value of  $s$  is attained as 300 for single suction and 400 for double suction pumps. The higher values of  $s$  are obtained for more rational construction and higher standard of workmanship for the pumps.

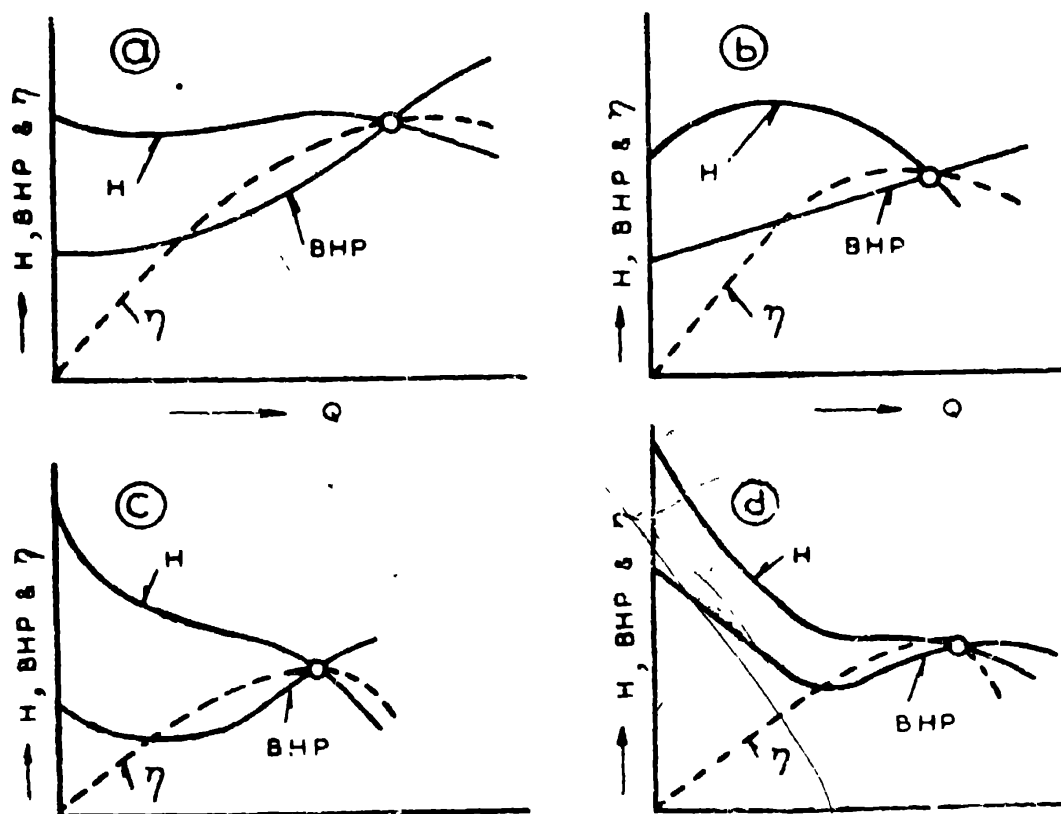


Fig 13.4 Typical Characteristic Curves (a) Single Stage Volute Pump (b) Multiple Stage Turbine Pump (c) Mixed Flow (Screw) Pump (d) Axial Flow (Propeller) Pump

**13.3 Characteristic Curves of Axial Flow Pumps**—Typical characteristics of an axial flow pump are shown in Fig 13.4d. It will be noticed that the head-discharge curve has a steeper slope than for a centrifugal pump of radial type. Unlike the latter, the BHP of an axial flow pump increases with head. Rising up of the BHP-discharge curve towards the left suggests that the motor may become overloaded if the lift were to be raised above the head corresponding to the maximum efficiency. The pumping set is, therefore, provided with a device to stop the motor before overloading. The efficiency-discharge curve indicates a maximum efficiency at less than half the shut off valve (*i.e.* when  $Q = 0$ ) and fall in efficiency for higher and lower heads.

Fig 13.4 a, b and c show the typical characteristic curves of a single-stage volute pump, multiple-stage turbine pump and mixed flow (screw type) pump respectively, which could be compared with the characteristic curves of a propeller pump (refer Fig 13.4 d).

\*Centrifugal and Axial Pumps by A. J. Stepanoff, (John Wiley & Sons), p. 268.

The speed of an axial flow pump may be varied widely without effecting efficiency or head. Since the discharge varies directly with speed (refer Eqn 12.51), variation of speed may be employed for regulating discharge.

**13.4 Uses of Propeller Pumps**—Propeller or axial flow pumps are widely employed for all types of low head pumping jobs such as land drainage, flood control, storm water disposal, irrigation, condenser, circulation in power plants, etc. This pump, however, is not satisfactory for developing pressures at throttled discharge.

**13.5 Screw Pumps or Mixed Flow Pumps** (refer Fig 12.7 b, 12.10 and 13.5)—Mixed flow pumps are often called angular or diagonal

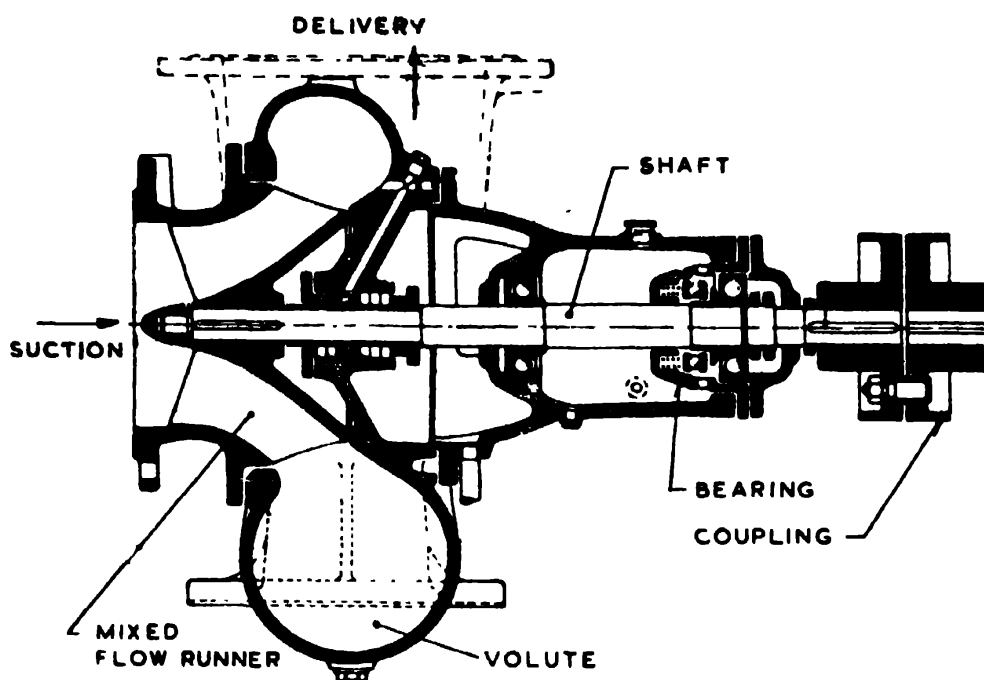


Fig 13.5 Section through a Screw (Mixed Flow) Horizontal Type Pump Manufactured by Sulzer Bros, Winterthur (Switzerland)

flow, or semi-axial pumps because the water flows through the impeller axially as well as radially. Like the axial flow type, these are also used for low head upto 20 m and high discharge, and have a limited suction lift. The specific speed of mixed flow impeller varies from 80 to 160 (refer Table 12.1). The head of liquid is developed partly by centrifugal action and partly by lifting action of vanes. Basically, these pumps have a construction similar to that of Francis or mixed flow turbines, but the operation is just the reverse. Typical characteristics of a mixed flow (screw type) pump are shown in Fig 13.4 c. The main difference between an ordinary centrifugal pump and screw pump is that the latter has an open impeller with blades of double curvature and the back shroud is generally inclined to the axis of the shaft.

Such pumps are also known as helical or helicoidal pumps.



Fig 13.6 Sulzer Screw Pump Delivering 14,000 lit/sec against a Head of 3.28 m

**Problem 13.2** Two centrifugal pumps A and B, each single stage, single suction, mixed flow type, when tested separately gave the following performance results—

**Pump A**

| <i>Q</i> lit/sec | 0    | 75   | 150  | 225 | 300  | 375 | 450 |
|------------------|------|------|------|-----|------|-----|-----|
| <i>H</i> in m    | 11.6 | 11.4 | 10.8 | 10  | 9.15 | 8.2 | 7   |

**Pump B**

| <i>Q</i> lit/sec | 0  | 75 | 150  | 225 | 300  | 375 | 450 |
|------------------|----|----|------|-----|------|-----|-----|
| <i>H</i> in m    | 18 | 17 | 15.5 | 14  | 11.9 | 9   | 4.9 |

Draw *H* vs *Q* curve of each pump. If the two pumps work in parallel, draw the combined *H* vs *Q* curve. Find the BHP of the combined pumping set when the system is delivering 700 lit/sec. Take the efficiency of the combined set as 65%. (UPSC)



**Solution**

$H$  vs  $Q$  curves of each pump is drawn in Fig 13.7. For pumps

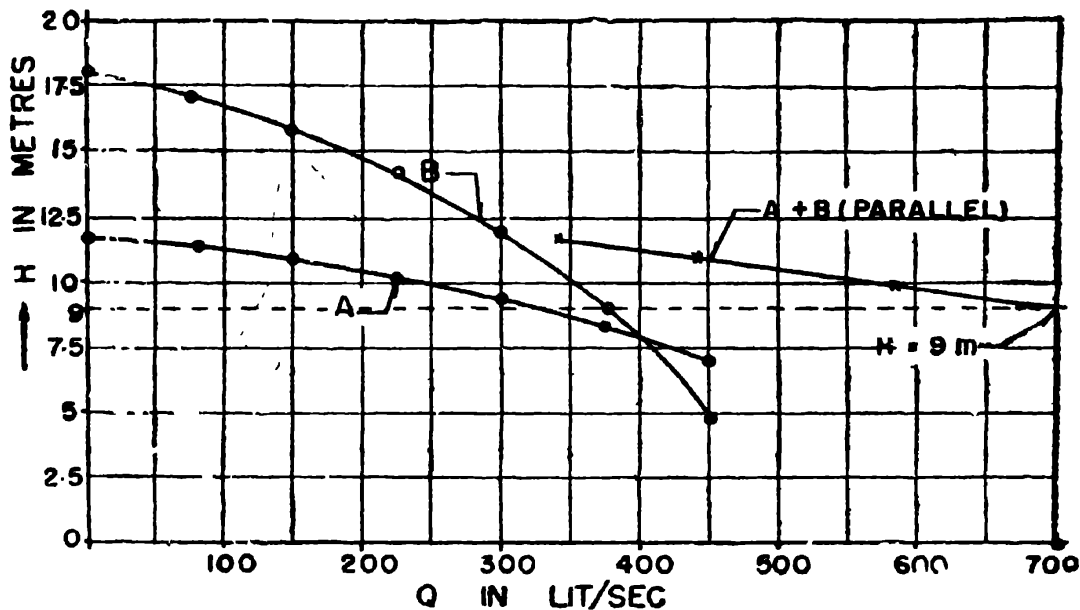


Fig 13.7  $Q$  vs  $H$  Curves of Two Mixed Flow Pumps Working in Parallel  
working in parallel add the  $Q$ 's of the two pumps at the same heads, thus—

| $H$ in metres | $Q$ in lit/sec |        | Total $Q$ ( $A+B$ )<br>lit/sec |
|---------------|----------------|--------|--------------------------------|
|               | Pump A         | Pump B |                                |
| 11.5          | 30             | 310    | 340                            |
| 11.0          | 120            | 320    | 440                            |
| 10.0          | 235            | 345    | 580                            |
| 9.0           | 325            | 375    | 700                            |

For the combined set, the head  $H$ , corresponding to  $Q = 700$  lit/sec is 9 m (from the new curve)

$$\therefore \text{BHP of the combined set} = \frac{(\gamma \cdot Q) \cdot H}{75 \eta} = \frac{700 \times 9}{75 \times 0.65} = 129 \text{ HP} \quad \text{Answer}$$

**OTHER WATER LIFTING DEVICES**

**13.6 Jet Pump** (Refer Figs 13.3 *a* and *b*)—Steam or water at a high pressure is forced through a fine aperture nozzle, thereby converting most of the pressure energy into kinetic form. It results in a lowering of pressure which causes suction to take place. Probably a part of the suction is due to the lowering of pressure resulting from condensation of steam.

Steam is generally used in jet pumps meant to feed boilers. The nozzle can lower the pressure to three fourths of atmospheric pressure i.e.

about 2.5 m suction head can be obtained. Steam also serves to preheat the water fed to the boiler.

With water at high pressure, a more perfect vacuum can be produced so that a suction lift of 5.5 to 6 m can be obtained. The jet should be near the surface of water. Several jets may be employed if a large quantity is to be pumped. This arrangement is often used in mines and also for pumping oil or petroleum. The capacity of a jet pump ranges upto about 50 lit/sec.

If  $Q_1$  be the rate of flow from supply reservoir and  $Q_s$  the quantity sucked per second, the delivery  $Q = Q_1 + Q_s$ .

If  $H_1$  be the height of supply reservoir above the jet,  $H_s$  the suction head and  $H_d$  the delivery head, then efficiency of the jet pump is given by—

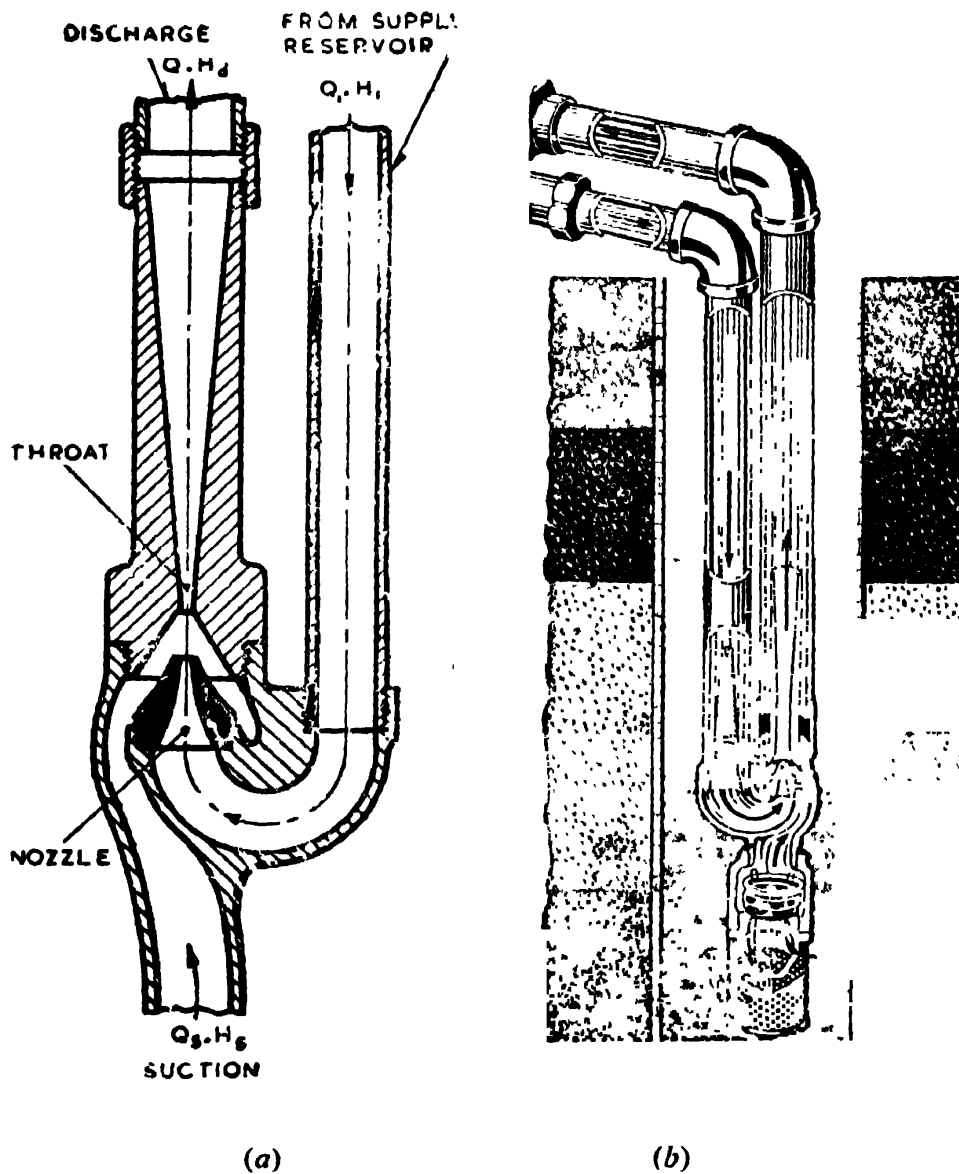


Fig 13.8 Jet Pump

$$\eta = \frac{Q_s (H_d + H_s)}{Q_1 (H_1 - H_d)} \quad \dots(13.9)$$

**Problem 13.3** A jet pump fitted 2.5 m above the suction reservoir and 18 m below the supply reservoir lifts water through a total height of 2.67 m. Determine the efficiency of the jet pump when it delivers 7.5 lit/sec of water while using 2.75 lit/sec from the supply reservoir.

**Solution**

$$H_1 = 18 \text{ m}$$

$$H_s = 2.5 \text{ m}$$

$$H_s + H_d = 2.7 \text{ m}$$

$$Q = 7.5 \text{ lit/sec}$$

$$Q_1 = 2.75 \text{ lit/sec}$$

$$Q_s = Q - Q_1 = 7.5 - 2.75 = 4.75 \text{ lit/sec}$$

$$\begin{aligned} \text{Jet pump efficiency } \eta &= \frac{Q_s (H_d + H_s)}{Q_1 (H_1 - H_d)} \\ &= \frac{4.75 \times 2.7}{2.75 \times (18 - 0.2)} = 0.262 \\ &= 26.2\% \quad \text{Answer} \end{aligned}$$

or

**13.7 Air Lift Pump—It**

utilises compressed air for raising water. Because the density of a mixture of air in water is much lower than that of pure water, if such a mixture is balanced against a water column, the former will rise much higher than the latter.

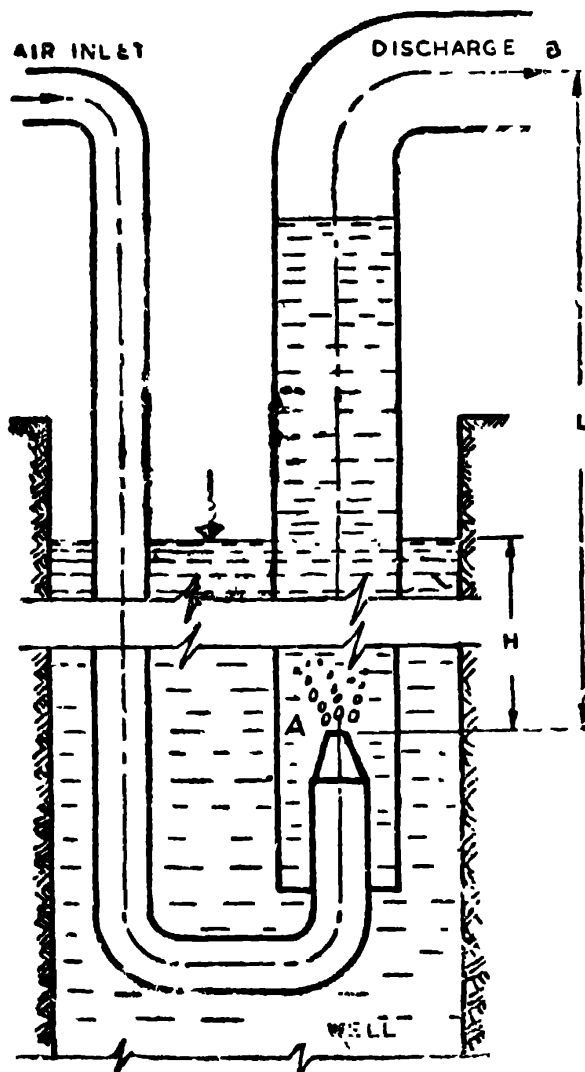


Fig 13.9 shows diagrammatically the simplest method of raising water from a deep well by means of compressed air supplied by a compressor unit installed on the surface. The rapid stream of air at A produce a jet and entraps the water in the immediate vicinity and carries it upwards to B. The air-water mixture can rise above the water level because its density is less. The air is introduced at a considerable depth below the water surface in the well, so that the actual difference  $(L - H)$  may be sufficient to initiate flow to B, the desired delivery level.

For best results, the lift of the pump  $(L - H)$  should be less than  $H$  (refer Fig 13.9).

Fig 13.9 Air Lift Pump

TABLE 13.1  
Lift vs Submergence

|                 |    |    |    |     |       |
|-----------------|----|----|----|-----|-------|
| Lift $(L - H)$  | 15 | 45 | 75 | 105 | 135 m |
| Submergence $H$ | 60 | 60 | 75 | 85  | 100 m |

**Advantages of air-lift pump :**

(i) No moving parts, no valves. No damage due to solids in suspension in water.

(ii) Raises more water for a given diameter than any other pump.

(iii) Can be used to drain out water in mines where the compressor units are already available.

**Disadvantages :**

(i) Efficiency is very low. The volume  $V$  of air in  $\text{m}^3$  at atmospheric pressure, required to lift one  $\text{m}^3$  of water through a height  $(L-H)$  depends upon the efficiency.

If  $\eta = 30\%$   $V \approx 1.75 (L-H) \text{ m}^3$

and if  $\eta = 40\%$   $V \approx 1.4 (L-H) \text{ m}^3$

(ii) Possibility of air leakage.

**13.8 Pulsometer Pump** (refer Fig 13.10 *a* and *b*)—Condensation of steam is utilised in a pulsometer pump for raising water. The pulsometer pump comprises two chambers arranged side by side. While water is rising in one chamber, discharge takes place from the other. There is a steam inlet at the top with connections to either chamber. A ball valve is used to shut off one chamber while the steam passes to the other.

There is a water inlet at the bottom connected to both chambers by simple non-return valves. As steam enters a chamber full of water, the surface of contact being small no appreciable condensation takes place. The steam pressure forces the water out through the delivery valve and continues to do so until the water level falls below that of the delivery valve. At this stage the exit is thrown open to steam and the violent escape of steam and the increased area of surface of contact result in sudden condensation. Immediately, the ball valve changes its seat cutting off steam supply, the delivery valve is closed and suction commences through a valve at the bottom. While the chamber is gradually filled with water, discharge takes place from the other chamber in a similar way. The two chambers work alternately and maintain the cycle. An air vessel is provided for cushioning.

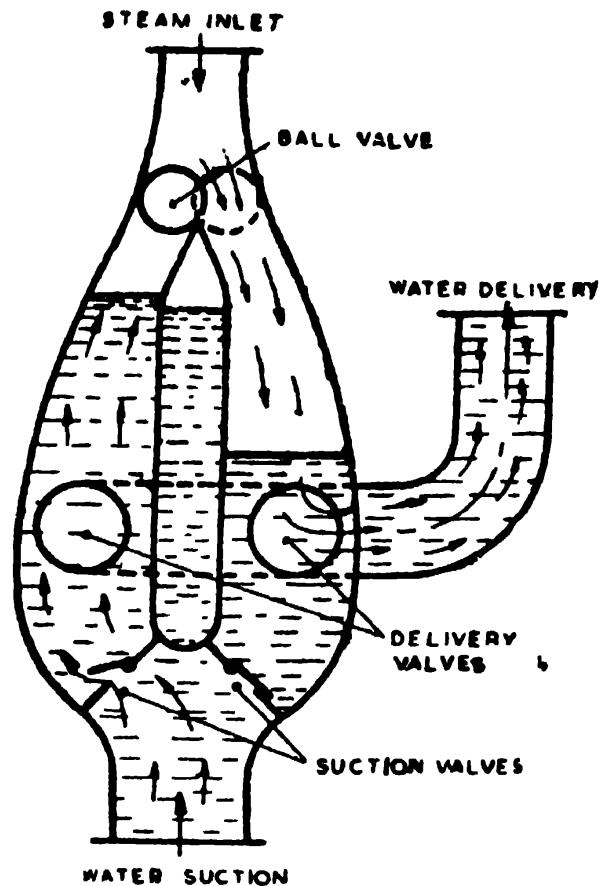


Fig 13.10 (a) Diagrammatic Sketch of a Pulsometer Pump

While the chamber is gradually filled with water, discharge takes place from the other chamber in a similar way. The two chambers work alternately and maintain the cycle. An air vessel is provided for cushioning.

**Practical Data**

Suction range = 2 to 4 m      Maximum suction lift = 8 m

Temperature of water at which suction stops and delivery begins  
=  $50^{\circ}\text{C}$

Delivery head per pulsometer = 50 m

Steam pressure = Delivery head in  $\text{kg/cm}^2 + 1.5$  to  $3 \text{ kg/cm}^2$ , i.e.,  
about 5% higher than water head required

Volume of steam required = 2 to 3 times the water delivered.

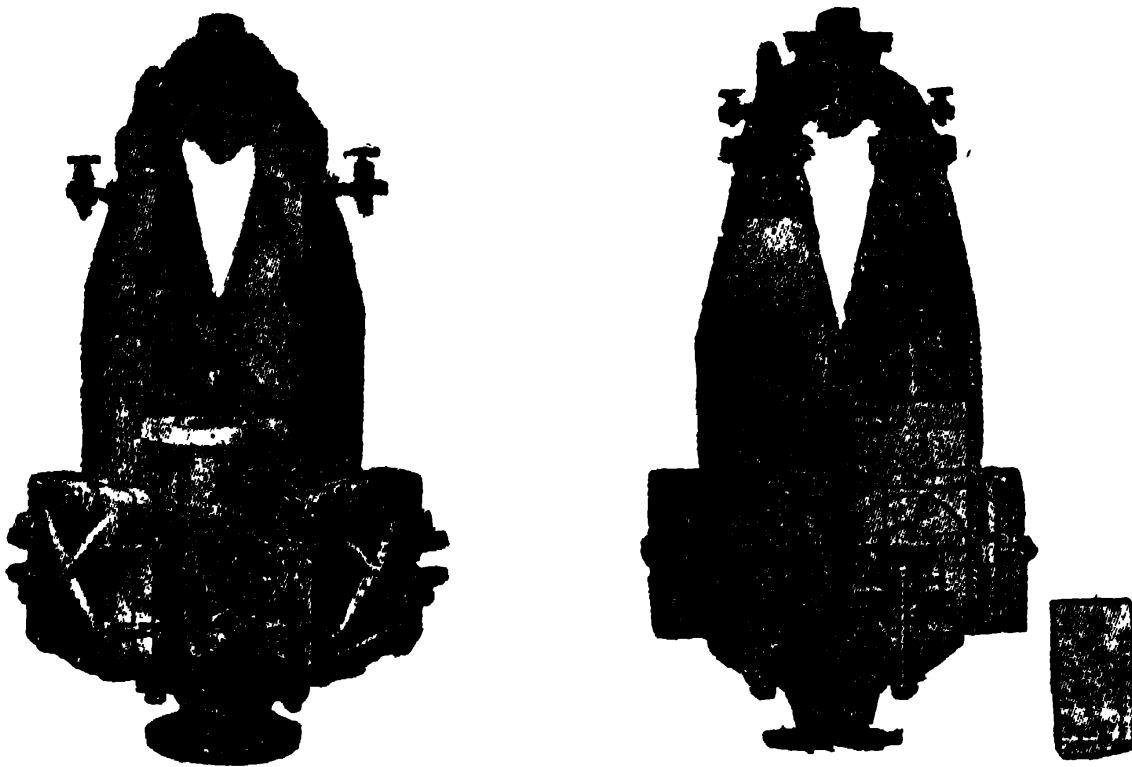


Fig 13.10 (b) Section Through a Pulsometer Pump

**13.9 Hydraulic Ram**—Hydraulic ram is a contrivance to raise a part of large amount of water available at some height, to a greater height. This is employed when some natural source of water like a spring or a stream is available at some altitude *e.g.*, in a hilly region. Work done by a large quantity of water in falling through a small height is used to raise a small part of it to a greater height. Action of water hammer makes it feasible. No external power is, therefore, required to work this machine. The first hydraulic ram was invented in 1775 by Join Whitehurst of Derby (England).

Fig 13.11 illustrates a typical ram installation. A is the source of supply connected to a supply pipe having a gate G. Water flows through

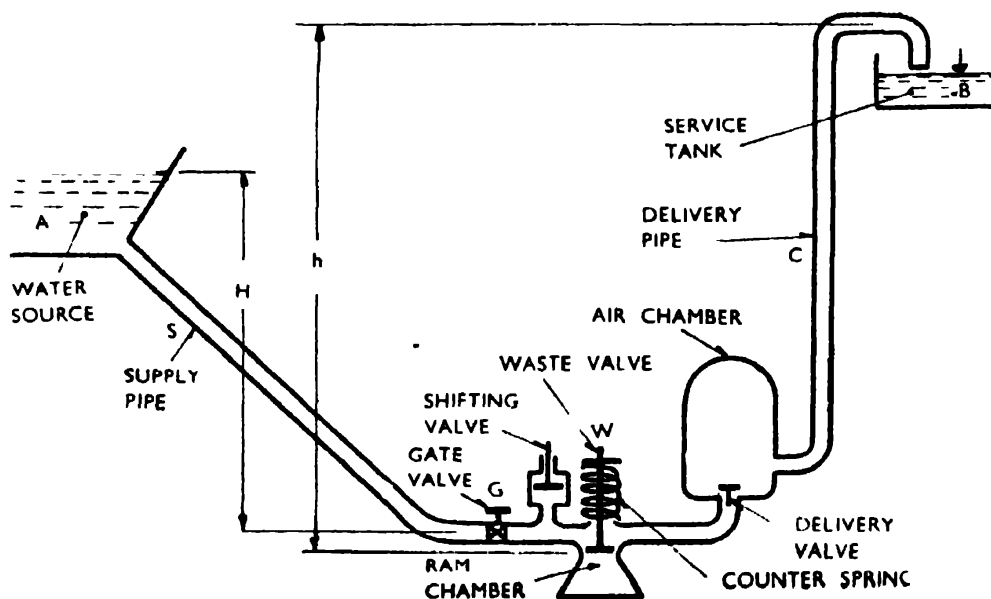


Fig 13.11 Hydranlic Ram

the supply pipe to a waste valve or impulse valve *W* which, when open, allows the water to escape. As this valve closes, water is suddenly brought to rest and the resulting water hammer forces open the delivery valve fitted in an air chamber. Water is pushed through the delivery pipe, connected to the air chamber, into the service tank *B*. A snifting or air valve is provided, which admits air for the air chamber to make up for the loss due to absorption in water. All these valves mentioned above are of non-return type.

**13.10 Ram Principle and Operation**—In principle, the hydraulic ram is an impulse pump. The impulse is developed at the expense of dynamic inertia possessed by a moving column of water. In other words, the momentum of a long column of water flowing through the supply pipe is made to force a part of the water to a level higher than that of the supply source itself. In order to develop maximum impulse, supply pipe should be as long as possible. Installation of ram too close to the source of supply will reduce the impulse and consequently the delivery head.

The cycle of operations is explained below. Initially, the water is at rest and the delivery pipe *C* and waste valve *W* are closed. Ram is started by opening the gate valve *G*, of the supply pipe, setting the water in motion. The column of water in the supply pipe rebounds a short distance creating partial vacuum in the ram chamber. The vacuum causes the snifting valve to open, admitting a small amount of air. This removes the pressure difference across the waste valve which then opens of its own weight. Water begins to escape through the waste valve. As flow accelerates, pressure on the waste valve rises and it begins to close. As it closes, pressure increases accelerating the rate of closing. Consequently, the valve closes suddenly, producing water hammer in the supply pipe. A very high pressure is momentarily produced in the ram chamber, by which the delivery valve is forced open. Opened delivery valve admits water to the air chamber. As the pressure gradually rises in the air chamber, inflowing water is brought to rest and the delivery valve is closed. Incoming water, however, compresses the air and as soon as the delivery valve is closed, the air pushes a part of the water into the delivery pipe. Water passing through the air chamber carries along with it the air admitted to the air chamber through the snifting valve at the beginning of the cycle. This ensures that there would always be some air in the air chamber. The cycle is complete and starts again at the beginning. The ram operation can be stopped by closing the gate valve *G* in the supply pipe.

This cycle of operation may be summed up as follows :

- (1) Delivery valve closed.
- (2) Vacuum is created by rebound and air is admitted through the snifting valve.
- (3) Waste valve opens and water starts flowing to waste through the valve.
- (4) Waste valve closes suddenly causing water hammer.
- (5) Momentary high pressure resulting from water hammer forces open the delivery valve.
- (6) Water enters the air vessel through the delivery valve.
- (7) Pressure rises in the air chamber, slowly bringing the incoming water to rest and closing the delivery valve, thus completing the cycle.

The operation of the hydraulic ram depends upon the successful creation and destruction of velocity in the supply pipe. The waste valve must close suddenly to enable kinetic energy to be utilized to a maximum. Theoretically, the pumping head remains constant during the operation.

### Advantages

- (a) No coal, oil or electricity is required as the motive power is the driving water.
- (b) It requires no lubrication and no packings as there are no piston glands.
- (c) Maintenance expenses are low and almost no labour is required for supervision.
- (d) Hydraulic rams can work continuously for 24 hours and thus give regular water supply.
- (e) These can be adjusted to work with any quantity from their maximum down to less than one-half and automatic adjustment is possible.
- (f) It has high efficiency and quiet working.

### 13.11 Some Hydraulic Ram Installations—

(i) Black's Hydram (Hydraulic Ram) manufactured by John Blake Ltd., England was installed at Taj Mahal, Agra in 1900 for raising water at the rate of 31.5 lit/sec and working satisfactorily till now with very low maintenance cost.

(ii) Blake's Hydram installed at Risalpur (Pakistan) in 1925 raises 56.5 lit/sec to a vertical height of 18.3 m and to a horizontal distance of 1,525 m.

### 13.12 Hydraulic Ram Calculations —

Let  $Q$  = the quantity of water wasted by ram, in  $\text{m}^3/\text{sec}$  ;

$q$  = the quantity of water pumped by ram, in  $\text{m}^3/\text{sec}$  ;

$h$  = the elevation of service tank above waste valve, in m ;

$H$  = the height of source above waste valve, in m ;

$L$  = the length of supply pipe from the source of supply to the waste valve, in m ;

$D$  = the diameter of supply pipe, in m ;

$H_{L_s}$  = the head loss in supply pipe, in m ;

$H_{L_d}$  = the head loss in delivery pipe, in m.

Size of the supply pipe should be so designed that the amount of liquid passing through per second is equal to three times the average quantity of liquid consumed by the ram.

$$D = \sqrt{0.5 (Q + q)} \text{ m} \quad \dots (13.10)$$

$$L = 1.5 \text{ to } 3 (h - H) \text{ m} \quad = (13.11)$$

Length of the supply pipe can also be determined as follows :

$$L = c \cdot h$$

where the co-efficient  $c$  which is a function of  $H$  may be obtained from the following table.

**TABLE 13.2**  
**Values of Pipe Length Co-efficient  $c$**

(i)  $h < 30$  m

|     |      |      |      |      |        |
|-----|------|------|------|------|--------|
| $H$ | 0.60 | 1.20 | 1.80 | 2.40 | 3.00 m |
| $c$ | 3    | 2.65 | 2.25 | 1.85 | 1.5    |

(ii)  $h > 30$  m

|     |      |      |      |      |       |
|-----|------|------|------|------|-------|
| $H$ | 0.60 | 1.20 | 1.80 | 2.40 | 3.0 m |
| $c$ | 3.5  | 3.0  | 2.6  | 2.25 | 2.0   |

Columbiana suggests the following values of lift and fall for maximum efficiency.

**TABLE 13.3**  
**Lift and Fall for Maximum Efficiency of Ram**

|       |    |     |    |    |    |
|-------|----|-----|----|----|----|
| $H$ m | 1  | 1.5 | 2  | 3  | 4  |
| $h$ m | 6  | 12  | 15 | 23 | 30 |
| $L$ m | 10 | 12  | 15 | 23 | 30 |

Volume of air chamber = Volume of delivery pipe

Dynamic pressure on waste valve

$$= 6.6 \times \frac{\gamma \cdot v^2}{2g} \text{ kg/m}^2 \quad \dots (13.12)$$

where  $\gamma$  = Specific weight of liquid in  $\text{kg/m}^3$  ;

$v$  = Velocity of water past waste valve in m/sec

Velocity of water  $t$  seconds after the opening of waste valve is given by \* —

$$v = c \cdot \frac{a}{a_0} \left\{ \frac{1 - e^{\frac{-2g}{k} t}}{1 + e^{\frac{-2g}{k} t}} \right\} \text{ m/sec} \quad \dots (13.13)$$

\*Hazra, L.F., Bulletin of the University of Wisconsin (1908) No. 205, P. 211.



where  $a$  = the cross-sectional area of supply pipe ;

and  $a_v$  = the effective area of delivery valve

= actual area  $\times$  co-efficient of discharge

$$\text{The co-efficient} = \sqrt{\frac{2g \cdot H}{H \cdot \frac{f \cdot l}{m} \cdot \frac{a_v^2}{a^2}}} \quad \dots (13.14)$$

$$\text{and} \quad k = 2l \cdot \frac{a_v}{a} \cdot \frac{1}{H \cdot \frac{f \cdot l}{m} \cdot \frac{a_v^2}{a^2}} \quad \dots (13.15)$$

where  $f$  = the frictional factor ;

$l$  = the length of pipe ;

$m$  = the hydraulic mean radius

=  $\frac{d}{4}$  for a pipe running full.

The area of delivery valve i.e., area of a port opening should be such that the maximum velocity of flow is about 1 m/sec.

Efficiency of ram is given by—

$$\eta = \frac{q(h + H_{L_d})}{(Q + q)(H - H_{L_s})} \quad \dots (13.16)$$

Clark gives the following figures for efficiency :

TABLE 13.4

$\frac{h}{H}$  vs  $\eta$

| Ratio of lift to fall $\left( \frac{h}{H} \right)$ | 4  | 8  | 12 | 16 | 20 | 24 | 26 |
|----------------------------------------------------|----|----|----|----|----|----|----|
| Efficiency $\eta$ in %                             | 72 | 52 | 37 | 25 | 14 | 4  | 0  |

Efficiency of ram is also expressed as—

$$(a) \quad \eta = \frac{g \cdot h}{(Q + q) H} \quad \dots \text{(by D' Aubisson)} \quad \dots (13.17)$$

(This is same as Eqn 13.16 neglecting frictional losses)

$$(b) \quad \eta = \frac{g \cdot (h - H)}{Q \cdot H} \quad \dots \text{(by Rankine)} \quad \dots (13.18)$$

**Practical Data for Ram**—The lowest head under which a ram can work is 60 cm. The delivery head  $h$  should be six to twelve times the fall.

Quantity of water delivered by the ram is about  $\frac{1}{10}$  to  $\frac{1}{15}$  of the amount supplied depending upon the ratio of lift to fall.

$$\text{Roughly } q = \frac{H \cdot Q}{2h} \quad \dots (13.19)$$

**Characteristic of Hydraulic Ram**—The delivery head  $h$  increases with the enhancement of beats of the ram.

The main losses of hydraulic ram are due to friction in pipes and valves, and the kinetic energy contained in waste water. These losses are directly proportional to the square of mean velocity of water in the supply pipe. Therefore the efficiency of hydraulic ram can be increased with the decrease of mean velocity of water, but this will result in decrease of delivery head  $h$ .

As shown in Table 13.4, the maximum efficiency of the ram is obtained if the ratio  $\frac{h}{H}$  is less. With  $\frac{h}{H} = 4$ , the efficiency will be 72 %.

**Problem 13.4** A hydraulic ram installation gave the following particulars :

|                                                                         |                         |
|-------------------------------------------------------------------------|-------------------------|
| Fall or supply head                                                     | $H = 3 \text{ m}$       |
| Length of supply pipe                                                   | $L = 6 \text{ m}$       |
| Diameter of supply pipe                                                 | $D = 25.0 \text{ mm}$   |
| Life of delivery head                                                   | $h = 7.5 \text{ m}$     |
| Length of delivery pipe                                                 | $l = 7.5 \text{ m}$     |
| Diameter of delivery pipe                                               | $d_d = 12.5 \text{ mm}$ |
| Time taken to pump 1.35 kg of water                                     | $= 33.8 \text{ sec}$    |
| Water wasted during 33.8 sec                                            | $= 6 \text{ kg}$        |
| Assuming a frictional factor 0.01, determine the efficiency of the ram. |                         |

**Solution**

Total quantity of water supplied by the reservoir

$$= 6 + 1.35 = 7.35 \text{ kg in } 33.8 \text{ sec}$$

$$\therefore Q + q = \frac{7.35}{33.8} \times \frac{1}{1,000} = 0.218 \times 10^{-3} \text{ m}^3/\text{sec}$$

Velocity of water flowing in supply pipe

$$v_s = \frac{Q + q}{a_s} = \frac{0.218 \times 10^{-3}}{\frac{\pi}{4} \times 0.025^2} = 0.45 \text{ m/sec}$$

$$\text{Water pumped } q = \frac{1.35}{33.8} \times \frac{1}{1,000} = 0.04 \times 10^{-3} \text{ m}^3/\text{sec}$$

Velocity of water flowing in delivery pipe

$$v_d = \frac{q}{a_d} = \frac{0.04 \times 10^{-3}}{\frac{\pi}{4} \times 0.0125^2} = 0.327 \text{ m/sec}$$

Head lost in friction in supply pipe

$$H_L = \frac{4fL}{D} \cdot \frac{v_s^2}{2g} = \frac{4 \times 0.01 \times 6}{0.025} \times \frac{0.45^2}{19.62} = 0.099 \text{ m}$$

Head lost in friction in delivery pipe

$$H_{L_d} = \frac{4 \times 0.01 \times 7.5}{0.0125} \times \frac{0.327^2}{19.62} = 0.135 \text{ m}$$

$$\begin{aligned}
 \therefore \text{Efficiency of hydraulic ram } \eta &= \frac{q(h + H_{Ls})}{(Q + q)(H - H_{Ls})} \\
 &= \frac{0.04 \times 10^{-3} \times (7.5 + 0.135)}{0.218 \times 10^{-3} \times (3 - 0.099)} = \frac{0.04 \times 7.635}{0.218 \times 2.9} \\
 &= 0.4825 \text{ or } 48.25\% \quad \text{Answer}
 \end{aligned}$$

**Problem 13.5** A hydraulic ram has a waste valve to 10 cm in diameter which begins to close when the velocity of flow past the valve itself is 2 m/sec. If the dynamic pressure on the valve per unit area is  $0.6 \frac{\gamma \cdot v^2}{2g}$  kg, find the necessary weight of valve.

**Solution**

Diameter of waste valve base  $d_w = 10$  cm

$$\text{Area of waste valve base } a_w = \frac{\pi}{4} \times 0.1^2 = 0.00785 \text{ m}^2$$

$$v = 2 \text{ m/sec}$$

$$\gamma = 1,000 \text{ kg/m}^3$$

$$\begin{aligned}
 \text{Dynamic pressure on the valve} &= \frac{0.6 \gamma \cdot v^2}{2g} \\
 &= \frac{0.6 \times 1,000 \times 2^2}{19.62} = 122 \text{ kg/m}^2
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Weight of valve} &= 122 a_w \\
 &= 122 \times 0.00785 \\
 &= 0.955 \text{ kg} \quad \text{Answer}
 \end{aligned}$$

**Problem 13.6** A hydraulic ram with a supply pipe of 75 mm diameter has a waste valve of 100 mm diameter and 1.6 kg weight. The length of supply pipe is 10 mm. The travel of the valve is 6 mm. The number of beats per minute is 110. Determine the discharge through the delivery pipe against a head of 6 m? Assume that there is no slip.

(Judavpur University)

**Solution**

$$D = 75 \text{ mm}$$

$$h = 6$$

$$d_w = 100 \text{ mm}$$

$$L = 10 \text{ mm}$$

Valve travel  $s = 6$  mm; Beats = 110 per min;  $w = 1.6$  kg

Pressure required to close the waste valve

$$\begin{aligned}
 &= \frac{\text{total valve weight}}{\text{area of base of waste valve}} = \frac{1.6}{\frac{\pi}{4} \times (0.1)^2} \\
 &= 204 \text{ kg/m}^2
 \end{aligned}$$

$$\text{Dynamic pressure on waste valve closing} = \frac{\gamma \cdot v^2}{2g} \text{ kg/m}^2$$

(neglecting factor 0.6 in Eqn 13.12)

where  $v$  = maximum velocity of water past waste valve just before closure.

$$\therefore \frac{\gamma \cdot v^2}{2g} = 204$$

or 
$$v = \sqrt{\frac{204 \times 19.62}{1,000}} = 2 \text{ m/sec}$$

Maximum velocity of water in the supply pipe  $v$

$$= \text{max velocity past waste valve} \times \frac{\text{area past waste valve}}{\text{area of supply pipe}}$$

Now, area past waste valve  $= \pi \cdot d \cdot s$

$$= \pi \times 0.1 \times 0.006 = 0.00188 \text{ m}^2$$

Area of supply pipe  $a_s = \frac{\pi}{4} \times (0.075)^2 = 0.00496 \text{ m}^2$

$$\therefore v = 2 \times \frac{0.00188}{0.00496} = 0.755 \text{ m/sec}$$

Kinetic energy of water column in the supply pipe at the time of closing the waste valve  $= \frac{1}{2} m \cdot v^2$

$$= \frac{(\gamma \cdot a_s \cdot L) \cdot v^2}{2g} = \frac{1,000 \times 0.00496 \times 10 \times 0.755^2}{2 \times 9.81} = 1.44 \text{ kg}\cdot\text{m}$$

$\therefore$  Neglecting slip, weight of water that could be lifted against a delivery head of 6 m

$$\frac{1.44}{6} = 0.24 \text{ kg}$$

This amount of water could be lifted for one beat of waste valve.

$\therefore$  Total amount of water lifted, or discharge

$$= 110 \times 0.24 = 26.4 \text{ kg/min}$$

or **26.4 lit/min** *Answer*

### UNSOLVED PROBLEMS

- 13.1 What is a propeller pump? Describe briefly such a pump with the help of a sketch.
- 13.2 Describe with the help of a sketch a mixed flow pump. What is the range of specific speed for such a pump?
- 13.3 Can axial flow pump be classified as a centrifugal pump?
- 13.4 Where would you employ a Kaplan pump? What is the approximate specific speed range for such a pump?
- 13.5 Explain the cause of cavitation in axial flow pump. With the aid of a sketch and suitable symbols indicate the factors which influence cavitation and methods of reducing its effects. (AMI Mech. E)
- 13.6 If a propeller pump delivers a discharge  $Q$  against a head  $H$  when running at a speed  $N$ , deduce an expression for the speed of a geometrically similar pump of such a size that, when working against unit head, it will transmit unit power to the water flowing through it. Show that this value is proportional to the specific speed of the pump.

## HYDRAULIC MACHINES

Explain why it is that, in general, the specific speed of a propeller pump is greater than that of a centrifugal pump. Then show that for stipulated conditions of head and discharge, a propeller pump is likely to be smaller than an equivalent centrifugal pump.

(AMI Mech E)

13.7 What kind of pump will be preferred for irrigation purposes ?

13.8 What do you know of the following ?—

(a) Hydraulic ram,

(b) Air lift pump,

(c) Jet pump,

(d) Pulsometer.

(AMIE)

13.9 Enumerate the advantages and disadvantages of air lift pump as compared with the usual reciprocating or centrifugal pump. Give a neat sketch of a good type of foot piece for an air-lift pump.

Derive an expression for the theoretical volume of air required to pump one cu m of water against a total head of  $h$  m by means of air lift pump when ratio of isothermal expansion of air during this operation is  $r$ .

13.10 Describe a hydraulic ram with the help of a line diagram.

(AMIE)

What are the sources of loss in such a machine ? Sketch curves to show how the efficiency depends upon the beat of the waste valve and the lift.

In what circumstances would you make use of such a machine and why ?

### Numericals

13.11 Four single stage axial flow pumps, manufactured by SULZER are working at RAINI Station (Ramganga River UP) each of them raises, 1420 lit/sec of water against a head of 4.7 m and is to be driven by an AC motor running at 730 rpm. Assuming tip speed  $2.1 \sqrt{2gH}$ ; boss diameter half the tip diameter. Overall efficiency 80%, mechanical losses 3%, uniform axial flow velocity and no whirl or shock at inlet, find :—

(a) the diameter of the propeller,

(b) the mean whirl velocity after leaving the propeller.

Draw the theoretical velocity diagrams at inlet and exit from the propeller blades at maximum radius.

Find also the h.p. required to drive the pump and the rating of A.C. driving motor. [(a) 525 mm (b) 2.37 m/sec ; 111.5 h.p. ; 120 h.p.]

(Jadavpur University)

13.12 In an axial-flow or propeller pump, the rotor has an outer diameter of 75 cm and an inner diameter of 40 cm ; it revolves at 500 rpm. At the mean blade radius, the inlet blade angle is 12 deg and the outlet blade angle is 15 deg. Sketch the corresponding velocity diagrams at inlet and outlet, and estimate from them (i) the head the pump will generate, (ii) the discharge or rate of flow in lit/sec, (iii) the shaft h.p. input required to drive the pump, (iv) the specific speed of the pump.

Assume a manometric or hydraulic efficiency of 88% and a gross or overall efficiency of 81%.

(19.8 m ; 705 lit/sec ; 230 HP ; 45). *Note* : Specific speed of 45 for a propeller pump is very low.)

[*AMIE Mech E (Lond)*]

- 13.13 An air lift pumping plant is required to discharge 115 lit/sec against a static head of 70 m. The rising main is 105 m long. The overall efficiency of the plant (i.e.,  $\frac{\text{WHP}}{\text{Compressor BHP}}$ ) is 0.32.

Estimate the volume of air per minute (at atmospheric pressure) that must be handled by the air compressor of 0.3 BHP is required to compress 30 litres of free air per minute to 10 kg/cm<sup>2</sup> pressure.  
(*Rajasthan University*)

- 13.14 A ram is used to lift 4 lit/sec through 150 m of 6 cm pipe into a reservoir at a height of 25 m above the ram, while drawing in 70 lit/sec of water from a supply tank 3 m above. Determine the efficiency.  $f = 0.014$ . (52.2%)  
(*Madras University*)

- 13.15 Describe the principle of operation of the hydraulic ram, and sketch graphs to show cyclical changes in pressure and velocity in the system. Discuss the limitations of this device, in relation to head and discharge.

A particular installation has a supply or drive pipe 7 m long the supply head (above waste valve) is 26.5 m the delivery head (above waste valve) is 8.8 m, and the waste valve closes when the velocity in the supply pipe is 2.5 m/sec. If the discharge of useful water lifted is to be 1.4 lit/sec, estimate : (i) the diameter of the supply pipe, (ii) the quantity of waste water per minute. (iii) the number of beats per minute. A simplified solution will serve, neglecting pipe friction, etc.  
[*AMI Mech E (Lond)*]

- 13.16 A ram working under a head of 4 m pumps 2 lit/sec into a steel tank placed 15 m above the ram. The amount of water used is 15 lit/sec. Taking a loss of head in the delivery pipe as 1.5 m of water, calculate the Rankine's efficiency of the ram.

(48%) (*Osmania University*)

- 13.17 (a) Explain with the help of diagrammatic sketch how a hydraulic ram works and also show how the pressure and velocity undergo changes.

- (b) The following particulars relate to a hydraulic ram installations—

|                                 |      |                    |
|---------------------------------|------|--------------------|
| Supply head                     | ...  | 3 m                |
| Drive pipe length               | ...  | 10 m               |
| Drive pipe diameter             | ...  | 6 cm               |
| Area past waste valve           | /... | 36 cm <sup>2</sup> |
| Head to close waste valve       | ...  | 0.5 m              |
| Delivery head above waste valve | ...  | 9 m                |

Compute the quantities of useful and waste water in litres per minute and find the Rankine efficiency of hydram.

(*Bombay University*)

## SECTION IV

# Testing and Characteristics of Hydraulic Machines

# 14

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## *Testing of Hydraulic Machines*

**14.1 Introduction**—Various tests which are required to be carried out on the turbines and pumps are given in this Chapter. These tests are basically meant to find the performance curves of the machines. The tests are made in accordance with the different specifications given in the test codes of various countries.

The measurement of discharge as well as the pressure (or head) are given in author's book on Hydraulics and Fluid Mechanics. The measurement of level of water and speed of machines are given in this chapter under 14C and 14D respectively. After knowing these data the power and efficiency of the turbine and pump are determined which are explained in this Chapter under 14B.

### **A. TESTING OF TURBINES AND PUMPS**

**14.2 Purposes of Tests**—There are generally five purposes for which the tests are made. The type of test will depend upon its purposes.

(1) *Investigation of Losses*—In order to find out the efficiency of units the different losses such as mechanical and hydraulic losses are determined first.

(2) *Design and Research*—These tests give valuable information to the designer and research worker regarding the performance of the units under different conditions. Such conditions include the varying of load, speed and head. The theory can be better understood by these tests, which are of two types viz. *model tests* and *acceptance tests*.

(3) *Determination of Results Under Specified Conditions*—These tests are carried out at site and are called acceptance or take-over tests. They are

undertaken to see that guarantees given by the manufacturers in the contract are fulfilled. Such tests comprise the determination of efficiency for given head and speed.

(4) *Determination of Best Operating Conditions*—These tests are employed to find out the best efficiency at different conditions of speed, load and head. They are, therefore, performed in the neighbourhood of best efficiency.

(5) *Sale of Machinery*—The tests are performed in the presence of the prospective buyer, to give him an idea of the performance of the machine. The results of these tests are recorded in booklets which serve as a handy reference to the sales engineer.

**14.3 Testing Codes**—Tests, whether in the field or in the laboratory, are based on the same fundamental principles. These principles and the rules of procedure are formulated in the standard test codes of various countries.

These codes have much in common. They give definitions of terms used ; the principles of testing ; the requirements and conditions of testing ; method of taking necessary measurements ; description of measuring instruments and notes on their calibration ; tolerances on measurements and advice for the solution of the other points of dispute which may come up between the purchaser and the manufacturer. Codes of different countries differ mainly in the extent and presentation of the matter. Each is a guide and reference to the Engineer. When referred to in the specifications given by the purchaser, it becomes a contract document. These codes can be modified by the purchaser and the manufacturer between themselves.

Testing codes may go a little further in apportioning the cost of these tests between the buyer and the supplier. The buyer sends his representative to the manufacturing works to watch the model tests conducted in their laboratories and the manufacturer has to send his engineering personnel and staff for testing at site.

On the testing of water turbines, the best known codes in the English language are :

(i) Swiss Rules for hydraulic turbines, published by the Swiss Electrotechnical Institution.

(ii) The British Standard Test Code for hydraulic turbines, published by the British Standards Institution.

(iii) The Testing Code of the Machinery Builders Society, U.K.

(iv) The Testing Code for hydraulic prime movers, published by the American Institute of Mechanical Engineers.

(v) The Standard Test Code for Hydraulic Power Plants issued by the authority of the Councils of the Institution of Civil Engineers and the Institution of Mechanical Engineers, London.

**14.4 Acceptance or Take-Over Tests** are those which are performed at the site before the unit is commissioned for service. They are undertaken to see that the guarantees given by the manufacturers in the contract are fulfilled. During erection, pressure tests are undertaken to check the strength of the individual parts. Before commissioning a turbine a number of tests, both mechanical and electrical are carried out on the turbine and its auxiliaries. The intake and penstock are cleared



of timber and other trash capable of causing damage. The governor and lubricating oil systems are cleared of any dirt or foreign matter that may have found its way in. All equipment is checked for operation and when found satisfactory, water is allowed to come in the turbine and the set is gradually brought up to full speed. This is done first by hand and then by the governor.

The set is subjected to a load test. The load may be a commercial one or a water-resistance. The load is varied according to the specifications and the behaviour of the set studied.

The governor performance of sudden rejection of load is studied. Loads at the fractions viz, 25, 60, 75, 100 per cent of the full-rated output are built-up and when conditions are steady, suddenly thrown off, the pressures and speeds being recorded during and after the rejection. The pressure rise and speed rise must remain within the specified tolerances.

The overspeed device, for shutting down in the event of racing is tested by manipulating the governor so as to run the machine upto the maximum speed, which may be as high as 2 to 3 times the rated speed.

**14.5 Model Tests**—Such tests are conducted in laboratories on small models of the big units. They are necessary to obtain complete data about the performance of turbines or pumps, before they are manufactured. Tests on actual-sized units are very much restricted in their scope. The limitations may be due to the costs of the tests and due to time, in case the plant is to be commissioned at an early date. In addition to these, there are hydraulic limitations. Head may not be variable, the speed may be constant and the load available may not be very steady which is desirable for accurate results.

Models are tested under a small head. This head is created by pumping units. It is desirable to use high head for impulse turbines and low head for reaction turbines. Generally the test turbines are of such a size that the BHP lies between 5 and 50.

The discharge is measured by any one of the methods explained in the author's book on "Hydraulic and Fluid Mechanics". The output is absorbed by any of the brakes to be explained in Chapter 15B. The prony brake still gives the best results at variable speeds and torques.

The test consists in running the turbine for a given position of its opening, from standstill to maximum runaway speed and measuring the data for each speed, when steady conditions prevail. The efficiency is calculated from the above data. The computation of different data and drawing of characteristic curves will be explained in Chapter 15 A. The various values for the actual-sized turbine are calculated by the principles of geometrical similarity explained in Chapter 9.

For pumps, the model tests are carried out for large units in which case their performance data are required before they are actually manufactured. The computation of such data and drawing of characteristic curves are given in Chapter 15 B.

**14.6 Test Beds**—The testing of model water turbines and pumps is carried out in a laboratory, specially constructed for the purpose in the manufacturer's works. Each type of turbine and pump require different type of test-beds. The type of test will depend on the purpose of test explained in Art 1.42. Some of the test-beds are given in the following articles. A few more test-beds such as aerodynamic test-beds, cavitation test-beds are also very important for the research engineer.

Aerodynamic test-bed is used for the Kaplan turbine employing a wind tunnel, where air is used instead of water. Such test-beds have become very common now-a-days, as it is very convenient to work with air because both pressure and velocity can be measured more easily at numerous points with which all-round observations are possible since the runner is not submerged in water as in the case of hydraulic experiments.

With cavitation test-bed the development of turbine runner blade profile is possible. Photographs are taken with the help of stroboscope, with an exposure upto  $\frac{1}{200,000}$  sec, to see whether the runner is effected by cavitation.

**14.7 Test Bed for Pelton Turbine**—Fig 14.1 shows the test bed used for testing Pelton turbine runner by Escher Wyss & Co Ltd, Zurich.

The pump 1 (refer Fig 14.1) of centrifugal type supplies the required head and quantity of water from sump 6. The quantity of water is

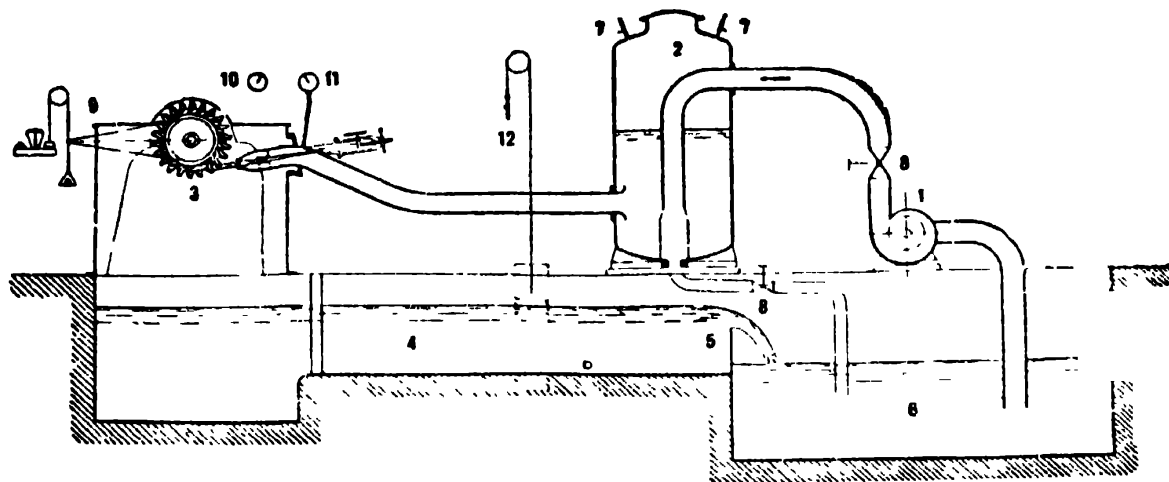


Fig 14.1 Test Bed for Pelton Turbine (Escher Wyss & Co Ltd)

- |                      |                                 |                         |
|----------------------|---------------------------------|-------------------------|
| 1. Centrifugal Pump  | 5. Measuring Weir               | 9. Dynamometer or Brake |
| 2. Air Vessel        | 6. Sump                         | 10. Tachometer          |
| 3. Pelton Wheel      | 7. Compressed Air Vents         | 11. Pressure Gauge      |
| 4. Measuring Channel | 8. Delivery or Regulating Valve | 12. Float               |

regulated by valve 8. The water is passed through a pressure vessel 2 which has compressed air on the top surface of water. The compressed air is supplied by a compressor (not shown in Fig) through the pipes 7. With the help of air vessel, working head of the turbine is made constant. The head is varied with the help of compressor. If the pump is connected to the turbine without air vessel, the fluctuation of pressure takes place. In this case the working head of the turbine is made constant by adjusting the delivery valve 8 of the pump or by varying the speed of electric motor driving the pump. The head before the water emerges out of turbine nozzle is recorded by a pressure gauge 11. The Pelton runner 3 is fitted on the open tank. The speed of wheel is measured by a tachometer 10 and the torque is measured by a brake 9. The water after giving the work to the Pelton runner falls in the channel on the tail side 4. It flows back to the pump sump over a weir 5. The head of water over the weir is measured by a float 12.

**14.8 Test Bed for Reaction Turbine**—Fig 14.2 shows the test bed used for reaction turbines, both Francis and Kaplan type, by Escher Wyss,

The pump 1 of centrifugal type supplies the required head and quantity of water from the pump sump 7. The water is regulated by the throttle

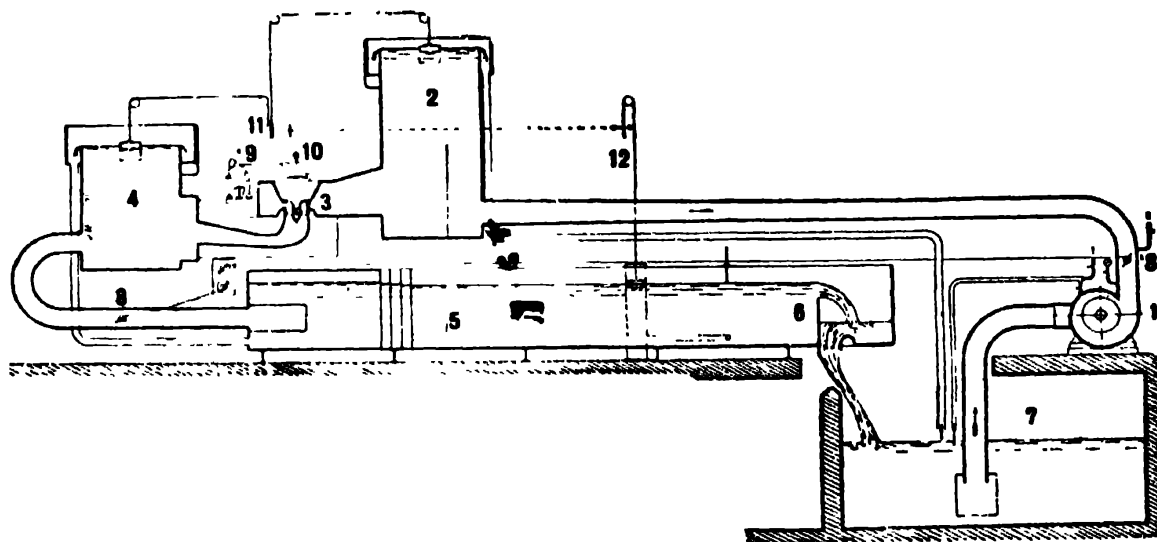


Fig 14.2 Test Bed for Reaction Turbines (Escher Wyes & Co. Ltd)

- |                     |                               |                    |
|---------------------|-------------------------------|--------------------|
| 1. Centrifugal Pump | 5. Measuring Channel with     | 9. Dynamometer or  |
| 2. Head Race Tank   | Stilling Devices              | Brake              |
| 3. Reaction Turbine | 6. Measuring Weir             | 10. Tachometer     |
| 4. Tail Race Tank   | 7. Sump                       | 11. Head Measuring |
|                     | 8. Delivery or Throttle Valve | Scale              |
|                     |                               | 12. Float          |

valve 8. The water enters the head race tank 2 which supplies water to the turbine 3. The speed of turbine is measured by tachometer 10 and the torque by brake 9. The water after doing the work passes on to the tail race tank 4 through the draft tube. The difference of water level in the two tanks 2 and 4, gives the head working on the turbine, which is measured by scale 11. The throttle valve regulates the supply for the tail race side. The water flows to the channel 15 which has stilling devices and at the end there is measuring weir 6. The head on the weir is measured by float 12.

**14.9 Test Bed for Pumps**—The pump sump is covered with a platform which is made generally of wooden sleepers and angle irons. The pump to be tested is fixed over the sleepers. The discharge is measured by calibrated tank if the pump is small. In case of large-sized pump, the water is passed through a channel having a measuring weir at its end. The head over the weir is measured by hook gauge (refer Art 14.27). The suction head is measured by piezometer tube dipped in a mercury trough. The delivery head is measured by dead weight pressure gauge or piezometer tube. The type of fluid used in piezometer tube depends upon the delivery head. If the head is more, mercury is employed, otherwise fluid whose specific gravity is near about two is preferred to give a fairly long column in the tube, in order to have smaller percentage error.

Driving motor of D.C. type or variable speed A.C. type is used for the tests, so that a wide range of speed variation can be obtained. The driving motor is provided, for the loading, with either mechanical dynamometer or fitted with ammeter and voltmeter. In the latter case the efficiency of electric motor must be known in order to get the input to the pump.

No change in pipe diameters or directions of flow should occur at the places where the gauges are located, otherwise the readings will not be correct. All readings on suction and discharge piping before and after pump flanges must be referred to centre line of pumps. When the suction diameter is larger than the discharge diameter, a correction factor,  $\frac{v_s^2 - v_d^2}{2g}$ , for the velocity head must be made at all capacities and must be added to the total pressure readings.

The speed of the pump is measured by a tachometer or RPM-counter (refer Art 14.29) from the shaft of driving motor.

#### 14.10 Determination of Total Head—

(a) **Turbine**—The net head for different types of water turbines is given in Fig 14.3 to 14.6, taken from the Swiss Rules for Hydraulic Turbines, SEI Publication No. 178e, 1947.

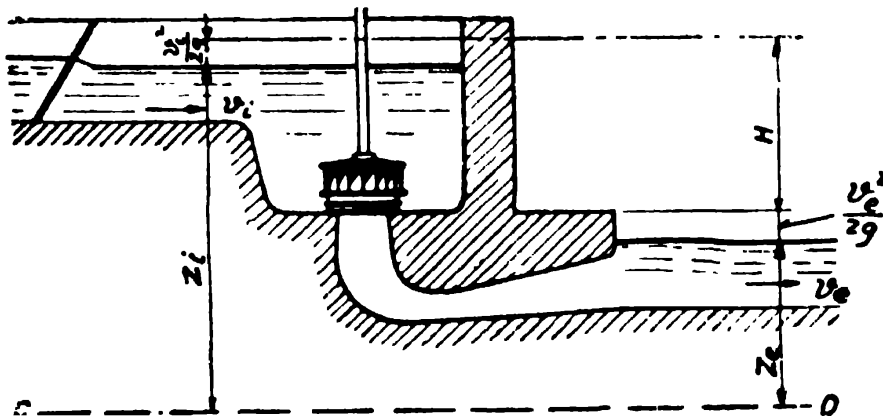


Fig 14.3 Reaction Turbine  
Open Flume, Vertical Shaft, Draft Tube Bend

$$\text{Net Head } H = z_i - z_e + \frac{v_i^2 - v_e^2}{2g}$$

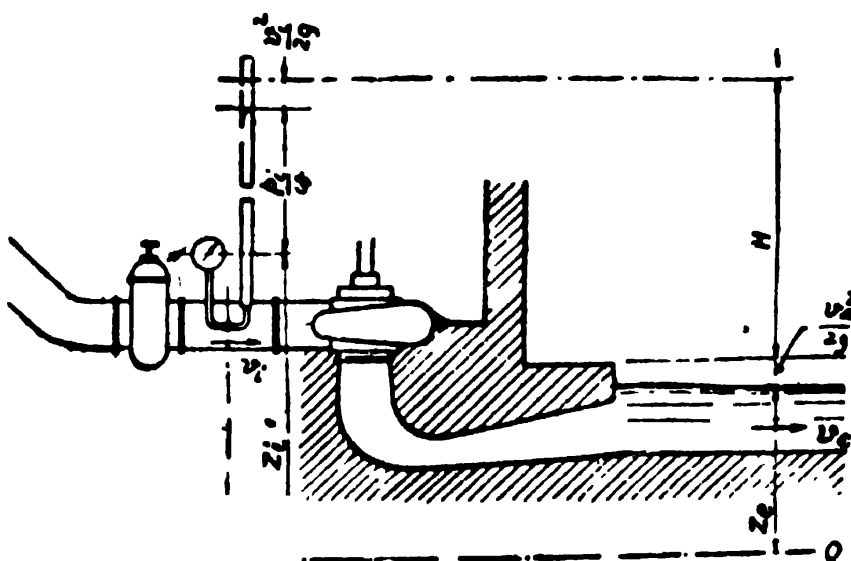


Fig 14.4 Reaction Turbine  
Spiral Casing, Vertical Shaft, Draft Tube Bend

$$\text{Net Head } H = z_i - z_e + \frac{p_i}{\gamma} + \frac{v_i^2 - v_e^2}{2g}$$

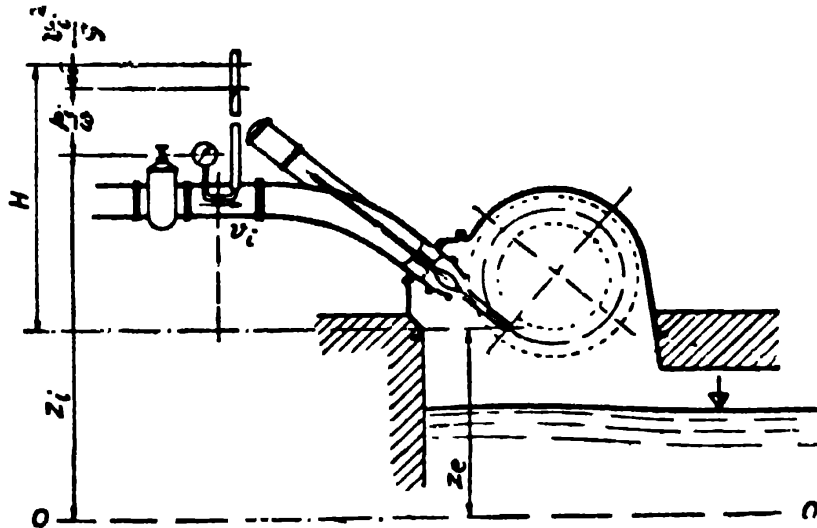


Fig 14.5 Impulse Turbine (Pelton)  
Single Nozzle, Horizontal Shaft

$$\text{Net Head } H = z_i - z_e + \frac{p_i}{\gamma} + \frac{v_i^2}{2g}$$

$$\text{Net Head } H = \frac{Q_1 \left( z_{i_1} - z_{e_1} + \frac{p_{i_1}}{\gamma} + \frac{v_{i_1}^2}{2g} \right) + Q_2 \left( z_{i_2} - z_{e_2} + \frac{p_{i_2}}{\gamma} + \frac{v_{i_2}^2}{2g} \right)}{Q_1 + Q_2}$$

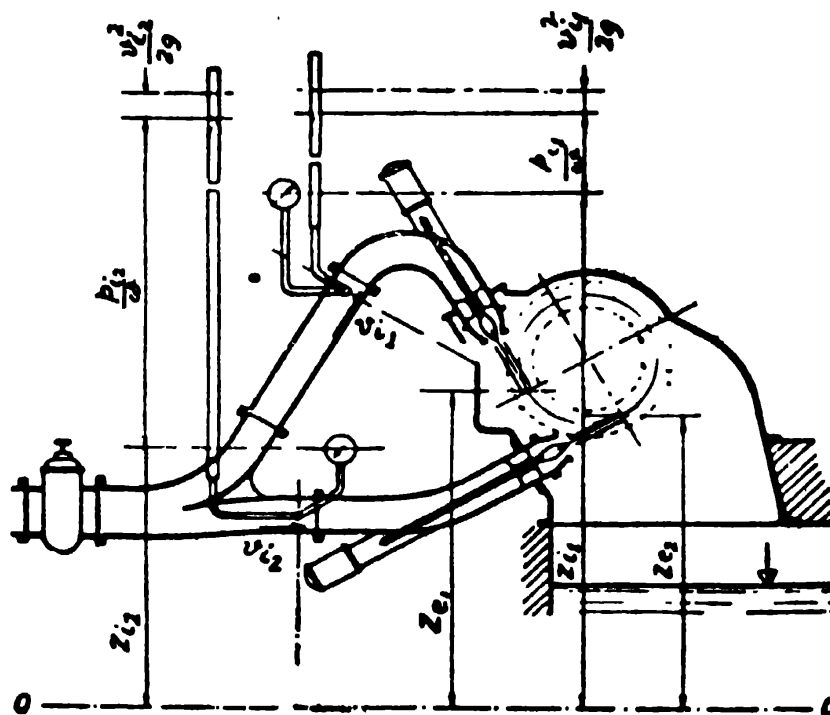


Fig 14.6 Impulse Turbine (Pelton)  
Twin Nozzle, Horizontal Shaft

(b) **Pumps**—The total dynamic head  $H$  for different types of connections of pressure gauges fitted to the suction and delivery ends of pump piping is given in Fig 14.7. The precautions must be taken that (a) the gauge connection pipes are full of water before the gauges are fitted as shown in Fig 14.7 *a* and *b*,

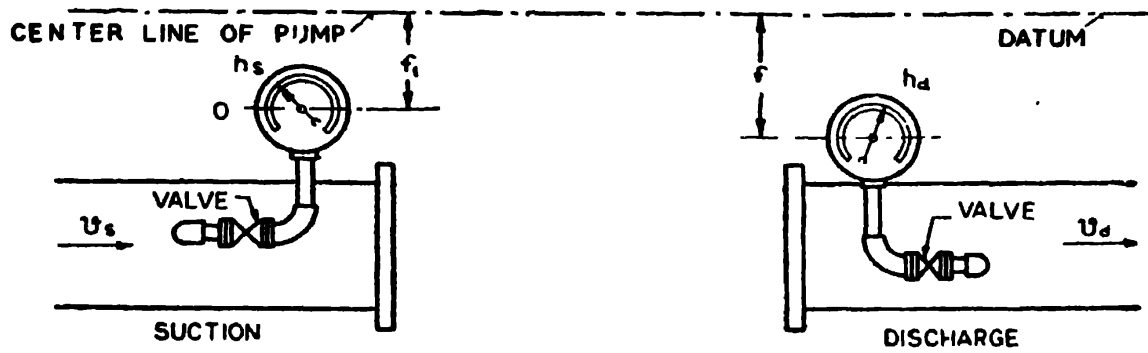


Fig 14.7 (a)  $H = h_d - h_s - f + f_1 + \frac{v_d^2 - v_s^2}{2g}$

Suction head at datum  $h'_s = h_s - f_1$

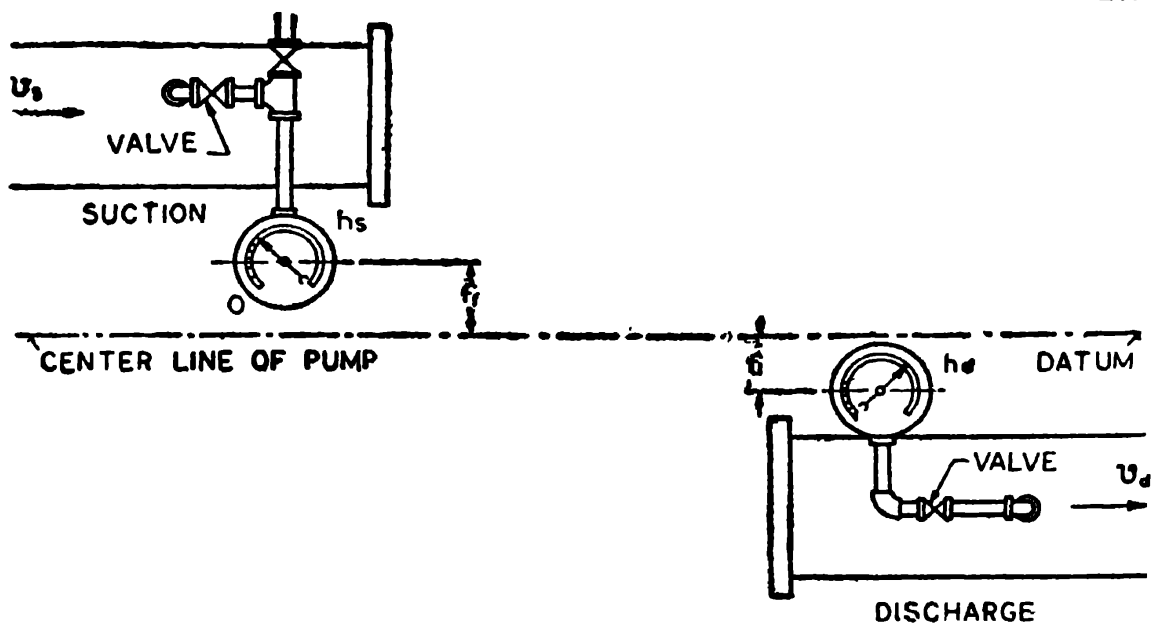


Fig 14.7 (b)  $H = h_d - h_s - f - f_1 + \frac{v_d^2 - v_s^2}{2g}$

Suction head at datum  $h'_s = h_s + f_1$

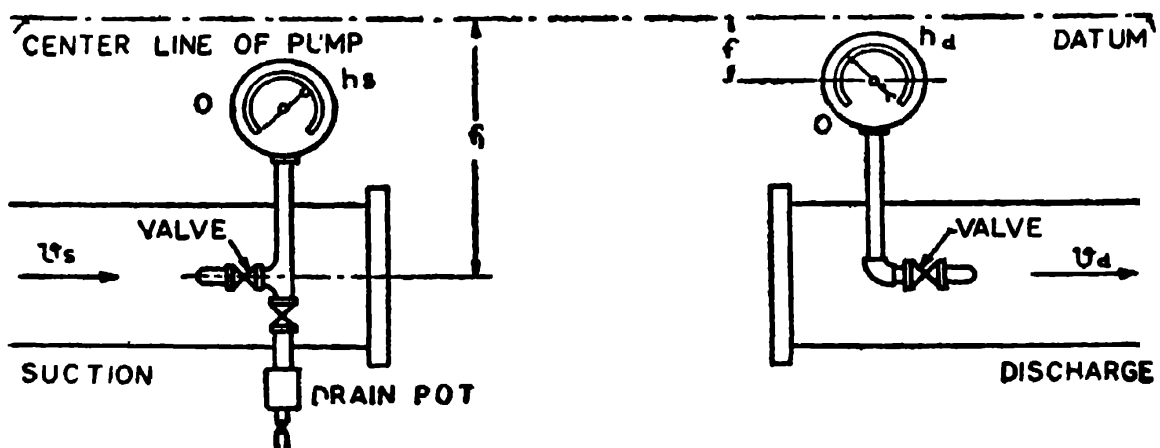


Fig 14.7 (c)  $H = h_d - h_s - f + f_1 + \frac{v_d^2 - v_s^2}{2g}$

Suction lift at datum  $h''_s = h_s + f_1$

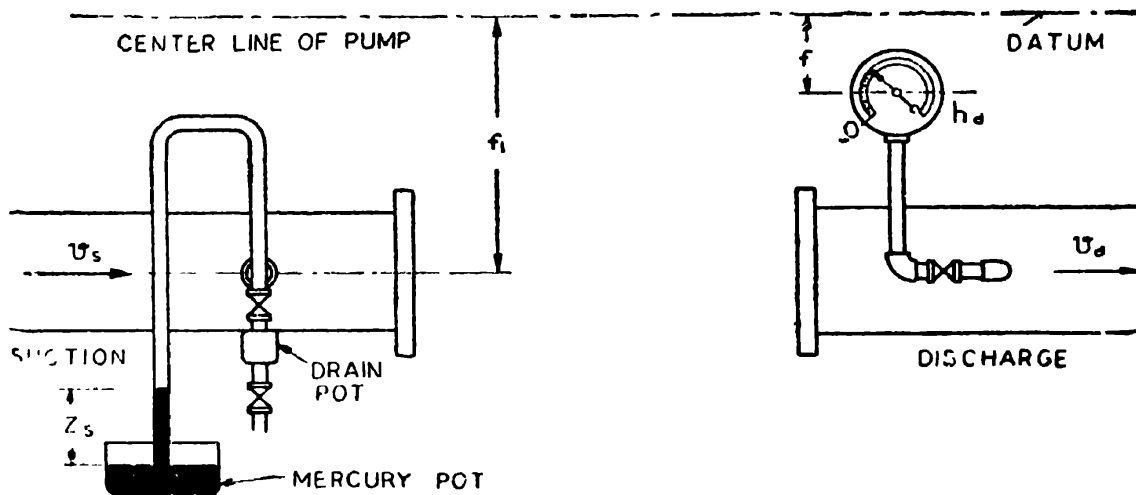


Fig 14.7 (d)  $H = h_d - s \cdot z_s - f + f_1 + \frac{v_d^2 - v_s^2}{2g}$

where  $s$  = sp gr of mercury

$z_s$  = m of mercury

Suction lift at datum  $h''_s = s \cdot z_s + f_1$

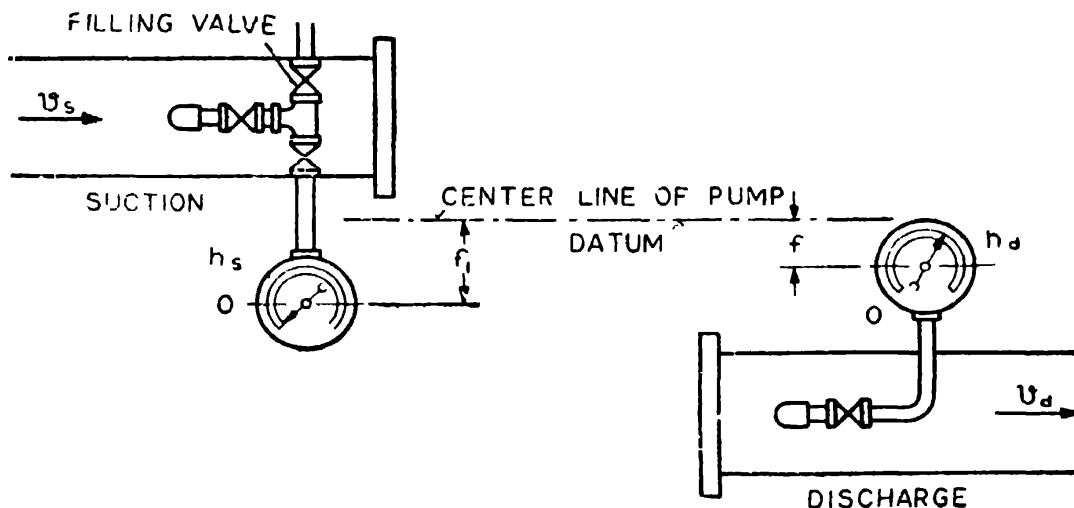


Fig 14.7 (e)  $H = h_d - h_s - f + f_1 + \frac{v_d^2 - v_s^2}{2g}$

Suction lift at datum  $h''_s = -h_s + f_1$

(b) in Fig 14.7c discharge gauge connection pipes are full of water and suction gauge connection pipes are filled with air and open drain valve on suction connections occasionally.

(c) in Fig 14.7d and e, the discharge gauge connection pipes are full of water.

**14.11 Procedure of Testing for Turbines in a Laboratory—** The turbines in the field must be supplied with natural flow of water ( $Q$ ) under some head ( $H$ ). However in a laboratory both  $Q$  as well as  $H$  are obtained by a suitable pumping unit. The pump draws water from a sump or underground reservoir of sufficient capacity, and delivers it under a specified pressure ( $H$ ) to the turbine as explained above.

The pump is started as explained in Art 14.15. The delivery valve 8 (refer Fig 14.1 and 14.2) of the pump is opened fully in order to supply

water to the turbine. The inlet valve to the turbine, if any, as well as the turbine nozzle (for Pelton turbine) or turbine gates (for reaction turbine) are opened. Let the turbine run for fifteen to twenty minutes before the readings are recorded.

Open the brake cooling water tap. Allowance for this water is made for the accurate calculations of discharge.

The turbine is set for a number of inlet openings, atleast four, say 25, 50, 75 and 100%. For each inlet opening, the turbine is first run on no-load and then it is loaded slowly by means of dynamometer till the turbine is at standstill. Not less than six different loadings for a single inlet opening, are to be recorded.

The head operating the turbine is kept constant as far as possible during testing. As explained in Art 14.7 and 14.8 it is done by the help of air vessel (for Pelton turbine) or water tanks (for reaction turbine). In the absence of air vessel or water tanks, the head is kept constant by adjusting the delivery or regulating valve of the pump or by varying the speed of the driving motor.

#### 14.12 Data to be Measured—

I. *The following data are measured while the experiment is being performed—*

(a) **Turbine Gauge Pressure**—This is obtained from the pressure gauge 11 (refer Fig 14.1 and 14.2) connected to the turbine. The pressure gauge gives the values in metres of water or in  $\text{kg/cm}^2$ .

(b) **Brake Load** from the meter connected for the purpose. For the rope brake (refer Art 14.20), measure the load from the spring balance.

(c) **Turbine Speed** is measured by means of tachometer or RPM-counter (refer Art 14.29) by inserting its knob in the centre hole made on the end of shaft. A tachometer may be connected to the turbine by means of belt.

(d) **Head over Weir Crest** is measured by means of hook gauge (refer Art 14.28 ii).

II. *The following data to be measured are required for the calculations explained in the next Art 14.13—*

(a) **Internal Diameter  $d$**  of the pipe at a place where turbine pressure gauge is connected. For a reaction turbine, internal diameter of the pipe at a place where vacuum gauge is connected, is also to be noted. These diameters are required to calculate the kinetic head of the flowing water in order to determine the effective head working on the turbine.

(b) **Vertical Distance  $Z_{\text{gauge}}$** , if any, from the centreline of the turbine inlet to the point where the pressure gauge is connected. This is also required to determine the turbine effective head. In case of reaction turbine the  $Z_{\text{gauge}}$  distance for the vacuum gauge is also to be taken into account.

(c) **Radius of Brake Arm  $l$**  (refer Fig 14.8) from the centreline of the turbine shaft to the point of application of weight. This is required to calculate the torque applied on the turbine. Sometimes the mechanical dynamometer is such that it is fitted with a meter showing directly the torque to be calculated.



(d) **Length of Weir Crest** is measured if the weir used is rectangular or trapezoidal (i.e., Cippoletti weir).

(e) **Weir Crest Level** is measured by the hook gauge after the experiment has been performed and no more water is flowing over the weir.

III. *The following instruments are to be calibrated before they are used to perform the experiment—*

(a) **Turbine Pressure Gauge and Vacuum Gauge**, the latter for reaction turbine only. It is required to calculate  $H_{cor}$  for the determination of effective head of the turbine.

(b) **Tachometer or RPM-Counter** to determine the correct speed.

(c) **Brake Load Meter, Torque Meter or Spring Balance.**

All the above data is tabulated for each inlet opening as shown in Table 14.1.

#### 14.13 Calculated Data at Constant Head—

(a) **Turbine Effective Head  $H$ —**

$$H = (H_{gauge} \pm H_{cor}) + (H_{vac} \pm H_{cor}) + Z_{gauge (press)} + Z_{gauge (vac)} + \frac{v_1^2 - v_2^2}{2g}$$

where  $H_{gauge}$  = Reading of pressure gauge in m of water.

$H_{vac}$  = Reading of vacuum gauge in m of water.

This will be required in case of reaction turbine.

The vacuum gauge is fitted to the outlet of turbine runner or inlet of draft tube.

$H_{cor}$  = Correction factor for the gauge, if any. This will be known after the gauge has been calibrated.

$v_1$  = Inlet velocity of water at the pipe cross-section where the pressure gauge is fitted.

$v_2$  = Exit velocity of water at the pipe cross-section where the vacuum gauge is fitted. This is required for reaction turbine only.

$Z_{gauge (press)}$  = Vertical distance from the centre line of turbine and the point where the pressure gauge is fitted.

$Z_{gauge (vac)}$  = Vertical distance from the turbine centre line to the point where vacuum gauge is fitted. This is required for reaction turbine only.

(b) **Discharge  $Q$ —**The discharge is calculated by means of weir or Venturimeter or by one of the methods explained in author's book on Hydraulics and Fluid Mechanics.

(c) **Torque on Turbine Moment  $M$ —**

$$M = F \cdot l \text{ kg-m}$$

where  $F$  = Brake load in kg from the meter connected to the brake.

In case of rope brake, this is equal to the difference of dead weight on brake and reading of the spring balance,

$$\text{i.e., } F = W - S$$

$l$  = Radius of brake arm.

In some cases a meter fitted on the brake will give directly the value of moment  $M$  in kg-m.

(d) **Brake Horsepower (BHP) from Brake**—The turbine brake horsepower is determined as follows—

$$P_t = \frac{M \cdot \omega}{75} = \frac{F \cdot l \cdot \omega}{75} = \frac{F \cdot l \cdot 2\pi \cdot N}{60 \times 75} \text{ BHP}$$

where  $N$  = Speed of turbine shaft in rpm

(e) **Available or Water Horsepower**

$$P_a = \frac{\gamma \cdot Q \cdot H}{75} \text{ HP}$$

$$(f) \text{ Turbine Overall Efficiency } \eta_t = \frac{P_t}{P_a}$$

Tabulate the above results as shown in Table 14.2.

TABLE 14.2

**Turbine Brake Test****Turbine Calculated Data at Constant Head**

Outlet Opening = .....%

Head.....m

|       | $H$ | $Q$                 | $N$ | $F$ | $M$  | $P_a$ | $P_t$ | $\eta_t$ |
|-------|-----|---------------------|-----|-----|------|-------|-------|----------|
| Point | m   | m <sup>3</sup> /sec | rpm | kg  | kg-m | HP    | BHP   | %        |
|       |     |                     |     |     |      |       |       |          |

**14.14 Calculated at Unit Head**—It will be seen that for different inlet openings of the turbine, the constant head (refer Table 14.2) has not got the same value. Therefore it is the usual practice to convert all the data given in Table 14.3 to a unit head which is then the constant head for all the values of inlet openings. Thus the different quantities given in Table 14.3 for the different inlet openings can then be easily compared with each other. To reduce the quantities to unit head, the following relations (refer Art 3.3) are used.

$$Q_1 = \frac{Q}{\sqrt{H}}, \quad N_1 = \frac{N}{\sqrt{H}}, \quad M_1 = \frac{M}{H},$$

$$P_{t_1} = \frac{P_a}{H^{\frac{3}{2}}}, \quad P_{t_1} = \frac{P_t}{H^{\frac{3}{2}}}$$

$P_{t_1}$  has a non-dimensional value, therefore it will not change.  
Tabulating the results—

TABLE 14.1  
Turbine Brake Test

|                                               |                                        |                                       |
|-----------------------------------------------|----------------------------------------|---------------------------------------|
| Type of Turbine.....                          | Dated.....                             |                                       |
| Turbine Specifications—                       |                                        |                                       |
| Manufacturers.....                            |                                        |                                       |
| Head $H = \dots\dots\dots$                    | Discharge $Q = \dots\dots\dots$        | Output $P_t = \dots\dots\dots$        |
| Overall Efficiency $\eta_t = \dots\dots\dots$ | Specific Speed $N_s = \dots\dots\dots$ | Working Speed $N_o = \dots\dots\dots$ |
|                                               |                                        | Runaway Speed $N_r = \dots\dots\dots$ |

---

(a) Internal diameter  $d$  of pipe at

|                                   |
|-----------------------------------|
| Pressure Gauge Connection = ..... |
| Vacuum Gauge Connection = .....   |

$$\therefore \frac{v_t^2 - v_o^2}{2g} = \frac{Q^2}{2g \times \left(\frac{\pi}{4}\right)^2} \left(\frac{1}{d_t^4} - \frac{1}{d_o^4}\right) = \dots Q$$

(b) Vertical Distance from Centre Line of Turbine to Gauge,  $Z_{gauge}$  (for pressure gauge) = .....

$Z_{gauge}$  (for vacuum gauge) = .....

$\therefore$  Effective Turbine Head  $H = (H_{gauge} \pm H_{cor}) + (H_{vac} \pm H_{cor}) - \frac{v_t^2 - v_o^2}{2g} + Z_{gauge} (vac) + Z_{gauge} (vac)$

$[H_{vac}, Z_{gauge} (vac)]$  and  $v_o$  are for reaction turbine only]

(c) Radius of Brake Arm  $l = \dots\dots\dots$

(d) Type of Weir = .....

Length of Weir Crest = .....

Crest Level of Weir = .....

$Q = \dots\dots\dots$  (Write the formula used)

(e) Any Other Type of Discharge Measurement

Inlet Opening = ..... = %

| Point | Head in m      |                    |              | Discharge in m <sup>3</sup> /sec |  | Dynamometer (Brake) |                                                | Speed           |             | Power in HP |             |
|-------|----------------|--------------------|--------------|----------------------------------|--|---------------------|------------------------------------------------|-----------------|-------------|-------------|-------------|
|       | Pressure Gauge | $\frac{v^2}{2g}$ m | Vacuum Gauge | $H$ m                            |  | Head Over Crest     | $Q$ m <sup>3</sup> /sec                        | Tacho-meter RPM | Correct RPM | $P_b$       | $P_t$ (BHP) |
|       |                |                    |              |                                  |  |                     |                                                |                 |             |             |             |
|       |                |                    |              |                                  |  |                     |                                                |                 |             |             |             |
|       | $H_{gauge}$ m  | $H_{cor}$ m        | $H_{sec}$ m  | $H_{cor}$ m                      |  |                     | $F$ kg<br>[For Spring Balance<br>$F = W - S$ ] |                 |             |             |             |

Tested by.....

**TABLE 14.3**  
**Turbine Brake Test**  
**Turbine Data at Unit Head**

| Inlet Opening = .....% |       |       |       |       |           | Head = 1 m |          |
|------------------------|-------|-------|-------|-------|-----------|------------|----------|
| Point                  | $Q_1$ | $N_1$ | $F_1$ | $M_1$ | $P_{a_1}$ | $P_{t_1}$  | $\eta_t$ |
|                        |       |       |       |       |           |            |          |

The characteristic curves (refer Art 15.3) are then drawn (refer Fig 15.1 to 15.4).

**14.15 Procedure of Testing for Pumps is a Laboratory—** Before the pump is tested check the following :—

(a) *The shaft of pumping unit is not jammed*—This is necessary so that the electric motor may not be burnt when it is switched on in case the shaft is jammed. It is checked by revolving the shaft-coupling by hand.

(b) *The delivery valve of the pump is closed*, so that pressure can be built up by the pump.

(c) *The pump is primed properly*—The pump will not build up its head if there is an air pocket in the pump impeller, volute or suction pipe. The priming is accomplished by opening the air valve and filling the pump with water until it overflows from the air valve. The pump is filled with water either by pouring it with buckets or from an overhead reservoir, through the delivery pipe.

Switch on the driving motor and bring it to speed gradually by moving the handle of the starter slowly. The air valve is kept open partially while the motor is brought to its required speed, so that any residual air may escape. In case there is air and the pump is not properly primed on opening the delivery valve, the delivery pressure (shown by the pressure gauge) will fall to a large extent and the suction pressure (shown by the vacuum gauge) will rise. On such occasions, it is advisable to switch the motor off and start it after priming it properly.

Note the readings of the pressure gauge, vacuum gauge, tachometer, voltmeter, ammeter, power factor meter (for A.C. motors only) and head over weir crest. In case the electric motor is coupled to a mechanical dynamometer, note the readings on the spring balance and measure the radius of the arm. All readings must be taken simultaneously. The axis of tachometer must be kept horizontal with the help of a level gauge while measuring the speed of priming unit. The reading of head over the weir should be taken after the water has travelled from the pump to the weir, say after about 3 minutes.

The delivery valve is opened gradually and several sets of all the above readings are taken at constant speed. The speed is kept constant by varying the voltage of the motor by moving the rheostat.

Change the speed of the motor and repeat the above process at constant speeds. Take such four or five different sets of readings at constant speeds which are required to draw the various characteristic curves of the pump (refer Fig 15.8 to 15.12).

**14.16 Data to be Measured**—Table 14.4 is used.

*I. The following data are measured while the experiment is being performed—*

(a) **Head from Pump Gauges**—This is obtained from the pressure gauge and vacuum gauge connected to delivery side and suction side respectively, both measured in m of water.

(b) **Pump Speed** is measured by means of a tachometer or RPM-counter.

(c) **Power Consumption by Electric Motor** by measuring voltage, amperage and power factor (for A.C. motors only). In case the driving motor is equipped with mechanical dynamometer (say rope brake), measure the load from the spring balance.

(d) **Head of Weir Crest** by hook gauge.

*II. The following data to be measured are required for the calculations explained in the next article (Art 14.17).*

(a) **Internal Diameters** of the pipes where the gauges are fitted.

(b) **Vertical Distance  $Z_{gauge}$**  between the centre lines of the pressure and vacuum gauges.

(c) **Radius of the Brake Arm  $l$**  from the centre line of the pump shaft to the point of application of weight.

(d) **Length of Weir Crest.**

(e) **Weir Crest Level** is measured when no more water is flowing over the weir, after the experiment has been performed.

All the instruments employed for the experiment *e.g.*, gauges, tachometer and electric meters, spring balance (for rope brake) must be calibrated before they are used.

All the above data is tabulated for each *constant* speed as shown in the Table 14.4.

**14.17 Calculated Data at Constant Speed—**

(a) **Pump Effective Head  $H$**  as well as, (b) **Discharge  $Q$**  are determined as explained in Art 14.13 *a* and *b* for turbines.

(c) **Input to Pump—**

$$\text{SHP} = \frac{A \cdot V}{1,000} \times \eta_{\text{motor}} \text{ KW} \quad \text{or} \quad \frac{A \cdot V}{736} \times \eta_{\text{motor}} \text{ HP}$$

where  $A$  = Amperage ;

$V$  = Voltage ;

$\eta_{\text{motor}}$  = Efficiency of the motor taken from the efficiency curve of motor supplied by the motor manufacturers or obtained earlier by performing the experiment on the motor.

Input to the driving motor is also obtained by mechanical dynamometer by calculating the torque.

$$T = F \cdot l \text{ m.kg}$$

and in case of spring balance

$$F = W - S$$

TABLE 14.4

**Pump Brake Test**

Type of Pump .....

*Pump Specifications :*

Manufacturers .....

Head  $H_{\text{mano}} = \dots\dots\dots$  Discharge  $Q = \dots\dots\dots$  Input  $\text{SHP} = \dots\dots\dots$

Overall Efficiency  $\eta_{\text{overall}} = \dots\dots\dots$  Speed  $N = \dots\dots\dots$  Specific Speed  $N_s = \dots\dots\dots$

*Driving Motor Specifications :* HP = ....., DC/AC, Type..... Voltage = .....

(a) Suction pipe Dia  $d_s = \dots\dots\dots$  }  $\therefore \frac{v_d^3 - v_s^3}{2g} = \frac{Q^2}{2g} \times \left(\frac{4}{\pi}\right)^2 \times \left(\frac{1}{d_d^4} - \frac{1}{d_s^4}\right) = \dots\dots\dots Q^2$

(b) Delivery Pipe Dia  $d_d = \dots\dots\dots$  }

(c) Vertical distance between Points of Connection of Suction and Delivery Gauges,  $Z_{\text{gauge}} = \dots\dots\dots$

$\therefore$  Effective Pump Head  $H_{\text{mano}} = (H_s \pm H_{\text{cor}}) + (H_d \pm H_{\text{cor}}) + Z_{\text{gauge}} + \frac{v_d^2 - v_s^2}{2g}$

(d) Radius of Brake Arm. of Mechanical Dynamometer  $l = \dots\dots\dots$

(e) Type of Weir = ..... Length of Weir Crest = .....  
Crest Level of Weir = .....  $Q = \dots\dots\dots$  (Write the formula used)

(f) Any other Type of Discharge Measurement

(g) Efficiency Curves of Driving Motor to get  $\eta_{\text{motor}}$ —(This curve must be ready before the experiment is performed)





where  $W$  = Dead weight ;

and  $S$  = Reading of spring balance.

$\therefore$  Output of driving motor or Pump Input

$$\text{SHP} = \frac{2\pi NT}{4,500} \text{ HP}$$

(d) **Output of Pump—**

$$\text{WHP} = \frac{\gamma \cdot Q \cdot H}{75} \text{ HP}$$

(e) **Overall Efficiency of Pump—**

$$\eta_{\text{pump}} = \frac{\text{Output}}{\text{Input}} = \frac{\text{WHP}}{\text{SHP}}$$

Tabulate the above result as shown in Table 14.5

**TABLE 14.5**  
**Pump Brake Test**  
**Pump Data at Constant Speed**  
 $N \dots\dots\dots \text{rpm} = \text{constant}$

| Point | $H_{\text{manometric}}$ |                     | $Q$     | $N$ | WHP | SHP | $P_{kw}$<br>(motor) | $\eta_{\text{overall}} = \frac{\text{WHP}}{\text{SHP}}$ |
|-------|-------------------------|---------------------|---------|-----|-----|-----|---------------------|---------------------------------------------------------|
|       | m                       | m <sup>3</sup> /sec | lit/sec | rpm | HP  | HP  | KW                  | %                                                       |
|       |                         |                     |         |     |     |     |                     |                                                         |

**Problem 14.1** In the test of a centrifugal pump with water, the rate of discharge was found to be  $0.25 \text{ m}^3/\text{sec}$ . Pressure gauge reads  $13.35 \text{ kg/cm}^2$  suction gauge reads  $150 \text{ mm}$  of Hg and the difference in elevation between the gauges was  $0.8 \text{ m}$ . The diameters of suction and delivery pipes are same. Find the efficiency of pump if input HP is 600. (AMIE)

**Solution**

$$Q = 0.25 \text{ m}^3/\text{sec} \quad H_d = 13.35 \text{ kg/cm}^2 \quad H_s = 150 \text{ mm of Hg}$$

$$\text{Difference in gauges elevation} = 0.8 \text{ m} \quad \text{HP} = 600$$

$$H_d = \frac{13.35 \times 100^2}{1,000} = 133.5 \text{ m of water}$$

$$H_s = \frac{150 \times 10}{760} = 1.98 \text{ m of water}$$

$\therefore$  Total head supplied by the pump

$$H = H_d + H_s + h = 133.5 + 1.98 + 0.8 = 136.28 \text{ m of water}$$

$$\text{Power delivered by the pump } P = \frac{\gamma \cdot Q \cdot H}{75}$$

$$P = \frac{1,000 \times 0.25 \times 136.28}{75} = 454.3 \text{ HP}$$

$$\therefore \text{Efficiency of pump } \eta = \frac{\text{output}}{\text{input}} = \frac{454.3}{600} \times 100$$

$$= 75.7 \% \quad \text{Answer}$$

### 14 B. Power and Efficiency Measurements

**14.18 Different Methods for Measuring Power and Efficiency**—There are several methods employed for the measurement of power output, and hence, for the calculation of efficiency of turbine models during tests in the laboratory or of actual turbines during takeover tests on the site. These are classified under two heads :

Methods using mechanical means and

Methods using electrical means.

The following types of mechanical equipment are used for the measurement of power and efficiency—

- (a) Prony Friction Brake ;
- (b) Rope Brake ;
- (c) Tesla Fluid-Friction Brake ; or
- (d) Froude Water Vortex Brake ;
- (e) Torsion Dynamometer.

**14.19 Prony Friction Brake**—The prime mover shaft is braked by means of friction and by measuring the *rpm* developed against a known frictional force, the output of power can be easily calculated.

Apparatus consists of a simple pulley keyed to the prime-mover shaft. Clamps held together by adjustable bolts and flynuts brake the pulley by means of mechanical friction through cork padding. To the clamp is attached a lever the other end of which is balanced against known weights as shown in Fig 14.8. This is used for small values of *N* (*rpm*) and higher values of *F*. It is suitable for tests in the laboratory.

If,  $F$  = the peripheral force on brake lever ;  
 $T$  = the tare ;  
 $G$  = the weight on the pan of the balance ;  
 then  $F = G - T$

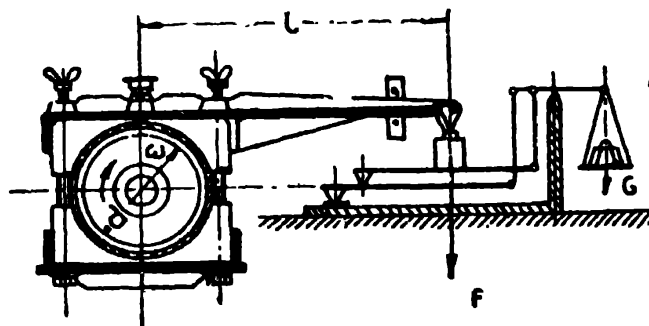


Fig 14.8 Prony Friction-Brake

Let  $l$  = length of brake lever which is known for a particular brake  
 then, torque =  $F \cdot l$

Further, the angular velocity of pulley

$$\omega = \frac{2\pi \cdot N}{60} \quad \text{where } N = \text{measured rpm of shaft,}$$

and Power  $P = (\text{Torque}) \cdot \omega = F \cdot l \cdot \frac{2\pi \cdot N}{60} \text{ kg-m/sec}$

$$\therefore \text{HP} = \frac{P}{75} = \frac{F \cdot l \cdot 2\pi \cdot N}{60 \times 75}$$

$$= \frac{F \cdot l \cdot \pi \cdot N}{4,500} \quad \dots(14.1)$$

Power in KW = Power in HP  $\times 0.736$  ...(14.1a)

**Determination of Brake Test Curves**—Power output expressed in HP,

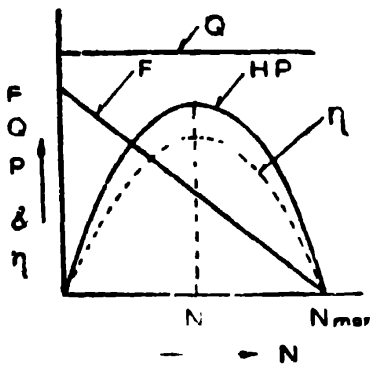


Fig 14.9 Brake Test Curves for Pelton Turbine  
 $F, Q, P$  &  $\eta$  vs  $N$

$$P_{\text{HP}} = \frac{2\pi \cdot N \cdot F \cdot l}{4,500} \text{ HP}$$

The factor  $\frac{2\pi \cdot l}{4,500}$  is a constant for a brake

$$\therefore P_{\text{HP}} = \frac{F \cdot N}{K_1}$$

By adjusting the position of flynuts, mechanical friction on the pulley is varied and a series of values for  $F$ , viz,  $F_1, F_2, F_3$  etc are chosen. Speed corresponding to each is measured with rpm counter. Value of HP for each point is calculated. These are tabulated and a curve is drawn (refer Fig 14.9). Also, since the power input  $P$  is known, efficiency  $\eta$  can be calculated.

The curve shown in Fig 14.9 belongs to a Pelton turbine. In this figure

$N_{\text{max}}$  = the runaway speed

and  $N_o$  = the speed for which the turbine is designed and where power  $P$  would be maximum.

**Problem 14.2** Find the values of  $F$ , the peripheral force on the lever on a Prony Brake,  $M$  the turning moment,  $P_t$  the turbine brake horsepower and  $\eta$ , the total (overall) efficiency of the turbine from the following experimental data obtained from the characteristics of a Pelton wheel :

Total head = 80.97 m,  $Q = 55.24 \text{ lit/sec}$ ,  $N = 755 \text{ rpm}$   
 $l = 1.43 \text{ m}$ , Tare = 7.5 kg, Weight in pan = 39.9 kg

**Solution**

$$F = G - T$$

$$= 39.9 - 7.5 = 32.4 \text{ kg} \quad \text{Answer}$$

$$M = F \cdot l = 32.4 \times 1.43 = 46.3 \text{ kg m} \quad \text{Answer}$$

$$P_t = \frac{2\pi \cdot N \cdot (F \cdot l)}{60} = \frac{2\pi \times 755 \times 46.3}{60} = 3,660 \text{ kg-m/sec}$$

Answer

$$P_a = \gamma \cdot Q H \quad \text{where } \gamma = \text{Specific weight of water,}$$

$$= 1 \text{ kg per lit}$$

$$= 1 \times 55.24 \times 80.97$$

$$= 4,490 \text{ kg-m/sec} \quad \text{Answer}$$

and  $\eta_t = \frac{P_t}{P_s} \times 100 = \frac{3,660}{4,490} \times 100 = 81.5\%$  *Answer*

#### 14.20 Rope Brake—

Rope brake consists of a doubly wound rope over the rim of a pulley on a flywheel keyed on the shaft transmitting the power to be measured (refer Fig 14.10). The rope is usually of cotton and for laboratory purposes, its diameter is not more than 10 to 20 mm depending upon the power of the machine to be tested. A few U-shaped blocks are provided to keep the rope in position and to guard against its slipping off the pulley. Dead weights are suspended from the bottom end of the rope and spring balance is connected to the upper end. A rod passing through slits in the weights is fixed to the frame of the prime-mover thus preventing the weights from flying off when the machine is being started or stopped.

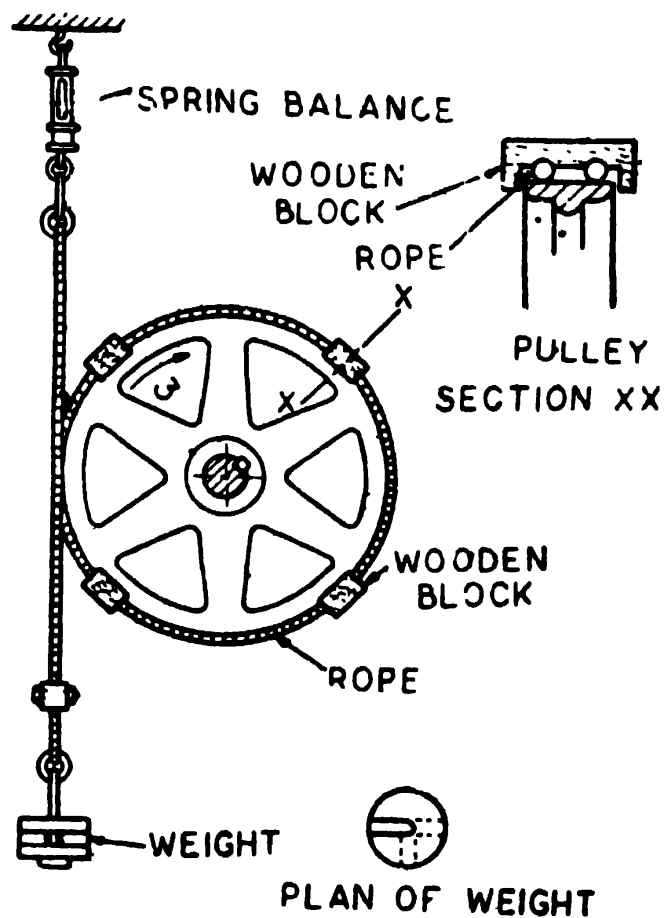


Fig 14.10 Rope Brake

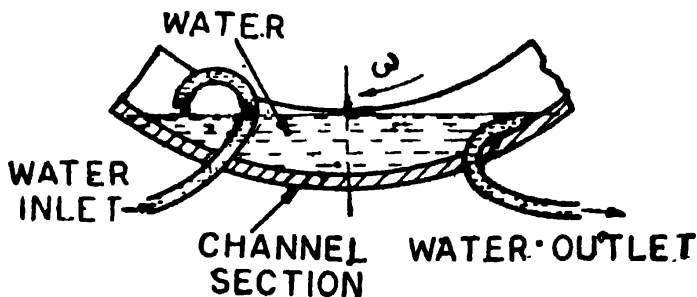


Fig 14.11 Cooling Arrangement in Rope Brake

A cooling arrangement may be provided if undue heat is being developed due to friction between the rope and the pulley. The pulley itself has a channel section with the flanges turned inside (refer Fig 14.11). Cooling water supplied by a pipe is circulated through the channel and discharged by

an outlet pipe with a flattened end which enables it to scoop the running water.

**Critical Speed of Pulley**—In order to hold the water in the top portion of the channel, the centrifugal force acting on any particle should be greater than its weight. The resultant force would then act upward and be equal to

$$\text{Centrifugal force—weight of water} = F - w$$

Now, 
$$F = \frac{w}{g} \cdot \omega^2 \cdot R$$

where,  $R$  = mean radius at which water is revolving and  $\omega$  the angular velocity  $= \frac{2\pi \cdot N}{60}$  ( $N$  is the speed of pulley in rpm).

$$\therefore F = \frac{w}{g} \left( \frac{2\pi \cdot N}{60} \right)^2 \cdot R \quad \dots(14.2)$$

For critical speed, which is the minimum speed holding the water,

$$F - w = 0$$

$$\text{or,} \quad \left( \frac{2\pi \cdot N}{60} \right)^2 \cdot \frac{R}{g} = 1$$

$$\text{i.e.,} \quad N_{\text{crit}} = \frac{60}{2\pi} \sqrt{\frac{g}{R}} \quad \dots(14.3)$$

If the speed falls below its critical value, water inside the rim would not rise to the top.

**Problem 14.3** The rim of a braking wheel of a turbine is of channel section and its internal diameter is 45 cm. Find the minimum speed in rpm at which the wheel will hold a layer of water 25 mm deep at the top of the rim.

**Solution**

$$R = 22.5 \text{ cm}$$

$$\text{Depth of water layer at top} = 2.5 \text{ cm}$$

$$\therefore \text{Minimum radius at top, } R' = 22.5 - 2.5 = 20 \text{ cm}$$

Now considering a water particle 20 cm vertically above the centre as a free body, for critical equilibrium :

Centrifugal Force = Gravitational Force

$$\text{i.e.,} \quad \frac{w}{g} \cdot \left( \frac{2\pi \cdot N}{60} \right)^2 \cdot R' = w$$

$$\text{or} \quad N = \sqrt{\frac{60^2}{4\pi^2} \cdot \frac{g}{R'}} = \frac{30}{\pi} \sqrt{\frac{g}{R}}$$

$\therefore$  Required minimum speed in rpm,

$$N = \frac{80}{\pi} \sqrt{\frac{9.81}{0.2}} = 63.8 \text{ rpm} \quad \text{Answer}$$

**14.21 Tesla Fluid Friction Brake**—The apparatus consists of a disc keyed to the shaft and moving with it, and a casing as shown in Fig 14.12. Space between the casing and disc contains water or any other

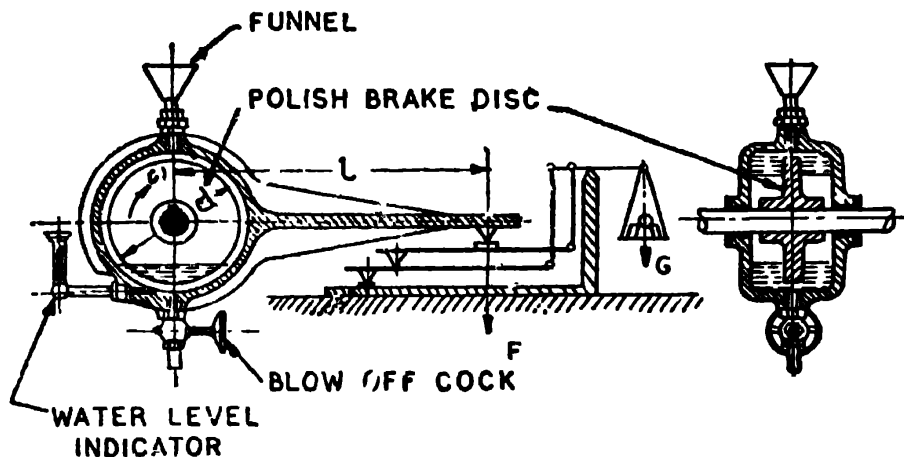


Fig 14.12 Tesla Fluid Friction Brake

liquid. Casing carries a lever, the end of which balances against weights and is, therefore, at rest. Relative motion between disc and casing causes viscous forces in the water which are utilised for braking the disc and the shaft.

if shear stress, 
$$f_s = \frac{F}{A} = \mu \cdot \frac{dN}{dy}$$

$$dF = da \cdot f_s$$

and area  $da = 2 \cdot (2\pi \cdot r \cdot dr)$  considering both sides of disc,

then, 
$$dF = \mu \frac{dN}{dy} \times 2 \times (2\pi r \cdot dr)$$

Now, Turning Moment or Torque,

$$dM_t = dF \cdot r = \mu \frac{dN}{dy} \times 2 \times (2\pi r \cdot dr) \cdot r \quad (14.4)$$

$$\therefore M_t = \int dM_t$$

and 
$$\text{Power} = M_t \cdot \omega$$

This arrangement is suitable for power measurement when the speed is high and the force  $F$  is small.

**14.22 Froude Water Vortex Brake**—It consists of a rotating disc provided with several semi-elliptical blades forming brake chambers. The disc is keyed on the driving shaft of the turbine and revolves inside a stationary casing equipped with counter blades projecting inside to form ring chambers. Fig 14.13 shows a cylindrical section through the blades of the runner and the casing. Water is supplied continuously through a flexible metal hose to the ring chamber of the casing at  $A$  and allowed to flow into the brake chambers of the disc,  $B$ . Air, which is supplied to the casing through a separate hose, is also passed on to the runner. As the disc rotates, vortices and eddy-currents are set up in the water enabling the mechanical power to be dissipated as heat.

Heated water emerges at  $C$ , the clearance runner and the casing (refer Fig 14.13). The casing carries a lever balanced at the end against known weights as shown.

Power delivered to the brakes by the prime-mover

$$P_t = \frac{2\pi \cdot N \cdot F \cdot l}{60 \times 75} \text{ HP}$$

$$\begin{aligned} \text{Heat generated per hour} &= \frac{P_t \times 60 \times 75 \times 60}{427} \text{ k cal} \\ &= 633 \text{ k-cal per HP} \end{aligned}$$

This is the amount of heat dissipated in water.

If the quantity of water supplied per BHP per hour is  $W$  kg, rise in temperature

$$= \frac{633}{W} \text{ } ^\circ\text{C}$$

Estimating the permissible rise in temperature the amount of water required per BHP per hour is predetermined.

This arrangement is generally, used for prime movers having more than 500 HP.

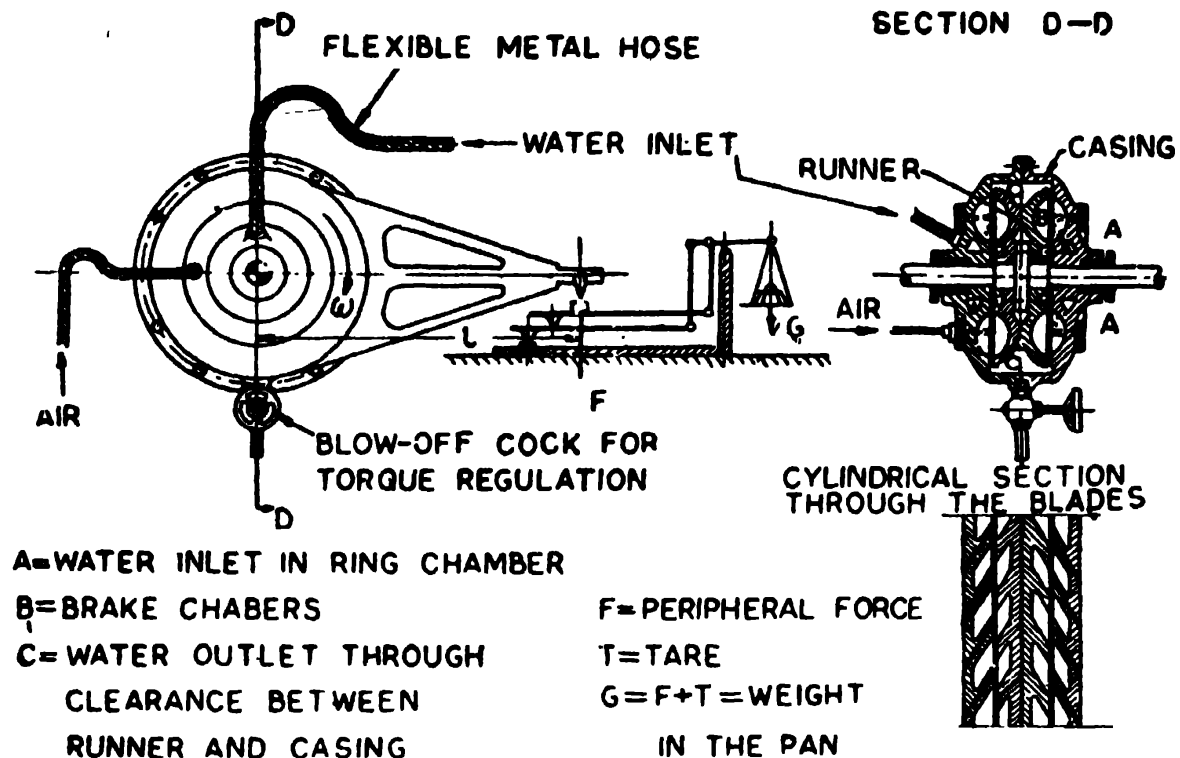


Fig 14.13 Froude Water Vortex Brake

**14.23 Torsion Dynamometer**—This instrument is suitable for measuring power transmitted at high speeds but against a nearly constant load.

In principle, it is a load measuring coupling inserted between the prime-mover and the driven machine. Torque is transmitted by a steel bar capable of elastic deformation. The angle of twist of this bar, which is read by an ingenious stroboscope device, is a measure of the power transmitted. Accuracy depends largely on the uniformity of speed and torque.

In practice, the elastic bar 1 (refer Fig 14.14) is rigidly connected at the ends of two flanges 3 and 4 which are bolted to similar flanges on the driver and driven shafts. The bar is carried inside a sleeve 2 which is integral with the flange 3 at one end but free at the other. When power

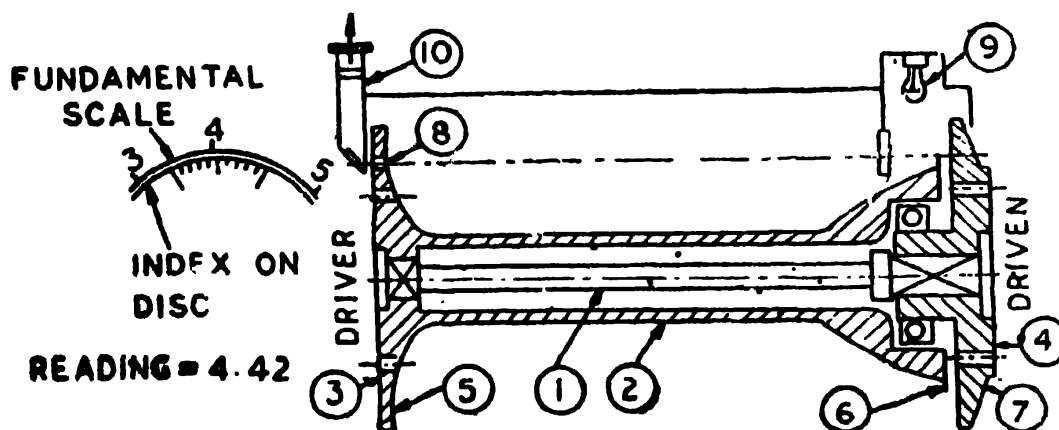


Fig 14.14 Amsler Torsion Dynamometer

is being transmitted, both the bar and the sleeve rotate with the same speed but the load is taken by the bar alone. There is, however, a clutch arrangement so that the sleeve can take the load when it exceeds the elastic limit set for the bar.

For measuring the angle of twist, three discs 5, 6 and 7 are provided, 7 on the bar and 5 and 6 on the sleeve. The disc 7 carried by the bar is engraved with a scale known as fundamental scale which has widely spaced divisions. The disc 6 on the sleeve just adjacent to the former disc 7 carries an index divided in tenths of a fundamental scale division as shown. The relative shift of the zero marks of the two scales 6 and 7 gives the angle of twist (see reading = 4.42 shown in Fig 14.14). To enable this shift to be read the sleeve 2 carries another disc 5 with a slot 8 in the line of the zero marks of the two scales when at rest. If a fixed source of light 9 illuminates the rest position of the two scales and a fixed observation instrument 10 is placed in front of the rest position of the slot 8 on the third disc 5, when the three discs rotate, the observer's eye will receive impulses of light of the same frequency as the rpm of torsion meter. If this frequency is sufficiently high, persistence of vision will produce a static image and displacement of index 6 with respect to fundamental scale can be easily read.

One precaution is necessary. The critical speed for vibration of the meter or the system of rotating parts as a whole must not fall in the working range of the apparatus.

Let  $\phi$  = the angle of twist ;  
 $l$  = the length of shaft ;  
 $I_p$  = the polar moment of inertia of section of shaft ;  
 $G$  = the Modulus of rigidity  $\left( = \frac{13}{15} \cdot E \right)$

Then  $M_t$ , the turning moment or twisting moment

$$= \frac{I_p \cdot G \cdot \phi}{l} \quad \dots(14.3)$$

For a solid shaft of diameter  $d$ ,  $I_p = \frac{\pi}{32} \cdot d^4$

or  $M_t = k \cdot \phi$

where  $k = \frac{I_p \cdot G}{l}$  is a constant for a particular shaft.

The value of  $k$  is determined by the *Gauss* method of least squares. If  $\delta$  be the error.

$$(M_{t_1} - k \cdot \phi_1)^2 = \delta_1^2 \quad \dots(1)$$

$$(M_{t_2} - k \cdot \phi_2)^2 = \delta_2^2 \quad \dots(2)$$

$$(M_{t_n} - k \cdot \phi_n)^2 = \delta_n^2 \quad \dots(3)$$

$$\text{Now, } \sum_{i=1}^{i=n} (\delta_i)^2 = f(k) \quad \text{and} \quad \frac{\sum (\delta_i)^2}{\delta k} = 0$$

whence  $k$  can be calculated.

**Problem 14.4** The horsepower of a turbine was found by observing that the angle of twist of a 20 m long attached shaft at 480 rpm was  $1.75^\circ$ . The shaft, which was solid, had a diameter of 0.6 m and it was known that



the modulus of rigidity of the material of the shaft was 840,000 kg/cm<sup>2</sup>. Neglecting the effect of the end thrust, determine the brake horsepower of the turbine.

**Solution**

$$l = 20 \text{ m} \quad N = 480 \text{ rpm} \quad d = 0.6 \text{ m}$$

$$\phi = 1.75^\circ \quad G = 840,000 \text{ kg/cm}^2$$

$$\phi = 1.75 \times \frac{\pi}{180} = 0.0305 \text{ radians}$$

Polar moment of inertia,

$$I_p = \frac{\pi}{32} \cdot d^4 = \frac{\pi}{32} (0.6)^4 = 0.01275 \text{ m}^4$$

$$G = 840,000 \times 10^4 \text{ kg/m}^2$$

$$\therefore \text{Torque } M_t = \frac{I_p \cdot G \cdot \phi}{l} = \frac{0.01275 \times 840,000 \times 10^4 \times 0.0305}{20}$$

$$= 163,000 \text{ m}\cdot\text{kg}$$

$$\omega = \frac{2\pi N}{60} = \frac{2\pi \times 480}{60} = 50.2 \text{ rad/sec}$$

Neglecting the effect of end thrust, the power

$$P = M_t \cdot \omega = 163,000 \times 50.2 \text{ m}\cdot\text{kg/sec}$$

$$= \frac{163,000 \times 50.2}{75} = 119,000 \text{ HP} \quad \text{Answer}$$

**14.24 Measurement of Power with the Help of Electrical Equipment**—A generator directly coupled to the turbine converts the mechanical energy delivered to the turbine shaft into electrical power. The output of generator is measured with the help of electrical meters. The turbine power is evidently equal to the sum of the generator output and the losses in the generator.

Now a days almost all electrical energy produced is AC 3-phase with delta or star connections. In India AC power is usually supplied at a frequency of 50 cycles per sec.

**Measurement of Electrical Power—**

**DC Power**— $W = E \cdot I$  watts

where  $E$  = voltage

and  $I$  = current in amperes

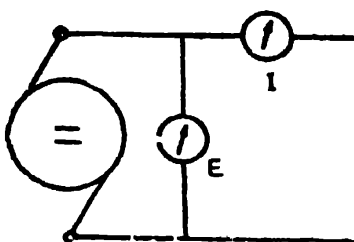


**Fig 14.15** Connection for DC Supply

**AC Power**— $W = \sqrt{3} \cdot E I \cos \phi$   
(for 3-phase supply)

where  $E$  = line voltage

and  $I$  = line current

$\cos \phi$  = line power factor

Voltage and current for DC are measured by a voltmeter and an ammeter respectively. Connections for the same are shown in Fig 14.15. AC can be measured with either two wattmeter or three-wattmeter methods. In practice the former is generally employed. For connections refer Fig 14.16.

The electrical method is usually applied where power and efficiency of actual turbines have to be measured. The operation is carried out at the site after the turbo-units have been installed. The first tests after installation are known as take-over tests. The manufacturer fulfills the guarantees offered to the customer at the time of submission of tenders. Since immediate consumers of electrical energy generated in take-over tests are not available, water resistance is employed as the load. Fig 14.17 shows the arrangement of water resistance for DC. Metal plate electrodes are lowered in a large tank kept full of water by supply pipes. For a 50 mm separation between adjacent plates, the impressed current density is about  $3.25 \text{ amp/m}^2$  at 220 V. Characteristics of water resistance shown in Fig 14.18 is a curve obtained by plotting  $I$  as a function of  $h$ , where  $I$  is the current consumed and  $h$  is the depth of plate under water. Connections for water resistance for AC are shown in Fig 14.19.

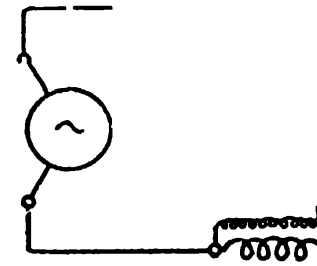


Fig 14.16 Connection for Two Wattmeter Method to Measure AC Supply

**14.25 Losses in a Generator**—Power losses in a generator are classified as follows —

(a) **Mechanical Losses**—

- (i) Bearing friction loss,  $\Delta P_{G_1}$
- (ii) Air ventilation or windage loss,  $\Delta P_{G_2}$

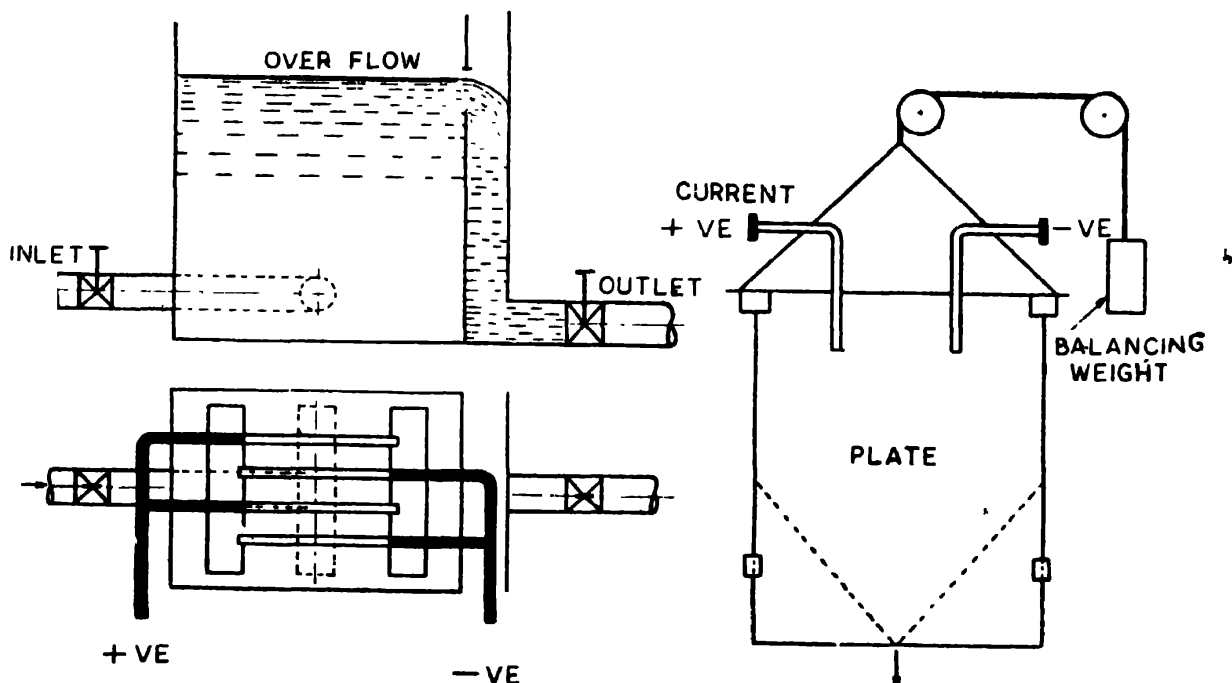


Fig 14.17 Water Resistance for DC Supply

(b) **Electrical Losses**—

- (iii) Excitation power in field coil,  $\Delta P_{G_3}$
- (iv) Copper loss, heating effect of current,  $\Delta P_{G_4}$
- (v) Iron losses or hysteresis loss,  $\Delta P_{G_5}$

Total generator loss,

$$\begin{aligned}\Delta P_G &= \Delta P_{G_1} + \Delta P_{G_2} + \Delta P_{G_3} + \Delta P_{G_4} + \Delta P_{G_5} \\ &= \sum_{i=1}^{i=5} \Delta P_{G_i}\end{aligned}$$

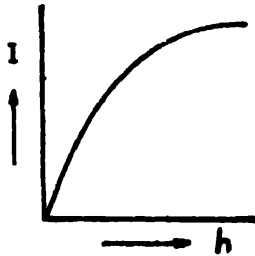


Fig 14.18 Characteristics of Water Resistance

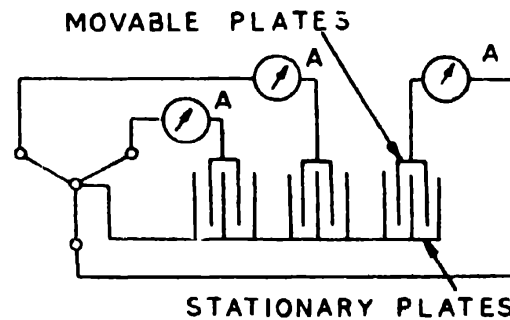


Fig 14.19 Connections for Y-Connected Three-Phase, AC Supply

If the output of the generator be  $P_G$  and power supplied to it by the turbine is  $P_{G_i}$ , then

$$P_{G_i} = P_G + \sum_{i=1}^{i=5} \Delta P_{G_i}$$

$$\text{Efficiency of generator, } \eta_G = \frac{\text{Output}}{\text{Input}} = \frac{P_G}{P_G + \sum_{i=1}^{i=5} \Delta P_{G_i}} \quad \dots(14.6)$$

$$\therefore P_{G_i} = \frac{P_G}{\eta_G}$$

TABLE 14.6  
Practical Data for Efficiency of Generator

| $P_G$ (KW)         | 100 | 500 | 1,000 | 5,000 | 10,000 |
|--------------------|-----|-----|-------|-------|--------|
| $\eta_G$ (percent) | 93  | 94  | 95    | 96    | 97     |

Table 14.6 has been prepared on the assumption that power factor  $\cos \phi = 1$ . In practice  $\phi > 0^\circ$  and  $\eta_G$  is less than what is given above. If the generator is coupled directly to turbine, there are no transmission losses between the two, and power given by the turbine on its shaft.

$$P_t = P_{G_i}$$

**14.26 Estimation of Turbine Efficiency**—Power output of the turbine can be measured by any of the above methods. To determine its overall efficiency the rate of flow of water and total available head should be known. Total head is the sum of the potential and kinetic heads. Potential head  $H'$  is equal to the height of the head race level above the

tail race level. If  $v_t$  and  $v_d$  be the velocities of water at the head and tail races respectively, kinetic head available =  $\frac{v_t^2}{2g} - \frac{v_d^2}{2g}$

$$\therefore \text{Net head } H = H' + \left( \frac{v_t^2}{2g} - \frac{v_d^2}{2g} \right) \quad \dots(14.7)$$

If water is conveyed through penstocks, the head loss due to friction is to be subtracted to obtain the net head.

Gross available power,  $P_a = \gamma \cdot Q \cdot H$

**Turbine Losses** — Energy losses in a turbine are divided under two heads :

(a) **Mechanical losses** —

- (i) Bearing friction loss,  $\Delta P_{t_1}$
- (ii) Ventilation and windage loss,  $\Delta P_{t_2}$

(b) **Hydraulic losses** —

- (i) Water head loss,  $\Delta P_{t_3}$
- (ii) Volumetric or water quantity loss,  $\Delta P_{t_4}$

Total power loss in the turbine

$$\Delta P_t = \sum_{i=1}^{i=4} \Delta P_{t_i}$$

$\therefore$  Power put into the turbine shaft,  $P_t = P_a - \Delta P_t$

Overall efficiency of turbine,

$$\eta_t = \frac{P_t}{P_a} = \frac{P_a - \Delta P_t}{P_a} = 1 - \frac{\Delta P_t}{P_a}$$

But the turbine losses  $\Delta P_t$  cannot be directly estimated. Therefore the generator power and its losses are measured, whence the turbine power,

$$P_t = \frac{P_G}{\eta_G}$$

$$\text{and then } \eta_t = \frac{P_t}{P_a} = \frac{P_G}{\eta_G \cdot P_a} \quad \dots (14.8)$$

can be determined,

Thus in order to calculate  $\eta_t$  the following must be measured :

$P_G$ ,  $\Delta P_G$ ,  $Q$ ,  $H'$ ,  $a_t$  and  $a_d$ .

From  $a_t$  and  $a_d$ , the velocities

$$v_t = \frac{Q}{a_t}, \text{ and } v_d = \frac{Q}{a_d}$$

can be calculated.

#### 14 C. Level Measurements

##### 14.27 Measurement of Level or Height of Free Surfaces—

The height of a free surface of liquid can be accurately measured by one of the following instruments :

- (i) Pointer Gauge, (ii) Hook Gauge, (iii) Floats.

(i) **Pointer Gauge**—A long rod with a fine point at one end move

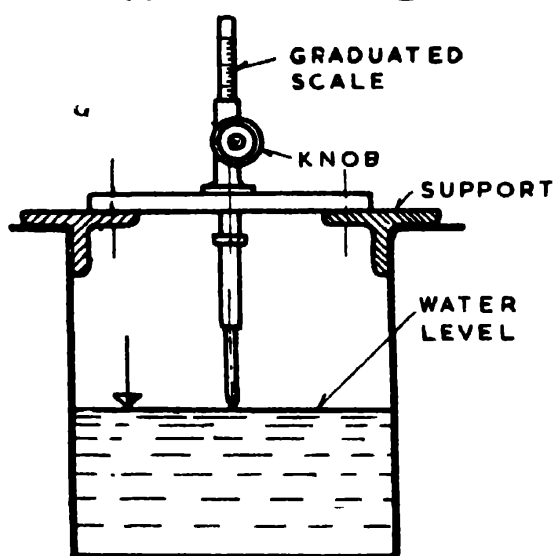


Fig 14.20 Pointer Gauge

vertically in a bearing (refer Fig 14.20). A scale with a vernier is fitted to the rod. As soon as the point touches the liquid surface the rod is screwed in its position and the height is read on the scale. It is difficult for the eye to judge correctly when the point is just touching the liquid surface. For accurate measurement, two points may be used and an electric circuit so arranged that as soon as the points touch the liquid, the circuit is completed and an ammeter placed in the circuit immediately shows a reading. When still water surface is being measured, a pinch of common salt dropped in the region will considerably improve its conductivity.

(ii) **Hook Gauge**—When the observer has to rely on judgment of the eye, it is often advantageous to use a hook gauge (refer Fig 14.21) instead

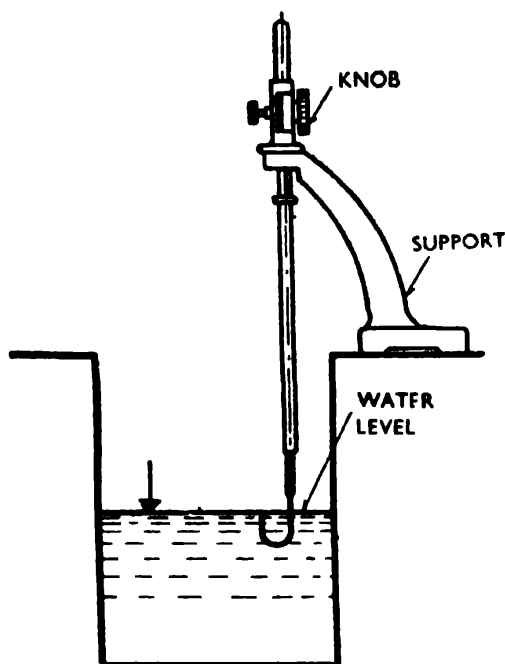


Fig 14.21 Hook Gauge

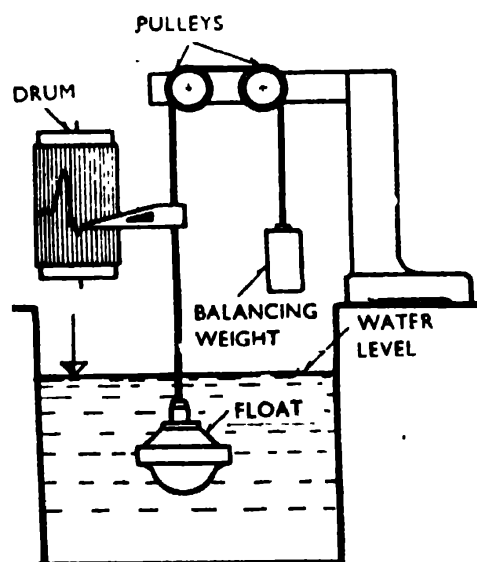


Fig 14.22 Float

of a pointer gauge. Height is read when the point of the hook just emerges above the liquid surface. This can be judged fairly accurately.

(iii) **Float**—Float in its simplest form is merely a hollow box or sphere of sheet metal, is allowed to float on the free surface of the liquid (refer Fig 14.21). A wire connected to it at one end passes over a pulley and carries suspended weight at the other end. Position of the weight or of any particular point of the wire can be used to find out the level of the liquid. If the friction at the pulley is small, this arrangement may yield accurate results.

## TESTING OF HYDRAULIC MACHINES

**14.28 Level Recorders** are now available. These work on a float principle. Fig 14.23 shows a typical level recorder which consists of a

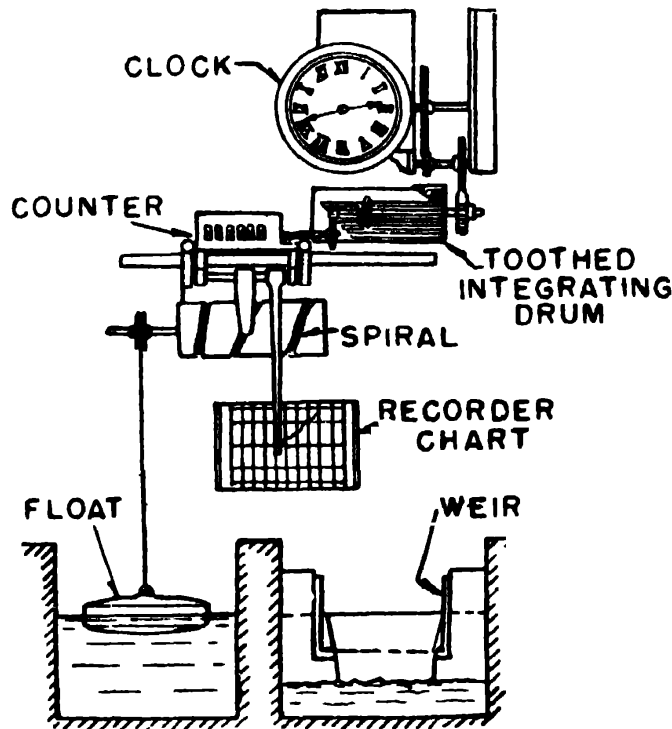


Fig 14.23 Level Recorder

float to which a chord is attached. The chord passes over a wheel fitted on a spindle. A cylinder is mounted on this spindle. On the cylinder, spiral grooves of uniform pitch are cut. The pen carriage carries an extension which engages in the spiral and is constrained to move horizontally and parallel with the cylinder axis. As the cylinder rotates with the vertical movement of float, the pen carriage undergoes a horizontal displacement. This movement is recorded on paper chart wrapped round a horizontal drum revolved by clockwork at constant speed.

### 14 D. Speed Measurement

**14.29 Measurement of Speed**—Revolutions per minute ( $N$ ) of the shaft are measured by one the following instruments :

- (i) Revolution Indicator or Revolution Counter,
- (ii) Tachometer,
- (iii) RPS or RPM Counter,
- (iv) Electrical or Optical Instruments.

In (i) revolutions are indicated on a scale at the end of the shaft. In (ii) no indication of time for the measurement of which a stop watch is needed.

Tachometer gives speed in rpm while it is being driven, generally, by means of a belt from the shaft. The actuating mechanism is (a) a centrifugal device or (b) an electrical device or (c) a resonant vibrating reed.

The centrifugal tachometers are made in multi-range units, the change from one range to another being made by a given gear train fitted inside the instrument. This change should *not* be effected while the instrument is in use.

RPM Counter is a combination of (i) and (ii). Depending on the design, it takes time upto about three seconds to attain full speed after which

the indicator needle stops, and the reading is preserved even if it is disconnected. Its working is similar to that of a stop watch.

**Electric Tachometer** comprises a generator with a constant permanent-magnet field and loaded only with high resistance voltmeter provides an output voltage which is nearly proportional to speed. These machines are made in both A.C. and D.C. types.

Another type of electric tachometer consists of a contact maker driven by the shaft. The pulses generated by the contact maker build up charge in a condenser, and a voltmeter reads the EMF across the condenser plates. Power supply or batteries are essential to operate this type of electric tachometer.

A Swedish firm, Nydqvist & Holm (NOHAB), has developed an electronic revolution counter having a counting capacity of 30,000 pulses per second. The pulses are produced by means of a rotating disc.

### UNSOLVED PROBLEMS

#### 14 A. Testing of Turbines and Pumps

- 14.1 What are the purposes of testing of the turbines and pumps ?
- 14.2 What are testing codes ? What do they contain and for what purposes are they used ?
- 14.3 What are acceptance or take-over tests ? What is the necessity of their performance ? What steps are taken to carry out such tests ?
- 14.4 Define model tests. What is the general size of a model turbine ? Describe briefly the procedure to carry out such tests for turbines and pumps.
- 14.5 What is the necessity of equipping the manufacturers' works with a test bed ? Why are different test beds needed for each type of turbine and pump ?
- 14.6 Explain briefly "Aerodynamic Test Bed" and "Cavitation Test Bed." What is the time of exposure for taking cavitation photographs ?
- 14.7 Describe briefly the layout of a test bed for —  
 (a) Pelton Turbine ;                      (b) Reaction turbine ;  
 (c) Centrifugal pump.
- 14.8 What is the correct factor which is applied for calculating the net head for a pump, if the suction pipe diameter is bigger than the delivery pipe diameter ?
- 14.9 Why are D. C. motors used for driving the test pumps ?
- 14.10 State the precautions taken while connecting gauges to the suction and delivery pipes.
- 14.11 The discharge through an experimental Pelton wheel installed in a laboratory is to be accurately measured. What type of meter or meters would you propose for the purpose ? If the discharge is to be 50 lit/sec, give an approximate size of the apparatus that you would recommend.

Give a neat sketch of the layout of an experimental Pelton wheel for a laboratory.  
(Madras University)

- 14.12 A test bed for testing centrifugal pumps for their characteristics at different speeds, heads and discharge, has to be fitted up with the necessary drive, measuring instruments etc. The ranges of speed, head and discharge are 600 to 1600 rpm, 6 to 30 m and 20 to 50 lit/sec respectively.

Make a neat sketch of the test bed you would propose with all the apparatus and give, in a tabular form the list of equipment and measuring apparatus you consider necessary.

Give the size and capacity of these items so that the bed will be able to take all pumps within the above range

(*Madras University*)

- 14.13 Describe the procedure of testing a water turbine in a laboratory
- 14.14 Which are the data required to be measured while the experiment is being performed? Give the information in a tabular form.
- 14.15 Which are the data required to be measured before the experiment is started? Explain how each of such information is used for calculating various other data.
- 14.16 Name the instruments which should be calibrated before the experiment is performed.
- 14.17 How is the effective or working head on the turbine determined? State the difference in calculation if the turbine is reaction or impulse.
- 14.18 State how is the overall efficiency of water turbine determined by brake test.
- 14.19 Why are the turbine data finally calculated at unit head?
- 14.20 Explain the procedure of testing pumps in a laboratory.
- 14.21 Name the items to be checked before the pump is tested.
- 14.22 Explain how the overall efficiency of a centrifugal pump is determined. Draw a table for such a pump test.

### **B. Measurement of Power and Efficiency**

- 14.23 What are the different equipments used to find the power and efficiency of a water turbine?
- 14.24 How would you measure the power and the efficiency of a turbine with the help of Prony brake?
- 14.25 Where do you employ torsion dynamometer and rope dynamometer to measure the power of prime movers?
- 14.26 Sketch and describe one form of torsion dynamometer and explain how the HP transmitted is calculated.
- 14.27 Which one of the two methods for power measurement is more accurate—(a) Electrical or (b) Mechanical?
- 14.28 Describe briefly how you will measure the power and the efficiency of power unit by means of electrical generator. Where would you use this method in practice?

### **Numericals**

- 14.29 A Prony brake of the type shown in Fig 14.8 was used to measure the power output of a turbine running at 180 rpm. The gross load on the scales during the test was 26 kg and the tare weight was 5 kg.



(a) What is the brake constant ? (b) What is the HP output of the turbine ? The brake pulley has a diameter of 50 cm and leverage is 55 cm. (0.000767, 2.84 HP)

14.30 A test of power and efficiency for a water turbine is made by connecting the prime mover to an electrical dynamometer which operates as a generator. The frame of stator of the dynamometer is mounted independently of its rotor in bearings which are co-axial with those of the rotor. In this way a direct measurement of the torque on the rotor developed by the generation of electric current may be made. The electrical energy generated is converted into heat by passing the current through electrical resistors. In the particular test involved the speed of the turbine is 500 rpm. The force measured during test is 10 kg and that due to the unbalanced weight of the stator alone with the motor disconnected is 1.9 kg. The turbine consumes 8.3 lit/sec while working under a head of 33.5 m. The leverage of the dynamometer is 45 cm. The turbine is directly connected to the dynamometer. Find the HP and the efficiency of turbine. (2.54 HP, 68.5%)

14.31 A Pelton wheel works under a head of 120 m and the mean diameter of the wheel is 1.15 m. During a brake test in which the needle valve is kept fully open it is found that when the wheel is brought completely to rest, the torque on the shaft is 300 m.kg. Estimate the speed and power of the wheel when running under its normal working conditions. (Madras University)

14.32 In a Pelton wheel the water is turned in the buckets through an angle of  $165^\circ$ , the diameter of the wheel is 0.85 m and the velocity of the jet 50 m per second. The Pelton wheel is tested with the needle valve fully open and at speeds varying from 0 to 200 rpm.

Plot graphs showing the relation between (a) the wheel speed and the thrust on the buckets and (b) the wheel speed and the efficiency, taking at least three different speeds within the range.

Calculate the speed and the power of the wheel when working under the condition of maximum efficiency.

(Madras University)

#### 14 C. Level Measurements

14.33 Explain the difference between a pointer gauge and a hook gauge.

14.34 Explain with the help of a sketch the working of a mechanical float.

14.35 Describe a level recorder with the help of a sketch.

#### 14 D. Speed Measurements

14.36 What are the different arrangements to measure the speed of a machine ?

14.37 What is the difference between a tachometer and a rpm counter ?

14.38 A centrifugal tachometer having range of 25 to 300, 250 to 3,000 and 2,500 to 30,000 rpm is to be used to determine the speed of a shaft which is believed to be in the vicinity of 3,000 rpm. What range setting should be tried first and why ?

14.39 How does the electronic revolution-counter work ?

14.40 Describe the principle of Electric Tachometer.

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# Characteristics of Water Turbines and Pumps

## A. Characteristics of Water Turbines

**15.1 Introduction**—Machines are always designed to work under a given set of conditions or a limited range of conditions. A turbine may be designed for some particular data say  $H_o$ ,  $Q_o$ ,  $N_o$ , and  $P_{t_o}$ , but in practice it may have to be used under conditions different from those for which it is designed. Therefore the exact behaviour of the prime-mover under varied conditions should be predetermined. This is graphically represented by means of curves known as *Characteristics* of the turbine. Practical data for these curves are obtained from experiments on models or actual-sized machines. The model tests are conducted in research laboratories and actual tests on the site. Field tests performed at the time of customer taking the plant over from the manufacturers are known as “take-over” tests. Performance of the machine should be upto the standard specified in the tenders and all guarantees must be fulfilled.

**15.2 Measurement of Characteristics Data**—The following data have to be obtained from the test :

- (1) Rate of flow of water,
- (2) Net head,
- (3) Available power,
- (4) Brake horse power,
- (5) Overall or final turbine efficiency.

(i) **Rate of Flow** : (refer authors book on Hydraulics & Fluid Mechanics for various methods of water measurements).

(ii) **Net Head** : (refer Fig 14.3 to 14.6) Different methods are adopted according to the magnitude of head.

(a) **Low Head Power Plants** : Potential head is determined by measuring simultaneously the head and tail race levels. Kinetic head is equal to the difference between heads by virtue of the velocities of approach and discharge respectively. Net head is obtained by adding the kinetic head to the potential head and subtracting the losses, if any.



Transmission losses may be caused by friction if water is conveyed through a penstock.

(b) **Medium and High Head Plants :** When these plants are closed-casing type, head can be measured with the help of a Piezometer tube, mercury manometer, Bourdon tube, pressure gauge etc.

(iii) **Available Power :**

$$P_a = \frac{\gamma \cdot Q \cdot H}{75} \text{ HP}$$

where  $H$  is the net head.

(iv) **Brake Horse Power  $P_t$  :** For different methods of measuring BHP see Chapter 14 B.

$$(v) \text{ Overall efficiency } \eta_t = \frac{P_t}{P_a}.$$

**15.3 Representation of Characteristics**—The behaviour of a water turbine may be exhibited by the following curves :

- (i) Main characteristics (constant head curves),
- (ii) Operating characteristics (constant speed curves),
- (iii) Muschel curves (constant efficiency curves).

(i) **Main Characteristics or Constant Head Curves**—The head is kept constant and speed varied by allowing a variable quantity of water to flow through the inlet opening. Thus a series values of  $N$  and  $Q$  are obtained. For each value, brake horsepower is measured by braking mechanically or coupling with a generator. All the measured and calculated quantities are tabulated for inlet opening as shown in Table 14.2.

The values of  $Q$ ,  $N$ ,  $M$  and  $P_t$  are reduced to unit head.

A new table containing these unit quantities is prepared for every inlet opening, as shown in Table 14.3.

The following curves are then drawn by plotting values from different tables :

$$\begin{array}{ll} Q_1 = f(N_1), & M_1 = f(N_1), \\ P_{t_1} = f(N_1), & \text{and } \eta_t = f(N_1). \end{array}$$

Fig 15.1 and 15.2 show the typical main characteristics of Pelton and reaction (Kaplan type) turbines respectively. The main characteristics of Francis type reaction turbine remain same except  $Q_1 = f(N_1)$  curves, shown in Fig 15.3.

Curves can also be drawn showing  $Q_1$ ,  $M_1$ ,  $P_{t_1}$  and  $\eta_t$  in percentage of their maximum values, as functions of speed  $N_1$  in percent of normal designed speed.

(ii) **Operating Characteristics or Constant Speed Curves**—When a turbine is working for the generation of power its speed must remain constant. The other operating condition viz.,  $Q$  and  $H$  may vary according to their availability.

# CHARACTERISTICS OF WATER TURBINES AND PUMPS

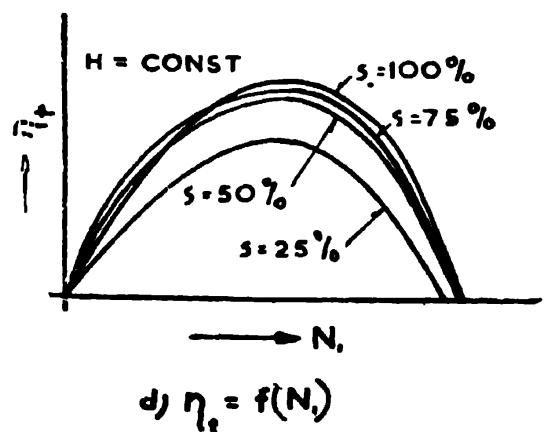
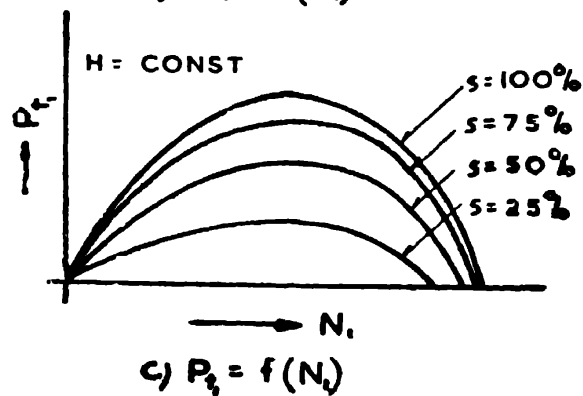
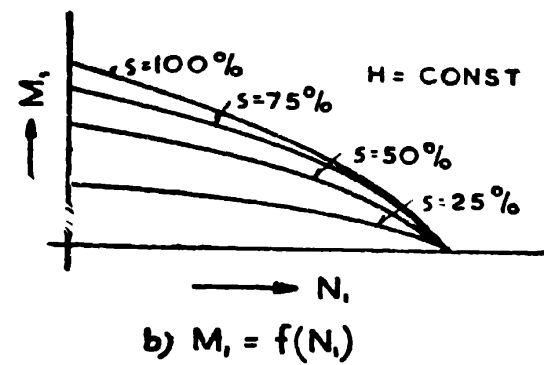
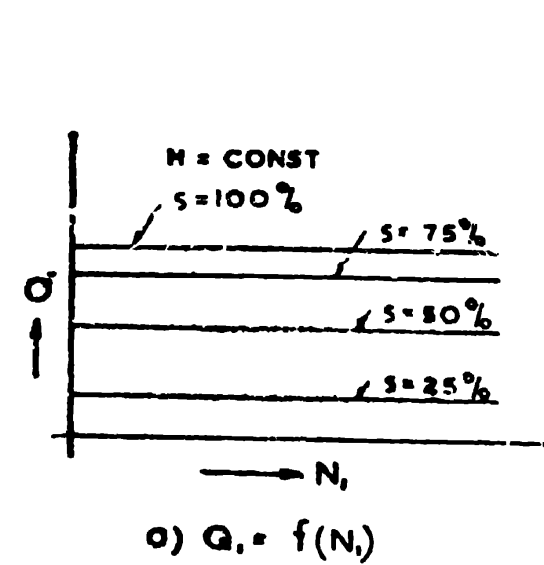


Fig 15.1 Typical Main Characteristics (Constant Head Curves) of a Pelton Turbine

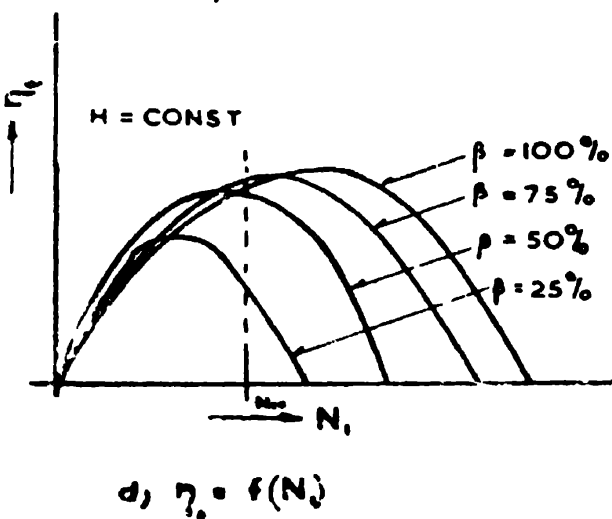
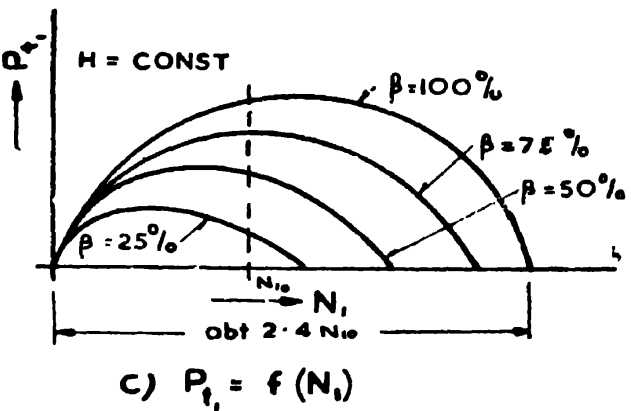
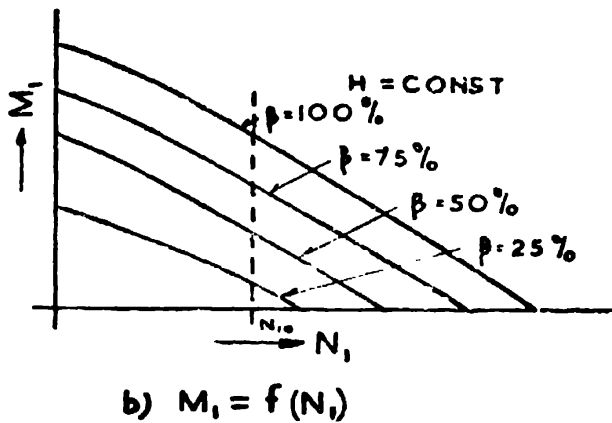
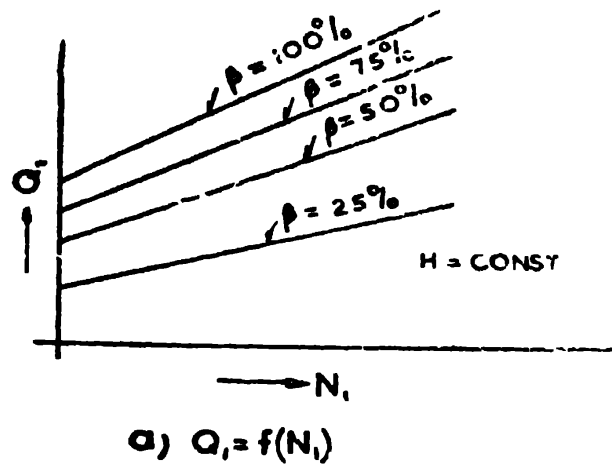


Fig 15.2 Typical Main Characteristics (Constant Head Curves) of a Reaction (Kaplan) Turbine

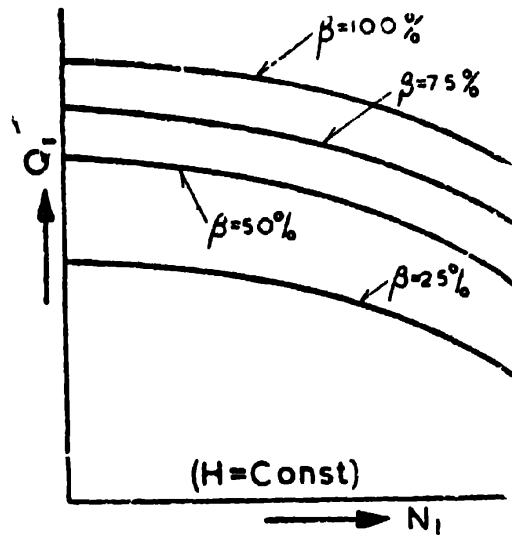
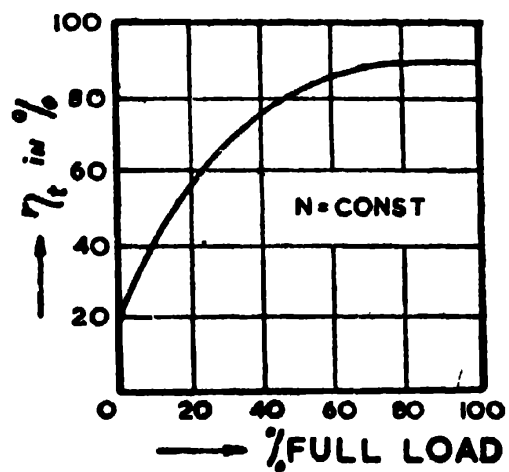
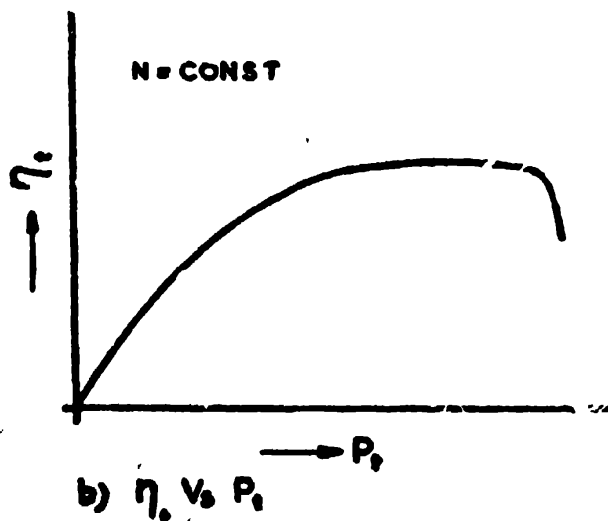
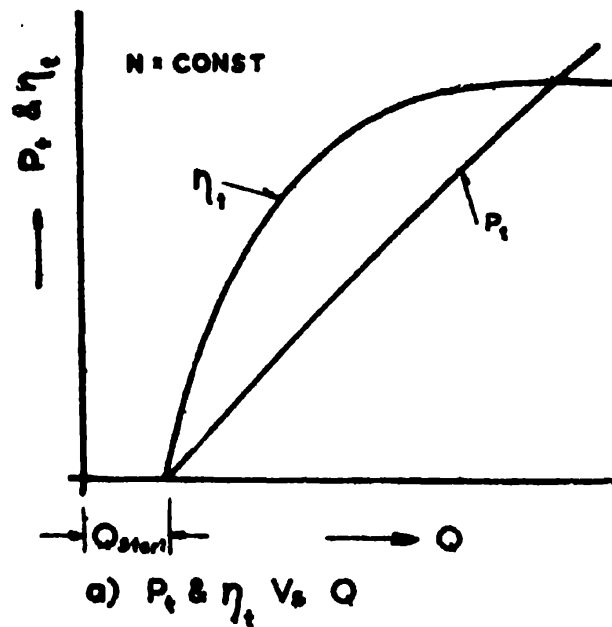
Fig 15.3  $Q_1 = f(N_1)$  for Francis Turbines

Fig 15.4 Typical Operating Characteristics (Constant Speed Curves) of a Water Turbine

Tables obtained from brake tests are used again and the following curves (refer Fig 15.4) are drawn :

- (a)  $P_t, \eta_t = f(Q)$   
 (b)  $\eta_t = f(P_t)$   
 (c) % of max  $\eta_t = f(\% \text{ full load})$

(iii) **Constant Efficiency Curves** (Muschel Curves)—Data for plotting these curves (refer Fig 15.5) are obtained from the constant head and constant speed curves. The primary purpose is to find out region of constant efficiency so that the turbine can always be operated

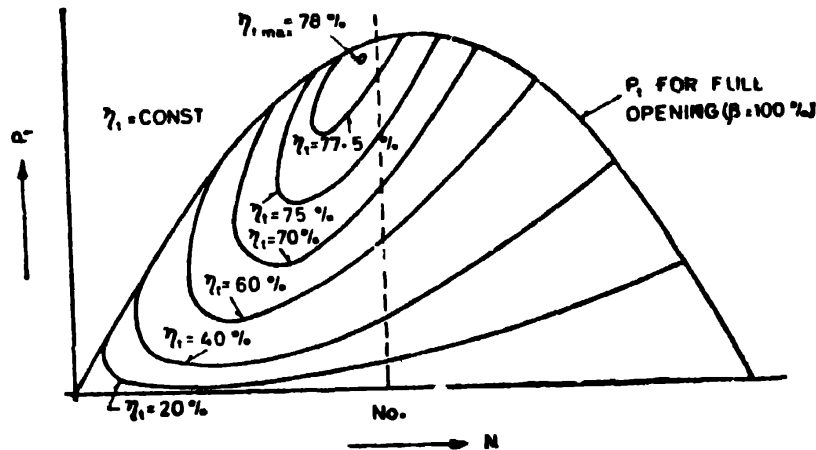


Fig 15.5 Typical Constant Efficiency (Muschel's) Curves for a Reaction Water Turbine

with maximum efficiency. The curves are also helpful to the sales engineers in showing to the customers the exact performance of the machine at different efficiencies.

**Problem 15.1** The following data are available from the main characteristics (constant head curves) of a Kaplan turbine :

Diameter of runner = 3 m

$$P_{t_1} = 7.02 \text{ HP}, \quad Q_1 = 1.7 \text{ m}^3/\text{sec}, \quad N_1 = 35.1 \text{ rpm}$$

Determine the runner diameter, the rate of flow and the speed of a geometrically similar turbine which works under a head of 90 m producing 2,700 HP.

### Solution

Given two turbines such that :

$$\begin{array}{lll} D_a = 3 \text{ m} & (P_{t_1})_a = 7.02 \text{ HP} & (P_t)_b = 2,700 \text{ HP} \\ (Q_1)_a = 1.7 \text{ m}^3/\text{sec} & (N)_a = 35.1 \text{ rpm} & H_a = 1 \text{ m} \\ D_b = ? & (D_b) = ? & (N)_b = ? \\ & & H_b = 90 \text{ m} \end{array}$$

Since the turbines are geometrically similar :

(i) Specific powers are equal,

$$\text{i.e.,} \quad \frac{(P_t)_b}{D_b^3 \cdot H_b^{\frac{3}{2}}} = \frac{(P_t)_a}{D_a^3 \cdot H_a^{\frac{3}{2}}} = \frac{(P_{t_1})_a}{D_a^3} \quad \dots(1)$$

(ii) Specific flows are equal,

$$\text{i.e.,} \quad \frac{Q_b}{D_b^3 \cdot \sqrt{H_b}} = \frac{Q_a}{D_a^3 \cdot \sqrt{H_a}} = \frac{(Q_1)_a}{D_a^3} \quad \dots(2)$$

(iii) Speed ratios are equal,

$$\text{i.e., } (K_{u_1})_a = (K_{u_1})_b \text{ or } \frac{D_b \cdot N_b}{\sqrt{H_b}} = \frac{D_a \cdot N_a}{\sqrt{H_a}} \\ = D_a \cdot (N_1)_a \quad \dots(3)$$

Now, from (1), by substituting the given values,

$$D_b = \sqrt{\frac{P_{t_b}}{(P_{t_1})_a} \cdot \frac{D_a^2}{H_b^{\frac{5}{4}}}} = \sqrt{\frac{2,700}{702} \times \frac{9}{90^{\frac{5}{4}}}} = 2 \text{ m Answer}$$

Similarly, from (2),

$$Q_b = (Q_1)_a \cdot \frac{D_b^3}{D_a^3} \cdot \sqrt{H_b} = 1.7 \times \frac{4}{9} \times \sqrt{90} \\ = 7.15 \text{ m}^3/\text{sec Answer}$$

And, from (3),

$$N_b = (N_1)_a \cdot \left( \frac{D_a}{D_b} \right) \cdot \sqrt{H_b} = 35.1 \times \frac{3}{2} \times \sqrt{90} \\ = 500 \text{ rpm Answer}$$

**Problem 15.2** A Francis turbine gives the following performance :

$$H = 5.3 \text{ m ; } Q = 1.525 \text{ m}^3/\text{sec ;}$$

$$\text{BHP} = 92.77 ; \quad \text{RPM} = 204.3$$

$$\text{Diameter} = 80 \text{ cm and part of gate opening} = 0.873$$

(a) Compute the efficiency and the speed ratio,

(b) Assuming the same wheel to operate at the same speed and same gate opening under a head of 15 m, what will be the new RPM, Q and HP? Check the results by recomputing from the efficiency and speed ratio.

**Solution**

$$(a) \quad P_t = \frac{\gamma \cdot Q \cdot H}{75} \cdot \eta_t$$

$$\therefore \eta_t = \frac{75 \times P_t}{\gamma \cdot Q \cdot H} = \frac{75 \times 92.77}{1,000 \times 1.525 \times 5.3} = 0.862$$

$$\text{or } 86.2\% \quad \text{Answer}$$

$$\text{Speed ratio } K_{u_1} = \frac{\pi \cdot D_1 \cdot N}{60 \times \sqrt{2gH}} \\ = \frac{\pi \times 0.8 \times 204.3}{60 \times 4.43 \times \sqrt{5.3}} = 0.84 \quad \text{Answer}$$

(b) New speed under 15 m head—

Specific speeds in both cases are equal,

$$\therefore N'_s = N_s \text{ or } \frac{N' \cdot \sqrt{P'_t}}{(H')^{\frac{5}{4}}} = \frac{N \cdot \sqrt{P_t}}{H^{\frac{5}{4}}}$$

$$\text{or } N' = N \left( \frac{P_t}{P'_t} \right)^{\frac{1}{2}} \left( \frac{H'}{H} \right)^{\frac{5}{4}}$$

Also, unit powers in both cases should be equal, as the only factor which changes is the head.

$$\therefore \frac{P'_t}{H'^{\frac{3}{2}}} = \frac{P_t}{H^{\frac{3}{2}}}$$

$$\text{or } P'_t = P_t \cdot \left( \frac{H'}{H} \right)^{\frac{3}{2}} = 92.77 \times \left( \frac{15}{5.3} \right)^{\frac{3}{2}} = 440 \text{ Hp} \quad \text{Answer}$$

And substituting the values of  $P_t$ ,  $P'_t$ ,  $H$  and  $H'$  in the above equation of  $N'$ ,

$$N' = 204.3 \times \left( \frac{92.77}{440} \right)^{\frac{1}{2}} \times \left( \frac{15}{5.3} \right)^{\frac{1}{4}} = 342 \text{ RPM} \quad \text{Answer}$$

Further, since, unit discharges are equal,

$$\frac{Q}{(H')^{\frac{5}{2}}} = \frac{Q}{H^{\frac{5}{2}}} \quad \text{or } Q' = Q \left( \frac{H'}{H} \right)^{\frac{1}{2}} = 1.525 \times \left( \frac{15}{5.3} \right)^{\frac{1}{2}} \\ = 2.56 \text{ m}^3/\text{sec} \quad \text{Answer}$$

$$\text{Check—} P'_t = \frac{\gamma \cdot Q' \cdot H'}{75} \cdot \eta_t = \frac{1,000 \times 2.56 \times 15}{75} \times 0.84 \\ \approx 440 \text{ HP (correct)}$$

$$\text{and } K_{u_1} = \frac{\pi \cdot D_1 \cdot N}{60 \times \sqrt{2gH}} = \frac{\pi \times 0.8 \times 342}{60 \times 4.43 \times \sqrt{15}} = 0.84 \text{ (correct)}$$

**Problem 15.3** The result of a test on water turbines operating at full gate opening under a head of 5.28 m are given as follows :

|                           |      |       |       |       |       |       |       |      |
|---------------------------|------|-------|-------|-------|-------|-------|-------|------|
| Unit Speed                | 65.4 | 72.6  | 76.3  | 79.9  | 83.5  | 87.2  | 90.75 | 94.4 |
| Unit Power                | 16.8 | 17.25 | 17.42 | 17.45 | 17.32 | 17.06 | 16.6  | 16.0 |
| $Q$ (m <sup>3</sup> /sec) | 3.74 | 3.66  | 3.6   | 3.5   | 3.51  | 3.45  | 3.4   | 3.32 |

Plot graphs of unit power and efficiency against unit speed. If the turbine runs at a real speed of 200 rpm, find the power and efficiency. If the head is increased to 5.5 m, the speed remaining the same, find the corresponding power and efficiency. (Panjab University)

**Solution**

$$\text{Turbine output } P_t = P_{t_1} H^{\frac{3}{2}}$$

$$\text{Turbine available power } P_a = \frac{\gamma \cdot Q \cdot H}{75}$$

$$\therefore \text{ Turbine efficiency } \eta_t = \frac{P_t}{P_a} = \frac{75 \times P_{t_1} \cdot H^{\frac{3}{2}}}{\gamma \cdot Q \cdot H} = \frac{75 \times P_{t_1} \cdot H^{\frac{1}{2}}}{\gamma \cdot Q}$$



$$\text{or } \eta_t = \frac{75 \times 5.28^{\frac{3}{2}}}{1,000} \times \frac{16.8}{3.74} = 0.771 \quad \text{or } 77.1\%$$

Similarly calculate the efficiency  $\eta_t$  for each unit power and  $Q$ , and tabulate the result as follows :

| $N_1$            | 65.4 | 72.6  | 76.3  | 79.9  | 83.5  | 87.2  | 90.75 | 94.4 |
|------------------|------|-------|-------|-------|-------|-------|-------|------|
| $P_{t1}$         | 16.8 | 17.25 | 17.42 | 17.45 | 17.32 | 17.06 | 16.6  | 16.0 |
| $Q$<br>$m^3/sec$ | 3.74 | 3.66  | 3.6   | 3.56  | 3.51  | 3.44  | 3.4   | 3.32 |
| $\eta_t$         | 77.1 | 80.9  | 82.7  | 84    | 84.7  | 84.7  | 83.85 | 82.5 |

Draw  $P_{t1}$  vs  $N_1$  and  $\eta_t$  vs  $N_1$  curves as shown in Fig 15.6

Now,  $N = 200$  rpm,

$$\therefore N_1 = \frac{200}{5.28^{\frac{1}{2}}} = 87$$

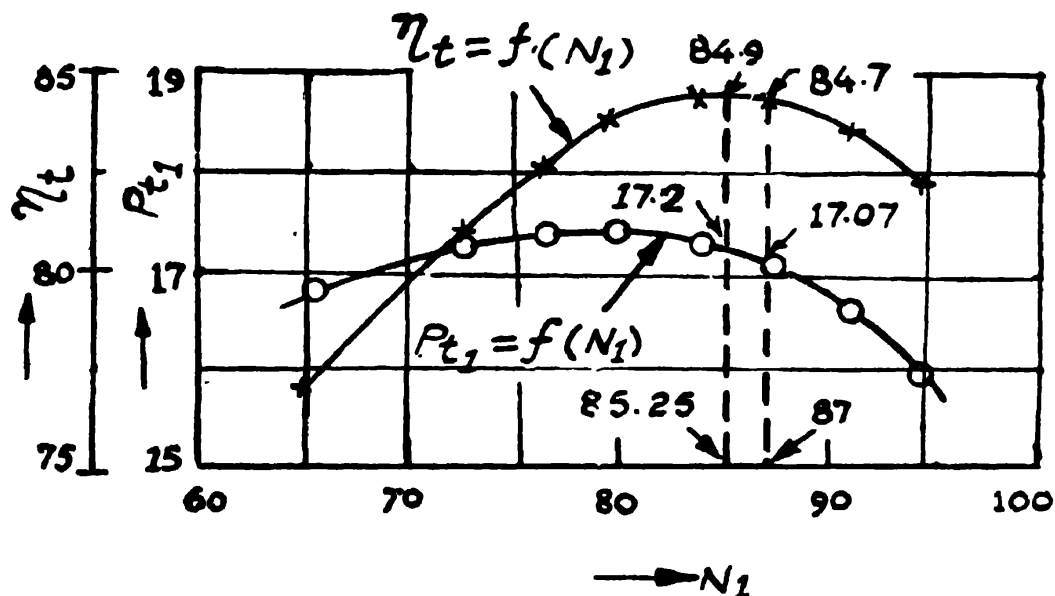


Fig 15.6  $P_t$  &  $\eta_t$  vs  $N_1$

Find the values of  $P_{t1}$  and  $\eta_t$  against  $N_1 = 87$  from Fig 15.6

$$\therefore P_{t1} = 17.07 \quad \text{or} \quad P_t = 17.07 \times 5.28^{\frac{3}{2}} = 207 \text{ HP} \quad \text{Answer}$$

$$\text{and } \eta_t = 84.7\% \quad \text{Answer}$$

$$\text{If } H = 5.5 \text{ m} \quad \text{and} \quad N = 200 \text{ rpm,}$$

$$N_1 = \frac{200}{5.5^{\frac{1}{2}}} = 85.25$$

$$\therefore P_{t_1} \text{ (from Fig 15.6) } = 17.2 \text{ units}$$

$$\text{or } P_1 = 17.2 \times 5.5^{\frac{3}{2}} = 221 \text{ HP and } \eta_t \text{ (from Fig 15.6) } = 84.9\%$$

*Answer*

**Problem 15.4** The head water surface and tail water surface levels in a water turbine installation are 51 m and 45 m above sea level respectively. The turbine is designed to run at a uniform speed of 75 rpm under all conditions.

The manufactures' tests under a head of 7.5 m show that the turbine develops 2,000 HP at its maximum efficiency of 87%. The computed values of unit speed  $N_1$  and unit power  $P_1$  in percentage of normal values and the efficiency  $\eta$  in percentage of the maximum efficiency are tabulated below :

|                                       |      |      |      |      |     |      |      |      |
|---------------------------------------|------|------|------|------|-----|------|------|------|
| Unit speed $N_1$ in % of normal value | 60   | 70   | 80   | 90   | 100 | 110  | 120  | 130  |
| Unit power $P_1$ in % of normal value | 73.5 | 82.2 | 90.0 | 96.5 | 100 | 99.5 | 99.0 | 90.0 |
| Efficiency $\eta$ in % of max value   | 85   | 88   | 94   | 98   | 100 | 99   | 94   | 85   |

Calculate the HP and Q of the turbine under normal working conditions.  
(UPSC)

### Solution

$$H_{test} = 7.5 \text{ m}$$

$$H_{normal} = 51 - 45 = 6 \text{ m}$$

$$P_{t(test)} = 2,000 \text{ HP}$$

$$N = 75 \text{ rpm (constant)}$$

$$\eta_{test} = 87\%$$

Designed speed  $N = 75 \text{ rpm}$  and tested head  $= 7.5 \text{ m}$

$$\therefore \text{ Tested Unit Power } P_1 = \frac{P}{H^{\frac{3}{2}}} = \frac{2,000}{7.5^{\frac{3}{2}}} = 97.7 \text{ HP} \cdot \text{m}^{-\frac{3}{2}}$$

$$\text{Tested Unit Speed } N_1 = \frac{N}{H^{\frac{1}{2}}} = \frac{75}{7.5^{\frac{1}{2}}} = 27.5 \text{ rpm} \cdot \text{m}^{-\frac{1}{2}}$$

$$\text{Normal working head} = 6 \text{ m}$$

$$\therefore \text{ Normal Unit Speed } N_1 = \frac{75}{6^{\frac{1}{2}}} = 30.8 \text{ rpm} \cdot \text{m}^{-\frac{1}{2}}$$

$$\therefore \text{ Percentage of tested } N_1 = \frac{30.8}{27.5} \times 100 = 112\%$$

Find the normal efficiency at 112% of  $N_1$  from graph (refer Fig 15.7) which is equal to 98.2% of  $\eta_{max}$

Also find the normal power at 112% of  $N_1$  from graph (refer Fig 15.7) which is equal to 99% of  $P_{1(tested)}$

$$\therefore P_{1(normal)} = P_{1(tested)} \times 0.99 = 97.7 \times 0.99 = 96.7 \text{ HP m}^{-\frac{2}{3}}$$

$$\text{and } \eta_{(normal)} = \eta_{max} \times 0.982 = 0.87 \times 0.982 = 0.854$$

$$\text{Hence } P_{(normal)} = 96.7 \times 6^{\frac{2}{3}} = 1,420 \text{ HP} \quad \text{Answer}$$

$$\text{and } Q_{(normal)} = \frac{P_{(normal)} \times 75}{\gamma \cdot H_{(normal)} \times \eta_{(normal)}} = \frac{1,420 \times 75}{1,000 \times 6 \times 0.854} = 20.8 \text{ m}^3/\text{sec} \quad \text{Answer}$$

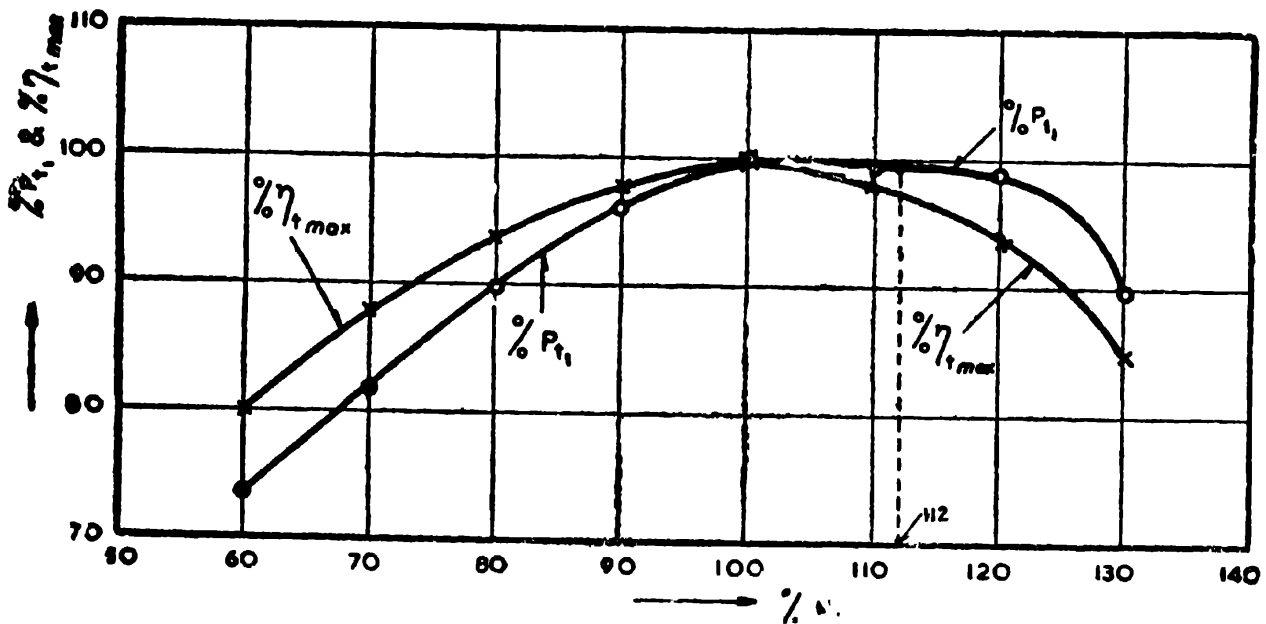


Fig 15.7 %  $P_1$  and %  $\eta_t \max$  vs %  $N_1$

## B. Characteristics of Centrifugal Pumps

**15.4 Characteristics of Centrifugal Pumps**—The characteristics of a pump is a graphical representation of its behaviour and performance under different circumstances. Characteristic curves show manometric head, brake horse power and efficiency of the pump as ordinates with its discharge as the abscissa. To determine the characteristics the pump should be tested after completion, before being despatched. Most pump manufacturers have their own test laboratories. Many pumps have also been standardized in some countries.

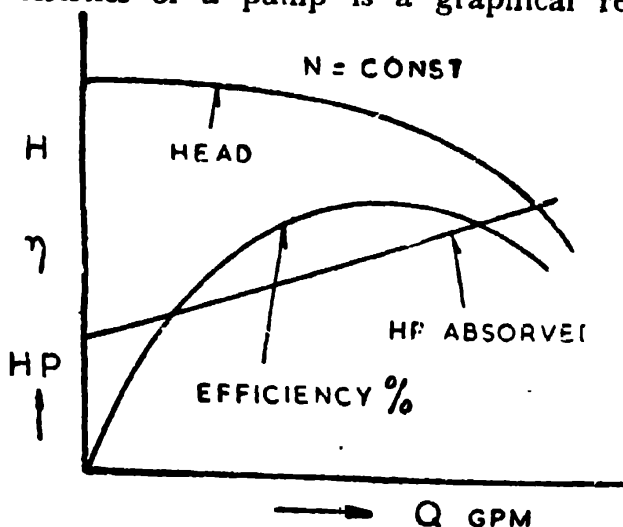


Fig 15.8 Operating Characteristics (Constant speed Curves) for a Radial Type Centrifugal Pump

performance under different circumstances. Characteristic curves show manometric head, brake horse power and efficiency of the pump as ordinates with its discharge as the abscissa. To determine the characteristics the pump should be tested after completion, before being despatched. Most pump manufacturers have their own test laboratories. Many pumps have also been standardized in some countries.

Pump characteristics can be divided into three categoristic:

- (i) Main characteristics including operating characteristics
- (ii) Muschel curves or constant efficiency curves and
- (iii) Constant head and constant discharge, curves.

Each characterises one aspect of the pump's performance.

**15.5 Main Characteristics**—These are obtained by fixing the pump speed at some arbitrary value and plotting separately  $H$ ,  $HP$ , and  $\eta$  against  $Q$  (refer Fig 15.8). The rate of flow  $Q$  is varied by means of a delivery valve and at several different values of  $Q$  the quantities  $H$ ,  $HP$  and  $\eta$  are measured or calculated. By plotting these sets of values a set of three curves is obtained. A number of different values of  $N$  are chosen and one such set of curves is drawn for each speed.

The pump is normally designed to run at the same speed as the driving unit which is generally an electric motor of the AC induction type. Speed of such motors is either 1,450 or 2,900 rpm for a frequency of 50 cycles per second. When electric power is not available, the pump may be driven by a diesel engine or a steam turbine. Pumps employed for irrigation purposes have to be installed in remote villages often having no electric power. Driving unit in such cases is a diesel engine. The pump may be coupled to the engine of a tractor which may be used for both ploughing and pumping. Smaller engines consume petrol while larger ones use diesel oil.

In such circumstances it is advantageous to know the performance of a pump at different speeds and this is best seen from the main characteristics.

A typical set of main characteristics is shown in Fig 15.9

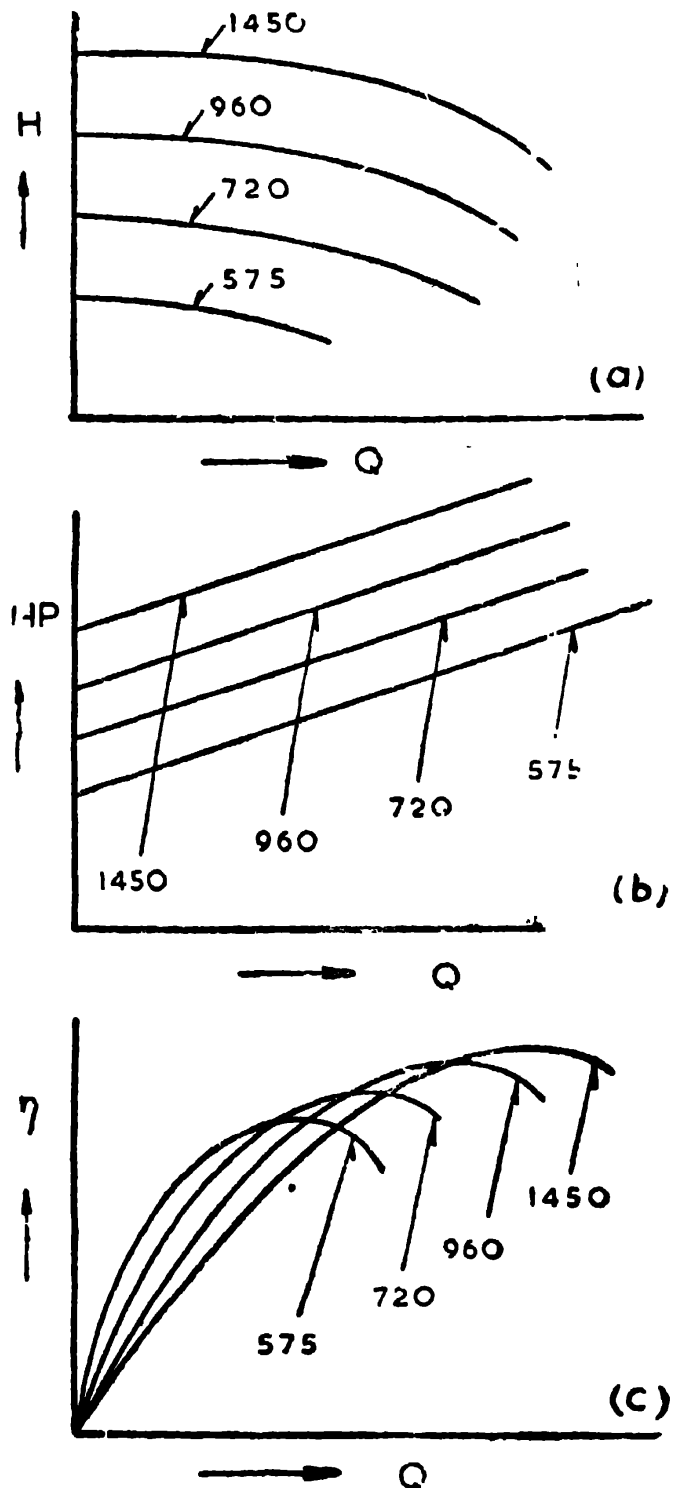


Fig 15.9 Main Characteristics of a Centrifugal Pump (a)  $H$  vs  $Q$ , (b)  $HP$  vs  $Q$  and (c)  $\eta$  vs  $Q$

**15.6 Operating Characteristics :** During operation the pump must run constantly with the speed of the driving unit. Normally, this is

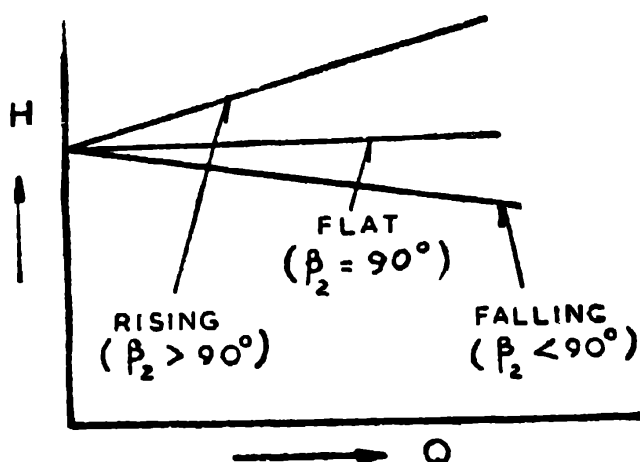


Fig 15.10 Rising, Flat or Falling  $H$  vs  $Q$  Curves of a Centrifugal Pump

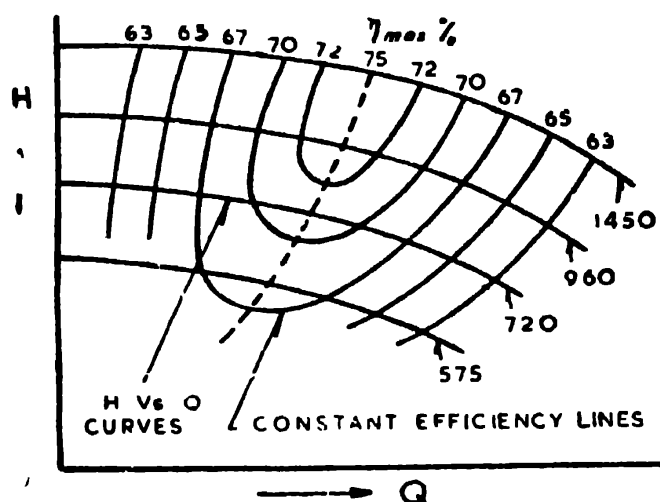


Fig 15.11 Constant Efficiency or Muschel Curves of a Centrifugal Pump

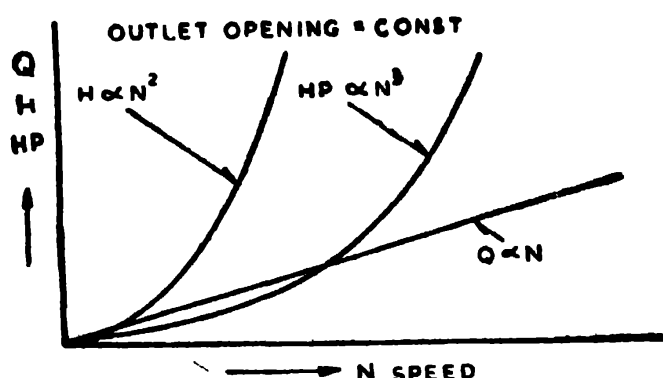


Fig 15.12  $Q$ ,  $H$  and  $HP$  vs  $N$  Curves of a Centrifugal Pump

the designed speed. That particular set of main characteristics which corresponds to the designed speed is mostly used in operation and is therefore known as the operating characteristics (refer Fig 15.8).

These curves are said to be rising, flat or falling characteristics respectively according as the head increases, remains more or less constant, or decreases with increase in rate of flow. It depends upon the outlet vane angle  $\beta_2$ . Ordinarily pumps have falling characteristics (refer Fig 15.10).

**15.7 Muschel Curves or Constant Efficiency Curves**—With the help of data obtained from the above curves a series of constant efficiency curves can be drawn (refer Fig 15.11). They facilitate the job of the salesman and enable the prospective customer to see directly the range of operation with a particular efficiency. They serve as a suitable basis for a comparison of pumps, specially from a commercial point of view.

**15.8 Constant Head and Constant Discharge Curves**—It is quite possible that a pump may be required to deliver water at a certain height, in which case it is fixed. If for some reason the speed varies, discharge will also be affected. In order to predetermine the performance of the pump under such conditions, it is necessary to draw a constant  $H$  curve by plotting  $Q$  vs  $N$  (refer Fig 15.12).

Similarly; to determine the speeds required to discharge a certain quantity at different pressures or to find the variation of  $H$  with  $N$ , it is convenient to draw constant  $Q$  curves showing  $H$  against  $N$  (refer Fig 15.12).

**Unsolved Problems****A. Characteristics of Water Turbines**

- 15.1 Draw the main characteristics of a reaction water turbine.
- 15.2 What is the difference between the main and the operating characteristics of a water turbine? Draw a set of such curves for a Pelton turbine.
- 15.3 Draw the following characteristics curves of a Kaplan turbine :  
(i)  $P_{t_1}$  vs  $N_1$  (ii)  $P_1$  vs  $Q_1$  (AMIE)
- 15.4 What are the different characteristic curves of a water turbine? Draw the following typical curves for a Pelton turbine?  
(i)  $P_t$  vs  $Q$  (ii)  $\eta_t$  vs  $Q$  (Jadavpur University)
- 15.5 Describe the method of preparing characteristic diagram for a turbine when the co-ordinates are 'unit speed' and 'unit power'. What is the use of such a diagram? (AMIE)
- 15.6 What are constant efficiency curves of a water turbine and where are they employed?
- 15.7 What is the difference between  $Q_1 = f(N_1)$  curves of Francis and Kaplan turbines?

**Numericals**

- 15.8 In a series of brake tests of a small Pelton wheel the following tabulated results were obtained :

|     |      |       |       |       |       |       |       |       |       |       |
|-----|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $W$ | 2.92 | 2.55  | 2.4   | 2.13  | 2.0   | 1.58  | 1.3   | 0.95  | 0.45  | 0     |
| $N$ | 960  | 1,360 | 1,480 | 1,800 | 1,900 | 2,240 | 2,520 | 2,800 | 3,280 | 3,580 |

Where  $W$  = effective load in kg on brake lever at 25 cm from the axis of wheel, and  $N$  = speed of wheel in rpm. The weight of water used in each test was 18 kg/min, and the pressure of water was 50 kg/cm<sup>2</sup> in the pipe behind the orifice. The diameter of orifice was 0.21 cm. Complete the above table by adding BHP and the efficiency in percent. Plot HP and efficiency on a speed base.

Scale HP

5 cm = 1 HP

$\eta$

1 cm = 10%

$N$

2 cm = 500 rpm.

State the maximum HP and maximum efficiency.

(1.342 HP ; 67.1%)

- 15.9 (a) When an impulse turbine runs at *constant* nozzle opening under *constant* head, its performance will be affected by the *speed* in rpm. Show these variations, for idealized conditions, by sketching graphs between rotational speed and (i) torque, (ii) efficiency, and rate of flow of water, for a range of speeds from zero to maximum.
- (b) In an actual Pelton wheel installation, the turbine gives a maximum efficiency of 86% when running at a speed of 500 rpm, under a head of 250 m. Assuming the speed and nozzle opening to remain unchanged, and the efficiency curve to be a parabola over

the speed-ratio range considered, make a close estimate turbine efficiency when the head is (a) 200 m and (b) 300 m.  
(84 and 85.3%) (Jadavpur University)

[Hint: The efficiency unit speed curve being a parabola, the maximum efficiency occurs at half the runaway speed.]

Operating unit speed

$$= N_1 = \frac{N}{\sqrt{H}} = \frac{500}{\sqrt{250}} = 31.7$$

Unit speed at which the efficiency is required

$$= \frac{500}{\sqrt{200}} = 35.4 \quad \text{and} \quad \frac{500}{\sqrt{300}} = 28.9$$

at point C and D in Fig 15.13.

Equation of parabola :  $y^2 = 4ax$

At point C,  $y = CP = 35.4 - 31.7 = 3.7$

$$\therefore 3.7^2 = (2 \times 31.7) \cdot x \quad (\because AB = 4a = 2 \times 31.7)$$

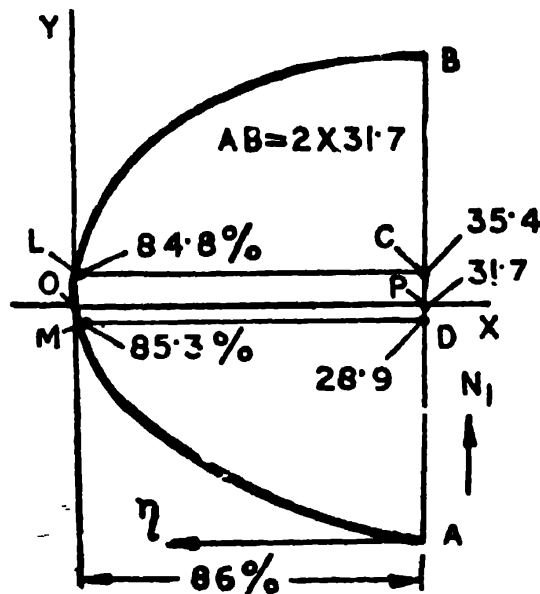


Fig 15.13 Efficiency vs Unit Speed (Parabola) Curve of Pelton Turbine

or

$$x = 0.216$$

$$CL = \frac{2 \times 31.7}{4} - 0.216 = 15.634$$

$$OP = \frac{2 \times 31.7}{4} = 15.85 \text{ units}$$

which corresponds to 86% (in Fig 15.13)

$\therefore$  Efficiency at 200 m head

$$= \frac{CL}{OP} \times 86\%$$

$$= \frac{15.634}{15.85} \times 86 = 84.8\%$$

Similarly find DM and then the efficiency at 300 m head.

**B. Characteristics of Pumps**

- 15.10 What are the characteristic curves of a centrifugal pump ?
- 15.11 Draw the operating characteristics of an ordinary centrifugal pump.
- 15.12 What are the constant efficiency curves of a centrifugal pump ? Are there similar curves for the turbines too ? For what purpose are such curves used ?  
(Jadavpur University)
- 15.13 What is the use of constant head and constant discharge curves ?
- 15.14 Show on a sketch of the constant speed characteristics of a centrifugal pump how an increase in the friction head due to the formation of scale in a long delivery pipe affects the performance of the pipe.  
[AMI Mech E (Lond)]

**Numericals**

- 15.15 A test on a single stage centrifugal pump when running at 1,000 rpm gave the following results :

|             |     |      |      |      |       |     |     |         |
|-------------|-----|------|------|------|-------|-----|-----|---------|
| Discharge   | 0   | 100  | 200  | 250  | 300   | 350 | 400 | lit/sec |
| Total head  | 37  | 35.6 | 33.5 | 32.3 | 29.25 | 25  | 19  | m       |
| Shaft input | 100 | 107  | 124  | 132  | 141   | 148 | 150 | HP      |

Plot curves of total head and efficiency to a base of discharge and find the specific speed at the best efficiency.

It pumps water from a river to reservoir through a pipe 30 cm diameter and 800 m long, having coefficient of friction 0.006. The difference of levels between the river and the reservoir is 20 m.

Plot the curve of total head of the main for the above static head against discharge base and so find the head, discharge and efficiency at the operating point.  
(Jadavpur University)

(Efficiency 0.445, 71.8, 81.7, 83 (max), 79, 67.7%)

$N_s = 43$  units

$$H_{total} = H_{stat} + H_L + 2g$$

|             |        |       |       |       |       |       |         |
|-------------|--------|-------|-------|-------|-------|-------|---------|
| $Q$         | 50     | 100   | 200   | 300   | 400   | 500   | lit/sec |
| $H_{total}$ | 20.568 | 22.27 | 29.08 | 40.43 | 56.32 | 76.75 | m       |

This  $Q$  vs  $H_{total}$  curve intersects the first  $H$  vs  $Q$  curve at the operating point where  $Q = 237$  lit/sec,  $H = 32.5$  m and  $\eta = 79\%$



- 15.16 A pumping installation consists of two identical constant speed centrifugal pumps working in parallel and forcing water through a single pipeline against a static or dead head of 56 metres. The head discharge characteristic curve of each pump can be represented by the equation

$$H = 128 + 14Q - 108Q^2$$

where  $H$  is the manometric head in metres and  $Q$  is the discharge per pump in  $\text{m}^3/\text{sec}$ .

The pipeline is 825 metres long and 0.6 metre in diameter ; its frictional co-efficient ' $f$ ' can be taken as 0.0045.

Estimate the discharge through each pump and through the pipe.

(Note : ' $f$ ' =  $gdh/2lv^2$ ,  $g = 9.81 \text{ m/sec}^2$ )

(0.691 and 1.382  $\text{m}^3/\text{sec}$ )

[*AMI Mech E (Lond)*]

- 15.17 Explain the head-discharge characteristics of two similar centrifugal pumps with different impeller blade angles at outlet only.

In a pumping station there are two units whose head—discharge characteristics at their constant working speeds can be represented by  $H = 132 + 15Q + 16Q^2$ , where  $H$  and  $Q$  have their significances. The pumps deliver into a single pipe line in which the static head is 45 metres ; the pipe is 0.6 m diameter and 800 m long and the value of the pipe co-efficient is 0.005. Compute the total discharge through the pipe, if the pumps were arranged—

(i) In parallel,

(ii) In series. Neglect velocity head. (*Madras University*)

## SECTION V

# Hydraulic Systems

# 16

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## *Hydraulic Systems*

**16.1 Hydraulic System**—Hydraulic System is a circuit in which force and power are transmitted through a fluid, generally an oil. Hydraulic systems may be divided into two groups, the hydrostatic and hydro-kinetic.

(a) **Hydrostatic System** is that in which the primary function of hydraulic fluid is the transmission of force and power by pressure. A hydrostatic system consists of two basic elements, a pumping unit to convert mechanical work into hydraulic energy and a hydraulic motor of reciprocating or rotary type to convert fluid energy into mechanical work. A lead which forms a circuit connects the two main components. The pumping unit transmits fluid pressure and is, therefore, known as *transmitter*. The hydraulic motor which receives force and power by means of fluid pressure is known as *receiver*.

**Example :** A pumping unit is operating a hydraulic press. In this case the pumping unit is a transmitter and hydraulic press is a receiver. The work done by the pump is utilised for displacement of oil against a force which arises from resistance to motion of the plunger in the hydraulic press.

(b) **Hydro kinetic or Rotodynamic System**—The purpose of a hydro kinetic or rotodynamic system is to transmit power and the required effect is obtained primarily by virtue of changes in velocity of flow of the working media and changes of pressure are avoided as far as possible.

A hydro-kinetic transmitter consists essentially of a centrifugal pump or impeller mounted on the driving shaft and an oil turbine or runner mounted on the driven shaft. Power is transmitted from the driving to the driven shaft through circulation of oil between the impeller and the runner.

**Example :** Hydraulic coupling and torque converter.

**16.2 Pumps and Motors**—Hydraulic pump and motors are basically energy conversion devices. A pump converts mechanical energy into hydraulic energy. Its input are mechanical torque and output are discharge and pressure (refer Fig 16.1). A motor similarly does the reverse of it, converting hydraulic energy to mechanical energy (refer Fig 16.2).

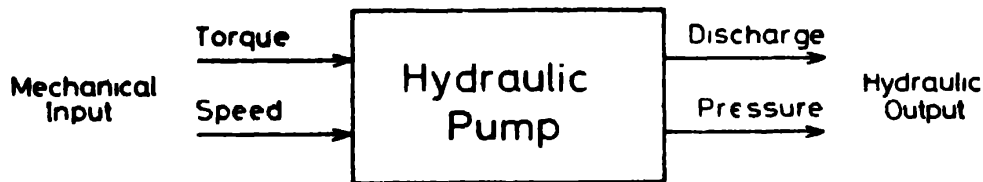


Fig 16.1 Transformation Through a Pump

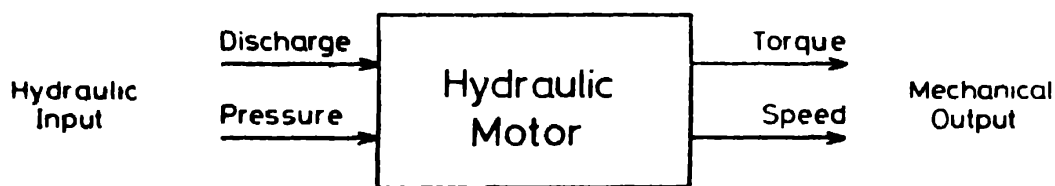


Fig 16.2 Transformation Through a Motor

**16.3 Rotary Positive Displacement Pumps for Hydrostatic Systems**—Reciprocating, centrifugal and other types of pumps dealt with in Chapters 11, 12 and 13 are usually employed as water lifting devices where large volumes have to be handled. In the oil hydraulic field, however, rotary pumps are used, for they develop very high pressure.

A rotary pump is a positive displacement pump with a circular motion. In outward appearance, it resembles a centrifugal pump. But it differs from a centrifugal pump in action. While it continuously scoops the liquid out of the pump chamber, the latter only imparts a velocity to the stream of the fluid.

**16.4 Theory of Pumps and Motors**—Ideally speaking a positive displacement pump delivers a given volume of fluid for every revolution, and similarly a positive displacement motor will accept a given volume of fluid for every revolution.

If the pump rotates at  $N$  rev./sec., then the discharge is given as

$$Q_{pt} = N_p V_p \text{ lit/sec} \quad \dots(16.1)$$

From a motor,  $Q_{mt} = N_m V_m \text{ lit/sec} \quad \dots(16.2)$

Since, for a motor the output variable is speed and not the discharge, it is expressed as,

$$N_m = \frac{1}{V_m} \cdot Q_{mt} \quad \dots(16.3)$$

Let  
 $a$  = Fluid passage of cross-section area  $A$  ;  
 $b$  = Piston of the machine moving in the passage ;  
 $c$  = Shaft of the machine.

Fig 16.3 shows the section of a fluid machine. Through the fluid pressure differential acting on the piston  $b$ , either the fluid does work and turns the shaft against a torque  $M$  acting on the shaft  $C$  or in turning the

shaft. The machine does work on the fluid, by displacing it against a pressure differential acting on the piston.

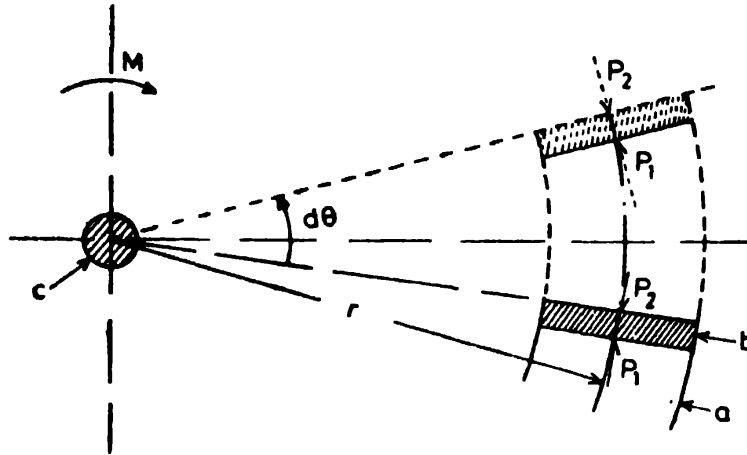


Fig 16.3 Working of a Fluid Machine

The work done in displacing the shaft through an angle  $d\theta$  (refer Fig 16.3),

$$M_t \cdot d\theta = \text{Force} \times \text{displacement}$$

$\therefore$  The work done per revolution is,

$$\int_0^{2\pi} M_t d\theta = \left\{ (p_1 - p_2) \cdot A \right\} \times 2\pi r$$

where  $(p_1 - p_2) \cdot A = \text{net force acting on the piston ;}$

$2\pi r = \text{mean distance covered by the piston in one revolution.}$

$$\begin{aligned} \text{or } M_t 2\pi &= (p_1 - p_2) \cdot A 2\pi r \\ &= (p_1 - p_2) \cdot V \end{aligned}$$

where  $V$  is the volume of the fluid displaced in one revolution.

$$M_t = \frac{V}{2\pi} (p_1 - p_2) \quad \dots(16.4)$$

Here, it is assumed that the torque and pressure differential remain constant.

$$\text{For a pump, } (p_1 - p_2) = \frac{2\pi}{V} M_t \text{ kg/cm}^2 \quad \dots(16.5)$$

$$\text{For a motor, } M_t = \frac{V_m}{2\pi} (p_1 - p_2) \text{ kg/cm}^2 \quad \dots(16.6)$$

In fact, none of the units is ideal. There are torque losses due to viscous and dry friction, and volumetric losses, due to leakage and compressibility. Therefore, the actual torque and discharge are given as

$$M = M_t \pm \Sigma M(\text{losses}) \quad \dots(16.7)$$

$$Q = Q_t \pm \Sigma Q(\text{losses}) \quad \dots(16.8)$$

Taking into account these losses, the expressions for torque and discharge for actual machine can be written as

$$M = \frac{V}{2\pi} \Delta p + C_f \cdot \frac{V}{2\pi} \cdot \Delta p \pm C_s \cdot V \cdot \mu \cdot N \quad \dots(16.7a)$$

$$Q = V \cdot N \mp \frac{C_s \cdot \Delta p}{2\pi \cdot \mu} \cdot V \quad \dots(16.8a)$$

where  $C_s$  = Coefficient of dry friction, non-dimensional ( $8 \times 10^4$  to  $5 \times 10^6$ );

$C_f$  = Coefficient of viscous friction, non-dimensional (0 to 0.2);

$C_s$  = Coefficient of slip, non-dimensional ( $1 \times 10^{-6}$  to  $1 \times 10^{-9}$ );

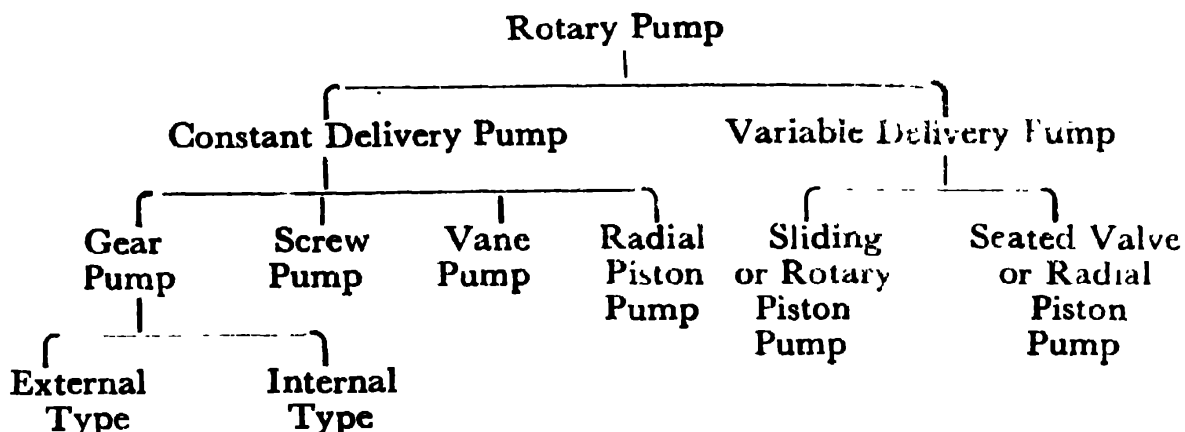
$V$  = Displacement revolution;

$\Delta p$  = Pressure differential;

$N$  = Shaft rotational speed;

$\mu$  = Dynamics viscosity of the fluid used.

### 16.5 Classification of Rotary Pumps—



**16.6 Constant and Variable Delivery Pump**—Constant delivery pump is one which has a continuous discharge of liquid at a uniform rate of flow whereas variable delivery pump has a continuous discharge of liquid at rates of flow varied as required. Selection of pump depends upon the nature of duty of the hydraulic receiver or motor. When the receiver or motor operates continually at varying speeds or with movement varying in extent, variable delivery pump is usually employed. The construction of variable delivery pump is complex and as such these pumps are more costly than constant delivery pumps. Therefore, constant delivery pumps are used to give variable delivery by the provision of variable speed control gear for the driving motor, or by regulating the flow of liquid by means of valves independent of pump.

### Constant Delivery Pumps

**16.7 External Type Gear Pump** (refer Fig 16.4 a, b)—A single-stage external type gear pump consists of two identical intermeshing spur wheels working with a fine clearance inside a suitably shape casing. One gear is keyed to the driving shaft of a motor and the other revolves idly. Oil is entrained in the spaces between the teeth and the casing and carried round between the gears from the suction port to the discharge port. It cannot slip back into the inlet side due to the meshing of gears.

If  $a$  = the area enclosed between two adjacent teeth and casing;

$l$  = the axial length of teeth;

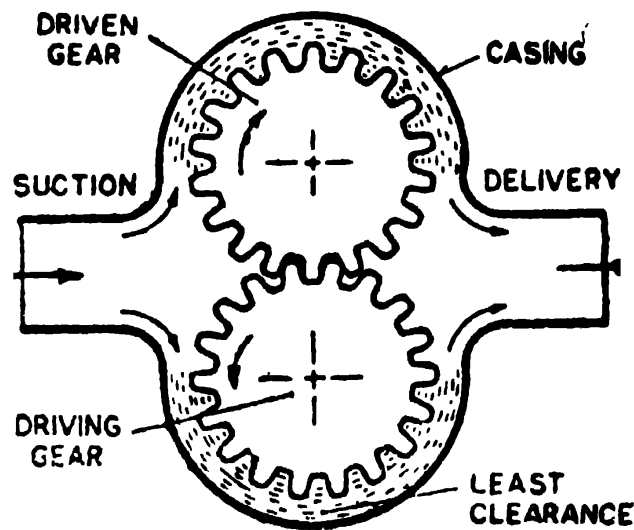


Fig 16.4 (a) Line Diagram of Gear Pump.

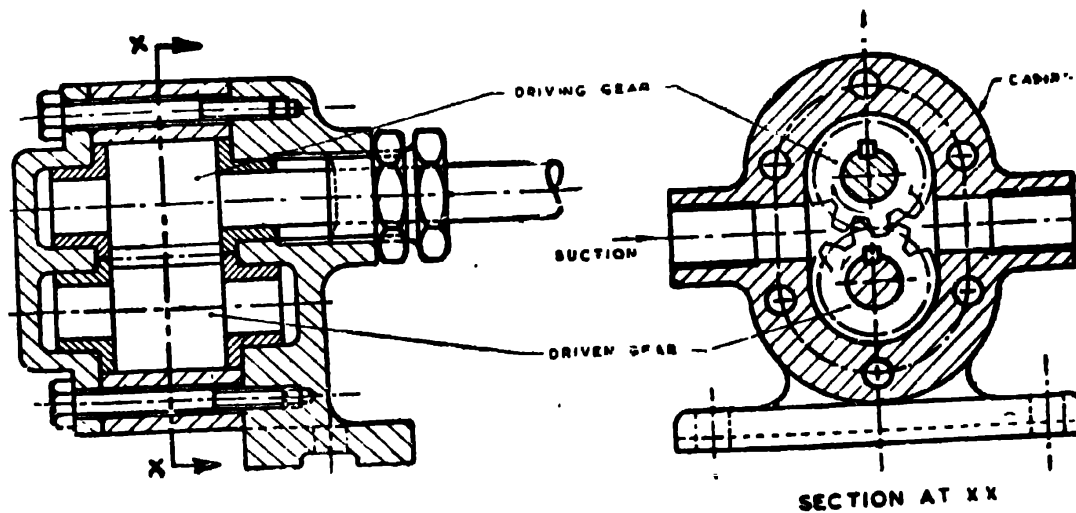


Fig 16.4 (b) External Type Gear Pump—Sectional View

$n$  = the number of teeth in each pinion ;  
 and  $N$  = the speed in rpm,  
 then, volume of liquid pumped in one revolution =  $2 . a . l . n$

$$\therefore \text{Total or ideal discharge} = \frac{2 . a . l . n . N}{60}$$

If  $\eta_Q$  be the volumetric efficiency, the actual discharge,

$$Q = \frac{2 . a . l . n . N}{60} . \eta_Q \quad (16.9)$$

The thicker the fluid, the higher is the efficiency  $\eta_Q$ .

If it is difficult to determine  $a$ , the area enclosed between the teeth and the casing, then the following empirical relation could be used for finding the volumetric displacement of pump per revolution :

$$q = 0.95 \pi . c . (D - c) . t \quad \dots (16.10)$$

where  $c$  = the centre to centre distance between axes of gears,

and  $D$  = the outside diameter of gears.

Further,  $n$  the number of teeth in the gear is related to ratio  $\frac{D}{c}$ .

TABLE 16.1

| $n$           | 7    | 10   | 13   | 18   |
|---------------|------|------|------|------|
| $\frac{D}{c}$ | 1.28 | 1.21 | 1.13 | 1.12 |

**Problem 16.1** A gear pump work against a total pressure of  $6.35 \text{ kg/cm}^2$  delivering  $6.5 \text{ lit/sec}$  of oil of specific gravity  $0.92$ , when running at  $720 \text{ rpm}$ . The volumetric efficiency of the pump is  $95\%$  and its overall efficiency is  $50\%$ . The length of the gear wheels is  $1.5$  times their outside diameter. Find the outside diameter of wheel and the horsepower input of the pump. Assume each gear has 12 teeth.

**Solution**

$$\begin{aligned}
 p &= 6.35 \text{ kg/cm}^2 & Q &= 6.5 \text{ lit/sec} & N &= 720 \text{ rpm} \\
 \text{Sp gr} &= 0.92 & \eta_Q &= 0.95 & \eta_{\text{overall}} &= 0.5 \\
 l &= 1.5 D & n &= 12
 \end{aligned}$$

Take the value of ratio  $\frac{D}{c} = 1.18$  from Table 16.1

$$q \text{ per revolution} = \frac{6.5 \times 60}{720} = 0.542 \text{ lit/res or } 542 \text{ cm}^3/\text{sec}$$

$$\begin{aligned}
 \therefore 542 &= 0.95 \times \pi \cdot c \cdot (D - c) \cdot l \\
 &= 0.95 \times \pi \times \frac{D}{1.18} \left( D - \frac{D}{1.18} \right) \times 1.5 D
 \end{aligned}$$

$$\text{or } D^3 = \frac{542 \times 1.18 \times 1.18}{0.95 \times \pi \times 0.18 \times 1.5} = 939 \text{ cm}^3$$

$$\text{or } D = 9.8 \text{ cm or } 98 \text{ mm} \quad \text{Answer}$$

A single gear pump can build up pressure as high as  $150 \text{ kg/cm}^2$ . However, due to the possibility of internal leakage it is generally used for a delivery pressure upto  $10 \text{ kg/cm}^2$  only. Gear pump finds its application in machine tools drive using oil at relatively low pressure, for combined lubrication and hydraulic control system of water turbines, steam turbines and other machines.

Fig 16.4 shows a pump having straight spur wheels, which make noise because the oil trapped between intermeshing teeth cannot easily escape. To avoid this difficulty single helical or double helical teeth gears are used. They run quieter at high speeds.

The gears of such pumps are generally external; however, internal gears are also used in exceptional cases.

**16.8 Internal Type Gear Pump**—Fig 16.5 shows an internal spur gear pump. The idler which is an internal gear meshes with the driving gear. The space between the outside diameter of driving gear and the inside diameter of idler is sealed by a crescent shape projection which forms a part of the cover.

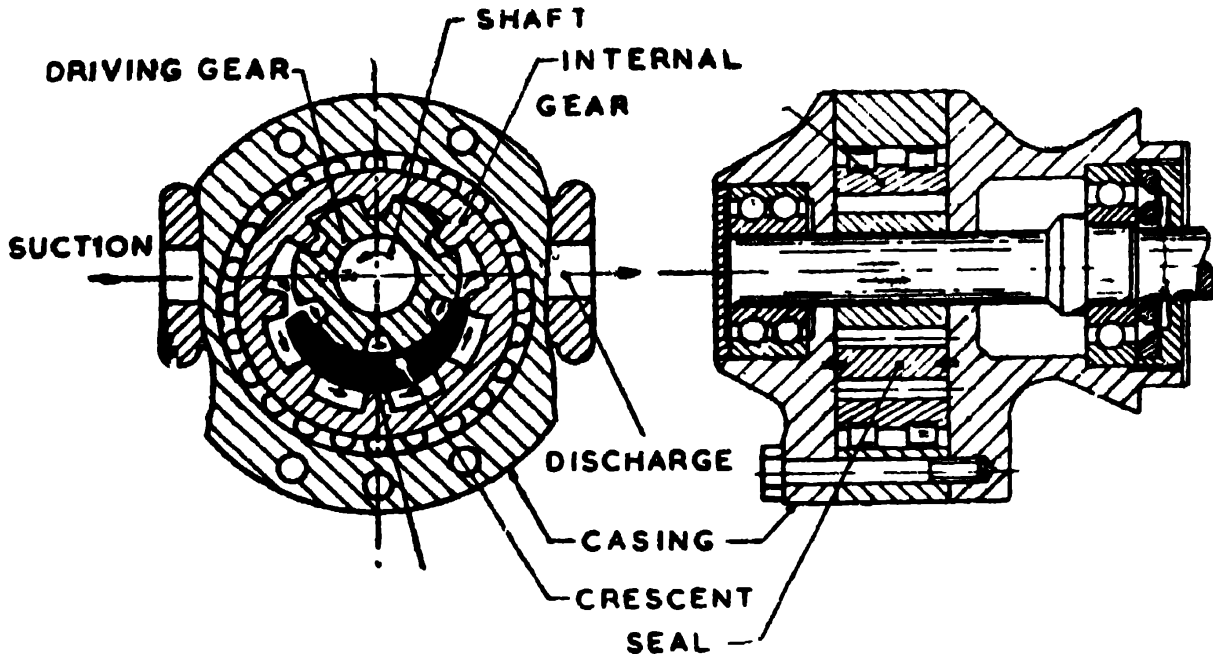


Fig 16.5 Internal Type Gear Pump

The driving gear gets its motion from the driving motor. As the teeth come out of the mesh, there is an increase in volume which creates a partial vacuum. Liquid is forced into this space by atmospheric pressure and stays in the spaces between the teeth of the driving gear and idler until the teeth mesh when, the liquid is forced from these spaces and out of the pump. The pump can be reversible if a provision is made for swinging the crescent through  $180^\circ$ .

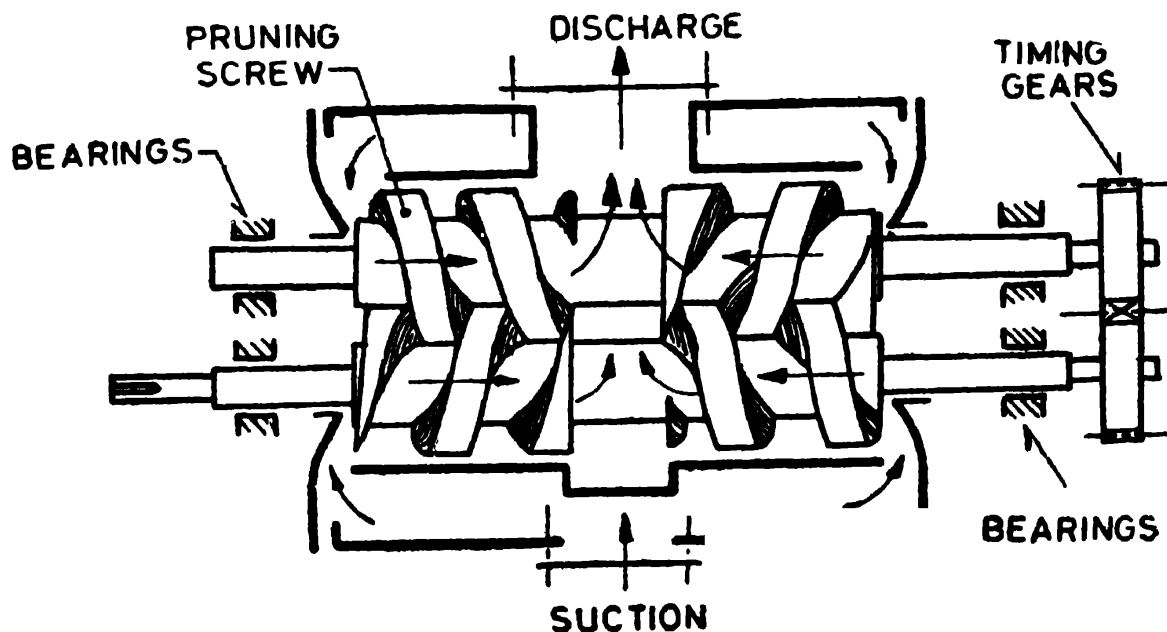


Fig 16.6 Screw Pump



**16.9 Screw Pump** (refer Fig 16.6)—Screw pump of a rotor or screw to which the source of power is directly connected. Unlike gear pump the rotors of screw pumps are extended and are of smaller diameter. The screw may be double helical (right and left hand) or single helical (one hand only). The advantage of double helical is that they are balanced axially. The fluid is carried forward to the discharge along the rotor in pockets formed between teeth and the casing.

There may be one, two or three screws. In a 2-screw pump one is a rotor and the other is idler. In a 3 screw pump there are two idlers on either side of the rotor. Two idlers act as seals to the power rotor and are driven by fluid pressure and not by metallic contact with rotor. The liquid entering the inlet passage divides and flows to the two ends of the rotor where it is trapped in pockets formed by the threads and is carried towards the discharge, like a nut on the power screw.

**Advantages :**

- (a) Screw pump operates upto a continuous working pressure of  $150 \text{ kg/cm}^2$ .
- (b) The screw pump is free from turbulence and pulsation.
- (c) The rotors of a screw pump are balanced dynamically, the pump is silent and free from vibration while running.
- (d) Screw pump is especially suitable for operation at high speed and thus can be directly driven by an electric motor or steam turbine running at 3,000 to 4,500 rpm.

**16.10 Vane Pump**—Vane pump consists of a rotor disc having a number of slots into which fit sliding blades or vanes (refer Fig 16.7). The rotor turns in a casing which is bored eccentric to the driving shaft. The vanes are free to slide radially with the help of springs and thus form the required seal between the suction and discharge connections. When the

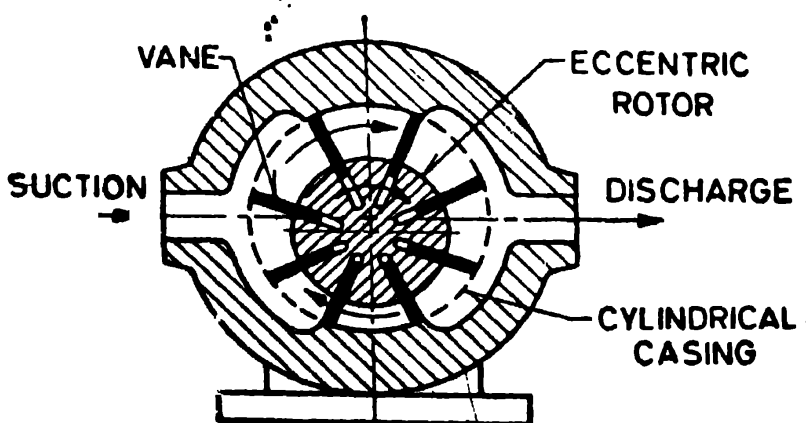


Fig 16.7 Vane Pump

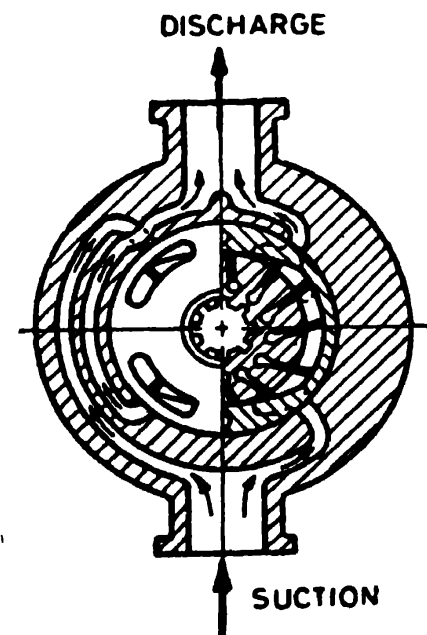


Fig 16.8 Vane Pump

rotor turns, the liquid flows in between its side and casing and is trapped by the vanes, which force it from inlet to discharge.

Fig 16.8 shows a modified form of vane pump. The rotor fitted with vanes is supported by bronze bushes. These are integral with plates constraining the end vanes axially. Drive is by means of the splined shaft carried on ball bearings. The sleeve machined oval internally, surrounds the vanes. The suction and delivery connections respectively, communicate with two ports, diagonally opposite in the plate valve. Thus the rotor is hydraulically balanced, and the pockets between the vanes undergo two cycles of expansion and contraction during each revolution. Therefore this type of vane pump will handle twice the amount of fluid handled by simple vane pump shown in Fig. 16.7.

#### Advantages ;

- (a) Single stage vane pump can be used for oil pressure of 17.5 to 70 kg/cm<sup>2</sup> continuous rating. Two stage pump can operate at 150 kg/cm<sup>2</sup>.
- (b) Wide range of application, especially for machine tools. For industrial equipment the vane pump is possibly the most used at the present time.

**16.11 Radial Piston Pump**—Radial piston pump consists of a number of cylinders disposed radially about an eccentric on the driving shaft. The rotation of shaft causes reciprocation of the pistons. The demand for operating pressures above 70 kg/cm<sup>2</sup> was responsible for the design of such a pump. Operating pressure upto 4,000 kg/cm<sup>2</sup> could be produced with a very high speed and volumetric efficiency. This is mainly due to the spherically seated valve used in the pump.

Fig 16.9 shows a radial piston pump manufactured by Automotive Products Co. Ltd. The oil enters the pump through inlet connection A and

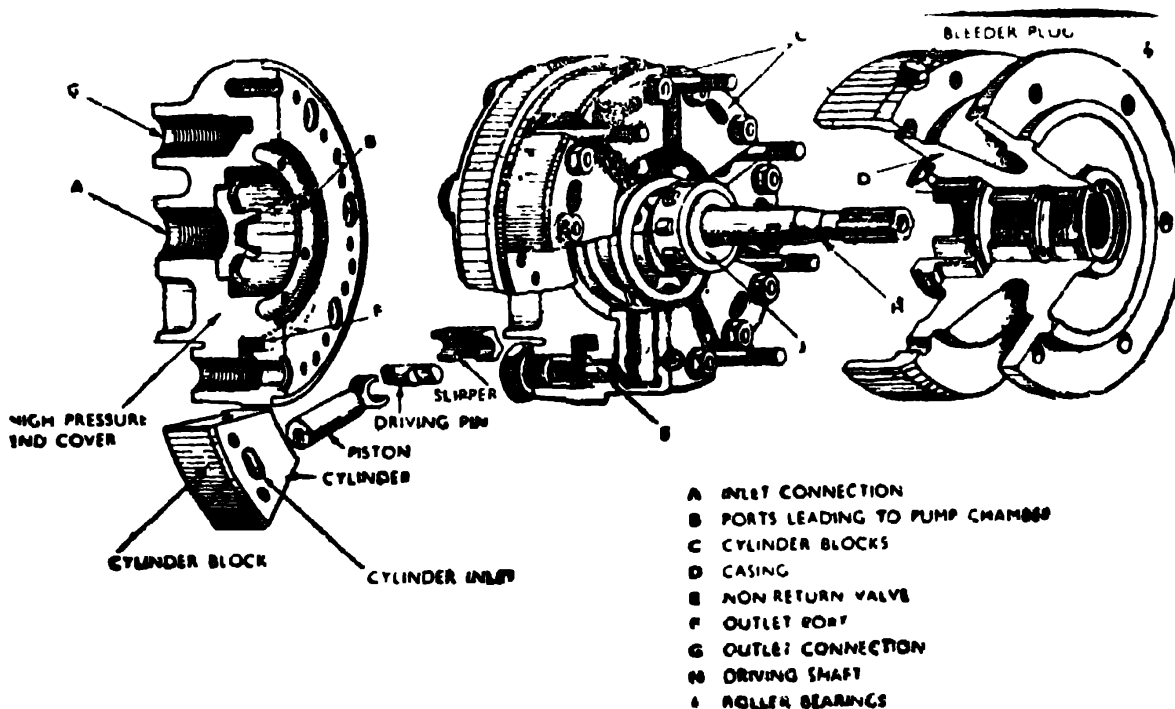


Fig 16.9 Radial Piston (Constant Delivery) Pump  
(Manufactured by Automotive Products Co. Ltd.)

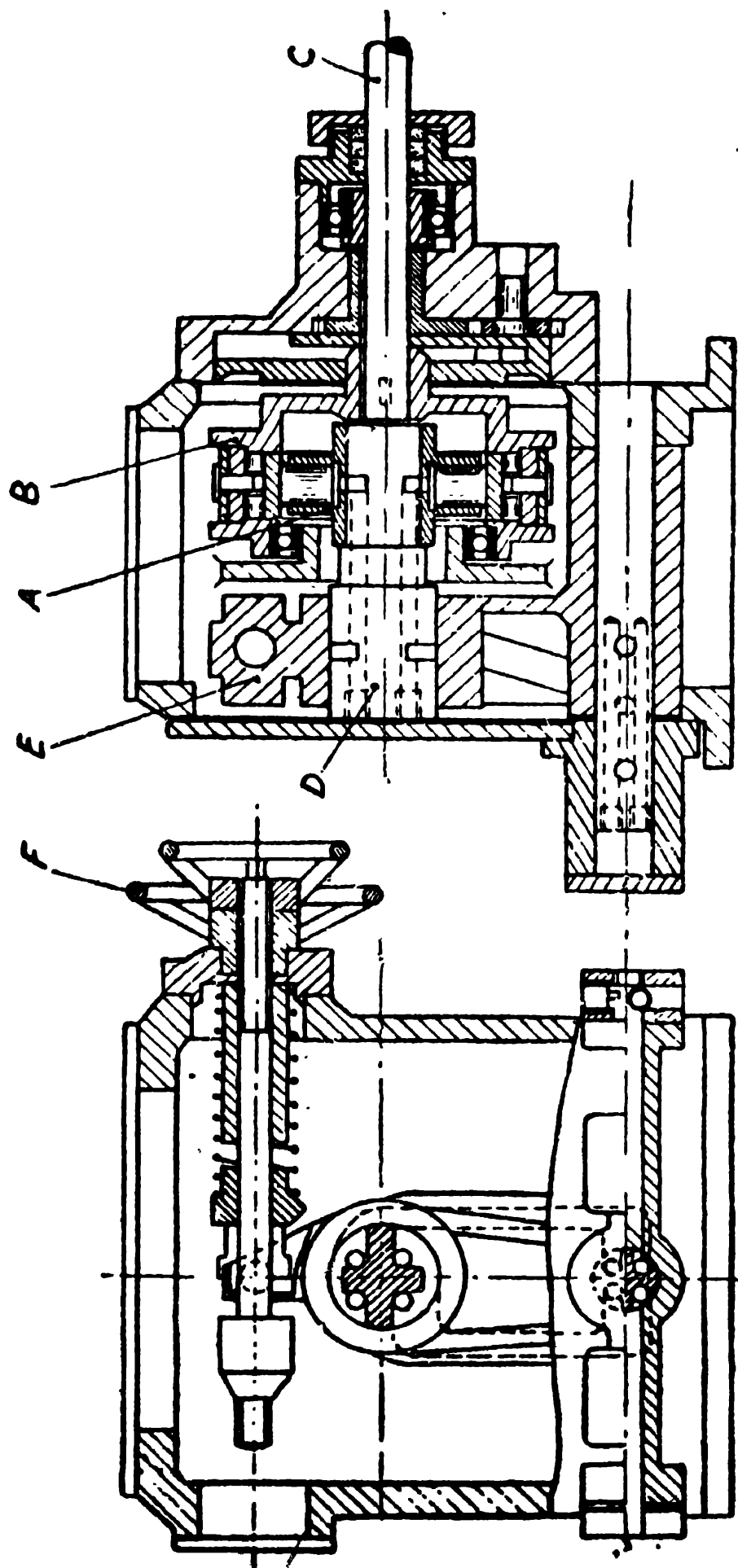


Fig 16.10 (a) Rotary Piston Pump

passes through inlet port *B* to reach an annulus in the end cover which forms a part of the pump chamber. The cylinder blocks *C* are bolted to the end cover and have inlet ports on the opposite side. Oil flooding the pump chamber reaches the inlet ports by way of annular clearing space inside the casing *D*. Each outlet port in the high pressure end cover leads to a spring-loaded non-return valve *E* communicating with the outlet connection *G*. The piston is moved through its suction stroke by two retaining rings which engage flats on the piston driving pins. These fit in slippers which transmit thrust to and from a bearing ring mounted on needle rollers on the eccentric. Two roller bearings carry the shaft. Output is about 1,200 cm<sup>3</sup>/min at 3,000 rpm.

### VARIABLE DELIVERY PUMPS

**16.12 Rotary Piston Pump**—Fig 16.10 (*a*) shows a rotary piston pump. It consists of a cylinder body *A* having a number of radial piston (in this case 5). The cylinder body rotates about a fixed control cylindrical valve axis *D*. The valve has suction and delivery openings. The pistons are connected to a floating ring *B* which is driven by the shaft *C*. The valve axis *D* is fixed to a swing arm *E* and can be adjusted to the piston corresponding to the desired output with hand wheel *F*. As in a vane pump, the fluid volume pumped depends upon the degree of eccentricity.

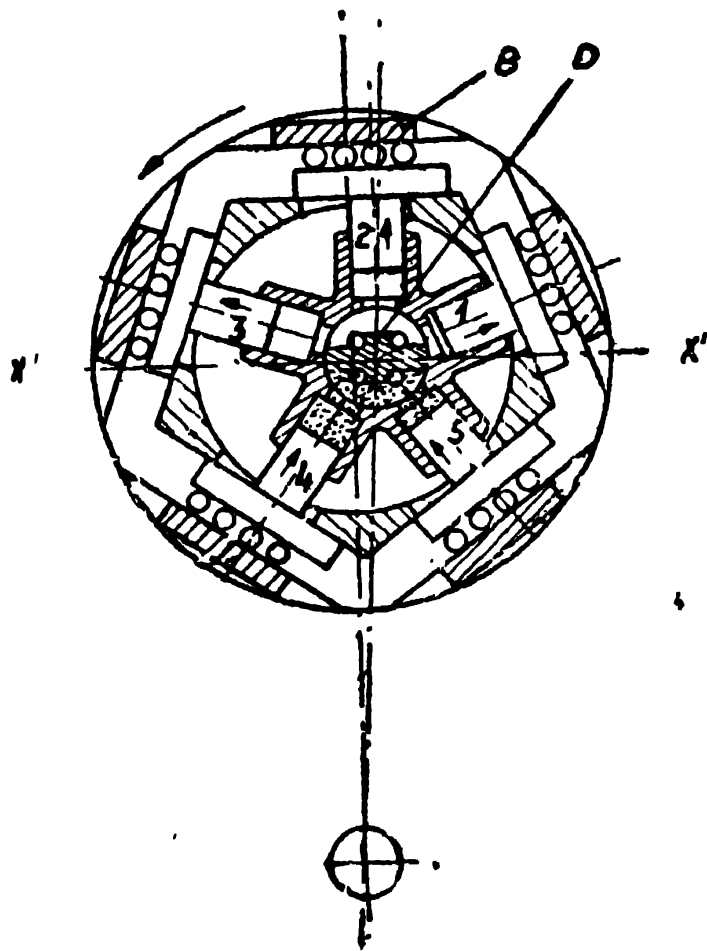


Fig 16.10 (*b*) Rotary Piston Pump

**Working Principle**—The floating ring *B* is driven over the shaft *C* in the direction shown in Fig 16.10 (*b*). As the axis *D* is made eccentric, the piston move to and fro. Piston 1 moves outward as shown by an arrow and sucks the oil. Suction takes place in all pistons above the centre line *X'X'*. Below the line *X'X'*, delivery takes place for the pistons 4 and 5 moving inward. The pressure generated is about 70 kg/cm<sup>2</sup> when running at 860 rpm.

**Advantages**—The oil flow can be regulated in both pistons positive as well as negative (reverse direction), which is very useful in machine tools.

**16.13 Radial Piston Pump**—Fig 16.11 (*a*) shows a radial piston pump for variable discharge. In this case the regulation of discharge is

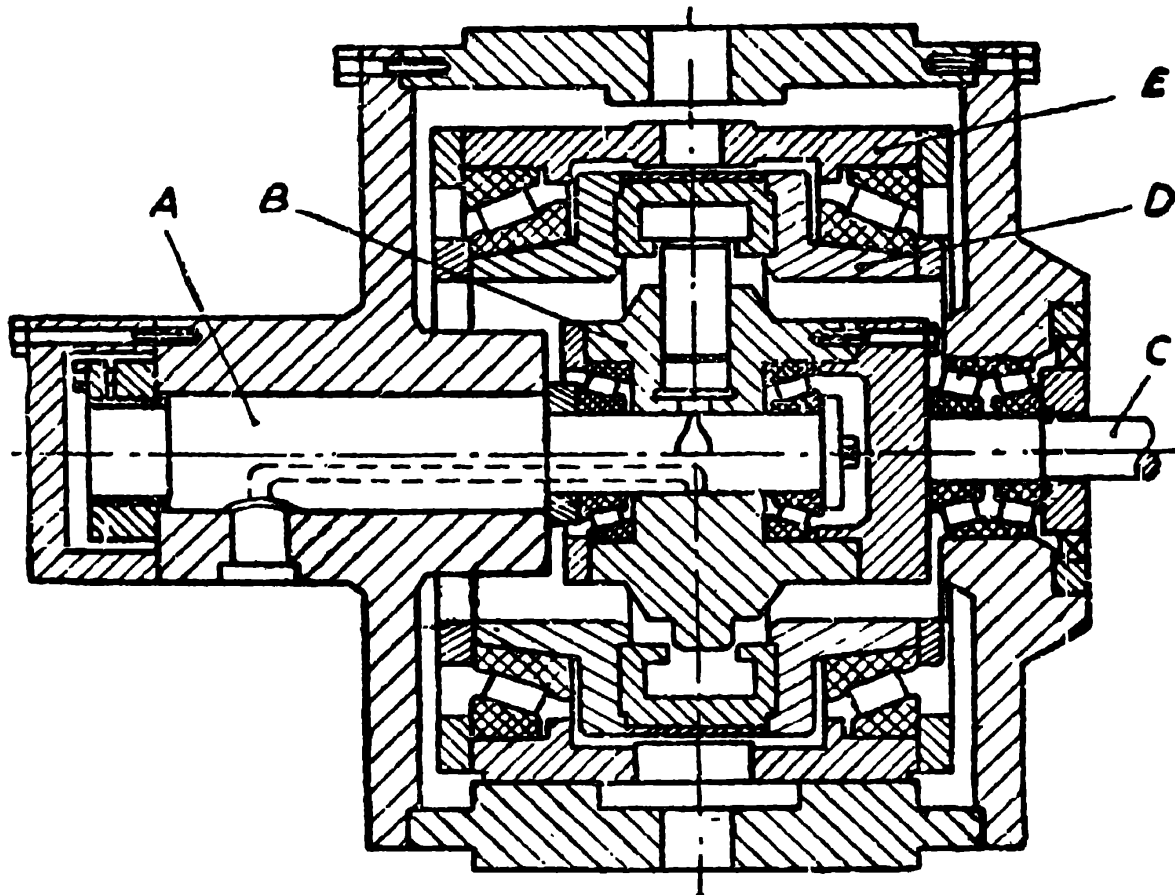


Fig 16.11 (a) Radial Piston (Variable Delivery) Pump

not effected by shifting the valve axis as in case of rotary piston pump (refer Art 16.12) but by the floating ring with which the pistons move eccentrically. The valve axis *A* is fixed to the casing and carries the cylinder body *B*. The suction and delivery pipes are bored axially in the valve axis.

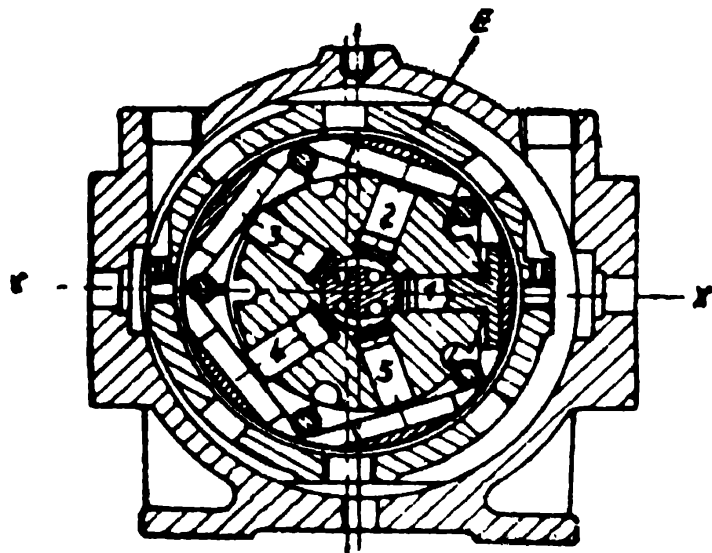


Fig 16.1 (b) Radial Piston (Variable Delivery) Pump

**Working Principle :**  
The cylinder body *B* is driven by the shaft *C*. The floating ring rotates in the outer ring which can be shifted horizontally in the casing to obtain the required eccentricity for the variation of discharge.

In Fig 16.11 (b), piston 1 has finished its delivery stroke and would move outwards by further rotating the floating ring (see piston positions 2 and 3; which means it would commence sucking. Below the centre line *XX* the motion is reversed and the piston movement is inward (see piston positions 4 and 5). The direction of flow depends upon the piston of outer ring *E*.

The maximum pressure generated would be about  $180 \text{ kg/cm}^2$ . The speed of the pump is 900 to 1,800 rpm according to its capacity.

## HYDROSTATIC SYSTEMS

**16.14 Functions of Hydrostatic System**—There are mainly three functions of a hydrostatic system :

- (a) Actuation of mechanical control,
- (b) Force multiplication and
- (c) Transmission and control of power.

(a) **Actuation of Mechanical Control**—Control of the motion of driving and driven mechanisms is achieved by means of transmission by the fluid of power and thrust. The development of power and force is of less importance than the nature of the motion imparted to driven unit. An example of an application of this type is use of a hydraulic relay. An arrangement in which oil forms a power-driven pump is used for control purposes constitutes a hydraulic relay. Prime movers such as water turbines and steam turbines are controlled by means of hydraulic relays. The control of governing of water turbines by hydraulic relay is described in Chapter 8.

(b) **Force Multiplication**—In this system, transmission of thrust is the important consideration. Transmission of power is also involved, example of application—Hydraulic Press (refer Art 16.16).

(c) **Transmission and Control of Power**—Power is transmitted for producing movement at a distance. The hydraulic system always includes the control of movement and speed of driven unit. Transmission of thrust also occurs, but is not so important as in case (b). Hydraulic drives in machine tools described in Art 16.23 are applications of this kind.

**16.15 Methods of Control**—The motion of hydraulic motor or receiver is governed by the flow of oil. Two methods are used for this purpose depending on the design of the pump :

- (a) Constant delivery system and
- (b) Variable delivery system.

(a) **Constant Delivery System**—The discharge of the pump is constant and the flow in the circuit is controlled by means of regulating valves. The system consists of a throttle valve fitted in the circuit between the motor and the receiver. As the pump discharges at a constant rate, a by-pass or relief valve is provided to return the excess oil to the tank, as well as to protect against overloads. The flow of oil passes through a four-way reversing valve which delivers the oil to the two sides of the piston as shown in Fig 16.12 (a). Constant delivery system is cheaper than the variable delivery system but, ~~the~~ delivery of excess oil back to the tank represents a loss of power.

(b) **Variable delivery system** (refer Fig 16.12 b)—The delivery of pump is variable and the flow in the circuit is controlled by the variable pump output. Variable delivery pumps are complex and costly ; however, there is no wastage of power by diversion of excess oil back to the tank. The pump discharge can be varied or direction of flow reversed by employing a rotary piston pump described in Art 16.9 (a). In this system the

pressure, against which the pump discharges, is governed by resistance to motion of the driven unit, and not by the rate of delivery of oil by the pump. It is possible to arrange for the delivery rate to vary automatically

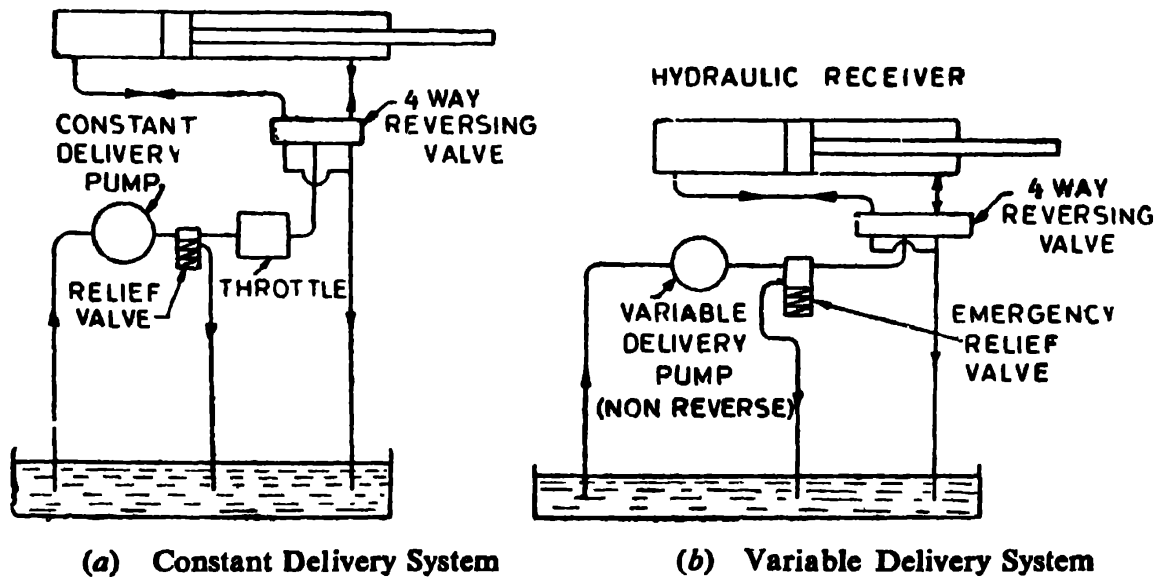


Fig 16.12 Methods of Control

and independently of back pressure. A relief valve is employed to protect the system against overload.

**16.16 Hydraulic Press**—Hydraulic press was first built in 1785 by Ernest Bramah. Since then, hydraulic power has been in use for heavy operations is cranes, lifts and capstans. With the development of electric power, the hydraulic applications were replaced until more recent times when hydraulic power has come into prominence again. As in a modern machine shop individual drive has replaced the line shaft, similarly in case of modern hydraulic drive the accumulator system has been replaced by self-contained hydraulic system, using a separate pump for each machine.

**Principle**—Mechanical advantage is obtained by a practical application of Pascal's Law of transmission of fluid pressure. Two pistons of different sizes operate inside two cylinders suitably connected with a pipe so that pressure in each is the same (refer Fig 16.13). If  $p$  is the pressure

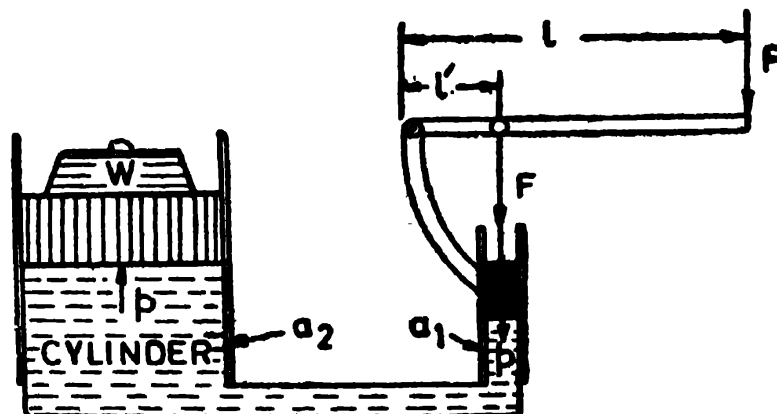


Fig 16.13 Principle of Hydraulic Press

and  $a_1$  and  $a_2$  are the cross-sectional areas of cylinders, then a force  $F_1$  applied to the smaller plunger will make available a large force  $F_2$  at the large plunger.

$$p = \frac{F_1}{a_1} = \frac{F_2}{a_2} \quad \dots(16.11)$$

$$\therefore \frac{F_2}{F_1} = \frac{a_2}{a_1} \quad \dots(16.11a)$$

If the volume of liquid is constant, the displacement of large piston will be proportionately smaller.

$$\text{Mechanical advantage of press} = \frac{a_2}{a_1} \quad \dots(16.12)$$

If force in the smaller piston is applied by a lever which has a mechanical advantage  $\frac{l}{l'}$  then total mechanical advantage of machine

$$= \frac{l}{l'} \cdot \frac{a_2}{a_1} \quad \dots(16.12a)$$

**Essentials of Hydraulic Press**—A hydraulic press consists of two stationary platens and a movable platen (refer Fig 16.14). The latter is carried by a plunger or ram which passes through the upper stationary platen and moves in the cylinder. The upper and lower stationary platens are joined by columns. Under hydraulic pressure, which is supplied by the pumps, the ram moves down and applies tremendous pressure upon any material placed between the movable and stationary platen. Fig 16.15 shows a hydraulic press used to compress plates. The pumping unit is shown on the right front. This can exert a force of 5,000 tonne. Fig 16.16 shows and autoclave vulcanising press.

Hydraulic press is employed to perform many odd jobs requiring tremendous pressures. Some examples are given below :

(i) Metal Press Work : Employed to press sheet metal to any desired shape (refer Fig 16.15),

(ii) Bakelite Press : Used to prepare moulds and casting of bakelite,

(iii) Autoclave Vulcanising Press, (refer Fig 16.16),

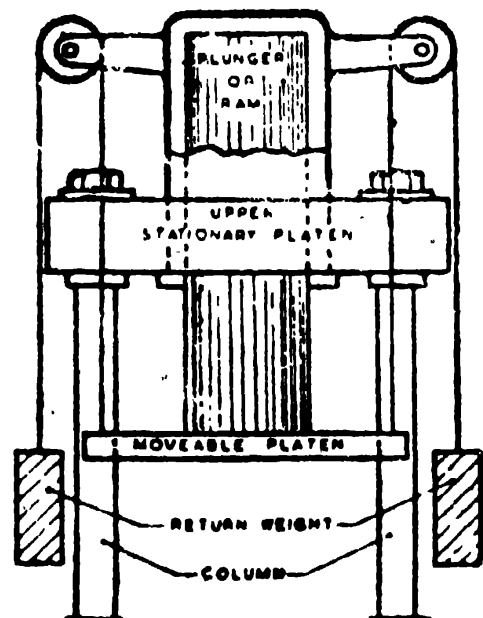


Fig 16.14 Elementary Inverted Press



- (iv) Packing Press,
- (v) Cotton Press.
- (vi) Metal Pushing Press,
- (vii) Filter Press,
- (viii) Forging Press,
- (ix) Plate Press,
- (x) Tablet making machine, and
- (xi) Drawing and Pushing rods, bending and straightening any metal piece.

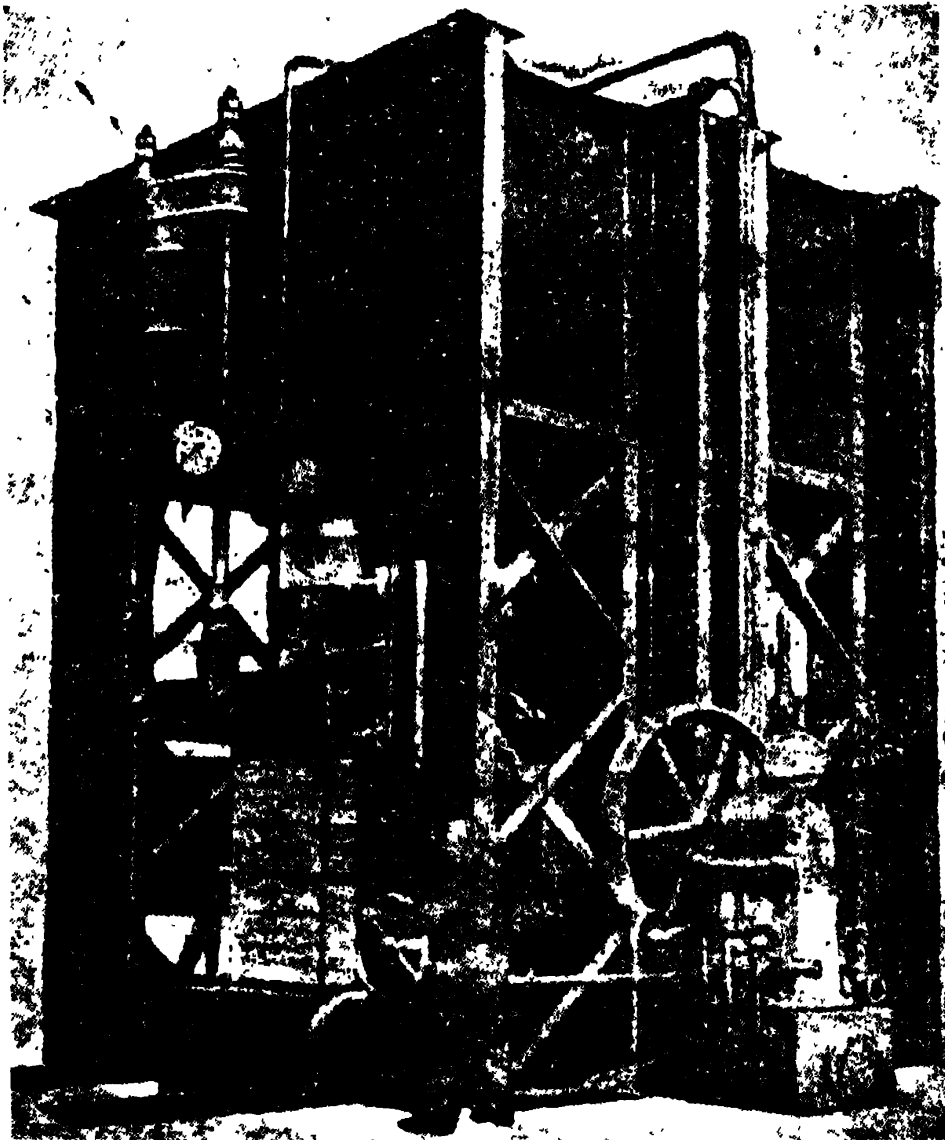


Fig 16.15 Hydraulic Plates Press for 5,000 Tonne Pressure with Pumping Unit. Total Weight 150,000 kg

Manufactured by Theoder Bell and Co. Ltd. Kriens (Switzerland)

These presses are generally worked by electrically driven pumps supplying oil under pressure.

**16.17 Hydraulic Crane** (refer Fig 16.17) consists of two main parts *jigger* and crane itself. The jigger attached to most of the cranes, is

made up of a cylinder and a ram, both having a set of pulleys, at their ends as shown in Fig 16.18. The water under pressure is forced into the cylinder, which pushes the ram vertically up. The ram descends by opening an outlet valve. The set of pulleys fixed to the cylinder remain stationary while the others fixed to the ram move vertically up or down. A wire rope with one of its ends fixed to a movable pulley and passing over all the pulleys, is stretched over the jib of the crane, down to the load  $W$ . The velocity ratio of crane hook to ram, depends upon the number of set of pulleys, thus a six-sheaf pulley block system will have a velocity ratio of 6 : 1, which means the load suspended to the hook of the crane will move six times the speed of the ram.

The crane consists of a vertical mast to which is attached a jib. The mast is supported on a pedestal, and can be revolved with its vertical axis. The jib swings with the mast and can be raised or lowered together with the load by means of wire rope when the ram moves in the cylinder.

Hydraulic crane finds its use in docks and warehouses. It can lift a load upto 250 tonne. However a hydraulic crane can hardly compete with the modern electric crane.

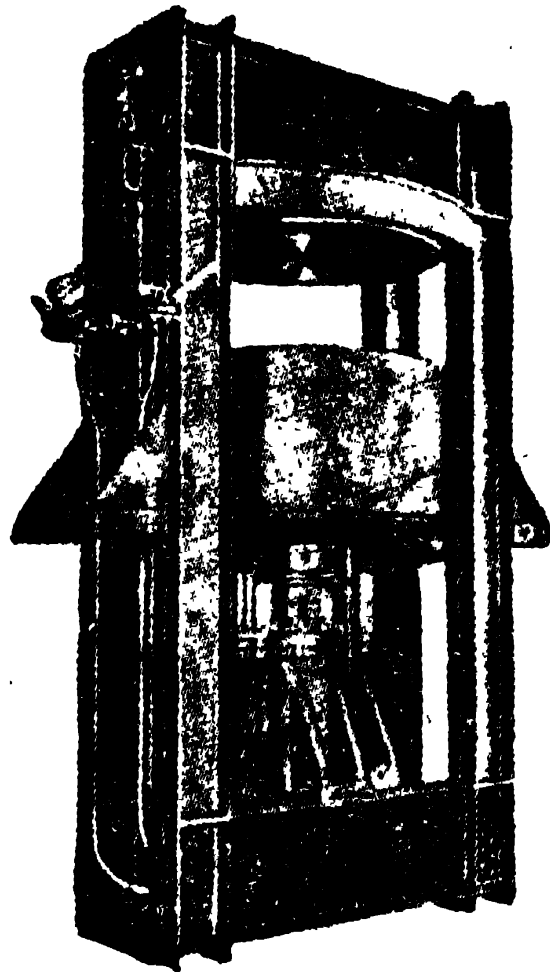


Fig 16 17 Autoclave Vulcanising Press for 150 Tonne Pressure Manufactured by Theodor Bell & Co. Ltd, Kriens (Switzerland)

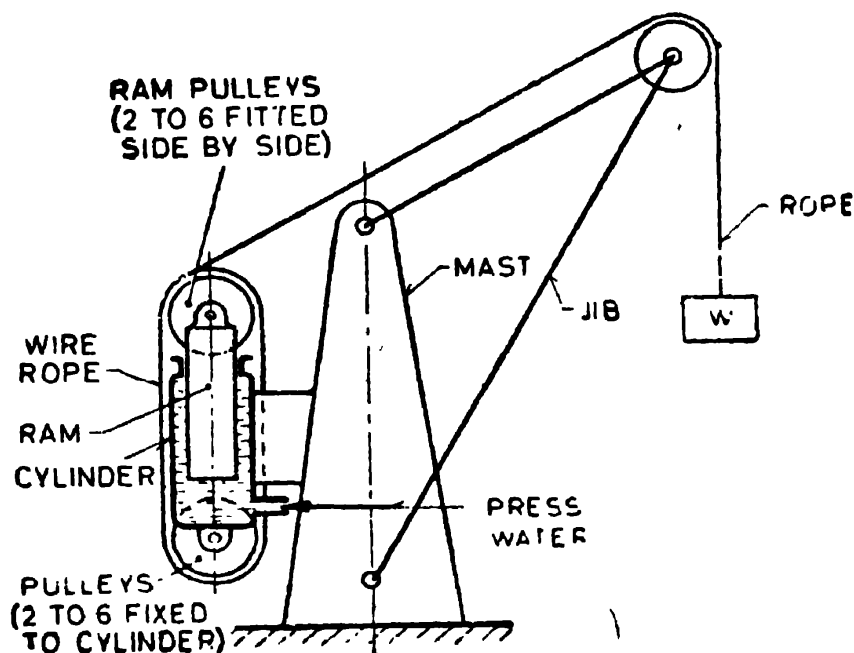


Fig 16.17 Hydraulic Crane

**16.18 Hydraulic Lift** (refer Fig 16.18) —A simple form of hydraulic lift consists of a cylinder and a ram. The water under pressure is forced into the cylinder, thus pushing the ram vertically upwards. A platform or cage is fitted to the top end of the ram. The platform can be made to stay, siding with the floors, so that one can walk over it. The platform or cage should move between guides.

The modified form of hydraulic lift is fitted with a jigger explained in Art 16.17, attached to hydraulic crane. The weight of cage in which the persons or load ascend, is balanced by balance weights also moves in guides.

Electric lifts have become quite common now-a days.

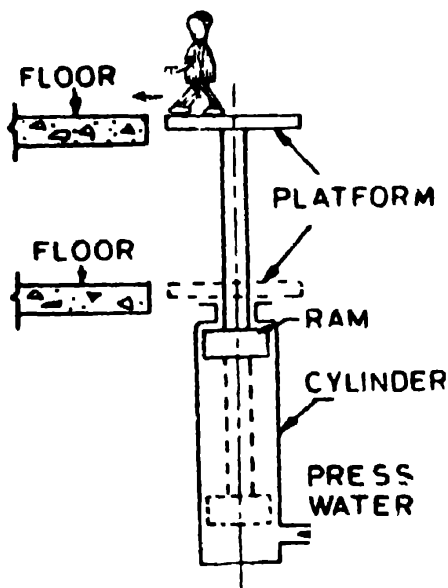


Fig 16.18 Hydraulic Lift

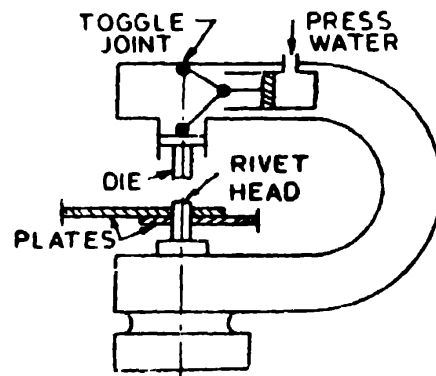


Fig 16.19 Hydraulic Riveter

**16.19 Hydraulic Jack**—This is based on the same principle as the hydraulic press. It is short-stroke hydraulic lift which is fed from hand pump. The hydraulic jack may be portable. This is extensively used for lifting automobiles usually to facilitate cleaning and repair.

**16.20 Hydraulic Riveter** (refer Fig 16.19)—It was very popular for making rivet heads but has lately been replaced by pneumatic riveters. A hydraulic riveter can exert a thrust of 50 tonne on the rivet.

At the time of closing, the cross-sectional area of rivet decreases which means a large pressure will then be required. This is supplied by means of toggle joint.

**16.21 Pressure Accumulator**—It is a device to accumulate liquid under pressure delivered by the pump when it is not required by the machine. The pressure can be later supplied to the machine as and when needed.

The various presses enumerated above require separate pumping units to furnish liquid at the desired pressure. Such pumps are known as *Press Pumps*. Normally, the pressure generated by these pumps ranges from 50 to 150 kg/cm<sup>2</sup> and is uniform throughout the supply period. However, the demand for liquid and its required pressure are variable. At

some instants the machine may not be doing any work at all, and to deal with such difficulties an arrangement to receive and store the liquid, being constantly supplied by the pump, is necessary. The device should be able to deliver the liquid back to the machine when needed. In some cases it may be desirable to obtain the stored liquid at a pressure higher than that provided by the pump itself. All this is done by pressure accumulator or the hydraulic accumulator.

The hydraulic accumulator (refer Fig 16.20) consists of a cylinder and a plunger generally known as a ram. One side of the cylinder is connected to the press pump and other to the hydraulic machine. Either the cylinder or the ram may be fixed. Generally the cylinder is fixed and the ram moves up and down to accommodate a variable quantity of liquid inside by the cylinder.

Accumulators may be dead load type or variable load type. In the former, dead weights are employed to press the plunger in, while the latter employs steam pressure. Main advantage of this type is that the pressure may be varied at will but, it is handicapped by the need of a boiler to supply steam. It can be used, however, on ships if steam is readily available.

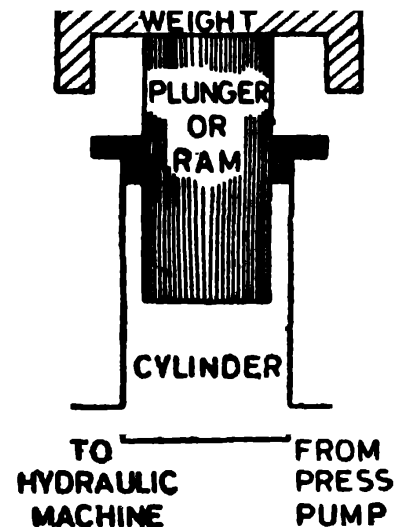


Fig 16.20 Hydraulic Accumulator

The accumulator also serves the purpose of a pressure regulator. A suitable arrangement can be easily made to switch off and start the motor after a certain travel of the ram.

**Capacity of Accumulator** is the maximum amount of energy which is stored by it. The storage capacity is equal to the potential energy of the lifted ram together with weight.

Let  $d$  = the diameter of ram ;  
 $s$  = the stroke or lift of ram ;  
 $p$  = the intensity of pressure of water supplied.

Total moving weight or weight of ram  $W = \frac{\pi}{4} d^2 \cdot p$

Work done in lifting ram or capacity of accumulator

$$= W \cdot s = \frac{\pi}{4} d^2 \cdot p \cdot s \quad \dots(16.13)$$

Volume of accumulator  $= \frac{\pi}{4} d^2 \cdot s$

$$\therefore \text{Capacity of ram} = p \times \text{volume} \quad \dots(16.13a)$$

Capacity of accumulator and that of ram are same.

**Problem 16.2** The weight of a 350 mm plunger of an accumulator is 4,500 kg. What additional weight is to be placed upon it to develop a hydraulic pressure of 42 kg/cm<sup>2</sup> ?

**Solution**

$$d = 350 \text{ mm}, \quad p = 42 \text{ kg/cm}^2, \quad w_1 = 4,500 \text{ kg}$$

$$\text{Cross-sectional area of plunger } a = \frac{\pi}{4} \times (0.350)^2 = 0.096 \text{ m}^2$$

Total weight to balance the hydraulic pressure

$$W = w_1 + w_2$$

where

$w_2$  = weight to be added

Now

$$W = p \cdot a$$

$$= 42 \times 0.096 \times 10^4 = 40,350 \text{ kg}$$

$\therefore$

$$w_2 = W - w_1 = 40,350 - 4,500$$

$$= 35,850 \text{ Answer}$$

**Problem 16.3** Find the length of stroke required for an accumulator having a displacement of 110 litres. The diameter of the plunger is 350 mm.

**Solution**

Displacement of accumulator is the volume displaced by the plunger per stroke

$$\text{Displacement} = 110 \text{ litres} = \frac{110}{1,000} = 0.11 \text{ m}^3$$

$$\text{Cross-sectional area of plunger} = \frac{\pi}{4} \times (0.35)^2 = 0.096 \text{ m}^2$$

$$\therefore \text{Length of stroke} = \frac{\text{displacement}}{\text{area of plunger}} = \frac{0.11}{0.096} = 1.145 \text{ m Answer}$$

**Problem 16.4** An accumulator has a ram 300 mm in diameter, an effective stroke of 6 m and is loaded with a total weight of 50 tonne. If the friction of ram amounts to 3% of the total load, find the total horse-power delivered to the hydraulic machine if the ram falls steadily through its full stroke in 2 minutes, while at the same time the pump delivers 7.5 lit/sec.

(Delhi University)

**Solution**

$$d = 30 \text{ cm} = 0.3 \text{ m}$$

$$S = 6 \text{ m}$$

Load

$$W = 50 \times 1,000 \text{ kg}$$

$$\text{Friction loss} = 3\%$$

$$\therefore \text{Net load} = 50 \times 1,000 \times 0.97 = 48,500 \text{ kg}$$

Time of ram fall = 2 min

$$Q_{\text{pump}} = 7.5 \text{ lit/sec} = 0.0075 \text{ m}^3/\text{sec}$$

Work is supplied to hydraulic machine by the pump which is working continuously as well as by the accumulator ram which is falling steadily through its weight.

(i) Work supplied by accumulator ram per minute

$$= 50 \times 1,000 \times 0.97 \times \frac{6}{2} = 145,500 \text{ m}\cdot\text{kg}/\text{min}$$

(ii) Intensity of pressure in accumulator

$$p = \frac{W \times 0.97}{\frac{\pi}{4} \times d^2} = \frac{50 \times 1,000 \times 0.97}{\frac{\pi}{4} \times (0.3)^2} = 687,000 \text{ kg/m}^2$$

Head  $H$  due to this pressure

$$= \frac{p}{\gamma} = \frac{687,000}{1,000} = 687 \text{ m of water}$$

Work supplied by the pump per minute

$$= (\gamma \cdot Q) H = (1,000 \times 0.0075 \times 60) \times 687 \\ = 309,000 \text{ m}\cdot\text{kg}/\text{min}$$

$\therefore$  Total work supplied to hydraulic machine

$$= \text{work supplied by ram} + \text{work supplied by pump} \\ = 145,500 + 309,000 = 454,500 \text{ m}\cdot\text{kg}/\text{min}$$

or Total HP supplied to hydraulic machine

$$= \frac{\text{Work done}/\text{min}}{75 \times 60} = \frac{454,500}{75 \times 60} = 101 \text{ HP Answer}$$

**Problem 16.5** In an installation of 6 hydraulic cranes, the working cycle of each of which takes 90 seconds in hoisting and lowering, each crane is fed with water at a pressure of 50 kg per sq cm and is required to lift a load of 5 tonne at a speed of 18 metres per minute, through a total height of 12 metres the jigger system giving a velocity ratio of 6. Estimate the stroke and diameter of the rams assuming an efficiency of 60%.

It is assumed that all the six cranes are making the working stroke at the same time. Calculate the minimum capacity of the pump feeding the installation and that of the accumulator.

(Madras University)

**Solution**

No. of cranes = 6

$t = 90$  seconds

$p = 50 \text{ kg}/\text{cm}^2$

$W = 5 \text{ tonne} = 5,000 \text{ kg}$

$v = 18 \text{ m}/\text{min}$

$h = 12 \text{ metres}$

velocity ratio = 6

$\eta = 60\%$

Load on each ram  $\times$  velocity of ram  $\times \eta$

= Load lifted  $\times$  velocity of lifting load

$\therefore$  Load on each ram =  $\frac{W \times \text{vel. ratio}}{\eta}$

$$= \frac{5,000 \times 6}{0.6} = 50,000 \text{ kg}$$

Area of ram

$$= \frac{\text{Load on ram}}{\text{pressure } p} = \frac{50,000}{50} = 1,000 \text{ cm}^2$$

$\therefore$  Dia of ram

$$= \sqrt{\frac{1,000}{\frac{\pi}{4}}} = 35.63 \text{ cm Answer}$$

Stroke of ram

$$= \frac{\text{Distance } h}{\text{velocity ratio}} = \frac{12}{6} = 2 \text{ m Answer}$$

Volume swept by ram

$$= \text{area of ram} \times \text{stroke} \\ = 1,000 \times (2 \times 100) \\ = 2 \times 10^6 \text{ cm}^3$$

Since there are 6 cranes, volume of water supplied by the pump

$$= 2 \times 10^5 \times 6 \text{ cm}^3$$

$$= 12 \times 10^5 \text{ cm}^3$$

∴ Minimum capacity of feeding pump

$$= \frac{\text{volume of water}}{\text{time taken}} = \frac{12 \times 10^5}{90} \times \frac{1}{1,000}$$

$$= 13.33 \text{ lit/sec}$$

$$\text{Operating time of cranes} = \frac{h}{v} = \frac{12}{18} \times 60 = 40 \text{ seconds}$$

$$\therefore \text{Idle time of crane} = 90 - 40 = 50 \text{ seconds}$$

But the pumps go on feeding the accumulator

$$\therefore \text{Accumulator Volume} = 13.33 \times 50$$

$$= 666.5 \text{ litres} \quad \text{Answer}$$

$$\text{Accumulator Capacity} = p \times \text{Volume}$$

$$= 50 \times (666.5 \times 10^3) \text{ kg-cm}$$

$$\text{or} \quad \frac{50 \times 666.5 \times 10^3}{100} = 3.33 \times 10^5 \text{ kg-m} \quad \text{Answer}$$

**Problem 16.6** An accumulator maintains a pressure of  $60 \text{ kg/cm}^2$  in a  $50 \text{ mm}$  diameter hydraulic main. A hydraulic crane situated at a distance of  $250 \text{ m}$  from the accumulator is supplied with pressure water from this main. The ram of the hydraulic crane is  $220 \text{ mm}$  diameter. Velocity ratio of crane hook to ram is  $4 : 1$ . A pressure of  $2.80 \text{ kg/cm}^2$  may be assumed on the ram to account for mechanical friction of ram, pulleys etc.

Assume a co-efficient of friction for the hydraulic main as  $0.01$ . Calculate the load lifted when it is raised with a speed of  $60 \text{ cm/sec}$ .

(Poona University)

### Solution

$$\text{Diameter of hydraulic main} = 50 \text{ mm}$$

$$\text{Diameter of ram} = 220 \text{ mm}$$

$$\text{Pressure in the accumulator} = 60 \text{ kg/cm}^2$$

$$\text{Pressure lost due to mechanical friction} = 2.80 \text{ kg/cm}^2$$

$$\text{Length of hydraulic main} = 250 \text{ m}$$

$$\text{Velocity ratio of crane hook to ram} = 4 : 1$$

$$\text{Co-efficient of friction for hydraulic main} = 0.01$$

$$\text{Velocity of crane hook} = 0.6 \text{ m/sec}$$

$$\text{Velocity ram} = \frac{0.6}{4} = 0.15 \text{ m/sec}$$

Let  $v$  = velocity of water in  $50 \text{ mm}$  main.

As quantity of water per second flowing through main equals quantity per second in the ram cylinder, then

$$v \times \frac{\pi}{4} \times 0.05^2 = 0.15 \times \frac{\pi}{4} \times 0.220^2$$

or

$$v = 2.9 \text{ m/sec}$$

$$\text{Head of water in accumulator} = \frac{60 \times 100 \times 100}{1,000} = 600 \text{ m of water}$$

$$\text{Head lost in friction in main } h_f = \frac{4 \times 0.01 \times 250}{0.05} \times \frac{2.9^3}{2 \times 9.81} \\ = 85.6 \text{ m of water}$$

Head lost due to mechanical friction of ram, pulley etc.

$$= \frac{2.80 \times 100 \times 100}{1,000} = 28 \text{ m of water}$$

$$\text{Net head available on the ram} = 600 - (85.6 + 28) \\ = 486.4 \text{ m of water}$$

$$\therefore \text{Net intensity of pressure on ram} = \frac{486.4 \times 1,000}{100 \times 10^6} \\ = 48.64 \text{ kg/cm}^2$$

$$\text{Load on the ram} = 48.64 \times \frac{\pi}{4} \times (22)^2 \times \frac{1}{1,000} = 18.4 \text{ tonne}$$

$$\therefore \text{Load lifted by the crane hook} = \frac{18.4}{4} = 4.6 \text{ tonne Answer}$$

**16.22 Pressure Intensifier**—Pressure intensifier, sometimes known as differential accumulator, is a device to multiply the pressure supplied by

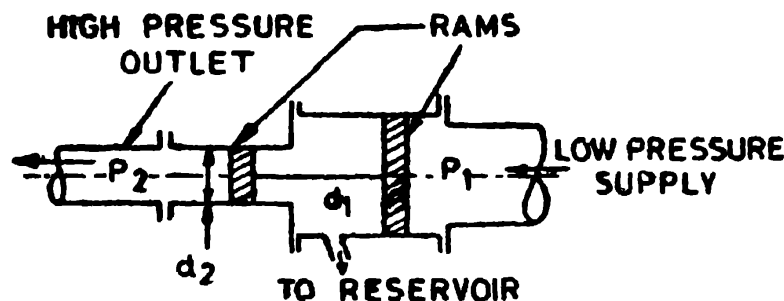


Fig 16.21 Pressure Intensifier

the pump to suit the requirements of a high-pressure machine. Often a fluid pressure machine requires a high pressure at a particular stage in its operation. It can be easily provided by the intensifier.

Normally, a simple intensifier (refer Fig 16.21) consists of two coaxial rams or pistons moving in cylinders as shown in Fig 16.21. Low pressure liquid is admitted to ram or piston of large cross-sectional area which then transmits force to a small ram or piston by a rod connecting the two rams or pistons. The piston on left hand side being of smaller cross-sectional area than the piston on the right hand side, the intensity of pressure of the liquid coming out will be high. The volume between the two pistons must be vented.

Let  $d_1$  and  $d_2$  be the diameters of the two rams and  $p_1$  and  $p_2$  the respective pressure of the liquid inside them. Then, if the ram moves slowly,

Forces

$$P_1 = P_2$$



$$\text{or} \quad p_1 \cdot d_1^2 = p_2 \cdot \frac{\pi}{4} \cdot d_2^2$$

$$\text{or} \quad p_2 = p_1 \cdot \frac{d_1^2}{d_2^2} \quad (16.14)$$

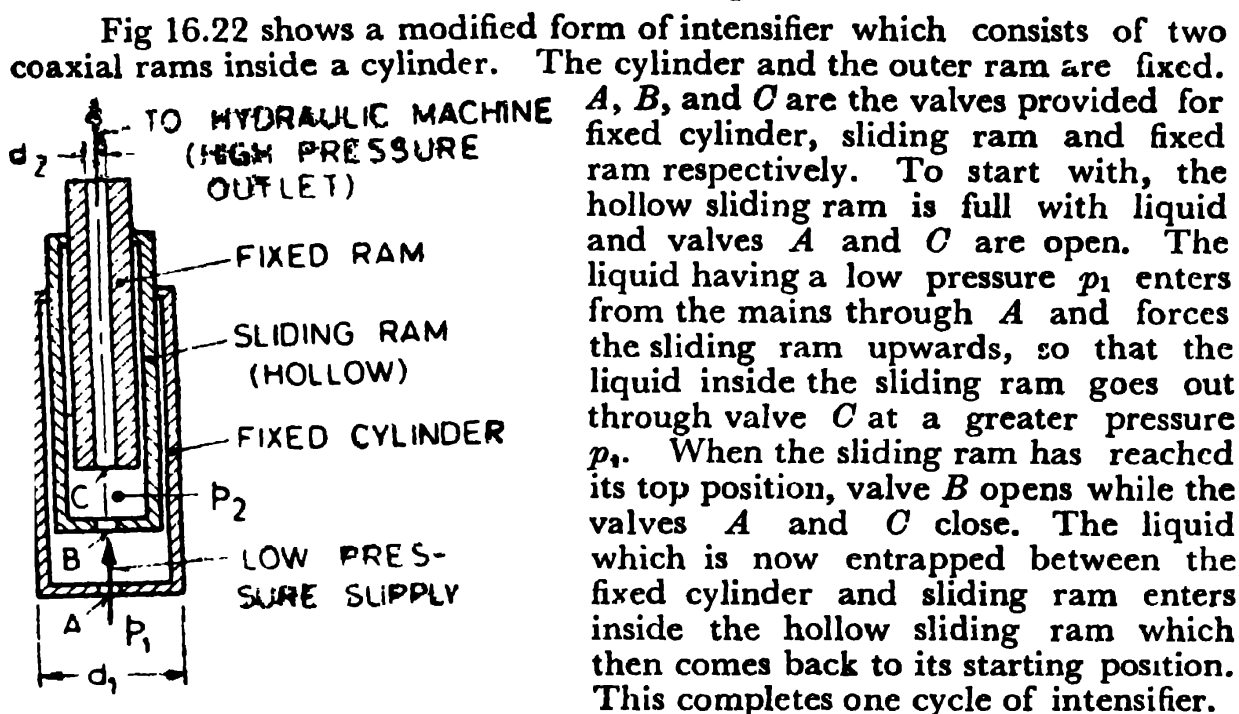


Fig 16.22 Modified Form of Intensifier

Sometimes, compressed air is supplied to the larger cylinder in place of low pressure hydraulic supply, in which case the intensifier is known as a **Hydro-Pneumatic Accumulator or Intensifier**.

Steam can also be supplied to the larger cylinder in place of low pressure hydraulic supply or compressed air. In such a case it is called **Steam Intensifier**.

**Problem 16.7** The diameters of small and large plungers of a pressure intensifier are 10 cm and 30 cm respectively. Find the pressure in the small cylinder, if the pressure in the large cylinder is 40 kg/cm<sup>2</sup>.

**Solution**

$$d_1 = 30 \text{ cm}, \quad d_2 = 10 \text{ cm}, \quad p_1 = 40 \text{ kg/cm}^2$$

$$\therefore \text{ Pressure in the small cylinder}$$

$$p_2 = p_1 \cdot \frac{d_1^2}{d_2^2} = 40 \times \left( \frac{30}{10} \right)^2 = 360 \text{ kg/cm}^2 \quad \text{Answer}$$

**16.23 Fluid Drives for Machine Tools** – Fluid pressure is widely employed for obtaining different types of movements of modern machine tools. Pressurised oil is supplied by one or more independent pumps. Rapid progress has been made in this field in recent years. The main advantages of fluid drive in machine tools are :

(i) Speed can be varied within wide limits, uniformly and gradually. With the conventional equipment speed variation in steps was unavoidable.

(ii) Mechanism is more durable. Life of machine tool using hydraulic drive is about 50% more than with mechanical drive.

(iii) Quick and shockless reversal of motion is made feasible.

(iv) Economy in power consumption.

(v) Fluid system is quiet and noiseless.

(vi) Simplicity and flexibility of design.

The movements which are obtainable by oil pressure are the following :

- (a) Straight line movement—tables and slides,
- (b) Rotary movement—spindles,
- (c) Auxiliary movement,
- (d) Straight line driver are employed in :
  - (i) Broaching machines in horizontal or vertical positions where draw head slide is travelled at the required cutting speed.
  - (ii) Shapping machines-- to move the tool.
  - (iii) Planing machines-- to move the tables.
  - (iv) Grinding wheel --to move horizontal or vertical spindle.
  - (v) Drilling, boring and reaming machines—to impart power and speed to spindle heads.
  - (vi) Milling machine—to move the tables.
  - (vii) Lathe—for sliding and surfacing motion.

(b) Rotary drive is mainly employed for spindles of lathes, cutter spindles and milling machines, rotary movement of honing machine spindle and for rotary tables on surface grinding machines. The drive however is not so common as straight line drive.

(c) Auxiliary movement :

- (i) Clamping the work in chuck, etc.
- (ii) Motion of tail-stock centre in lathes,
- (iii) Tool slide advancement in lathes.

Some of the simple hydraulic circuits employed in machine tool operation are given in the following articles.

**16.24 Hydraulic Shaper**—The hydraulic circuit for the operation of a shaper is shown in Fig 16 23. It consists of a variable delivery pump for pressure generation. If in place of a variable delivery pump, a constant delivery pump is used, a control valve is to be fitted in the circuit to obtain the required speeds. For low pressure requirements upon 4 to 5 HP, a constant delivery pump is generally economical and a variable delivery pump is recommended above 10 to 12 HP. For horse powers between 5 to 10, any of the two types can be used. A relief valve is provided in the circuit to take care of the pump and the system against possible overloads.

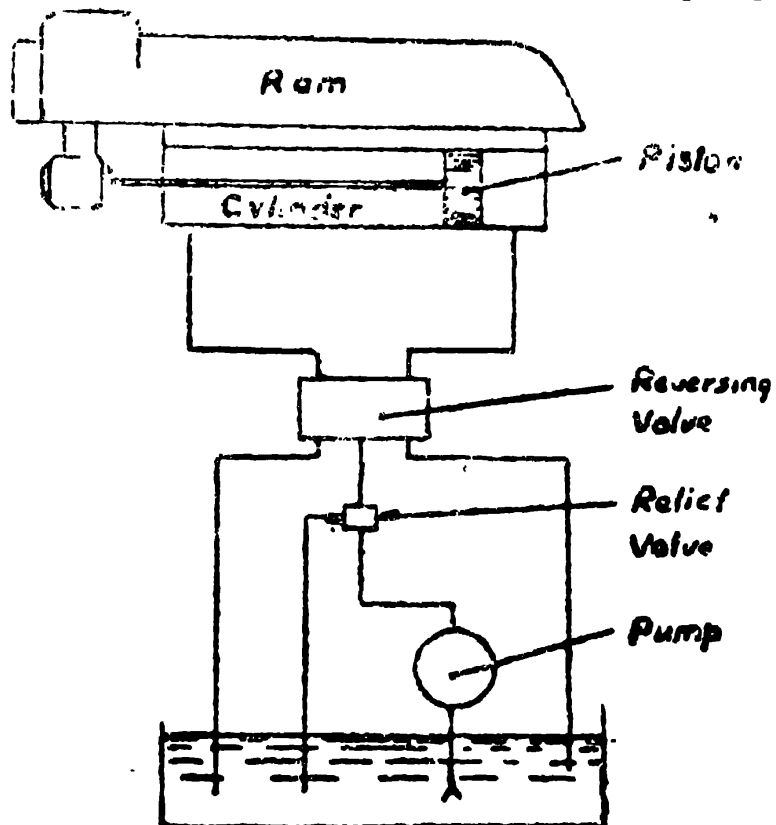


Fig 16-23 Hydraulic Shaper Circuit

**16.25 Surface Grinder**—The hydraulic circuit for the operation of

the table of Albwerk surface grinder is shown in Fig. 16.24. The gear pump supplying oil through valve *B* to double piston valve *C*. The latter distributes oil to cylinder through pipes *A* and *B*. *G* is the throttle valve which controls the table speed. Part of the oil goes through the pipe *H* to control valve which is operated either by hand lever or automatically.

It controls the auxiliary piston *D* which is coupled with piston valve *C*. The piston *D* controls the automatic transverse motion of the cross slide through the rack and pinion arrangement. Throttle valve is adjusted to provide shockless reversal of the table by regulating the flow of oil expelled during the movement of piston *D*, and consequently the speed at which the piston moves. Excess oil is by-passed through a relief valve.

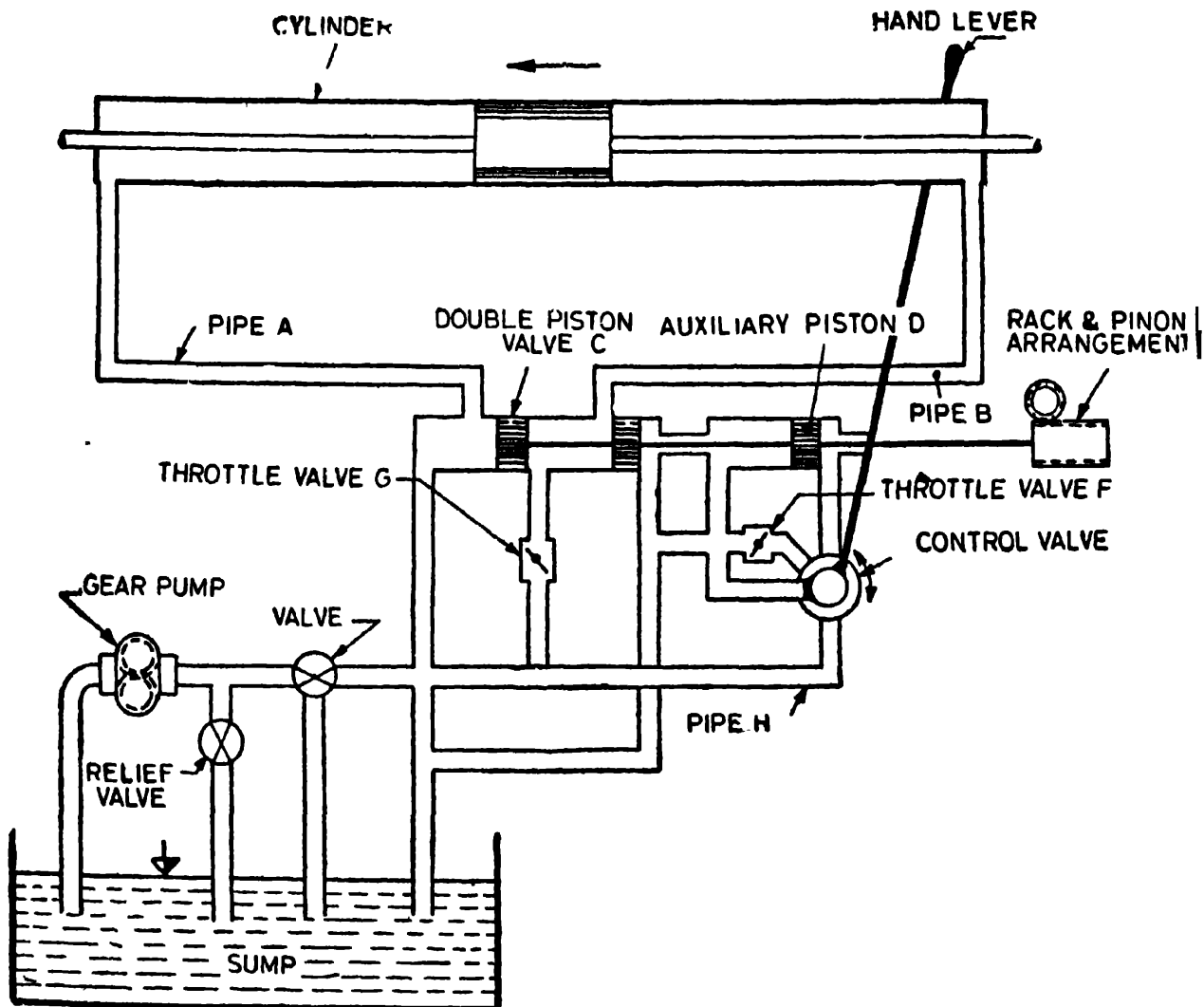


Fig. 16.24 Circuit for Albwerk Surface Grinder

**16.26 Milling Machine**—Fig 16.25 shows the hydraulic circuit of a hydraulic Cincinnati make milling machine. The variable delivery pump is placed between the forward and back pressure lines. By setting this pump, feed water is controlled. A booster pump is connected to the forward pressure line to make up for leakage losses, and to maintain pressure on both forward and back pressure lines. The differential relief valve determines the relationship between the forward and backward pressures. Sum of forward and backward pressures remains constant and a variation of one is compensated for by the other. A large capacity pump is meant for rapid traversing in either direction. The selector valve controls the direction and rate of travel and it is adjusted by a pilot valve.

Finally, there is a start-stop by-pass valve from where two pipe lines are connected to the cylinder having a piston moving to and fro inside.

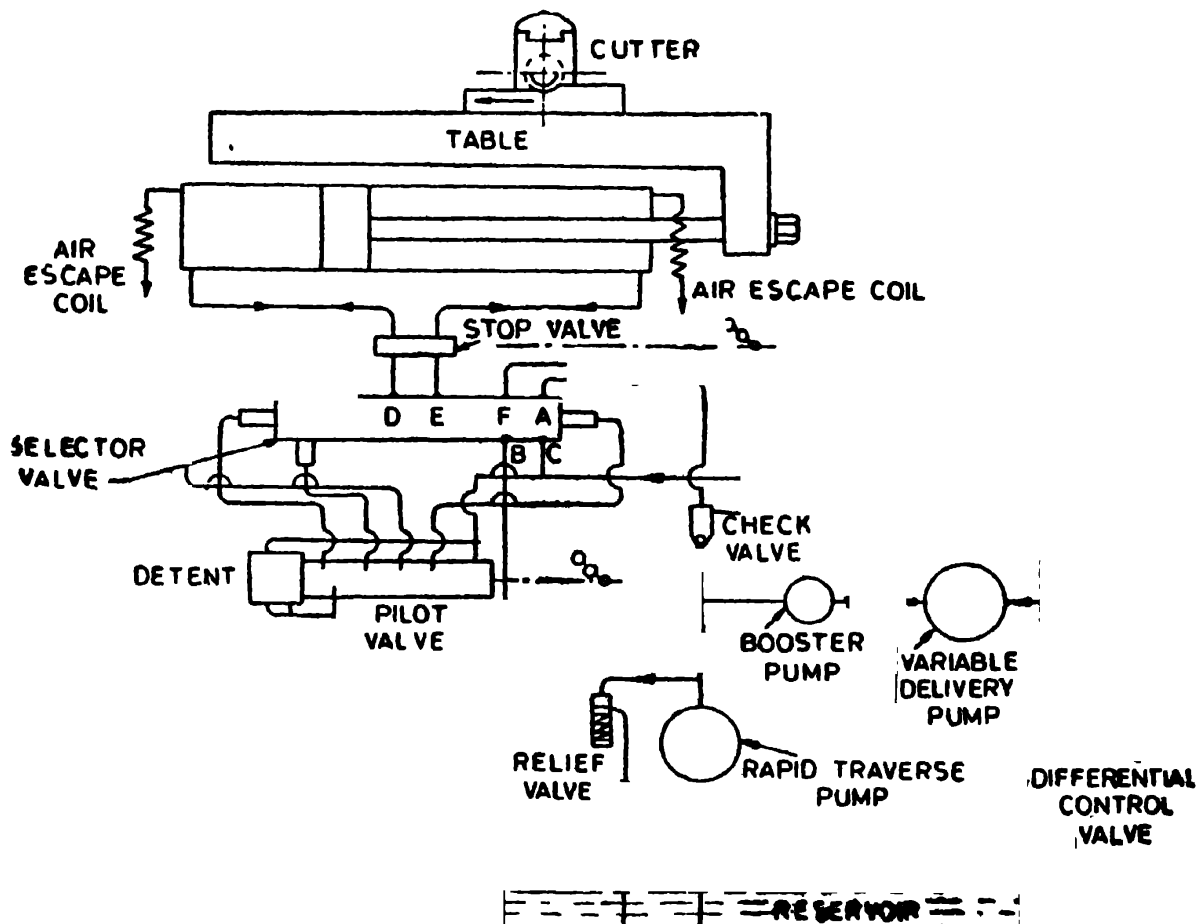


Fig 16.25 Locked Hydraulic Circuit of Hydromatic Milling Machine (Cincinnati)

**16.27 Tail Stock**—Fig 16.26 shows the hydraulic tail stock operation employed on Drummond make lathes. The centre and barrel are moved towards and away from the job by means of double acting piston  $P_1$ . Rack teeth are cut on the piston  $P_1$  as well as the barrel and a common pinion works both of them. Either of the two ends of the cylinder  $C_1$  are connected to the pressure supply pipe by a rotary reversing valve operated by hand lever  $L$  through bevel gearing. The opposite end is connected to exhaust. When the barrel is moved to the left in order to engage the centre with the job, momentarily, pressure is built up in pipe  $W$ . When a predetermined pressure is reached, the spring loaded valve  $V$  is opened and pressure

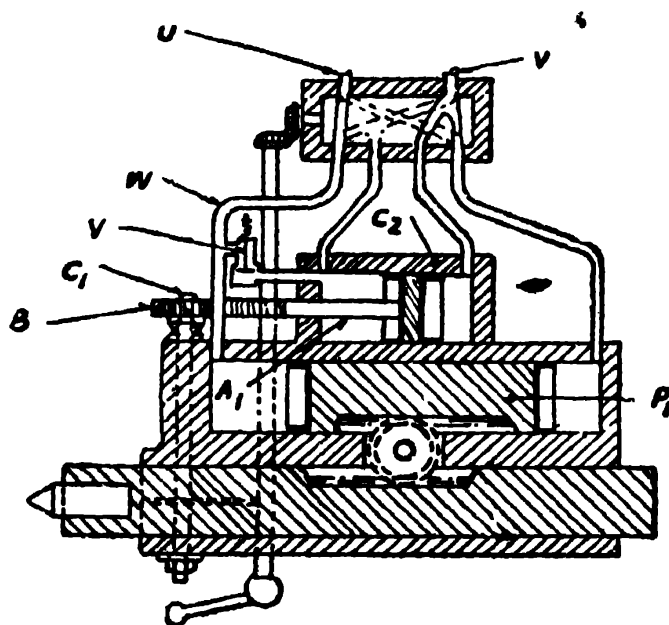


Fig 16.26 Diagrammatic section of Tailstock with hydraulically Operated Centre

is admitted in cylinder  $C_2$  causing the piston  $P_2$  (in cylinder  $C_2$ ) to move towards right. The rack teeth are cut on the piston rod  $A_1$  which meshes with pinion  $B$ . The pinion  $B$  serves as a nut to the clamping screw  $C_1$  and the rotation of pinion clamps or releases the tail stock barrel according to the direction of movement of the piston  $P_2$  (in cylinder  $C_2$ ).

**16.28 Some More Applications of Hydrostatic Transmission**—There are so many applications of hydrostatic transmission that it is difficult to present a general picture which will adequately cover them all but, perhaps the following machines are worth mentioning in addition to what has already been explained in the chapter.

(i) **Hydraulic Deck Machinery**—Hydraulic power is transmitted to deck machines through pipes by means of constant delivery pumps generally situated in the ship engine room, driven either by auxiliary diesel engines or by constant speed electric motors. The deck machines workable by hydraulic pressure are cargo winches, anchor windlasses, capstans and warping winches, deck cranes, mast winches and dredgers.

(ii) **Agricultural Machinery**—The hydraulic power transmission system in agricultural machinery is employed in tractor external fittings through which the required services are applied to the implements. Mostly the hydraulic applications are found in connection with bulldozers and scrapers. However, there is a great scope for future developments in the extension of hydraulic services to control the tractor or implement components. Another line of development is the transmission of power from the engine to the rear wheel of the tractor.

(iii) **Rod and Rail Traction**—Experiments were carried out in the past in connection with the use of hydraulic transmission of power for road and rail vehicles which achieved a fair degree of technical success, but their use was not made properly. Recently, there has been a revival of interest in the applications of past experimental results.

(iv) **Lifting Equipment**—Though hydraulic driven cranes and lifts have been replaced by electric ones, yet modern types of transmission are used in a large number of comparatively novel lifting and tilting applications such as hydrostatic lifting and tilting gear for fork-lift trucks, tilting furnaces, X-ray tilting table etc. In electric cranes the hydraulic control is applied to give stable creeping speeds when required, by automatically applying brakes to the required extent. Winches of helicopters and flight refuelling gear are often worked by hydraulic motors. Operation of penstock sluices and bulk-head gates as some more applications of the same.

## HYDRO-KINETIC SYSTEMS

**16.29 Hydro-Kinetic Systems and its Applications**—The hydro-kinetic system is based upon changes in velocity of working fluid whereas the hydrostatic system is based upon differences of pressure. Hydro-kinetic system is used to transmit power. It is simply a combination of a centrifugal pump and a turbine. The former acts as driver and the latter as driven. The two distinct elements are built into a single unit with a closed hydraulic circuit. As there is no mechanical connection between the driver and the driven, impulsive shocks and periodic vibrations are prevented by using a fluid coupling.

The principle of hydro-kinetic system can be easily demonstrated by means of two ordinary electric fans which are set facing each other. If one of them is switched on, air currents will be generated moving the blades of the other which is not connected to electric mains.

Hydro-kinetic transmissions are of two types—hydraulic coupling and hydraulic torque converter. The coupling provides for power transmission with the same torque on driving and driven shaft, whereas the converter provides for transmission with torque multiplication and could be compared with reduction gearing. Both the coupling and the converter are outgrowths of the original Foettinger transformer of Vulcan Co (Hamburg, W. Germany), a torque converter developed before World War I in 1905 for marine use as a reduction gear between the steam turbine and propeller.

**16.30 Fluid or Hydraulic Coupling**—Fluid coupling consists of a radial pump impeller keyed to a driving shaft *A* (refer Fig. 16.27), and a reaction (radial type) turbine runner keyed to driven shaft *B*. There is no mechanical or rigid connection between the driving and driven shafts. The impeller and turbine runner together form a casing completely filled with oil as the fluid with which the shafts *A* and *B* are supposed to be coupled. The fluid is mostly ordinary mineral lubricating oil. Now if the shaft *A* is made to revolve slowly, the oil due to a forced vortex will flow out from the impeller and will strike the turbine runner blades. After sufficient head has been built up by increasing the speed of *A*, the fluid will drive the turbine runner and thus set the shaft *B* in motion. When at full speed, the two shafts are supposed to be rotating at the same rate, but in practice, owing to slip, the driven shaft speed is about 2% less. Thus, the efficiency of the coupling would be 98%. If both driver and follower rotate at the same speed, the circulation of oil cannot take place. It is the difference of centrifugal forces set up in the driver and the follower which causes oil circulation. The necessary reduction in speed of the driven thus maintains the continuous flow of oil from impeller to the turbine runner. The blades of impeller and runner are generally of straight radial type.

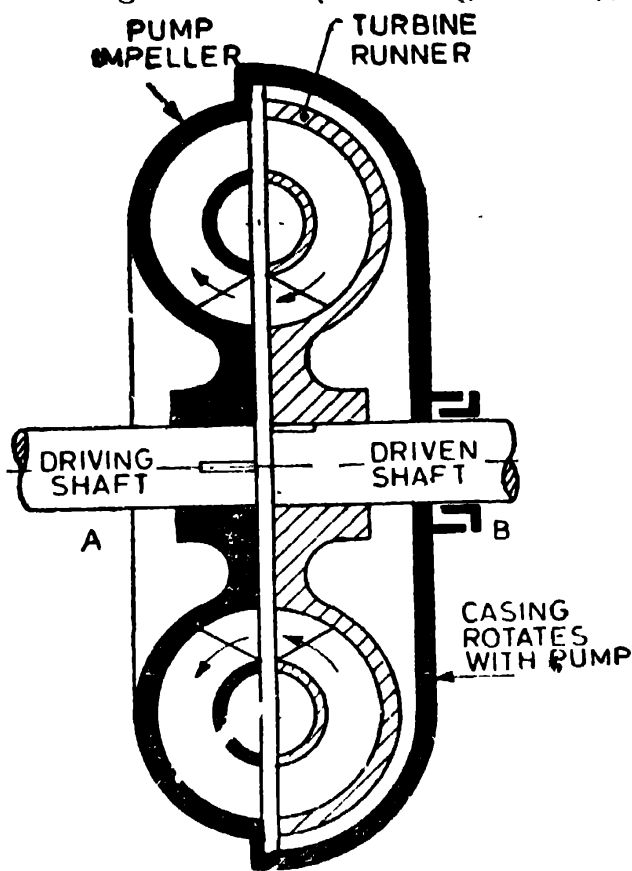


Fig 16.27 Fluid or Hydraulic Coupling

The necessary reduction in speed of the driven thus maintains the continuous flow of oil from impeller to the turbine runner. The blades of impeller and runner are generally of straight radial type.

**16.31. Theory of Hydraulic Coupling**—The actual torque  $T_1$  which is applied to the driving shaft is slightly greater than that applied by the pump to the circulating fluid due to viscous resistance offered by the rotor surfaces on the fluid. Similarly the torque  $T_2$  applied by the driven shaft to the load is slightly less than that exerted on the turbine by the fluid.

Neglecting friction,

$$T_1 = T_2$$

... (16.15)

Let the rotational speeds of driving and driven shafts be  $N_1$  and  $N_2$ , then the *efficiency* of the coupling is given by

$$\begin{aligned}\eta_{\text{coupling}} &= \frac{\text{Power Output}}{\text{Power input}} = \frac{T_2 \omega_2}{T_1 \omega_1} \\ &= \frac{T_2 N_2}{T_1 N_1} = \frac{N_2}{N_1} \quad \dots (16.16)\end{aligned}$$

Thus the efficiency of an ideal coupling is the ratio of the speeds of driven shaft to that of driving shaft. This is represented by the Fig 16.28.

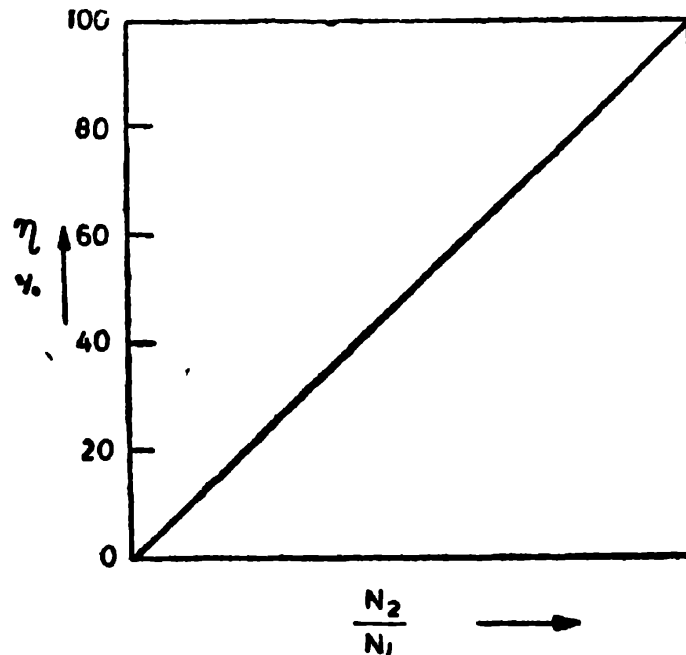


Fig 16.28 Efficiency of Ideal Coupling versus Ratio of Speeds of Driven to Driving Shafts

The *slip* of torque coupling is defined as the ratio of difference between the two speeds to the speed of the driving shaft.

$$\text{Thus} \quad S = \frac{N_1 - N_2}{N_1} \quad \dots (16.17)$$

$$\begin{aligned}\text{or} \quad S &= \frac{N_1}{N_1} \\ &= 1 - \eta_{\text{coupling}}\end{aligned}$$

$$\text{er} \quad \eta_{\text{coupling}} = 1 - S \quad \dots (\text{for ideal coupling}) \quad \dots (16.18)$$

The actual efficiency of the coupling is slightly less than that given by the Equation 16.18.

The slip for a fluid coupling varies between 2 to 4%

The *stall* is the condition when the speeds of the driving and driven shafts are same. For such a condition the slip is equal to unity. Then the efficiency is equal to zero. It shows the coupling will not work as stated in Art 16.30.

**16.32 Characteristics of Fluid Coupling**—Fig 16.29 is the characteristics diagram of a fluid coupling showing the variation of driving shaft torque input with the driving shaft speed for different values of slip.

These are parabolic curves. It shows that for a given slip the input torque increases with the cube of driving shaft speed. Thus the power transmitted will also vary with the cube of the speed for a given slip. In order to reduce the input torque, the slip must be of a smaller value.

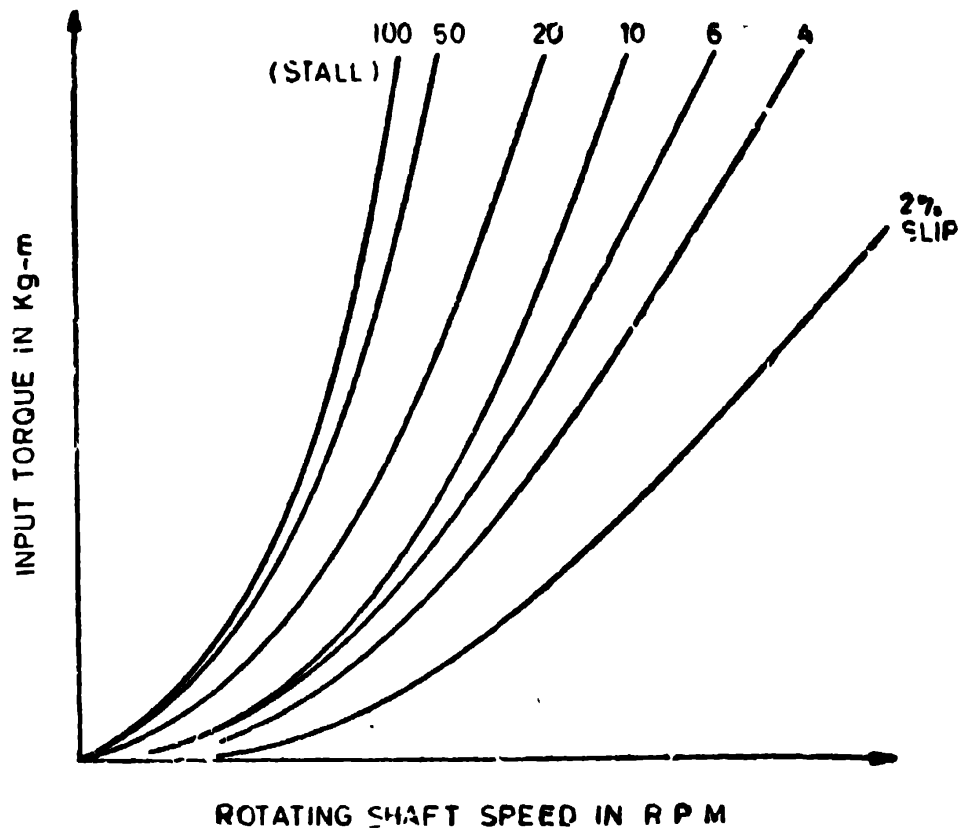


Fig. 16.29 Characteristics Diagram of Fluid Coupling

From the characteristics diagram it can also be seen that with the increase in slip, the torque will increase. If the slip increases, the efficiency of coupling will fall as indicated by Eqn 16.18.

When the help of dimensional analysis for the two geometrically similar fluid couplings, the following relation will hold good.

$$\phi \left[ \frac{T_1}{\rho N_1^3 D^5}, \frac{N_2}{N_1}, \frac{N_1 D^2}{\nu} \right] = 0 \quad \dots (16.19)$$

Where  $D$  is the diameter of impeller of pump and  $\nu$  is the kinematic viscosity of fluid used. The third term in Eqn 16.19 relates to Reynolds number ( $R_e$ ) and as the value of  $R_e$  is very high, its effect is negligible. The second term of Eqn 16.19 gives the efficiency of coupling (refer Eqn 16.16). The first term of Eqn 16.19 will be actual behaviour of the fluid coupling and is known as *torque coefficient*.

$$\text{Thus } \frac{T_1}{\rho N_1^3 D^5} = \psi \left( \frac{N_2}{N_1} \right) = \psi' (S) \quad \dots (16.20)$$

Where  $\psi$  and  $\psi'$  are the 'functions of'.

From this equation it is seen that for two geometrically similar couplings having the same speeds and slip, the power transmitted varies as fifth power of the diameter.

**16.33 Types of Fluid Coupling**—There are generally two types, fixed casing and rotatable casing coupling. The fixed casing type is shown



in Fig 16.30. In such a type the pump impeller and the turbine runner revolve in a stationary casing.

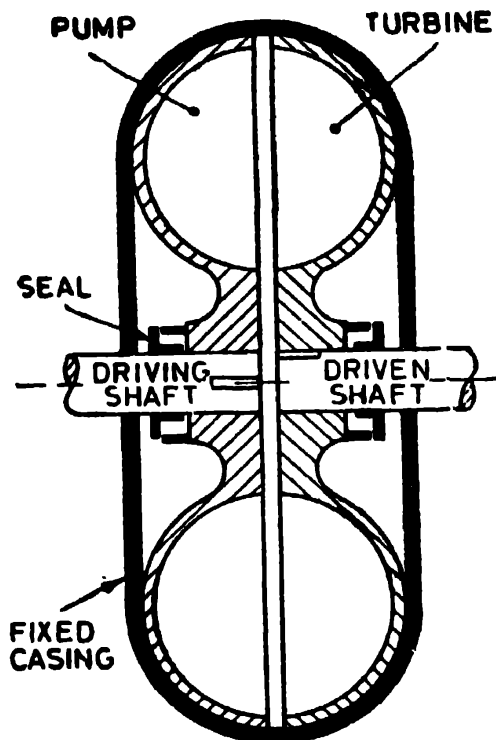


Fig 16.30 Fixed Casing Type Fluid Coupling

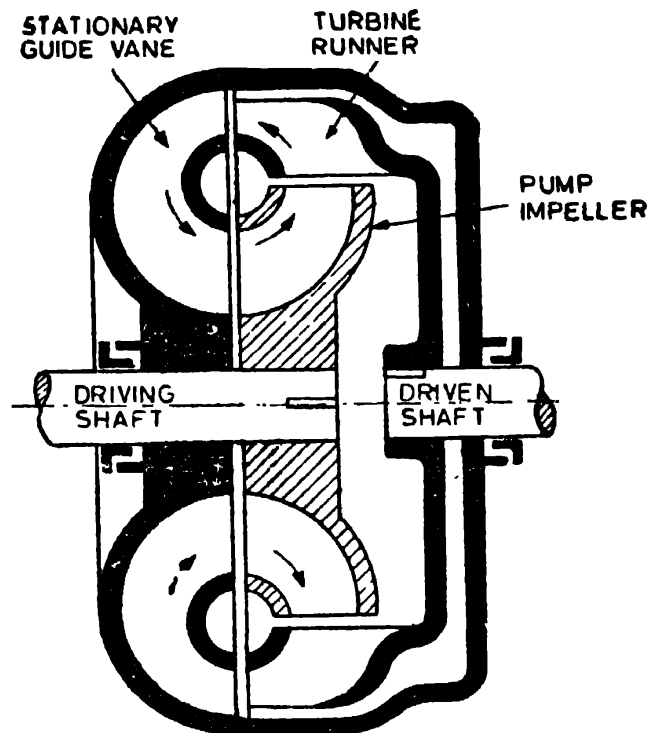


Fig 16.31 Fluid or Hydraulic Torque Converter

In rotatable casing type (refer Fig 16.27), the casing turns as a unit with the impeller. This type has the advantage over the former that the rotation of casing assists convective transfer of heat to the surrounding air, thus increasing its efficiency, as the loss of mechanical energy converted to heat is less.

**16.34 Uses of Fluid Couplings**—The fluid couplings are employed in rail road and automobiles to transmit power from the internal combustion engine to the moving wheel. Also, they are widely used for power driven excavations.

Fluid couplings having sizes from 1 to 36,000 HP have been manufactured. The larger couplings are built for the diesel engine driven warships.

**16.35 Fluid or Hydraulic Torque Converter**—Fluid torque converter differs from the fluid coupling in that the torque output of the latter is equal to the torque input, whereas the former is used to multiply or reduce the torque. It is accomplished by inserting a third member generally known as a reactionary member or stator (refer Fig 16.31) between the pump impeller and the turbine runner. The function of the reactionary member is to change the direction of the fluid. It consists of a series of fixed guide vanes. As the fluid flowing out of the impeller rim strikes the turbine runner and blades mounted on the stationary housing, it releases its energy in the form of torque and speed. As a result of the fluid reacting upon the turbine and stationary blades, the input or engine torque is increasingly multiplied, and the speed of the output falls. Thus the stationary blades by redirecting the fluid flow multiply the torque delivered by the engine and form a *torque converter*. Efficiency of a torque converter is very high at low speeds and may be as high as 85 to 87%. Fig 16.31 shows a fluid torque converter having a fixed speed ratio. However, with several stages of turbine runners and stationary vanes,

a variable speed ratio can be obtained. When a converter having a variable speed ratio is fitted to a road or rail vehicle driven by an internal combustion engine, it acts as a variable gear box. The required gear ratio is obtained by opening the throttle. This principle has also been applied in motor car drives.

### 16.36 Characteristics of Fluid Torque Converter—Fig 16.32

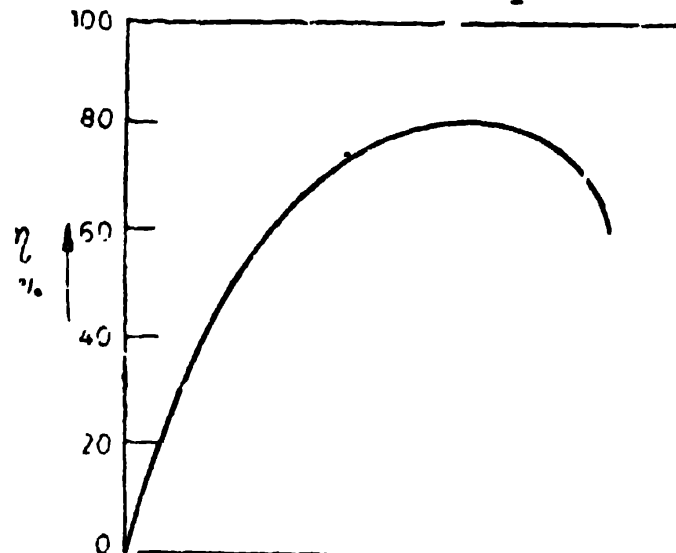


Fig 16.32 Efficiency of Fluid Torque Converter versus Ratio of Speeds of Driven to Driving Shaft

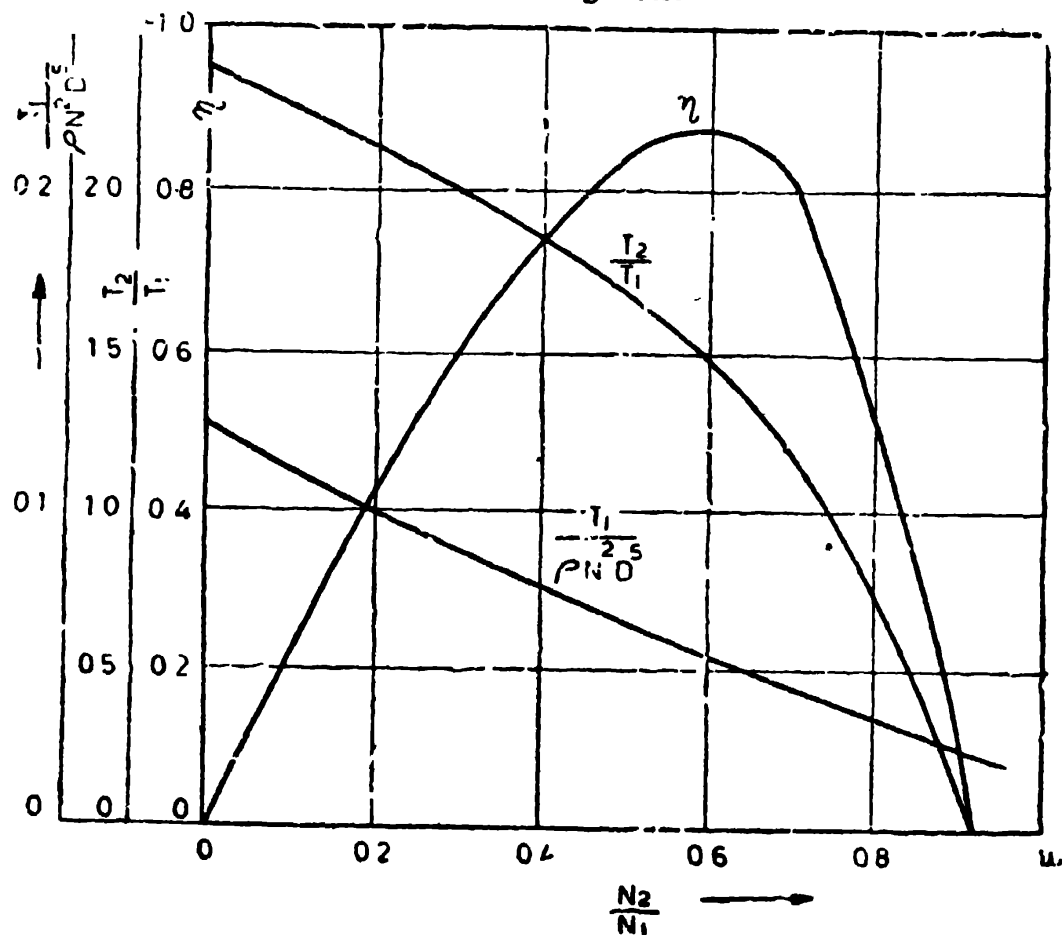


Fig 16.33 Characteristic Curves of Fluid Torque Converter

shows the variation of fluid torque converted efficiency with the ratio of speeds of driven shaft to that of driving shaft.

From the curve drawn, it is seen that the maximum efficiency occurs at about  $\frac{1}{2} \frac{N_2}{N_1}$ .

Comparing the efficiency of torque converter with that of fluid coupling (refer Fig 16.27). it will be seen that it is economical to use torque converter than the coupling for low transmission ratio  $\frac{N_2}{N_1}$ . For a high value of transmission ratio the efficiency of coupling is more as the former droops down whereas the latter represents a straight line rising graph. However for higher value of transmission ratio the converter is able to supply increased torque.

Fig 16.33 shows variation of input torque coefficient  $\frac{T_1}{\rho N^2 D^5}$ , torque ratio  $\frac{T_2}{T_1}$  and converter efficiency  $\eta$  with ratio of speeds of driven to driving shafts.

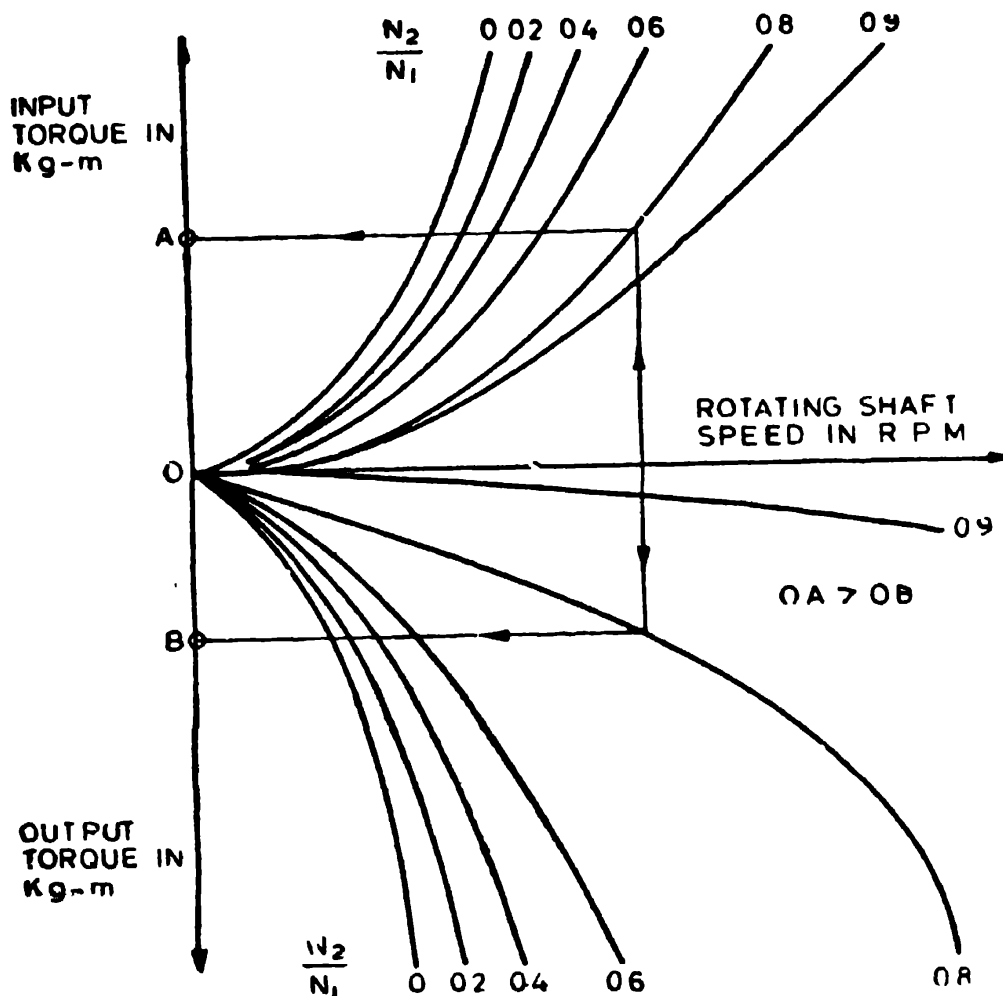


Fig 16.34 Characteristics Diagram of Fluid Torque Converter

Fig 16.34 shows the characteristics diagram similar to one (refer Fig 16.29) drawn for fluid coupling. It will be seen that the torque of the driven shaft is less than the torque of the driving i.e., shown in the diagram.

**16.37 Uses of Fluid Torque Converter**—The fluid torque converter is used as a variable gear box for the automobile and locomotive drives. It provides for smooth starting and absorbs torsional vibrations and shocks.

In bulldozers and scrapers, it is required to run the engine at full torque output while the vehicle is stalled against an obstacle. This is possible by the use of hydraulic torque converter.

**16.38 Fluid Transmission of Power in Automobiles**—With the advent of fluid couplings and torque converters, the traditional system of power transmission by means of clutch, gear box and rear axle has been dispensed with in modern cars. Now-a days a pump or a series of small pumps enclosed in a casing attached to the engine have replaced the above mentioned gear drive. Engine drives the pumps which transmit high pressure oil to the rear wheels where it is used to cause rotation on the turbine principle. To obtain neutral gear, oil may be suitably by-passed without driving the rearwheels. Different gear ratios are achieved with the aid of two sets of fluid drives, one on each rear-axle shaft and three rotary sliding vane pumps of different capacities. To obtain different speeds and power, different combinations of three pumps and two drives may be used.

Modern automatic systems employ fluid torque converter in conjunction with three or four speed gear boxes. Mechanical pedal clutch may be dispensed with. Automatic control is effected by accelerator and foot brake pedal.

**Fluid Brakes in Automobiles**—Liquid medium is used to transmit and multiply the force applied for braking the wheels. The essential parts of the brake-system are the following :

- (i) Master cylinder with reserve tank,
- (ii) Brake or wheel cylinder,
- (iii) Brake-pedal link mechanism, and
- (iv) Piping for liquid transmission.

When the brake pedal is depressed, it moves the piston inside the master cylinder and displaces liquid to the four wheel cylinders which brake their respective wheels. As the pressure on foot pedal is released, brake shoes return to their original positions by means of releasing springs.

### UNSOLVED PROBLEMS

- 16.1 What are hydraulic systems? Describe briefly a hydrostatic and hydro-kinetic system and cite examples.
- 16.2 What is the difference between a constant delivery and a variable delivery pump? Describe one from each type.
- 16.3 What are rotary pumps and how are they classified?
- 16.4 What is a gear pump and state the difference between an internal and an external type of such pump.

- 16.5 Describe with sketches the working of  
 (a) Hydraulic Jack (c) Hydraulic Crane  
 (b) Hydraulic Press (d) Hydraulic Lift  
*(Madras University)*
- 16.6 Describe with the aid of neat sketches the working of a hydraulic intensifier.  
*(Jadavpur University)*
- 16.7 Write brief notes on—  
 (a) Intensifier (b) Hydraulic Accumulator  
 (c) Application of fluid pressure in machine tools.
- 16.8 What are the advantages of hydraulic drive in machine tools? Describe one such drive.
- 16.9 What is the difference between a hydraulic coupling and a hydraulic torque converter? Where are they used in practice?
- 16.10 Show that the efficiency of an ideal fluid coupling is the ratio of speeds of driven shaft to that of the driving shaft.
- 16.11 Define slip and stall of fluid coupling. Find the efficiency of the coupling in terms of slip.
- 16.12 State and draw the characteristics of fluid coupling and torque converter.

### Numericals

- 16.13 A gearwheel type of positive rotary pump has two identical rotors or gearwheels each 16 cm long with a pitch diameter of 8 cm. When running at a speed of 500 rpm, it delivers 200 litres of oil per minute against a total head of 38 m.

Draw graphs, approximately to scale, showing how this pump would behave as follows : (i) between speed in rpm and discharge in lit/sec, for the range 300 to 800 rpm against a constant head of 38 m (ii) between discharge in lit/sec and total head in metres, when running at a steady speed of 500 rpm.

Then estimate the size of a pump geometrically similar to the specified pump, such that it would deliver 650 litres of oil per minute against a head of 15 m when running at 360 rpm. The rotor pitch diameter and length should be stated.

*[AMI Mech E (Lond)]*

- 16.14 The ram for a hydraulic crane has diameter of 15 cm and the ratio between the movement of the load and ram is 6 to 1. Water is supplied through a 40 mm diameter pipe having a length of 4.0 m, the pressure at the inlet end of the pipe being 80 kg/cm<sup>2</sup>. The co-efficient of friction for the pipe is 0.01. A pressure of 6 kg/cm<sup>2</sup> on the ram is required to overcome mechanical losses.

Determine (a) the maximum speed with which a load of 1,000 kg can be lifted and (b) the load and speed of lifting which correspond to the maximum power obtained from the crane.

(1.722 m/sec ; 1,362 kg ; 1.410 m/sec) *(London University)*

- 16.15 An intensifier has a ram diameter of 150 mm and sliding cylinder piston diameter of 750 mm. Calculate the pressure of water on

the low pressure side of the intensifier if the pressure of water on the high pressure side is to be  $210 \text{ kg/cm}^2$ . The loss due to friction at each of the packings of the intensifier is 5% of the total pressure on each piston.  $(9.25 \text{ kg/cm}^2)$  (*Madras University*)

- 16.16 Describe the working of an accumulator. The ram of an accumulator is loaded with 150 tonnes. It supplies power through a 100 mm diameter pipe, 1,500 m long. The loss in the pipe is estimated to be 2% of the power supplied by the accumulator. The co-efficient of friction in the pipe is 0.01. The diameter of the ram, working frictionless is 0.75 m. What is the maximum horse power it can supply?  $(16.56 \text{ HP})$  (*Poona University*)
- 16.17 230 HP is to be conveyed to several hydraulic machines at a distance of 3 km from the accumulator by means of pipes 150 mm diameter. The water pressure in the accumulator is  $50 \text{ kg/cm}^2$ . Find the least number of pipes if the efficiency of transmission is not to be less than 92%. Take  $f = 0.007$  for the pipes. (2) (*Aligarh University*)
- 16.18 Calculate the displacement of the ram of a hydraulic crane, whose efficiency is 50% in order that with a water pressure of  $50 \text{ kg/cm}^2$  it may raise a load of 25 tonne. Find the diameter of the ram, if the stroke is 6 times the diameter.  $(0.2142 \text{ m}^3 ; 357 \text{ mm})$  (*Annamalai University*)
- 16.19 (a) Describe with sketches any type of hydraulic accumulator or intensifier.
- (b) An accumulator has a ram 0.3 m diameter and a stroke of 5 m. It is loaded with 80 tonne but friction of the gland consumes 1.5% of this load. If the ram falls steadily in 2 minutes while the pumps are delivering 15 lit/sec, what HP is being delivered to the machinery by the hydraulic system?  $(267 \text{ HP})$  (*Bombay University*)
- 16.20 A hydraulic lift raises a load of 8.75 tonne at a speed of 0.5 m/sec through a height of 13.5 m once in 1.6 minutes, being worked from an accumulator which is fed by a pump having an efficiency of 80%. If the pressure of the water is  $70 \text{ kg/cm}^2$  and the efficiency of the lift 70%, find the HP required to drive the pump and the minimum capacity of the accumulator.  $(29.4 \text{ HP} ; 121,500 \text{ m}\cdot\text{kg})$  (*Madras University*)
- 16.21 An intensifier of 100 mm diameter ram and one metre diameter piston is connected to a press having a ram 300 mm diameter. Water is supplied to the intensifier from a tank 15 m above the intensifier through 50 mm diameter pipe, 200 m long,  $f = 0.008$ . Calculate the speed of advance of the press ram in metres per hour when exerting a force of 60 tonnes.  $(\text{one metre/hr})$  (*Madras University*)
- 16.22 A hydraulic crane lifts 20 tonne when working at a pressure of  $55 \text{ kg/cm}^2$ . The rate of lifting is 20 m per minute and total lift is 10 m. The crane works once every two minutes. The efficiency of the crane is 0.8. Calculate the minimum HP of the pump and minimum capacity of the accumulator which is also provided. What is the volume of the crane cylinder? (*Gujarat University*)

- 16.23 Describe with the aid of suitable sketches the principle of working of a hydraulic crane. Under what circumstances would you recommend the use of such a crane ?

A hydraulic crane has a ram 225 mm diameter and water is supplied to the crane at  $55 \text{ kg/cm}^2$  at the rate of 0.9 lit/sec. The pulley system of the crane has a velocity ratio of 6. If the height to which the load is to be lifted is 15 m, find the load that can be lifted and the time taken to lift the same.

Assume an overall efficiency of 71%.

(2,570 kg ; 110 sec) (*Roorkee University*)

- 16.24 Describe with sketches the working of the hydraulic accumulator.

Certain machinery is worked from a weight loaded accumulator through a pipe line 900 m long and 100 mm diameter. The accumulator has a ram of 375 mm diameter and 3.6 m stroke. It is loaded with 47 tonnes and is supplied with water by a three throw pump having a diameter of 100 mm, stroke 200 mm and speed 50 rpm. The slip may be estimated at 3% and the pipe co-efficient is 0.007. If the machinery absorbs 60 WHP, calculate the longest period during which it may be operated continuously.

(55.5 seconds) (*Bombay University*)

- 16.25 In an installation of hydraulic cranes, the cranes are supplied with water at a pressure of  $50 \text{ kg per cm}^2$ . Each crane is required to raise a load of 5 tonnes at a speed of 18 metres per minute through a total height of 12 metres. The system of ropes and pulleys provides a velocity ratio of 6. Assuming an efficiency of 60% estimate the diameter and stroke of the crane hydraulic motor.

There are five cranes in the installation and the working cycle of each crane occupies 90 seconds. If it be assumed that all the five cranes be making their working strokes simultaneously, calculate the minimum capacity in litres per second of the pump feeding the installation and the minimum capacity in litres of the accumulator required.

(35.7 cm dia ; 2 m stroke ; 11.11 lit/sec ; 555.5 litres)

(*Bombay University*)

- 16.26 In a testing machine the force is applied to the specimen by hydraulic pressure on a ram of 250 mm diameter. The maximum force required is 100 tonnes, and the frictional resistance at the ram may be taken as an additional 5 tonnes. The water is supplied from an accumulator consisting of a vertical cylinder with a loaded plunger, 112.5 mm diameter. Find the necessary load on the plunger if friction there causes a resistance of 2 tonnes to its motion. If the water for the accumulator is supplied by a pump with an efficiency of 0.85 and the friction force at the plunger has same value when the accumulator is receiving water, calculate the overall efficiency of the arrangement. Find this efficiency when the force required is 50 tonnes.

(23.26 tonnes, 0.681 and 0.34)

- 16.27 The ram of a hydraulic accumulator is 450 mm in diameter. The stroke is 7 m and the water pressure is  $80 \text{ kg/cm}^2$ . If the useful work given by the accumulator during one full downward stroke is utilized in raising  $W$  tonnes to a height of 15 m by means of a hydraulic crane, whose efficiency is 60%, find the value of  $W$ . If this work is done in 3 minutes, what is the gross horsepower of the crane ?

(35,700 kg ; 66 HP) (*Utkal University*)

## SECTION VI

# Fluidics

# 17

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## *Fluidics*

**17.1 Birth of Fluidics**—Fluids are being used for transmitting force and energy in mechanical system for many years. It was in 1959 when three engineers of USA Diamond Ordnance Fuse Lab came up with some unique idea involving the use of flowing fluids to accomplish control functions. This idea combined the two functions *viz* fluid amplification and fluid logic giving a new name—*fluidics*. The concept came by permitting one stream of fluid to impinge on another and thus changing its direction of flow and the tenacity of a fluid to stick to a wall. Till recently the predominance of electronic technique in computation and in control system was such that it was thought that such system could be carried out electronically only. With fluidics the techniques have now been developed for using fluid as medium for computation and for carrying significant information in control system which hitherto was the function in the province of electronics. Thus *fluidics is defined as a control technology which makes use of fluids interaction to produce useful signals*. Fluids can have a major impact on fluid power control.

The term fluidics was decided upon by terminology task group of the National Fluid Power Association Committee on standards, by contraction of the words “fluid” and “logic.” Before considering in some details the nature and uses of fluidics devices, it might be worthwhile to take a look at some of the pioneering work in fluid devices which presaged the fluidics technology.

In 1904, L. Prandtl the famous German aerodynamist, suggested that in a wide angled diffuser the flow separation could be controlled by applying suction to the boundary layer. Had Prandtl been a Boolean algebraist, it is likely that he would have been the inventor of the first logical NOT device.

In 1916, Nikola Tesla filed a patent for a “Valvular Conduit” which was granted in 1920. While Tesla claimed to have invented the first fluid device having no moving parts, he could equally well have claimed to have invented the first fluid diode.

Prior to 1938, Henri Coanda, a Rumanian engineer discovered that a free jet would follow an adjacent curved or inclined surface. If an imaginative computer experimenter had been available in the 1930's the discovery of the “Coanda effect” could easily have led to the development of a means of controlling a fluid stream in terms of Boolean logic.



In 1958, Moore and Klive, working with wide angled diffusers, discovered that a jet could have two or three stable stages, depending on the angle of the diffuser through which the jet was flowing.

**17.2 A New Approach to Control Systems**—The ever rising demand for logic systems has been mainly satisfied by using electrical devices. It has, however, become very apparent that there is a need for a simple element having no moving parts, absolute reliability, infinite working and shelf life and immunity from damage through overload. In the search of such an element there has been a natural gravitation towards the use of compressed air in view of its inherent ability to comply with these conditions.

Fluid logic has now emerged as the probable answer to this quest after considerable research and development.

Upto the present time it has been necessary to employ both electrical and pneumatic elements in most control systems in which the former catered for the logic and sensing and the later for the power requirements. With the new approach the need for dual power system no longer exists. Thus not only there is a saving in the cost of installing the power media, but saving in the cost of electro-pneumatic conversion units alone is considerable. Additionally this type of system is readily understood and maintained by mechanical engineers. Providing logic and sensing functions is a major advance in the field of 'low cost automation'.

The control circuit element has an unlimited life, is easily inter-connected, cannot be damaged by incorrect connections or excessive pressures, power consumption is very low and maintenance a negligible factor.

The use of air as a means of performing logic functions is not new, and as far back as 1933 the control of air by an apparatus having no moving parts was achieved in a device known as "wall attachment" by Coanda who applied for French patents in 1935. In 1940, Braithwaite, Wilcox and Todd were experimenting with interacting jet devices and a patent was assigned to Metropolitan Vickers to cover a type of fluid operated relay.

Developments in the past few years led to significant steps forward in the design and manufacture of fluid logic elements for industry. Most of these have an effectively infinite working life and are very tolerant of environmental variations.

**17.3 Fluidics Terminology**—Before discussing fluid amplifiers it is necessary to define the word 'amplifier' and also explain the difference between digital and analogue amplifiers. It is also necessary to define certain terms used in fluidics.

(a) **Amplifier** can be either electric, hydraulic, pneumatic, or fluidic.

If a device gives a large change in output either pressure, flow or both, as a result of a small change in control input, the device is said to have 'gain', or in other words it is an amplifier. (It amplifies its input signal).

(b) **Types of Elements**—These Fluidic Devices fall into two basic categories, Digital or Analogue

|                                |     |     |     |          |
|--------------------------------|-----|-----|-----|----------|
| (i) Wall attachment            | ... | ... | ... | Digital  |
| (ii) Turbulance                | ... | ... | ... | Digital  |
| (iii) Vortex Feedback          | ... | ... | ... | Digital  |
| (iv) Stream or Beam Deflection | ... | ..  | ... | Analogue |
| (v) Impact Modulator           | ... | ... | ... | Analogue |
| (vi) Vortex                    | ... | ... | ... | Analogue |

(c) **Digital**—A digital element has two outputs, and flow takes place from one or other output depending upon the presence or absence of a control signal. The flow never splits between the two outputs ; it is either fully out of one or other. It can be compared to a simple ON-OFF Switch.

(d) **Analogue**—An analogue element on the other hand varies its output continuously as a function of the control output signal and gives proportional control. Similar to an accelerator pedal of a car. (Analogue Proportional)

The types of devices that are of most importance for industrial applications at the present time are digital devices and these are readily available on the commercial market.

These digital devices split into two categories *i.e.* Bistable or Monostable.

(e) **Bistable**—This means that the device is stable in any one of its two states either in the presence or absence of an applied signal. A device having bistable properties is also said to have memory.

(f) **Monostable**—In the absence of a signal the device always selects one of its two output states. The other state can only be selected by applying and maintaining the signal *i.e.* when the applied signal is removed the device returns to its former state.

Fluidic devices fall into two further categories *i.e.* Active and Passive.

(g) **Active**—If the device is active it means that the power nozzle of the device is continuously supplied with an air pressure source.

(h) **Passive**—This is a group of devices which intermittently receive pressure signals at the power nozzle, and this signal usually comes from the output of preceding element in the circuit. In other words the air only passes momentarily through the device.

**17.4 Types of Fluid Logic Element**—There are three main types of such devices *i.e.*

- (a) Mechanical ;
- (b) Interacting jet ;
- (c) Coanda or Wall Attachment

(a) **Mechanical Devices** (refer (i) to (iv) below) are those where physical movement of certain components within the device will control a flow or jet.

(i) The first a 'Spool Valve' (refer Fig 17.1) is a form of control valve which is readily recognised by engineers. The spool is a friction fit in the valve body so that a momentary signal  $A$  from the bottom will cause the spool to move upwards and remain in the top position. An output, say,  $S_2$  will result without the application of further signals. The application of a signal  $RP$  from top will give another output, say,  $S_1$  while the application of signal  $A$  will overcome  $RP$ , so that the application of both signals will give output  $S_2$ .

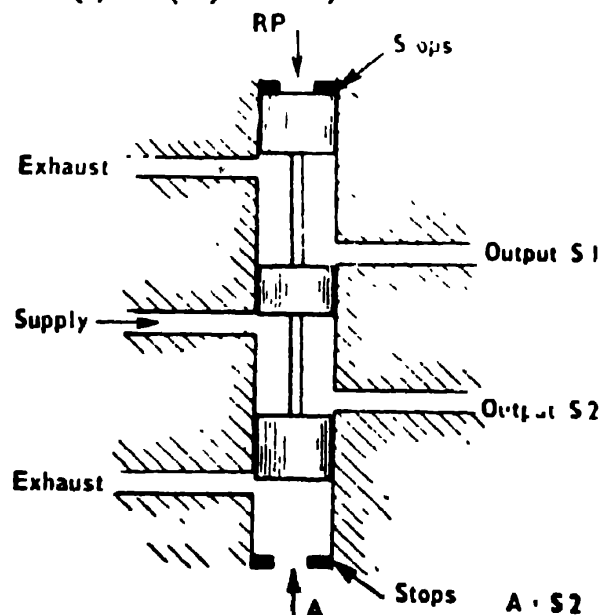


Fig 17.1 Spool Valve

(ii) **Positioning Jet**—is another device (refer Fig 17.2) performing the same function as the spool valve with added advantage of proportional control. With no signals applied i.e. when the spring attached to the piston is at no tension position, an output, say,  $S_1$  is obtained. When a signal

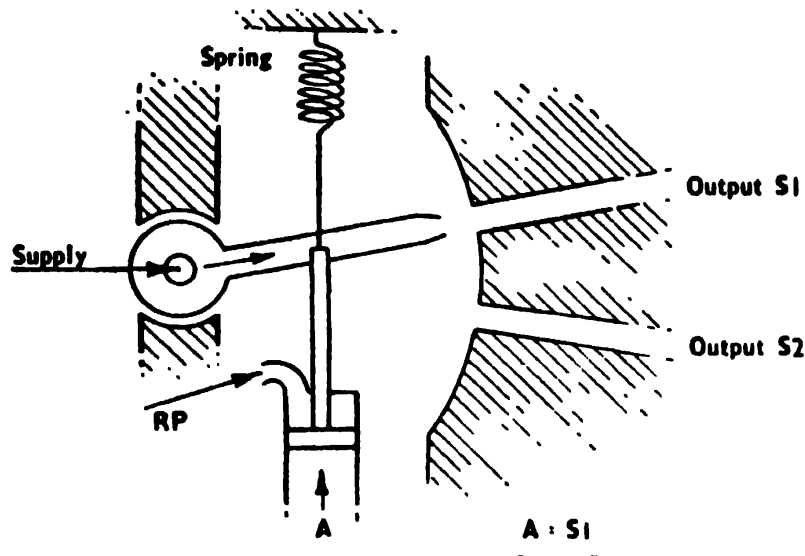


Fig. 17.2 Positioning Jet (Askania)

$RP$  is applied to the spring another output  $S_2$  is obtained, but the application of signal  $A$  equal in magnitude to output  $S_1$  and opposite in direction to signal  $RP$  will overcome  $RP$  and output  $S_1$  will again result. Thus by varying the signal pressures the jet can be positioned in relation to the output collectors to give varied outputs.

(iii) **Diaphragm Flapper Nozzle Value**—Fig 17.3 shows a simple form of signal amplifier. The nozzle is greater in area than the restrictor

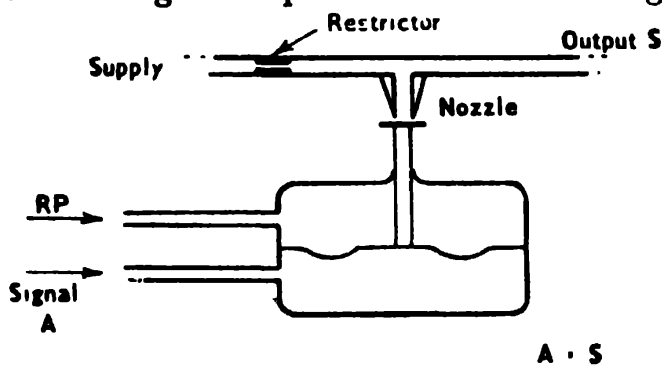


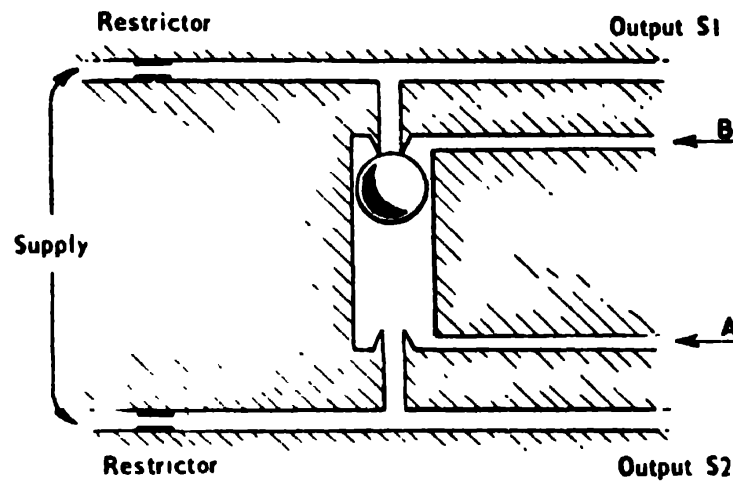
Fig 17.3 Diaphragm Flapper Nozzle Valve

therefore with the diaphragm in position shown, there is a continual bleed of air from the gap at the nozzle. A signal applied at ' $A$ ' will cause the diaphragm to lift and seal off the nozzle; this causes a rise in pressure which results in output ' $S$ '. ' $RP$ ' and ' $A$ ' can be adjusted so that ' $A$ ' can overcome ' $RP$ ' at selected pressure ranges.

(iv) **Moving Ball (Kearfott)**—Fig 17.4 shows a bi-stable device which operates through the closure of, or application of a signal to ports ' $A$ ' or ' $B$ ' which are normally vented to the atmosphere. The effective areas of the ball is greater than that of the nozzle. So by momentarily blocking or applying pressure to ' $A$ ' the ball is forced upward into the position shown by the rise in effective pressure from the lower supply. This blocks the top nozzle, and providing part ' $B$ ' is not blocked, the ball will remain indefinitely in this upward position, giving output  $S_1$ .

By blocking port ' $B$ ' the normal leakage around the ball to atmosphere is prevented and the ball will be in a state of balance; therefore air from

the top nozzle will move the ball downwards thus exposing its effective area to the top supply. The ball will move to block the bottom nozzle so giving output 'S 2'.



A - S1 (Ball Up). B - S2 (Ball Down)

Fig 17.4 Moving Ball (Kearfott)

The restrictors can be adjusted to give precise control over the movements of the ball and so control the value of the outputs. Thus continuous output from either 'S1' or 'S2' can be obtained by brief control pulses at 'A' or 'B'.

(b) **Interacting Jet Device**—This is the type (refer Fig 17.5) in which a flow or signal is switched by opposing pressures. Input jet 'A' placed perpendicularly to air supply jet will cause air stream to be deflected and pass out to output S 2 instead of axial output S1, simply by conservation of momentum.

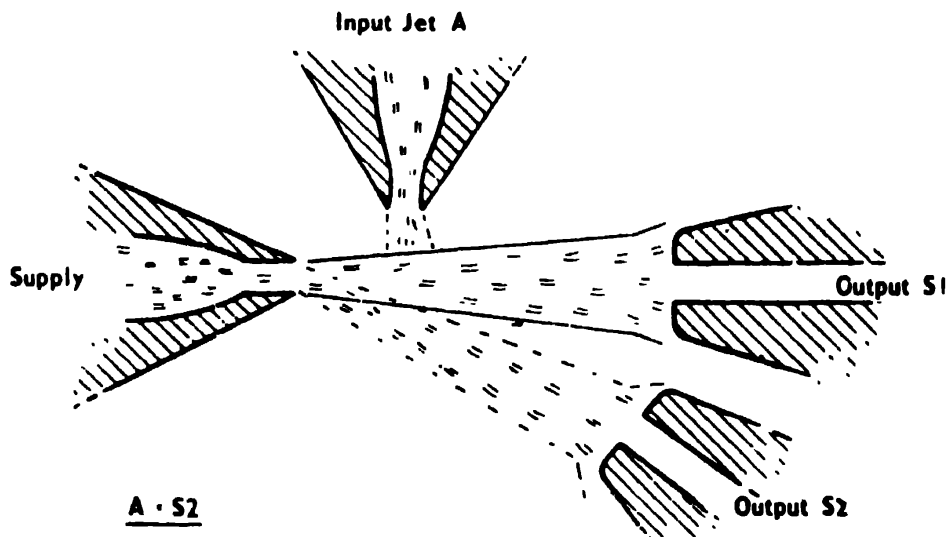


Fig 17.5 Jet Interaction Device

(c) **Coanda or Wall Attachment**—In this device (refer Fig 17.6) a flow of fluid will follow the contour of the wall of the device making a permanent memory. Fluid flow can be made to dislodge from one side of the passage and follow the contour of the other by a brief control pulse.

In this a brief pulse  $A$  will cause an output  $S1$ . This output will remain until 'Switched over' by another pulse at second input.

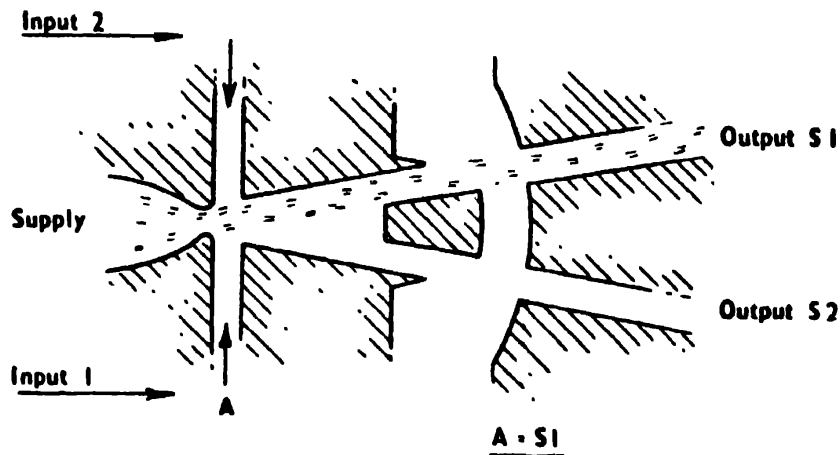


Fig 17.6 Wall Attachment Device

**17.5 Turbulence Amplifier**—The turbulence amplifier is a special form of interacting jet device which utilizes the fact that a laminar flow stream projected across a gap can be very easily disturbed and made turbulent. It has a very low operating pressure of a few centimetres of water, uses only about one-tenth of  $m^3$  of air per hour and has a power consumption of the order of 1 watt per unit.

The ability to transform the laminar stream into a turbulent one by means of a very small disturbance, means that a switching action can be achieved which gives a large initial amplification of the disturbing force. From Fig 17.7 it can be seen how the laminar stream is disturbed by the input jet  $A$ .

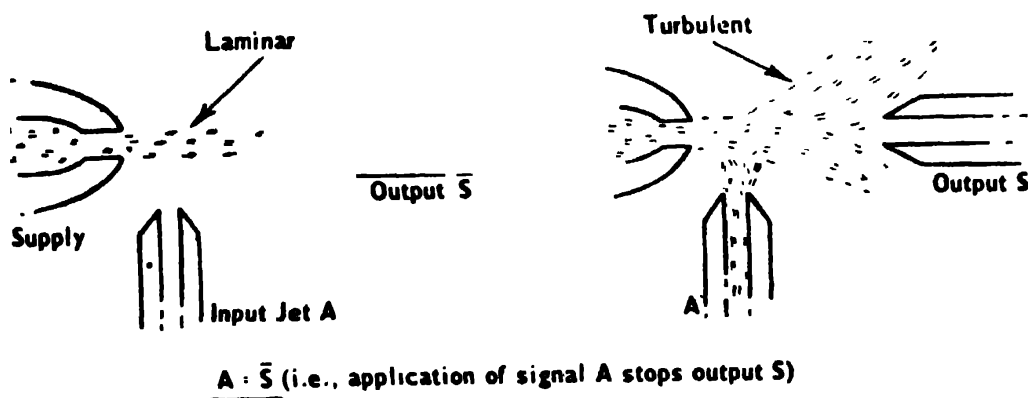


Fig 17.7 Turbulence Amplifier

The power needed to turn off a turbulence amplifier is much less than the power output, one turbulence amplifier can in fact control six others. This gives a good ratio of amplification in logic systems. The normal pressure output of a turbulence amplifier is about 10 cm of water ( $1/100 \text{ kg/cm}^2$ ) and turn off input pressure required is about one cm of water supply pressures used are in the range of 20 to 30 cm of water. The time needed to 'turn off' a turbulence amplifier is only about 2 milliseconds and the 'turn-on' time is 5 milliseconds. It has, therefore, a reversal time of approximately of milliseconds. This is fast enough for most control systems.

**17.6 Industrial Type Turbulence Amplifier**—The normal arrangement of this type (refer Fig 17.8) has four input tubes in relation to the supply and output tubes. The advantages are as follows :

**Industrial Type Turbulence Amplifier**

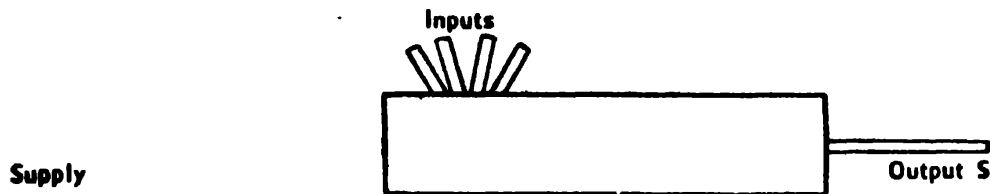


Fig 17.8 Industrial Type Turbulence Amplifier (vent not shown)

- (i) Any one of the input tubes will 'turn off' the amplifier.
- (ii) Each 'turn off' signal is completely isolated, so that one signal will not affect the other signal lines in the input tubes by feed back.
- (iii) There is no pressure rise within the amplifier when a signal is applied.

**17.7 Vortex Amplifier**—Vortex amplifiers utilize the properties of a vortex to regulate flow. This vortex flow is confined within a flattened, cylindrical chamber. Usually the fluid is uniformly fed into the chamber around the periphery of the outer cylindrical wall. If no disturbing influences are present the fluid flows along a radius towards the center and then out through a hole at the center of one of the flat surfaces. The fluid is usually introduced at the periphery of the chamber through a porous outer cylindrical wall (refer Fig 17.9)

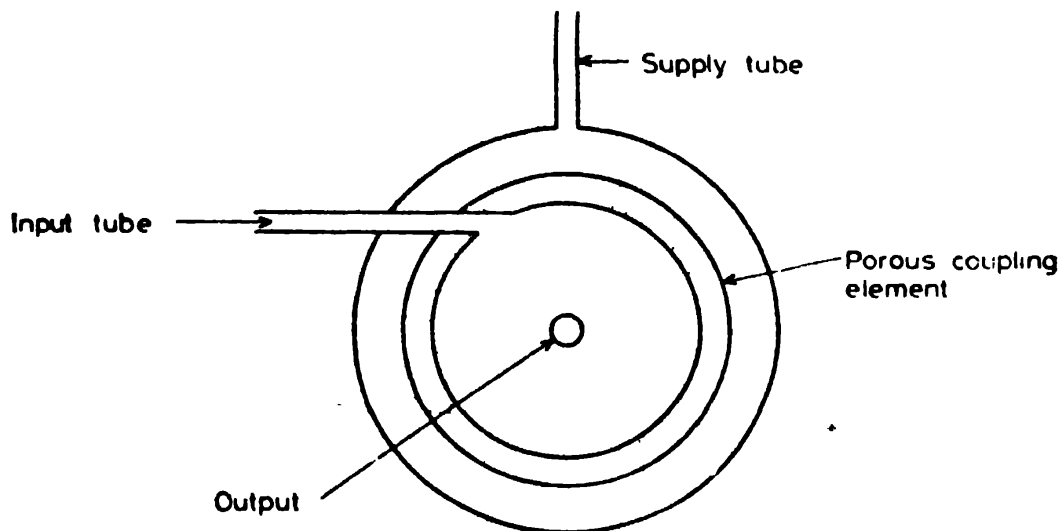


Fig 17.9 Vortex Amplifier

Flow in the chamber described above is radial. To introduce a vortex motion in the chamber, a tangential control port is inserted through the porous ring (refer Fig 17.10). Use of several control ports in parallel produces a greater uniformity in the flow field.

**Vortex Flow :** Consider an element of fluid which enters the vortex chamber through the porous coupling element. As this element flows towards the output port its radial velocity must increase. Again consider

the same fluid element as it enters through the porous coupling with tangential velocity imparted to it as it leaves coupling. To conserve angular

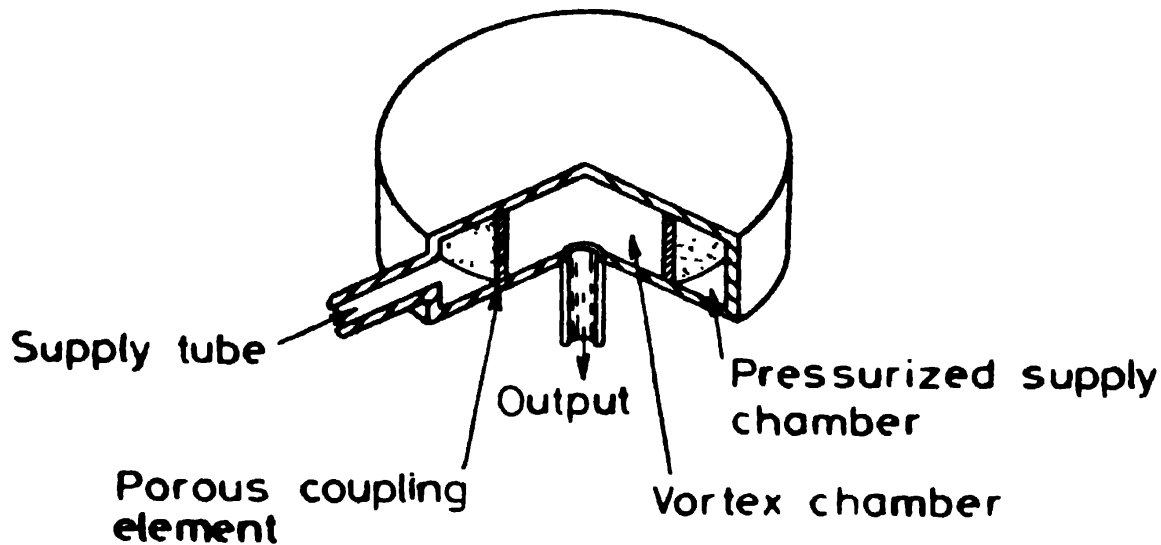


Fig 17.10 Vortex Motion

momentum, angular velocity of the fluid must increase as it approaches the output port. Thus there are two amplifying properties of vortex amplifier: Increase in radial velocity and increase in angular velocity.

When the supply pressure is regulated, if the vortex motion of the fluid produces a centrifugal pressure drop across the vortex chamber, there is less pressure at the exit to expel the fluid. Thus, less fluid flows from the exit. The introduction of vortex motion into the chamber effectively throttles the fluid flow through the chamber.

Typical curves of output flow as a function of the control pressure applied to the vortex chamber are drawn. It will be seen that the flow is reduced as control pressure is increased.

For sufficiently high control pressures, flow from the supply is zero or even slightly reversed. Centrifugal pressure increases the pressure at the inside surface of the porous coupling element so that the total pressure is equal to or slightly higher than the supply pressure. Thus the control flow can cut off completely the flow from the supply chamber.

However, there may still be substantial flows out the exit—the flow which is being introduced through the control port. When this condition is reached, further increases in the control pressure increase output flow from the exit tube. Thus for high control pressures the flow versus Control pressure curves become tangent to the line of zero supply flow.

For any supply pressure a certain minimum amount of control pressure must be applied before any of the flow is throttled. If the supply pressure is  $4 \text{ kg/cm}^2$  for example then at least  $4 \text{ kg/cm}^2$  must be applied to the control port to force any flow into the chamber and thus cause vortex motion. This characteristic is in contrast to other types of fluidic amplifiers in which small control pressures can be applied to devices which are operating with large supply pressures.

**17.8 Function of Digital Devices**—Fluidic devices are used for amplifying small control signals. They can perform switching functions and can be arranged to carry out logical sequence of operations based

upon receipt of essential information which must be delivered in the correct sequence if the final operation is to be carried out.

In pneumatic circuits the parts that will essentially be replaced by fluids will be the electric and electronic components, such as relays and contacts, transistors, electronic timers etc.

Components such as directional control power valves, cylinders etc. will still be required.

Fluidic devices are therefore the basis of the control system (the "brain" not the muscle) and serve to provide operating instructions to the power side of the circuit.

The main function of a digital device whether it be fluidic, pneumatic, hydraulic, electric or electronic is to provide a control function. Before the control function can be completed decisions have to be made in a logical sequence, and this implies that the problem must be fully understood.

For years pneumatic engineers have been designing digital control systems without fully understanding or analysing the thought process which they had gone through in order to evolve the solution to this problem. This thought process is called logic.

A logical proposition is a sentence meaningful in itself about which it can be said "that sentence is true" or "that sentence is false". No half truths or shades of meaning are permitted.

For example a simple problem :

I am here talking to you and I wish to draw on the blackboard a diagram. The blackboard is on my left and the chalk is the other side of the room, on my right. I can either move the board to the chalk or the chalk to the board. It is obviously not desirable to move the blackboard, so I must walk to the chalk and not the blackboard.

The decision as to how this problem is to be solved has been made, so I must proceed in a logical sequence to carry out the operation.

1. I walk towards the chalk.
2. I arrive at the chalk 'AND' stop.
3. I pick up the chalk with either right 'OR' lefthand.
4. I must turn round 'AND NOT' drop the chalk before starting towards the blackboard
5. I arrive at the blackboard 'AND' stop.
6. I am now ready to start drawing, if the chalk is in my right hand, but 'NOT' if it is in the left hand.

You will notice throughout this operation I have constantly used the terms of AND, NOT, OR, as well as the combining term AND NOT. This was basically a thought or logic process utilizing what are commonly known as logic terms.

**17.9 Logic States**—The common algebraic expressions for the logic states are explained as under :

A. B means A and B.

$A \vee B$  means A or B,



a bar over a letter means the complement or negative value of that letter *i.e.*  $\bar{A}$  means not A, S signifies the output obtained from a circuit or logic function.

The common Logic States are as follows :

| Function  | Input Signal                                                                           | Output                                               |
|-----------|----------------------------------------------------------------------------------------|------------------------------------------------------|
| NOR       | neither input A nor B nor C nor D<br><i>i.e.</i> , $\bar{A}.\bar{B}.\bar{C}.\bar{D}$ . | =an output<br>=S                                     |
| NOT       | one input A                                                                            | =no output<br>or $\bar{S}$                           |
| OR        | any input A or B or C or D<br><i>i.e.</i> , $A \vee B \vee C \vee D$                   | =an output<br>=S                                     |
| AND       | all inputs A and B and C and D<br><i>i.e.</i> , $A.B.C.D$ .                            | =an output<br>=S                                     |
| NAND      | all inputs A and B and C and D<br><i>i.e.</i> , $A.B.C.D$ .                            | =no output<br>= S                                    |
| FLIP-FLOP | brief input A<br>brief input B                                                         | =maintained output $S_2$<br>=maintained output $S_1$ |

These can be achieved by the following arrangements of Turbulence Amplifiers :

(a) **Logic State 'NOR' or 'NOT'** (refer Fig 17.11) — This is achieved simply by one Turbulence Amplifier. Signals from neither 'A' nor 'B'

LOGIC STATE 'NOR' or 'NOT'

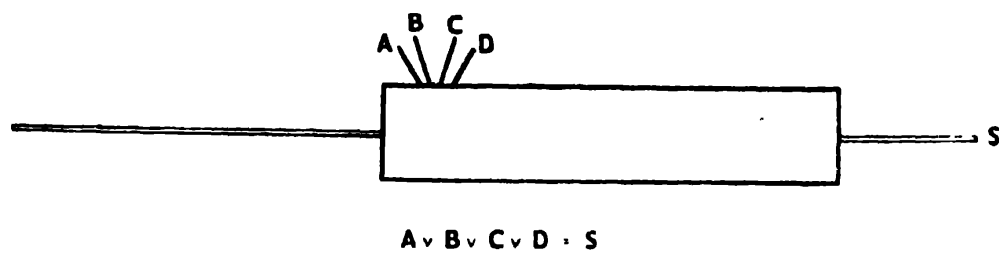
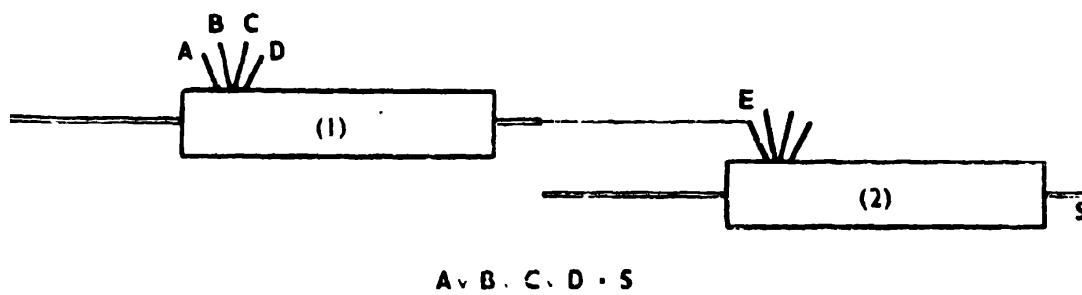


Fig 17.11 Logic State NOR or NOT

nor 'C' nor 'D' can be present if output 'S' is required. If one output signal only is used, the result is NOT or signal inversion, *i.e.*  $A = \text{not } S = \bar{S}$ .

(b) **Logic State 'OR'** (refer Fig 17.12) — This is achieved by two Turbulence Amplifiers. If a signal is applied at either 'A', 'B', 'C' or 'D'

LOGIC STATE 'OR'



17.12 Logic State OR

Turbulence Amplifier (1) will be 'off' thus the supply to Turbulence Amplifier (2) will be 'off' while Turbulence Amplifier (2) will be 'on' so that 'A' or 'B' or 'C' or 'D' will give output from Turbulence Amplifier (2).

(c) **Logic State 'AND'** (refer Fig 17.13). This is achieved by three Turbulence Amplifiers where both (1) and (2) are connected to (3). If

#### LOGIC STATE AND

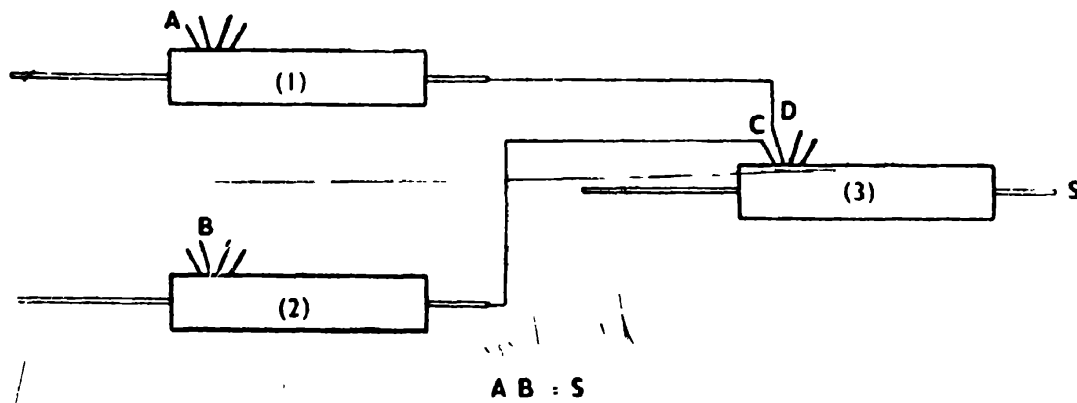


Fig 17.13 Logic State AND

either input signal 'A' or 'B' is not applied, Turbulence Amplifiers (1) or (2) respectively will be 'on', thereby sending a signal to either 'C' or 'D' and holding Turbulence Amplifier (3) 'off'. Therefore 'A' and 'B' signals must be present to obtain output 'S' from Turbulence Amplifier (3).

(d) **Logic State 'NAND'** (refer Fig 17.14) — This is achieved by connecting Turbulence Amplifiers (1), (2) and (3) to (4) which in turn is connected to Turbulence Amplifier (5). An output 'S' is

#### LOGIC STATE 'NAND'

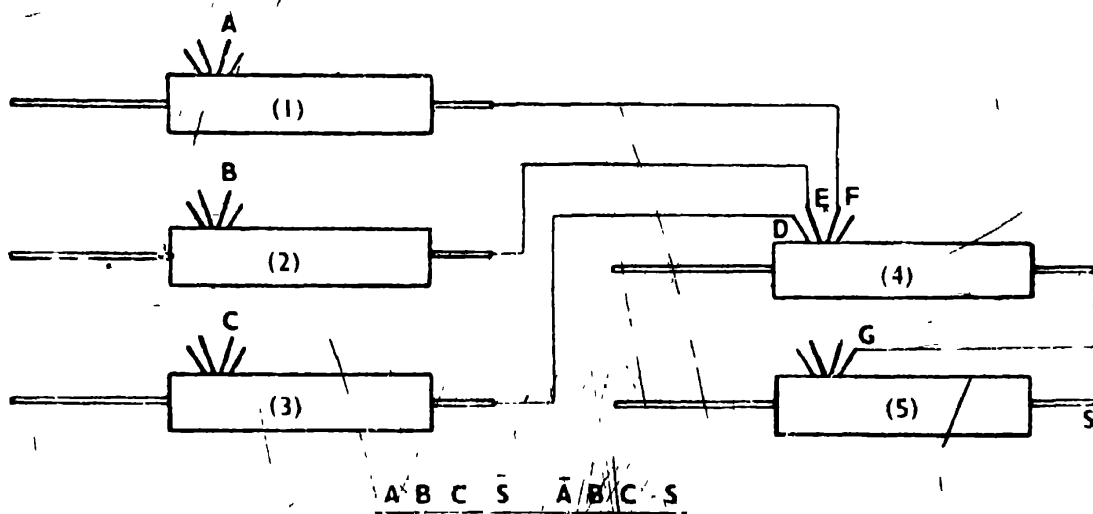


Fig 17.14 Logic State NAND

obtained unless all input signals 'A', 'B' and 'C' are present holding Turbulence Amplifiers (1), (2) and (3) 'off'. There are therefore no signals at 'D', 'E' and 'F'.

**Example :** If signal 'A' only is absent Turbulence Amplifier (1) would signal to 'F' and hold (4) off thus, with no signal to 'G' Turbulence Amplifier (5) would give output 'S'. This is an inverse 'AND'.

(e) **Logic State Flip-Flop** (refer Fig 17.15) If an input pulse is given to input Tube 'B', Turbulence Amplifier (2) will be 'turned-off' and

LOGIC STATE 'FLIP FLOP'

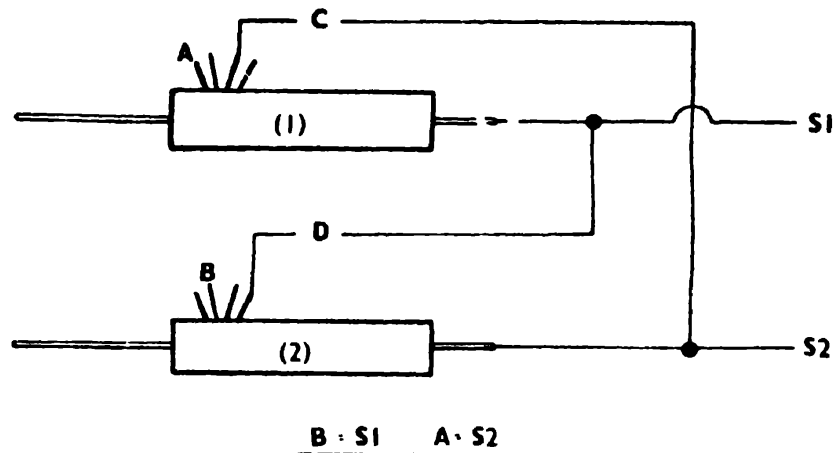


Fig 17.15 A Memory or FLIP-FLOP

therefore no signal will be present at 'C' and output 'S1' will be obtained Turbulence Amplifier (1). Now output 'S1' also feeds control tube 'D' which continues to hold (2) off.

A signal input at A will turn off Turbulence Amplifier (1) and then Turbulence Amplifier (2) will reset to provide output 'S2' which will in

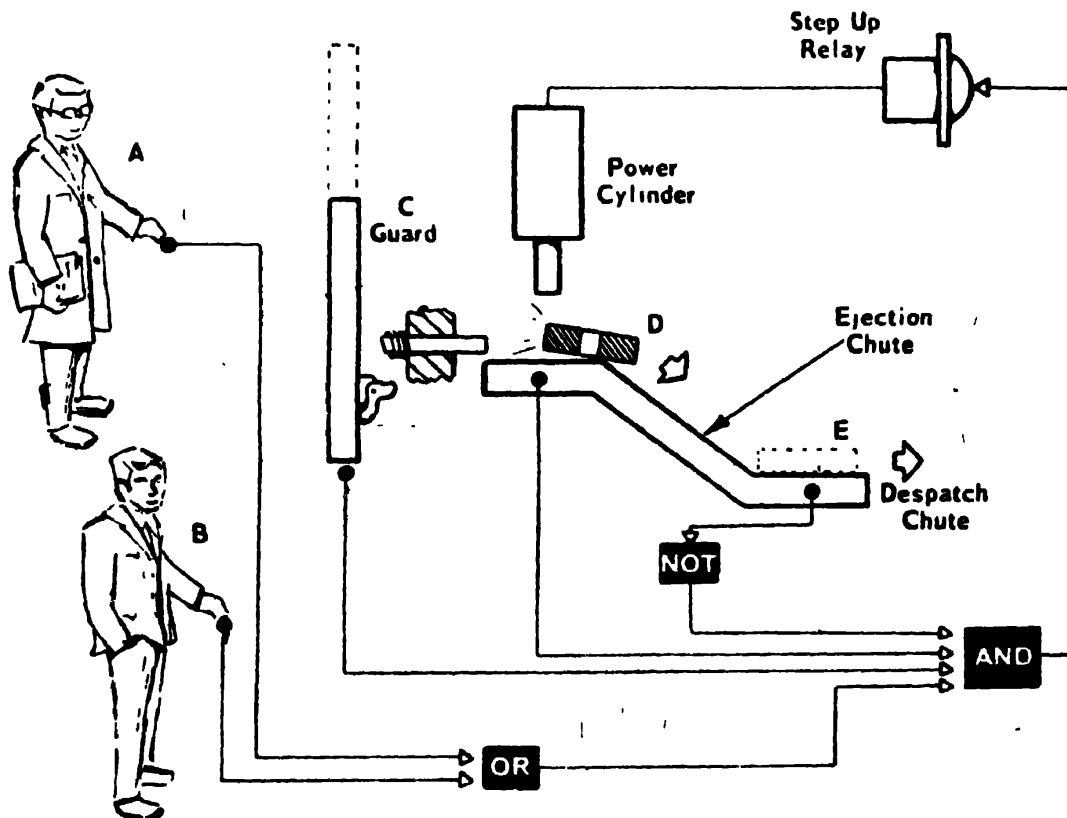


Fig 17.16 Example of Machine Control

turn continue to hold off Turbulence Amplifier (1). This condition is known as a permanent state or unlimited memory.

**Circuits**—These simple combinations of Turbulence Amplifiers which are linked to perform various logic functions can in turn be interconnected into a complete control or logic system. Simple examples of this logic sequence and Turbulence Amplifier inter-connection are shown

**Example of Machine Control**—The conditions under which a machine can be operated (refer Fig 17.16) are as follows: Operators A and B are able to operate the machine from different positions providing the following conditions are satisfied:—

The guard 'C' must be lowered and component 'D' must be in the machining position. The despatch point 'E' must not be occupied by the previous component. Mechanical despatch of the component is achieved by raising the guard 'C'.

These conditions can be interpreted into technical terms as follows:—

Either of signals 'A' or 'B' can be used to start the machine. Guard 'C' and component 'D' must be in position. The ejection path must not be occupied by 'E'. The logic symbols are shown in Fig 17.17.

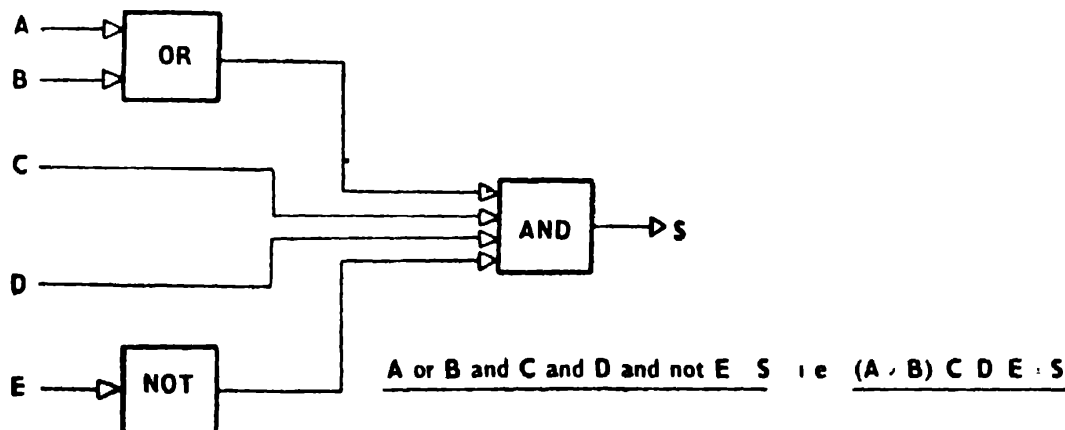


Fig 17.17 Logic Symbols for Machine Control

In the Turbulence Amplifier circuit shown in Fig 17.18 supply lines are not shown, only the inputs and outputs. Supplies are ignored to

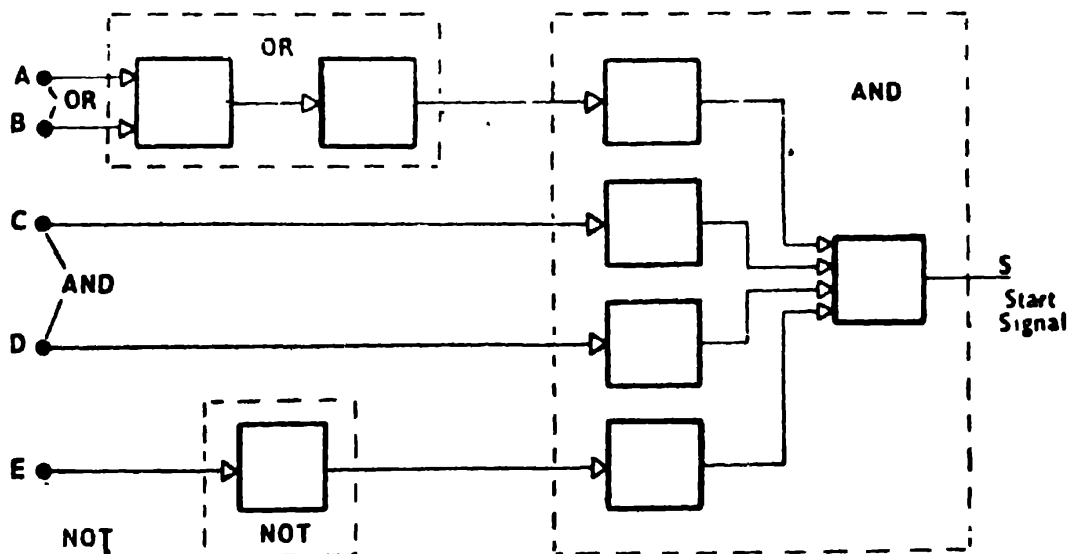


Fig 17.18 Turbulence Amplifier Symbols

simplify the layout. The piping circuit is as shown below in Fig 17.19.

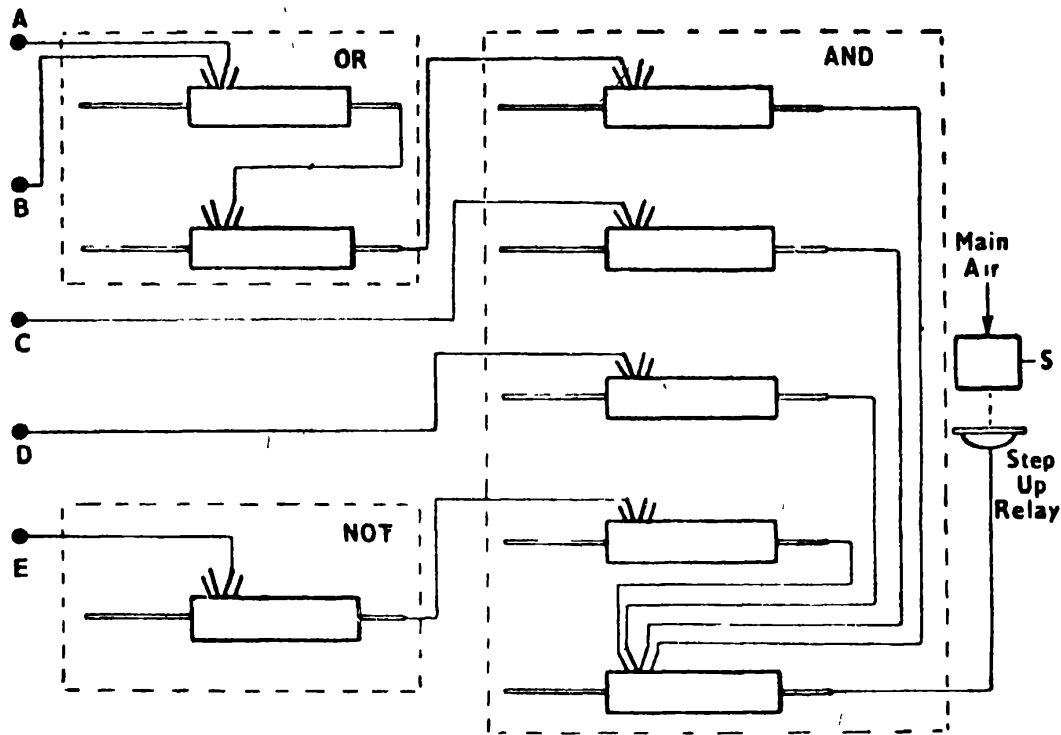


Fig 17.19 Piping Circuit

**17.11 Methods of Obtaining Input Signals and Power Outputs**—As can be done in a piping circuit, a step up relay is used to convert the logic output signal into a pneumatic output of much higher pressure. It could also be a pressure switch for electrical output. The input signals are obtained from the devices described in Figs 17.20, 17.21 and 17.22. The intermediate equipment provides the means of obtaining initial input signals to the logic circuits from outside sources and also provides the means of converting the logic output signals into power outputs.

(a) *Input Sources* : Input control can be achieved by two main methods i.e., Interruption of flow as achieved by the Interruptible Jet (refer Fig 17.20) and the Air Stream Detector (refer Fig 17.21) or Pressure Build up as achieved by Sensing Heads (refer Fig 17.22).

(b) *Interruptible Jet*—This is form of 'Open Turbulence Amplifier' which can detect objects physically impinging on and therefore disturbing

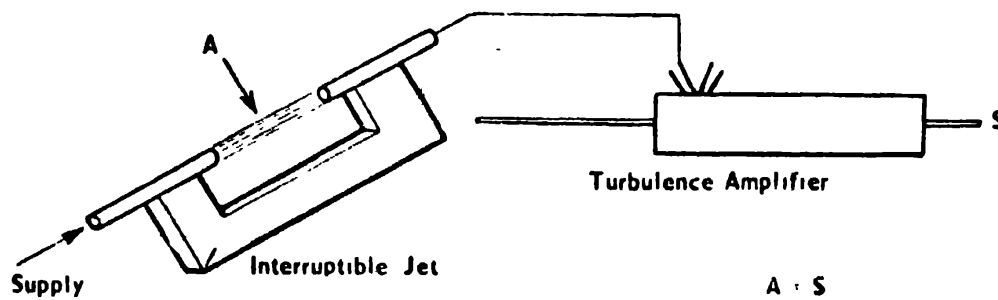


Fig 17.20 Interruptible Jet

the laminar stream. An object or component 'A' disturbing the open flow of the Interruptible Jet without physical contact, will interrupt the signal.

to the Turbulence Amplifier. Thus, the interruptible Jet is normally holding the Turbulence Amplifier 'off' until interrupted.

(c) *Air Stream Detector*—This is similar in construction to Turbulence Amplifier but has one large input. It is very sensitive and will 'turn off'

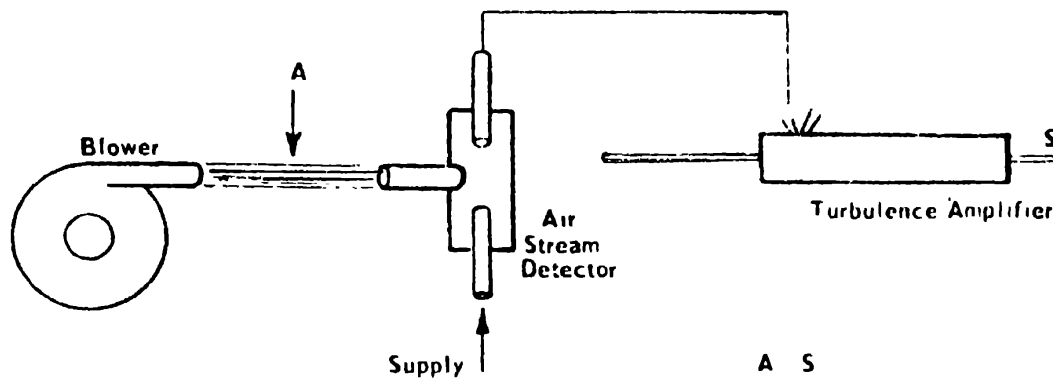
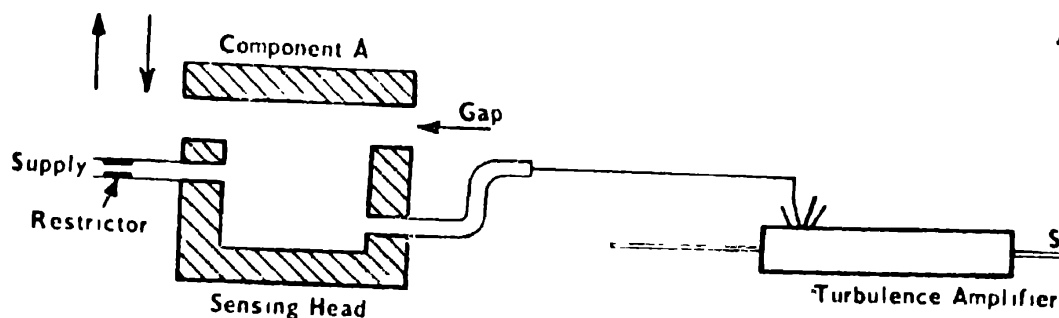


Fig 17.21 Air Stream Detector

when a very low pressure air stream is directed to the input. In Fig 17.21 an interruption 'A' into the path of air from the blower will cause a signal input to the Turbulence Amplifier. Thus the Air Stream Detector is normally 'off' and the Turbulence Amplifier 'on' until the air stream is interrupted. This equipment provides a means of detecting objects in the same way as a photocell but with the added advantage that the emitter and receiver performance will remain constant, regardless of adverse conditions such as dust laden atmosphere or changes in light intensity.

(d) *Sensing Head*—When component A (refer Fig 17.22) closes the gap between itself and the sensing head, pressure build-up will cause the

#### Sensing Head



Logic functions AND - MEMORY - NOT - NOR

Fig 17.22 Sensing Head

Turbulence Amplifier to be switched off. As the gap opens the pressure in the sensing head will fall and the Turbulence Amplifier will be 'on'. Sensitivity can be controlled by the restrictor so that this system can be used as a basis for automatic gauging of components etc.

**17.12 Application Examples**—The following are examples of the application of the Turbulence Amplifier to some industrial problems.

- |                    |                          |
|--------------------|--------------------------|
| (a) Sensing        | (b) Counting             |
| (c) Discrimination | (d) Timing               |
| (e) Dispensing     | (f) Liquid Level Sensing |

(a) **Sensing**—Probe contact with component not necessary for signal.

Machine Process—Positioning—guaging.

Illustrates inverse function, sensitivity and power output of Turbulence Amplifier.

Logic function NOT.

When the component or moving probe (refer Fig 17.23) decreases the gap between itself and the sensing head, the pressure output from the

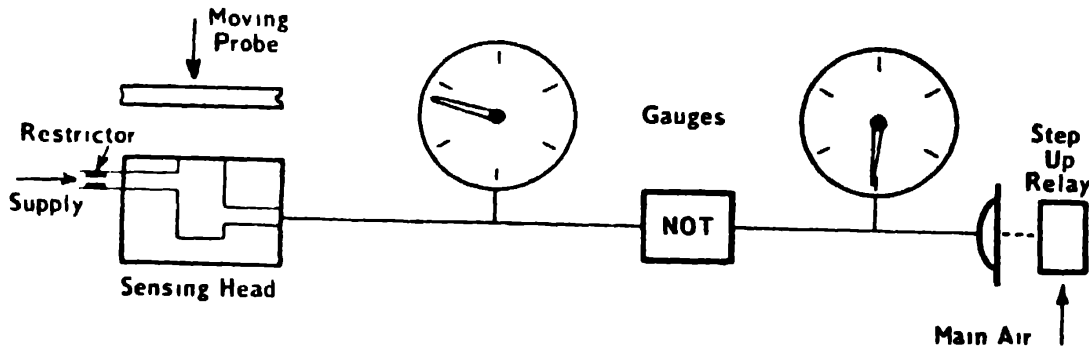


Fig 17.23 Sensing

latter will increase as indicated on the left hand guage. This output can control a Turbulence Amplifier as shown by the NOT symbol. So that with the probe close to the sensing head, the rise in pressure will 'switch-off' the Turbulence Amplifier giving no output as indicated on the right hand guage.

Thus the step up relay could be passing a signal to advance a machine slide until the component decreases the gap between itself and the sensing head. There is no need for the component to touch the sensing head as a signal will be obtained while the component is still approximates 0.0125 cm away. The signal can be obtained with a positional accuracy of at least 0.0025 cm. The sensing head can be adjusted by varying the restrictor setting. The rise in pressure caused by the presence of the component will 'turn off' the step up relay and end the machining operation.

(b) **Counting**—Versatile means of obtaining timing or counting signals with fast power output and no contact good response speed—good

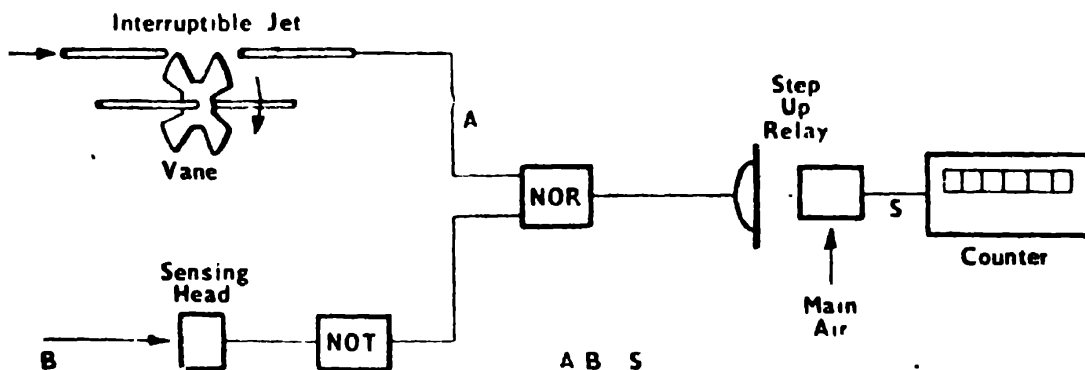


Fig 17.24 Counting

repeatability. Logic function NOT—NOR. In Fig 17.24 two methods of obtaining initial input signals are used, *i.e.* the interruptible jet and the pressure build-up or sensing head as described earlier.

With no signals applied, the NOT element (one Turbulence Amplifier) continually supplies a signal to the NOR element (one Turbulence Amplifier) and therefore no signal can be supplied to the step up relay.

When signal 'B' is applied, the NOT element will be switched off, and thus with the vane revolving, signal A from the interruptible jet can switch the NOR element on and off. This will control the step up relay which operates the counter.

(c) **Discrimination**—Location or detection of components illustrates selection of components with no contact. Logic function NOT—NOR and non-implication.

This method (refer Fig 17.25) of discriminating between two components of different sizes, is achieved by using two interruptible jets as the method of obtaining the initial input signals.

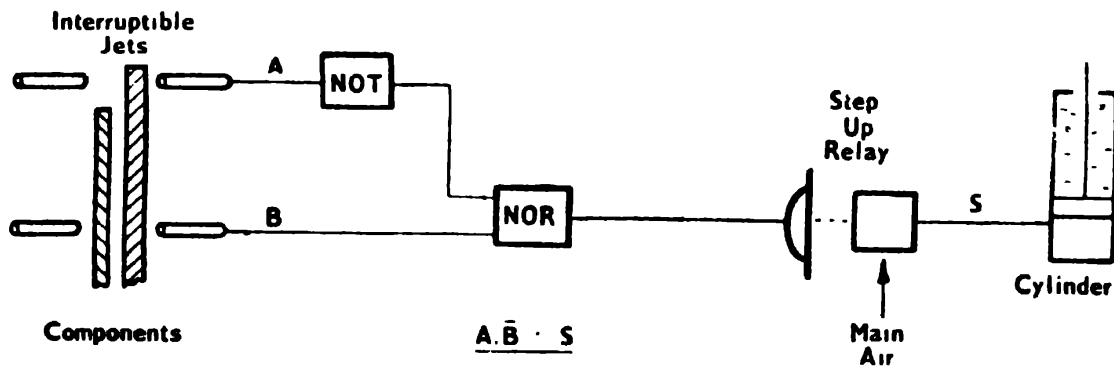


Fig 17.25 Discrimination

When a tall component passes through both jets, signals 'A' and 'B' are blocked, thus the NOR element is held off by the NOT.

When a short component is present signal 'A' will hold off the NOT element and signal 'B' will be blocked. So that no signals will be applied to the NOR element which results in an output to the step up relay thus giving cylinder operation.

With no component present both signals 'A' and 'B' will be applied, and 'B' will hold off the NOR element.

Thus, the short components can be marked or rejected with no physical contact. The jet pressure required is very low so that glass or very delicate components can be detected easily and with complete safety. While this method has all the advantages of a photocell, it has none of the disadvantages in that it is unaffected by smoke, transparent components, etc.

(d) **Timing**—Shows case of conversion of standard apparatus for signal output.

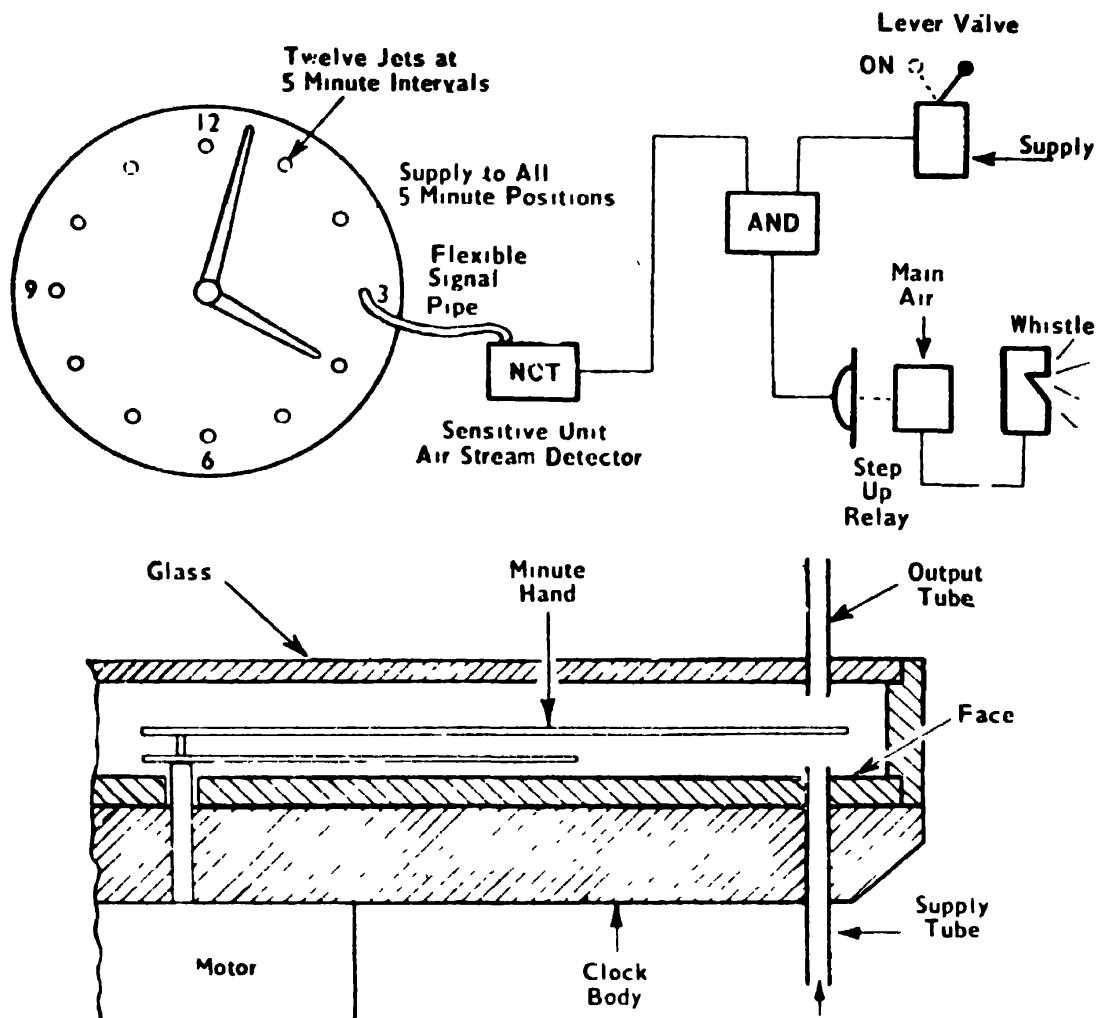
No contact—No load—No explosive hazard.

Logic functions NOT--AND

This is an adaption, (refer Fig 17.26) of a standard piece of apparatus in which the interruptible jet is used as a means of obtaining initial input signals.



The supply tubes pass through the clock from the rear and are connected to the twelve interruptible jets placed at the 5 minutes positions.



With Flexible Signal Pipe placed at 3 and Lever Valve ON, the AND function is complete when Minute Hand interrupts Jet

Fig 17 26 Timing

The output tubes are positioned in the clock glass directly opposite the supply tubes so that there is a small gap between them sufficient to allow movement of the minute hand. The jets are interrupted by the passage of the minute hand.

A common air supply is made to all twelve supply tubes and to any one of the output tubes a flexible signal pipe can be attached. This signal pipe can therefore be used to connect any of the 5 minute positions to the NOT part of the circuit. The NOT element is normally held off by the air stream and it resets when the minute hand reaches the selected 5 minute position and so interrupts the air stream.

The AND function is not completed until the lever valve is supplying one signal and the NOT element the other. When both signals are supplied, the AND unit will operate the step up relay which will blow the whistle. So that manual control of the apparatus is by the lever valve and signal selection is by the position of the flexible pipe.

This arrangement in no way interferes with the clock performance, there is no load imposed upon it and no physical contact of any kind.

It will be appreciated that this example is applied to selection of 5 minute intervals as a simple demonstration. There is no reason why 1 minute intervals should not be selected, or by the use of rotating dials, any intermediate position. It would also be possible to use selector tubes operating on both the hour and minute hands for the selection of any time in a 12 hour period.

(e) **Dispensing**—Means of controlling solid objects with extreme sensitivity and easy selection. Blow and press signals must both be present. Any selection can be made. More than one selection at any one time is not possible.

Memory will maintain cylinder stroke until full extension is reached and press signal released, which then allows further selection. Logic functions AND—MEMORY—NOT—NOR.

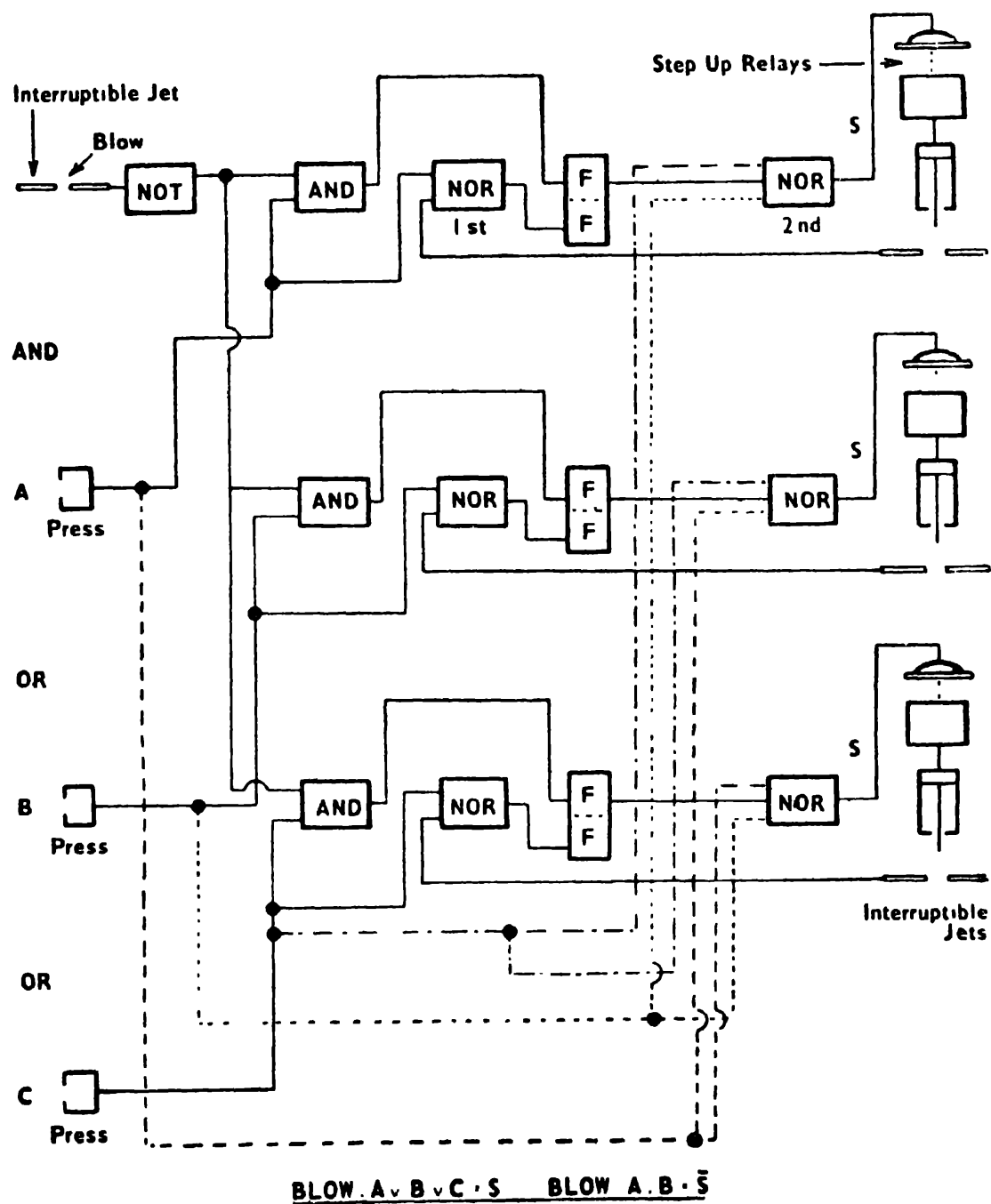


Fig 17.27 Dispensing

Fig 17.27 illustrates a means of dispensing solid objects from a magazine by air cylinders. For convenience it is assumed that the forward stroke of the cylinder will dispense the selected article. The interruptible jet and pressure build-up methods are used to obtain initial input signals.

If the interruptible jet is disturbed by a person blowing into the air stream, it will allow the NOT element to reset and send one input signal to each of the AND units. Simultaneous selection and operation of any of the sensing heads 'A', 'B', or 'C' by light finger pressure will cause a pressure build up to occur, thus providing the other input signal to the AND unit selected. This completes the AND condition for one selection. This AND unit can now trigger its respective FLIP—FLOP which allows the second NOR element to reset and operate the cylinder through a step up relay. When the cylinder completes its outward stroke it interrupts another jet which normally holds the first NOR element off. Provided the signal at 'A', 'B' or 'C' is removed, the first NOR element can reset FLIP—FLOP and so retract the cylinder. Thus the cylinder cannot unless the applied signal which can be 'A' 'B' or 'C' has been ed, and the cylinder has completed its full stroke.

An interlock is arranged which demonstrates the advantage of the complete isolation of Turbulence Amplifier inputs when either 'A', 'B', or 'C' is selected, the signal obtained is also used to hold off the other 2nd NOR elements as indicated by the dotted signal lines. Therefore, selection of 'A' will prevent a further selection of 'B' or 'C'. Simultaneous selection of more than one sensing head will render all cylinders inoperative.

(f) **Liquid Level Sensing**—Means of controlling liquid levels by surface contact with very low pressure air jet. No load—No explosive hazard. Both pumps stop independently when high level is reached.

When both high levels are reached both discharge valves dump mixture. When low levels are reached discharge valves shut independently. With low levels reached in both containers, pumps again start.

Lever valve overrides pump control.

Logic functions AND—MEMORY—NOT—NOR.

The circuit shown in Fig 17.28 uses the pressure build-up principle of obtaining initial input signals. This time, however, the pressure build-up is obtained by using the level of a liquid to cause a pressure signal.

There are two liquid containers and these are filled to a certain level and then discharged into a common vessel below. This illustrates a method of mixing liquids by volume measurement.

Manual control of the automatic cycle is achieved by the use of the lever valve. The use of this will stop the cycle when both lower levels are reached by preventing the re-start of the pumps.

When the lever valve is moved to the 'ON' position both low level detectors are emitting no signal to the input of the NOR element, thus the NOR can now send a signal through the valve to both FLIP—FLOPS controlling their respective pumps. Both FLIP—FLOPS start the pumps through the NOT stages. The NOT stages in this case are for convenience of control only. The level of liquid will now rise in both containers. It will continue to rise until the high level detectors are blocked. When this happens the rise in pressure will send a signal to the pump control FLIP—FLOPS and the pumps will stop independently. When both high levels are reached, the rise in pressure in both detectors will send signals to the

two inputs of the AND unit. This unit will now send a signal to both FLIP—FLOPS controlling the discharge valves. Both discharge valves

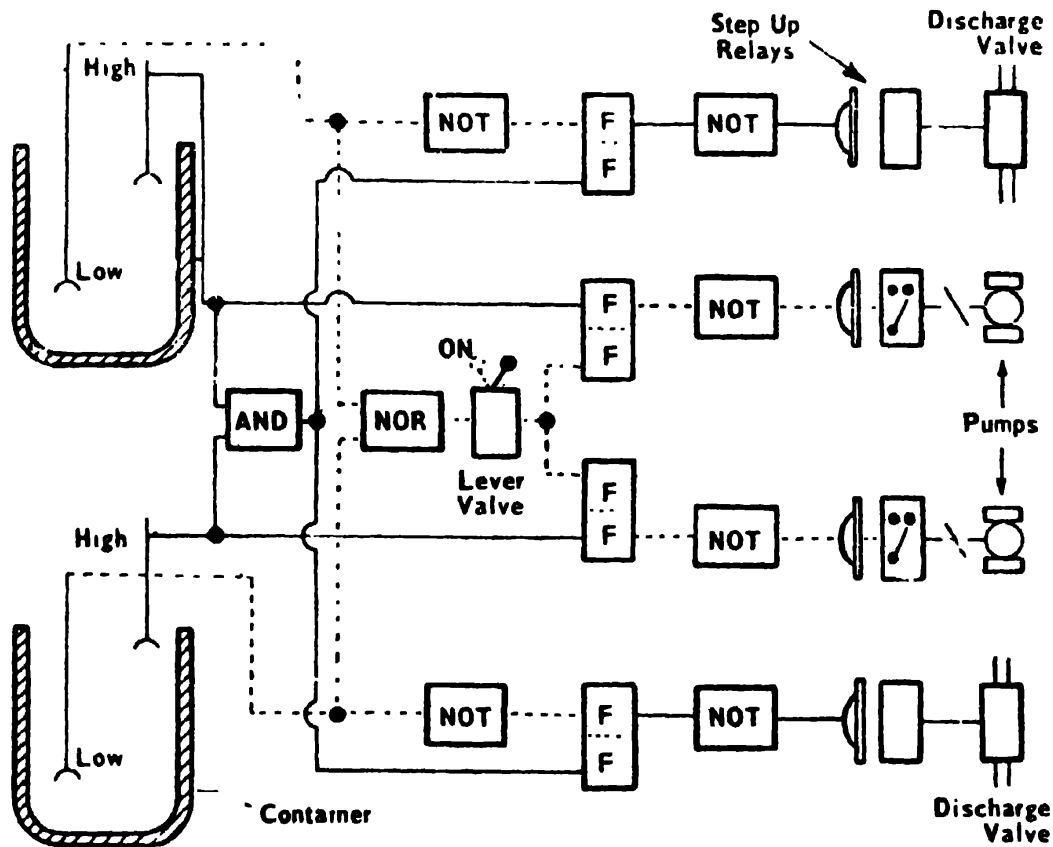


Fig 17.28 Liquid Level Sensing

will open and the liquid will be released into the vessel below. Once again the NOT element or inverter is used for convenience only. The liquid level will continue to fall until the lower detectors are uncovered. This will cause a drop in signal pressure from each, so that the NOT element controlling the discharge FLIP—FLOPS will reset and both discharge valves will shut independently.

When both low levels are reached, and providing the lever valve is 'ON' the drop in pressure will allow the NOR element to again reset and send a signal through the lever valve to the pump FLIP—FLOPS. The pumps will again begin to fill both containers.

The system will cycle automatically and illustrates how a continuous mixing process can be achieved by using air as a means of detecting liquid levels.

**17.13 Third Generation Fluidics**—The third generation fluidic modules utilize an exclusive element image specifically designed to allow a power nozzle aspect ratio approaching one. This low ratio of nozzle depth to width permits low cost mass production of single plane fluidic elements using precision photofabrication techniques. Photofabrication manufacturing techniques in turn allow unparalleled flexibility in the design and production of multi-element functional modules and special integrated circuits.

Each of the third generation elements incorporates internal multiple inputs, eliminating the need for external input expanders. A passive

input gate to each active element provides virtually complete control isolation between adjacent input ports. This isolation eliminates intra-circuit impedance matching problems and simplifies circuit design and construction.

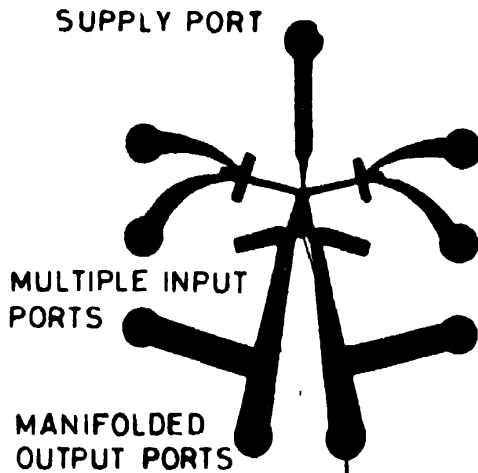


Fig 17.29 Third Generation Fluidics  
by Norgren Fluidics U.K.  
(Advanced Element Design)

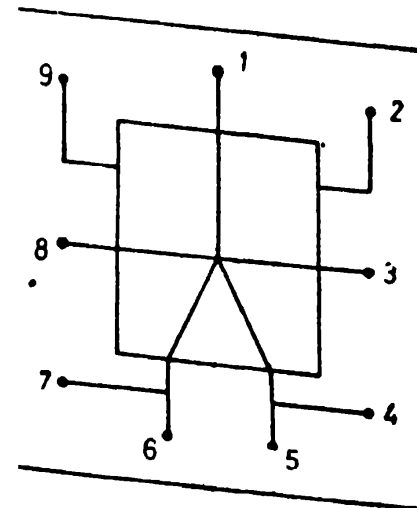


Fig 17.30 Advanced Package Design  
(cover) manufactured by  
Norgren Fluidics U.K.

All elements include output manifolds providing two direct connections to each output leg (refer Fig 17). These manifolded output permit desired inter-element connections and virtually eliminate the number of tees and crosses required to fabricate complex circuits.

This advanced element design is a result of continuing efforts in the development of industrially oriented fluidic modules.

#### 17.14 Advanced Package Design (refer Fig 17.30)—

**Cover :** The heavy aluminium cover provides positive physical protection for the element package. The symbol for the particular element in the package is printed on the cover. Each connection is shown (refer Fig 17.30) with the corresponding pin number from the underside of the module or from any of the optional sub-bases.

**Element Package :** The *element package* is the heart of the fluidic module. This package is made up of several planes permanently bonded together to eliminate internal leakage.

**Manifold Gasket :** The *manifola gasket* is a Neoprene gasket included in the module assembly to insure a positive seal between the Element Package and the Manifold.

**Vent Filler :** All fluidic elements draw in outside air through their *vents* under certain conditions of operation.

**Manifold :** The *manifold* provides the base for assembling all of the components included within the module.

**Subbase Gasket :** A *subbase gasket* is included with each sub base to positively seal the fluidic module to the subbase.

**Subbase :** Two different types of molded subbase are available for added convenience in building and servicing fluidic systems.

# APPENDIX I

## Conversion Factors

| Measure                  | FPS                                       |                                       | Metric                                            |                                       |
|--------------------------|-------------------------------------------|---------------------------------------|---------------------------------------------------|---------------------------------------|
|                          | Unit                                      | $\longrightarrow$<br>FPS<br>To Metric | Unit                                              | $\longrightarrow$<br>Metric<br>to FPS |
| Length<br>(or<br>Linear) | inch (")                                  | $\times 25.4$                         | mm = $\frac{1}{1000}$ m.                          | $\times 0.03937$                      |
|                          | foot (') = 12"                            | $\times 0.3048$                       | m                                                 | $\times 3.28$                         |
|                          | yard = 3'                                 | $\times 0.9144$                       | m                                                 | $\times 1.09361$                      |
|                          | mile = 1,760 yd                           | $\times 1.6093$                       | km = 1,000 m                                      | $\times 0.621$                        |
|                          | nautical mile                             | $\times 1.852$                        | km = 1,000 m                                      | $\times 2.540$                        |
| Area<br>(or<br>square)   | sq in.                                    | $\times 6.451$                        | cm <sup>2</sup>                                   | $\times 0.1550$                       |
|                          | sq ft                                     | $\times 0.0929$                       | m <sup>2</sup>                                    | $\times 0.764$                        |
|                          | acre                                      | $\times 0.4047$                       | hectare                                           | $\times 2.471$                        |
| Volume<br>(or<br>cubic)  | cu in.                                    | $\times 16.3871$                      | cm <sup>3</sup>                                   | $\times 0.0610$                       |
|                          | Imp gallon                                |                                       | litre = (dm) <sup>3</sup>                         | $\times 0.220$                        |
|                          | (= 1.2 US gallons)                        | $\times 4.546$                        | m <sup>3</sup>                                    | $\times 220$                          |
|                          | Imp gallon (= 10lb)                       | $\times 0.004546$                     |                                                   |                                       |
|                          | US gallon                                 | $\times 3.785$                        | litre                                             | $\times 0.2642$                       |
|                          | Petroleum barrel                          |                                       | hl                                                | $\times 0.63$                         |
|                          | = 42 US gallons                           | $\times 1.59$                         | litre                                             | $\times 0.0353$                       |
|                          | cu ft                                     | $\times 28.316$                       | m <sup>3</sup>                                    | $\times 35.38$                        |
| Weight                   | cu ft                                     | $\times 0.0283$                       |                                                   |                                       |
|                          | grain = $\frac{1}{7000}$ lb               | $\times 0.0648$                       | gr                                                | $\times 15.48$                        |
|                          | ounce = $\frac{1}{16}$ lb                 | $\times 28.35$                        | gr                                                | $\times 0.035$                        |
|                          | pound = 1 lb                              | $\times 0.4536$                       | kg                                                | $\times 2.205$                        |
|                          | cwt = 112 lb                              | $\times 50.802$                       | kg                                                | $\times 0.0197$                       |
|                          | short ton                                 |                                       |                                                   |                                       |
|                          | or US ton                                 |                                       |                                                   |                                       |
|                          | = 2,000 lb                                | $\times 0.907$                        | tonne                                             | $\times 1.102$                        |
| Pressure<br>and<br>Head  | long ton = 2,240 lb                       | $\times 1.016$                        | tonne                                             | $\times 0.98421$                      |
|                          | lb/sq in.                                 |                                       | kg/cm <sup>2</sup>                                |                                       |
|                          | (= $\frac{\text{ft of water}}{2.31}$ )    | $\times 0.0703$                       | (= $\frac{\text{m of water}}{10}$ )               | $\times 14.22$                        |
|                          | lb/sq in                                  | $\times 0.068$                        | atm                                               | $\times 14.7$                         |
|                          | long ton/sq in.                           | $\times 157.5$                        | kg/cm <sup>2</sup>                                | $\times 0.00635$                      |
|                          | in mercury                                |                                       |                                                   |                                       |
|                          | (= $\frac{\text{ft of water}}{1.13}$ )    | $\times 345$                          | mm of water                                       | $\times 0.0029$                       |
|                          | in. of mercury                            | $\times 25.4$                         | mm of mercury                                     | $\times 0.03937$                      |
|                          | 1 atm (= 34 ft of water = 14.7 lb/sq in.) |                                       | 1 atm. (= 10 m of water or 1 kg/cm <sup>2</sup> ) |                                       |
|                          |                                           | $\times 1.033$                        |                                                   | $\times 0.967$                        |

| Measure                | FPS                                                           |                                           | Metric                                                           |                                           |
|------------------------|---------------------------------------------------------------|-------------------------------------------|------------------------------------------------------------------|-------------------------------------------|
|                        | Unit                                                          | $\xrightarrow{\quad}$<br>FPS<br>to Metric | Unit                                                             | $\xrightarrow{\quad}$<br>Metric<br>to FPS |
| Density                | lb/cu ft                                                      | $\times 16.02$                            | kg/m <sup>3</sup>                                                | $\times 0.0625$                           |
|                        | lb/cu ft                                                      | $\times 0.01602$                          | kg/dm <sup>3</sup> or                                            |                                           |
|                        | lb/in. <sup>3</sup>                                           | $\times 27.6799$                          | kg/litre                                                         | $\times 62.424$                           |
|                        |                                                               |                                           | kg/cm <sup>3</sup>                                               | $\times 0.0362$                           |
| Moment<br>of Inertia   | lb ft <sup>2</sup> ( $WR^2$ )                                 | $\times 0.1686$                           | kg cm <sup>2</sup> ( $GD^2$ )                                    | $\times 5.933$                            |
| Velocity               | ft/sec                                                        | $\times 0.3048$                           | m/sec                                                            | $\times 3.28$                             |
|                        | ft/min                                                        | $\times \frac{5.08}{1000}$                | m/sec                                                            | $\times 196.85$                           |
| Velocity               | ft units $\left( = \frac{\text{lb sec}}{\text{ft}^2} \right)$ | $\times 479$                              | Poise<br>$\left( = \frac{\text{dyne. sec}}{\text{cm}^2} \right)$ | $\times \frac{2.083}{1,000}$              |
| Kinematic<br>viscosity | ft units ( $=\text{ft}^2/\text{sec}$ )                        | $\times 929$                              | Stokes<br>( $=\text{cm}^2/\text{sec}$ )                          | $\times \frac{1.076}{1,000}$              |
| Tempera-<br>ture       | $^{\circ}F$                                                   | $\times \frac{5}{9}(^{\circ}F - 32)$      | $^{\circ}C$                                                      | $\times \frac{9}{5}^{\circ}C + 32$        |
| Rate of                | cu ft/sec (cusec)                                             | $\times 0.0283$                           | m <sup>3</sup> /sec (cumec)                                      | $\times 35.38$                            |
| Discharge              | cu ft/sec (cusec)                                             | $\times 28.316$                           | lit/sec                                                          |                                           |
|                        | cu ft/min                                                     | $\times 0.472$                            | ( $=\text{dm}^3/\text{sec}$ )                                    | $\times 0.0353$                           |
|                        | gal/min (gpm)                                                 | $\times 0.0757$                           | lit/sec                                                          | $\times 2.22$                             |
|                        | gal/min (gpm)                                                 | $\times \frac{0.757}{10,000}$             | ( $=\text{dm}^3/\text{sec}$ )                                    | $\times 13.22$                            |
|                        |                                                               |                                           | m <sup>3</sup> /sec                                              | $\times 13,220$                           |
| Energy                 | ft lb                                                         | $\times 0.1382$                           | m kg                                                             | $\times 7.28$                             |
| or Power               | HP ( $=5.0$ ft lb)                                            | $\times 1.014$                            | HP                                                               | $\times 0.986$                            |
|                        | 1 KW                                                          |                                           | ( $=75$ kgm/sec)                                                 |                                           |
|                        | ( $=1.341$ HP)                                                | $\times 1$                                | 1 KW                                                             | $\times 1$                                |
| Heat                   | BTU                                                           | $\times 0.252$                            | k cal                                                            | $\times 3.968$                            |
|                        | BTU                                                           | $\times 107.6$                            | m kg                                                             | $\times \frac{9.29}{1,000}$               |
|                        | BTU/sq ft                                                     | $\times 2.7126$                           | k cal/m <sup>2</sup>                                             | $\times 0.3687$                           |
|                        | BTU/sq ft $^{\circ}F$                                         | $\times 4.882$                            | k cal/m <sup>2</sup> $^{\circ}C$                                 | $\times 0.205$                            |
|                        | BTU/lb                                                        | $\times 0.556$                            | k cal/kg                                                         | $\times 1.800$                            |
|                        | BTU/ft <sup>3</sup>                                           | $\times 8.899$                            | k cal/m <sup>3</sup>                                             | $\times 0.112$                            |

## **APPENDIX II**

### **Mensuration of Surfaces and Solids**

|                                                                       |                                                                                                                   |
|-----------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------|
| <b>Circumference of a circle</b>                                      | $= \pi \times \text{diameter}$                                                                                    |
| <b>Area of circle</b>                                                 | $= \frac{\pi}{4} \times (\text{diameter})^2$                                                                      |
| <b>Area of sector of circle</b>                                       | $= \frac{\text{radius}}{2} \times \text{length of arc}$                                                           |
| <b>Area of square, parallelo-gram, rectangle, rhombus or rhomboid</b> | $= \text{base} \times \text{height}$                                                                              |
| <b>Area of trapezium</b>                                              | $= \frac{1}{2} \text{ sum of two parallel sides} \times \text{height}$                                            |
| <b>Area of regular polygon</b>                                        | $= \frac{1}{2} \text{ radius of inscribed circle} \times \text{length of one side} \times \text{number of sides}$ |
| <b>Area of triangle</b>                                               | $= \frac{1}{2} \text{ base} \times \text{altitude}$                                                               |
| <b>Area of parabola</b>                                               | $= \frac{2}{3} \times \text{base} \times \text{altitude}$                                                         |
| <b>Area of ellipse</b>                                                | $= \frac{\pi}{4} \times \text{major axis} \times \text{minor axis}$                                               |
| <b>Area of cycloid</b>                                                | $= 3 \times \text{area of generating circle}$                                                                     |
| <b>Surface area of sphere</b>                                         | $= \pi \times (\text{diameter})^2$                                                                                |
| <b>Volume of sphere</b>                                               | $= \frac{\pi}{6} \times (\text{diameter})^3$                                                                      |
| <b>Volume of paraboloid</b>                                           | $= \frac{1}{2} \text{ volume of circumscribing cylinder}$                                                         |
| <b>Volume of prism</b>                                                | $= \text{area of base} \times \text{altitude}$                                                                    |
| <b>Volume of cylinder</b>                                             | $= \frac{\pi}{4} (\text{diameter})^2 \times \text{height}$                                                        |
| <b>Volume of pyramid</b>                                              | $= \frac{1}{3} \times \text{area of base} \times \text{height}$                                                   |
| <b>Volume of one</b>                                                  | $= \frac{1}{3} \times \frac{\pi}{4} \times (\text{diameter of base})^2 \times \text{height}$                      |
| <b>Length of arc</b>                                                  | $= \text{Radius of circle} \times \text{angle subtended at the centre in radians } (= r\theta)$                   |
| <b>One radian</b>                                                     | $= 57.5^\circ$                                                                                                    |



## APPENDIX III

### (a) Bernoulli's Theorem with regard to Absolute Motion

This important theorem is proved and derived in several ways. The following proof is based on consideration of momentum.

Consider a small element of fluid of length  $ds$ , breadth  $b$  and thickness  $l$  and  $(l+dl)$  at the two ends (refer Fig III.1). This element is a part of a stream tube.

Forces on the element are the fluid pressure and gravitation.

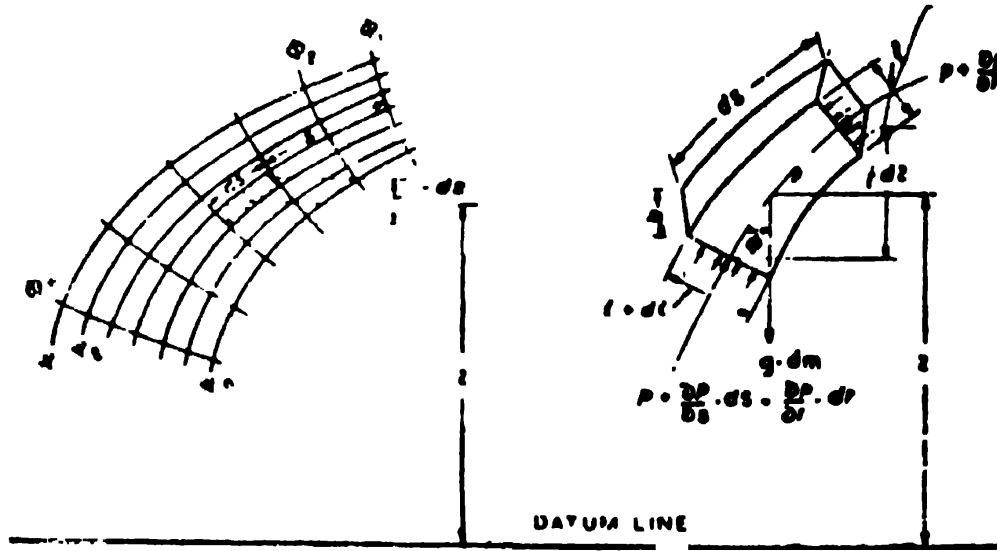


Fig III.1 Absolute Motion of a Fluid Particle

If  $p$  be the initial pressure at the right end at any given time, then after a time  $dt$ .

$$\text{Pressure at right end} = p + \frac{\partial p}{\partial t} \cdot dt$$

$$\text{Pressure at left end} = p + \frac{\partial p}{\partial s} ds \times \frac{\partial p}{\partial t} \cdot dt$$

$$\text{For steady flow, } \frac{\partial p}{\partial t} = 0,$$

$$\therefore \text{Pressure at right end} = p + 0 = p$$

$$\text{Pressure at left end} = p + \frac{\partial p}{\partial s} \cdot ds + 0$$

$$= p + \frac{dp}{ds} \cdot ds = p + dp$$

Acceleration of fluid may be easily calculated

Let,  $v = f(t, s)$

Then,  $dv = \frac{\partial v}{\partial t} dt + \frac{\partial v}{\partial s} ds$

$$\therefore \frac{dv}{dt} = \frac{\partial v}{\partial t} + \frac{\partial v}{\partial s} \frac{ds}{dt}$$

$$= \frac{\partial v}{\partial t} + v \cdot \frac{\partial v}{\partial s}$$

For steady flow,  $\frac{\partial v}{\partial t} = 0$

$$\therefore \frac{dv}{dt} = v \cdot \frac{\partial v}{\partial s} = v \cdot \frac{dv}{ds}$$

Net external force on the element in the direction of motion

$$= \gamma \cdot b \cdot l \cdot ds \cdot \cos \phi + p \cdot b \cdot l - (p + dp) \cdot b \cdot (l + dl)$$

$$= \gamma \cdot b \cdot l \cdot ds \cdot \cos \phi - dp \cdot b \cdot l \dots \dots \dots \left( \text{neglecting } \frac{dl}{l} \right)$$

This must be equal to the rate of change of momentum  
= mass  $\times$  acceleration

$$= \frac{\gamma}{g} \cdot (b \cdot l \cdot ds) \cdot v \cdot \frac{dv}{ds}$$

$$\therefore \frac{\gamma}{g} \cdot b \cdot l \cdot ds \cdot v \cdot \frac{dv}{ds} = \gamma \cdot b \cdot l \cdot ds \cos \phi - dp \cdot b \cdot l$$

$$\text{or } \frac{v \cdot dv}{g} = ds \cdot \cos \phi - \frac{dp}{\gamma}$$

$$= -dz - \frac{dp}{\gamma} \dots \dots \dots (\text{since } ds \cdot \cos \phi = -dz)$$

Integrating,

$$\frac{v^2}{2g} + \frac{p}{\gamma} + z = c \quad (\text{Assuming that } \gamma \text{ is constant, i.e. fluid is incompressible}). \quad \dots (\text{III. I})$$

This holds good for potential flow of ideal fluid. In practice, frictional losses have to be considered, and then

$$\frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 = \frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - H_{L(1-2)} \quad \dots (\text{III. Ia})$$

### (b) Bernoulli's Equation for Relative Motion

Consider the motion of fluid inside a turbine runner or an impeller of a centrifugal pump. Let its angular velocity  $\omega$  be constant and let relative velocity be  $w$ .

Again take an element similar to the one chosen in the previous case.

Let the angle which the directions of vertical gravitational force and radial centrifugal force make with the tangent to the stream line at the point under consideration be denoted by  $\phi$  and  $\psi$  respectively.

Let the weight density i.e. specific weight of fluid be denoted by  $\gamma$  and the angular velocity  $\omega$ .

For steady flow,

$$\frac{\partial w}{\partial t} = 0; \quad \frac{\partial \omega}{\partial t} = 0$$

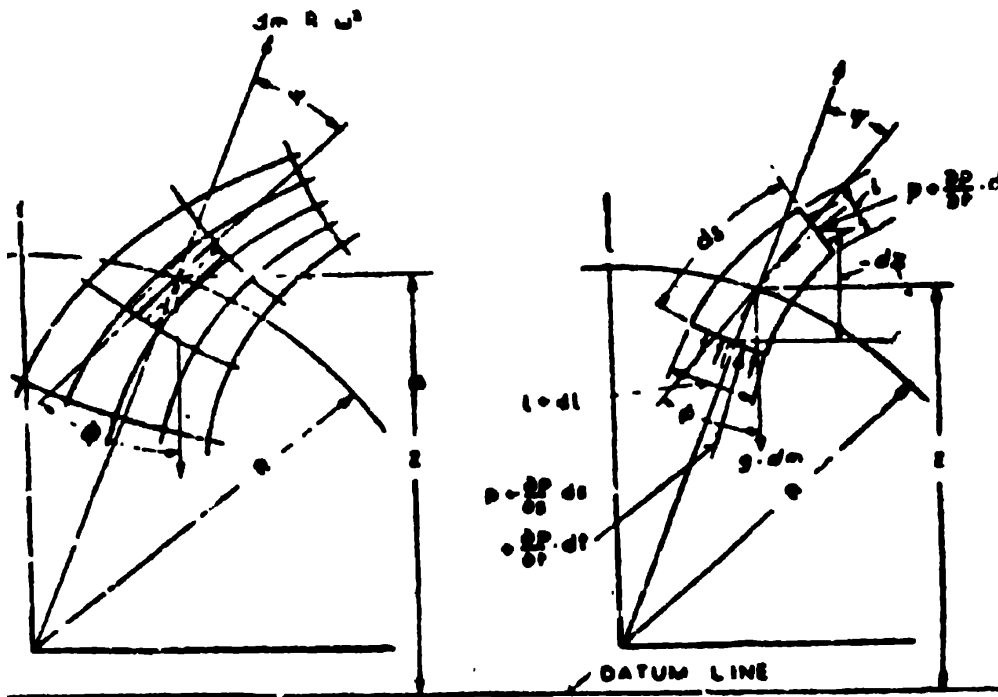


Fig III.2 Relative Motion of a Fluid Particle etc.

Acceleration in the direction of relative velocity  $w$ ,

$$\begin{aligned} \frac{dw}{dt} &= \frac{\partial w}{\partial t} + w \frac{\partial w}{\partial s_w} \\ &= w \cdot \frac{\partial w}{\partial s_w} = w \cdot \frac{dw}{ds_w} \end{aligned}$$

$$\text{Mass of fluid accelerated} = \frac{\gamma}{g} \cdot (b \cdot l \cdot ds_w)$$

Forces acting on the element are :

$$(a) \text{ weight} = \gamma \cdot b \cdot l \cdot ds_w$$

$$(b) \text{ centrifugal force} = \frac{\gamma}{g} \cdot b \cdot l \cdot ds_w \cdot R \cdot \omega^2$$

$$\begin{aligned} (c) \text{ pressure difference} &= b \cdot l \cdot p - (p + dp) \cdot b \cdot (l + dl) \\ &= -b \cdot l \cdot dp \dots \dots \left( \text{neglecting } \frac{dl}{l} \right) \end{aligned}$$

Now, resultant force in the direction of stream line

$$= \text{mass} \times \text{acceleration}$$

$$\therefore \gamma \cdot b \cdot l \cdot ds_w \cos \phi - b \cdot l \cdot dp - \frac{\gamma \cdot b \cdot l \cdot ds_w \cdot R \cdot \omega^2}{g} \cdot \cos \psi$$

$$\frac{\gamma}{g} \cdot b \cdot l \cdot ds_w \cdot w \cdot \frac{dw}{ds_w}$$

Simplifying,

$$w \cdot \frac{dw}{g} = ds_w \cdot \cos \phi - \frac{dp}{\gamma} - \frac{R \cdot \omega^2}{g} \cdot ds_w \cdot \cos \psi$$

Substitute,

$$ds_w \cdot \cos \phi = -dz; \text{ and } ds_w \cdot \cos \psi = -dR$$

$$\text{Then } \frac{w \cdot dw}{g} = -dz - \frac{dp}{\gamma} + \frac{R \cdot \omega^2}{g} \cdot dR$$

$$\text{or } w \cdot \frac{dw}{g} + dz + \frac{dp}{\gamma} - \frac{R \cdot \omega^2 dR}{g} = 0$$

Integrating,

$$\frac{w^2}{2g} + z + \frac{p}{\gamma} - \frac{R^2 \cdot \omega^2}{2g} = c$$

Since  $R \cdot \omega$  as circumferential velocity  $u$ , final form of Bernoulli's Equation for relative motion would be

$$\frac{w^2}{2g} + z + \frac{p}{\gamma} - \frac{u^2}{2g} = c \quad \dots(\text{III II})$$

for ideal fluid and ideal flow.

In practice, considering frictional and other losses,

$$\frac{w_2^2}{2g} - \frac{u_2^2}{2g} + \frac{p_2}{\gamma} + z_2 = \frac{w_1^2}{2g} - \frac{u_1^2}{2g} + \frac{p_1}{\gamma} + z_1 - H L_{(1-2)} \dots(\text{III.IIa})$$

Bernoulli's theorem thus proved applies strictly to a single stream line. In problems of practical importance, as flow in pipes, however, the stream is made up of an infinite collection of stream lines or stream tubes. In general the velocity varies from point to point in a cross-section of such a stream and the expression usually yields the average or the mean velocity.

Even under such circumstances Bernoulli's theorem can often be applied without introducing any grave error because the kinetic head, in practice, is small compared to other heads.

If kinetic head is considerable, it is expressed as  $\alpha \times \frac{(v)^2}{2g}$ , where  $v$  is the average velocity  $= \frac{Q}{A}$ , and  $\alpha$  is a correction factor varying from 1 to 2 (refer Fig III.3)

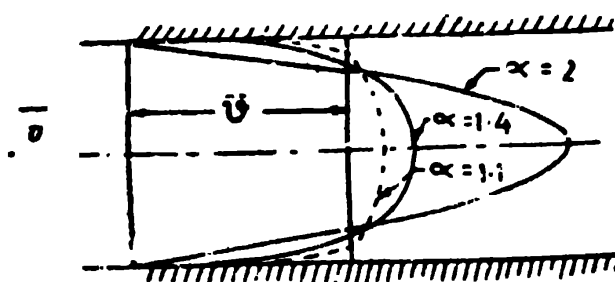


Fig III.3 Velocity Distribution Over the Section of Pipe

$\alpha = 1$ , when velocity is constant over the cross-section.

$\alpha = 2$ , when velocity varies parabolically from zero at the boundary to a maximum at the centre, as in pipes.

## APPENDIX IV

### Combined Formulae for Turbines and Pumps

| Turbine (Reaction Type)                                                                                                                                                                          | Pump (Centrifugal Type)                                                                                                                                                                                                                                                                               |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| (1) Generally inward flow (centripetal type)                                                                                                                                                     | Outward flow (centrifugal type)                                                                                                                                                                                                                                                                       |
| (2) Work done/sec<br>$\frac{\gamma \cdot Q}{g} (v_{u_1} \cdot u_1 - v_{u_2} \cdot u_2)$                                                                                                          | Work consumed/sec<br>$= \frac{\gamma \cdot Q}{g} (v_{u_2} \cdot u_2 - v_{u_1} \cdot u_1)$                                                                                                                                                                                                             |
| (3) $\alpha_2 = 90^\circ$ (i.e. radial discharge)                                                                                                                                                | $\alpha_1 = 90^\circ$ (i.e. radial inlet)                                                                                                                                                                                                                                                             |
| (4) $\therefore$ W.D./sec = $\frac{\gamma \cdot Q}{g} v_{u_1} \cdot u_1$                                                                                                                         | $\therefore$ W.D./sec = $\frac{\gamma \cdot Q}{g} v_{u_2} \cdot u_2$                                                                                                                                                                                                                                  |
| (5) W.D./sec/kg of water = $\frac{v_{u_1} \cdot u_1}{g}$<br>(Fundamental equation of reaction turbine)                                                                                           | W.D./sec/kg of water = $\frac{v_{u_2} \cdot u_2}{g}$<br>(Fundamental equation of centrifugal pump)                                                                                                                                                                                                    |
| (6) Head utilised by turbine = $H$                                                                                                                                                               | Head supplied by pump = $H$<br>i.e. Head shown by gauge                                                                                                                                                                                                                                               |
| (7) $H = H_{static} - \Sigma H_L$<br>(Losses outside the turbine)                                                                                                                                | $H_{mano} = H + \Sigma H_L$<br>(Losses outside the pump)                                                                                                                                                                                                                                              |
| (8) $H - \Delta H = \frac{v_{u_1} \cdot u_1}{g}$<br>( $\Delta H$ = Losses inside the turbine)                                                                                                    | $H_{mano} + \Delta H = \frac{v_{u_2} \cdot u_2}{g}$<br>( $\Delta H$ = Losses inside the pump)                                                                                                                                                                                                         |
| (9) $\eta_H = \frac{v_{u_1} \cdot u_1}{gH}$                                                                                                                                                      | $\eta_{mano} = \eta_H = \frac{g \cdot H_{mano}}{v_{u_2} \cdot u_2}$                                                                                                                                                                                                                                   |
| (10) $Q = \pi \cdot D_1 \cdot B_o \cdot v_{m_1}$<br>(neglecting thickness of vanes)<br>$= (\pi \cdot D_1 - z_s \cdot t) \cdot B_o \cdot v_{m_1}$<br>(taking the thickness of vanes into account) | $Q = \pi \cdot D_1 \cdot B_1 \cdot v_{m_1}$<br>$= \pi \cdot D_2 \cdot B_2 \cdot v_{m_2}$<br>(neglecting thickness of vanes)<br>$= (\pi \cdot D_1 - z \cdot S_1) \cdot B_1 \cdot v_{m_1}$<br>$= (\pi \cdot D_2 - z \cdot S_2) \cdot B_2 \cdot v_{m_2}$<br>(taking the thickness of vanes into account) |

If Positive (+) sign is used for Pumps

Negative (-) sign is used for Turbines.

(a) Head  $H = H_{stat} \pm \Sigma H_L \quad H_{th} = H \pm \Delta H$

(b) Power  $P_t = \frac{\gamma}{75} (Q \pm \Delta Q) \cdot H_{th} \pm \Delta P_{mech}$

(c) Efficiencies  $\eta_{th} = \left( \frac{H}{H_{th}} \right) \pm 1$

$$\eta_{mech} = \left( \frac{P_h}{P_t} \right) \pm 1 = \left( \frac{P_t + \Delta P_{mech}}{P_t} \right) \pm 1$$

$$\eta_t = \left( \frac{\gamma \cdot Q \cdot H}{75 \cdot P_t} \right) \pm 1$$

## APPENDIX V

### SI Units

International System of units (SI) is the refined and extended form of the traditional metric system. It was in 1950 when International Electrotechnical Commission (IEC) adopted 'ampere'—the unit of electrical current as the fourth basic unit, besides metre, kilogram and second, giving the MKSA system. The General Conference on Weights and Measures (CGPM—Conference Generale des Poids et Mesures) at its 10th meeting in 1954 adopted the rationalized and coherent system of units based on MKSA units besides degree Celsius (for temperature) and 'candela' for luminous intensity. In 1960, CGPM at its 11th meeting formally gave this system the title "*System International d'Unites*", with the abbreviation 'SI'.

The British Government decided to adopt SI units on 24th May 1965. It is expected that the U.S.A. shall also adopt this system soon. India has taken a statutory decision in December 1956 (Act of Parliament No. 89, 1956) to change over to metric system of weights and measures. The definitions of various units given in the Act conform to the definitions of the SI units. Many other countries of World are switching over to this system. Thus SI is going to be the only legally accepted universal language for science, engineering, technology and commerce.

#### Main Features of SI Units :

1. **Basic Units**—This system is based on six basic units (refer Table A). However, 'Mole' as the basic seventh unit for the amount of substance is likely to be formally adopted in the field of Chemistry.

2. **Supplementary Units**—There are two supplementary units radian (rad) and steradian (sr) for the measurement of the plane angle and solid angle respectively.

Other units of measurements are derived from these basic and supplementary units such as force, volume, velocity etc. Some of them which are used in Hydraulics are given in Table D with their conversion factors.

3. **Unit of force** which is a derived unit is **newton (N)** in place of **kilogram force ( $kg_f$ )**. Unlike kilogram force or weight, newton is independent of earth's gravitation. This is one of the main departure from the traditional metric system.

*Newton* is defined as that force which, when applied to a body having a mass of one kilogram, gives it an acceleration of one metre per second squared.

4. **Energy** whether it is mechanical, thermal, magnetic, electrical or chemical is basically the product of force and distance. Hence the

unit of energy of all forms is 'joule' abbreviated as 'J' (1 joule = 1 newton  $\times$  1 metre) i.e. there is only one unit of energy.

5. **Power** is essentially the rate of generation or dissipation of energy hence, whatever be the form of power, it will have one and only one unit i.e. newton per second or joule per second. It is given a special name watt (1W = 1J/s = 1N m/s).

Thus horsepower will not be used as unit of power. The two types of calorie (the big calorie and the small calorie), kilowatt-hour, British and French or metric horsepower, BTU and CHU are all discarded.

6. **Electrostatic and Electromagnetic Units** are replaced by SI electrical units. This simplifies the various electrical units. Here these are not mentioned, because these are not related with Hydraulics or Fluid Mechanics.

7. **Multiple or Sub-multiples of Units** are normally to be restricted to steps of a thousand i.e.  $10^{\pm 3n}$ . They are formed by means of prefixes (refer Table C).

8. **Comprehensive, Coherent, Rational and Practical System** is the 'Système International d' unit' (SI).

In a coherent system of units, the product or quotient of any two unit quantities in the system is the unit of resultant quantity, for example unit velocity is obtained when unit length is divided by unit time.

9. **Decimal Relationship** between units of the same quantity facilitates the conversion work. It is estimated that the time for calculation of a typical problem in mechanics using SI units will only be one-sixth the time required for solving the same problem in old system of units i.e. FPS, CGS etc. According to Dr. E.G. Tagg, Director of the Accelerated Teaching of Higher Level Mathematics Project at the University of Lancaster, U.K., each year the time saved by using SI units might be estimated at about  $2 \times 10^9$  child-hours or about  $6 \times 10^7$  teacher-hours in schools, which would roughly work out to about £100 million a year.

### Definition of Basic SI Units

(1) **Unit of Length**—metre (m)—the metre is the length equal to 1 650 763.73 wave lengths in vacuum of the radiation corresponding to the transition between the levels  $2p_{10}$  and  $5d_5$  of the krypton—86 atom.

(2) **Unit of Mass**—kilogram (kg)—The kilogram is the unit of mass and is equal to the mass of the international prototype kilogram.

(3) **Units of Time**—second (s)—The second is the duration of 9 192 631 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the caesium—133 atom.

(4) **Unit of Electric Current**—ampere (A)—The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed one metre

apart in vacuum, would produce between these conductors a force equal to  $2 \times 10^{-7}$  newton per metre length.

**(5) Unit of Thermodynamic Temperature—kelvin (*K*)**—The kelvin, unit of thermodynamic temperature, is the fraction  $\frac{1}{273.15}$  of the thermodynamic temperature of the triple point of water.

*Notes :* (i) The degree Celsius ( $^{\circ}\text{C}$ ) is a unit of the International Practical Temperature Scale on which the thermodynamic temperature of the zero point is 273.15 K.

(ii) The degree Celsius may also be used for expressing a temperature interval. ( $1^{\circ}\text{C} = 1\text{K}$ ).

**(6) Unit of Luminous Intensity—candela (*cd*)**—The candela is the luminous intensity in the perpendicular direction, of a surface of  $\frac{1}{600000}$  square metre of black body at the temperature of freezing platinum under a pressure of 101 325 newtons per square metre.

**TABLE A**  
**Basic SI Units**

| Sl. No. | Physical Quantity         | Name of SI Unit | Symbol      |
|---------|---------------------------|-----------------|-------------|
| 1       | Length                    | metre           | m           |
| 2       | Mass                      | kilogram        | kg          |
| 3       | Time                      | second          | s (not sec) |
| 4       | Electric Current          | ampere          | A (not amp) |
| 5       | Thermodynamic temperature | kelvin          | K           |
| 6       | Luminous intensity        | candela         | cd          |

*Note*—Symbols for units do not take a plural form. They shall be printed in Roman upright type and written without final full-stop.



**TABLE B**  
**Supplementary SI Units**

| Sl. No. | Physical Quantity | Name of Unit | Symbol |
|---------|-------------------|--------------|--------|
| 1       | Plane angle       | radian       | rad    |
| 2       | Solid angle       | steradian    | sr     |

**TABLE C**

|   |           | Prefix | Symbol |            | Prefix | Symbol |
|---|-----------|--------|--------|------------|--------|--------|
| { | $10^{12}$ | tera   | T      | $10^{-1}$  | deci   | d      |
|   | $10^9$    | giga   | G      | $10^{-2}$  | centi  | c      |
|   | $10^6$    | mega   | M      | $10^{-3}$  | milli  | m      |
|   | $10^3$    | kilo   | k      | $10^{-6}$  | micro  | $\mu$  |
|   | $10^2$    | hecto  | h      | $10^{-9}$  | nano   | n      |
|   | 10        | deca   | da     | $10^{-12}$ | pico   | p      |
|   | 1         | —      | —      | $10^{-15}$ | femto  | f      |
|   |           |        |        | $10^{-18}$ | atto   | a      |

*Note :* Bracketed values are preferred.

**TABLE D**  
**Various Units & Conversion Factor for MKS and SI Units**

| Sl. No. | Quantity                                                          | Symbol            | Metric Unit                                                                |                   | Conversion factor   | SI Unit                |                                        |
|---------|-------------------------------------------------------------------|-------------------|----------------------------------------------------------------------------|-------------------|---------------------|------------------------|----------------------------------------|
|         |                                                                   |                   | Name of Unit                                                               | Symbol            |                     | Name of Unit           | Symbol                                 |
| 1       | length                                                            | $l$               | micro metre<br>nautical mile                                               | $\mu m$<br>n mile | $10^{-6}$<br>1852   | metre                  | m                                      |
| 2       | mass                                                              | $m$               | metric slug<br>(i.e. metric technical unit of mass $\frac{kg, sec^2}{m}$ ) | msl               | 9 80665             | Kilogram               | kg                                     |
| 3       | force                                                             | $F$               | kilogram force                                                             | kgf               | 9·806 65            | newton                 | $N (= \frac{1 kg \cdot m}{s^2})$       |
| 4       | work energy                                                       | $W$<br>$E$        | kilogram force metre                                                       | kgf.m             | 9·806 65            | Joule<br>(1 J = 1 N.m) | J (= N.m)                              |
| 5       | power                                                             | $P$               | kilogram force metre per second<br>metric horse power                      | kgf.m/s<br>HP     | 9·806 65<br>735·499 | watt                   | $W (= J/s)$<br>$= \frac{N \cdot m}{s}$ |
| 6       | Moment of force,<br>bending moment<br>torque, moment<br>of couple | $M$<br>$M$<br>$T$ | kilogram force metre                                                       | kgf.m             | 9·806 65            | newton metre           | N.m                                    |

|    |                                     |                    |                                                                                                   |                                                                 |                                     |                                      |           |
|----|-------------------------------------|--------------------|---------------------------------------------------------------------------------------------------|-----------------------------------------------------------------|-------------------------------------|--------------------------------------|-----------|
| 7  | density<br>(mass density)           | $\rho$             | tonne per cubic metre                                                                             | $t/m^3$                                                         | $10^3$                              | kilogram per<br>cubic metre          | $kg/m^3$  |
| 8  | specific weight<br>(weight density) | $\gamma$           | kilogram force<br>per cubic metre                                                                 | $kg, m^3$<br>or $kg/m^3$                                        | 9.806 65                            | newton per<br>cubic metre            | $N/m^3$   |
| 9  | pressure                            | $p$                | kilogram force<br>per square centimetre                                                           | $kg/cm^2$                                                       | 98 066.5                            | newton per<br>square metre           | $N/m^2$   |
| 10 | viscosity<br>(dynamic-viscosity)    | $\eta, (\mu)$      | centipoise<br>dyne second per square centi-<br>metre<br>kilogram force second per<br>square metre | cP<br>P<br>dyn.s<br>/cm <sup>2</sup><br>kg.s<br>/m <sup>2</sup> | $10^{-3}$<br>0.1<br>0.1<br>9.806 65 | newton second<br>per square<br>metre | $N.s/m^2$ |
| 11 | kinematic<br>viscosity              | $\nu$              | Square centimetre per second<br>Stoke                                                             | cm <sup>2</sup> /s<br>St                                        | $10^{-4}$<br>$10^{-4}$              | square metre<br>per second           | $m^2/s$   |
| 12 | surface tension                     | $\sigma, (\gamma)$ | dyne per centimetre                                                                               | dyn/cm                                                          | $10^{-3}$                           | newton per<br>metre                  | $N/m$     |









